Examining Relationships among Elementary Mathematics Teachers’ Efficacy and Their Students’ Mathematics Self-efficacy and Achievement

Yu-Liang (Aldy) Chang  
National Chiayi University, TAIWAN

*Received 11 March 2015* • *Revised 8 April 2015* • *Accepted 20 April 2015*

The purpose of this study was to examine the effects of fifth-grade mathematics teachers’ efficacy (MTE) on their students’ mathematics self-efficacy (SMSE) and mathematical achievement (SMA) in classrooms. Two mathematics efficacy instruments were administered to 58 classes (i.e. 58 teachers and 1244 fifth-graders respectively) to gather data associated with SMA scores in school. Statistical analyses were applied to the obtained data. The findings revealed that MTE beliefs significantly influenced both SMSE and SMA. Additionally, SMSE had a mediative impact on the effect of MTE on SMA. Subsequent to a discussion of the findings, suggestions were proposed for further improvement of mathematics teachers’ efficacy and, in turn, for enhancing fifth-graders’ mathematics self-efficacy and mathematical achievement in the future.

*Keywords:* mathematics teachers’ efficacy, students’ mathematics self-efficacy, students’ mathematical achievement, elementary school

INTRODUCTION

Teacher efficacy influences teaching performance and students’ learning

Contemporary educational reforms in many countries focus on advancing the quality of teaching and learning in every classroom (Goddard, Goddard, & Tschannen-Moran, 2007; Moolenaar, Sleegers, & Daly, 2012). Grounded in Bandura’s (1977) social cognitive theory and his construct of self-efficacy (SE), teacher efficacy (TE) has been recognized as “a variable accounting for individual differences in teaching effectiveness” (Gibson & Dembo, 1984, p. 569) and has a strong relationship to student learning and achievement (Cantrell, Young, & Moore, 2007).
true.

2003; Gibson & Dembo, 1984; Ross, 1998). Tschannen-Moran, Woolfolk Hoy, and Hoy (1998) defined TE as “the teacher’s belief in his or her capability to organize and execute courses of action required to successfully accomplish a specific teaching task in a particular context” (p. 223). Actually, from research in 1970s (e.g. Armor et al, 1976), “teacher efficacy was first conceptualized as teachers’ general capacity to influence student performance” (Allinder, 1995, p.247). Further, Ross (1998) indicated that most researchers treated “teacher efficacy as a type of self-efficacy” (p. 50). Since then, TE has been viewed as “self-efficacy beliefs directed toward a teaching context” (Knoblauch & Woolfolk Hoy, 2008, p.167); that is, teacher efficacy is referred to as “their belief in their capability to have a positive effect in student learning” (Ashton, 1985, p.142).

The concept of self-efficacy consists of two kinds of expectation, efficacy expectation and outcome expectancy. A teacher’s efficacy expectation influences her/his thoughts and feelings, her/his selection of instructional activities, the amount of effort s/he spends in teaching, and the degree of her/his persistence while confronting difficulties (Bandura, 1981). The outcome expectancy refers to her/his own estimate of the likely consequences of teaching performance at the expected level of competence (Bandura, 1981). Applying this construct to the subject of mathematics, the “Mathematics Teaching Efficacy Beliefs Instrument (MTEBI)" was originated by Enochs, Smith, and Huinker (2000) to measure pre-service teachers’ efficacy beliefs. Later, the current researcher (Chang & Wu, 2004; Chang & Wu, 2009) adapted the MTEBI to assess efficacy beliefs of elementary in-service mathematics teachers in Taiwan, resulting in the development of the “Elementary Mathematics Teacher Efficacy Instrument (EMTEI)”. Consistent with the above description, the EMTEI includes two cognitive dimensions: personal mathematics teaching efficacy (PMTE) and mathematics teaching outcome expectancy (MTOE). The EMTEI was employed in this study to obtain efficacy ratings of participating mathematics teachers.

Bandura (1997) argued that SE, defined as “belief in one’s capabilities to organize and execute the courses of action required to produce given attainments” (p. 3), had a great influence on one’s task choices, effort, persistence, and achievement. Based on this concept, a student’s self-efficacy refers to “belief in her/his capabilities to organize and execute the courses of learning”. Thus, students who are self-efficacious in learning are likely to make more effort, persist longer while facing obstacles, and eventually attain higher levels of achievement. As to the domain of mathematics, students’ mathematics self-efficacy (SMSE) beliefs have a powerful

**State of the literature**

- Teacher efficacy has been recognized as “a variable accounting for individual differences in teaching effectiveness” and has a strong relationship to student learning and achievement.

- Empirical evidence has revealed that self-efficacy begins to decline in grade 7 or earlier; this is particularly evident in mathematics at the transition to middle school. Fifth and sixth grade children confront dramatic psychological, physiological, and social changes.

- A student’s mathematics self-efficacy is predictive of her/his mathematics achievement. In addition, teacher efficacy significantly influences students’ learning. However, less empirical evidence exists to support the effect of teacher efficacy on students’ achievement, especially for elementary mathematics.

**Contribution of this paper to the literature**

- The findings of this study indicate that the more efficacious a mathematics teacher the better her/his students’ mathematics self-efficacy, and that, in turn, promotes their mathematical achievement.

- Mathematics teacher educators must devote extensive efforts to establish a positive and collaborative working and in-service learning environment that promotes mathematics teacher efficacy.

- Successful teaching attainments help develop a positive and robust belief in a mathematics teachers’ efficacy, which then contributes to the expectation of future proficient performance. Students’ successful learning attainments in mathematics are likely to be affected by their teachers’ effective teaching performance, which are partially reinforced by the teachers’ efficacy belief attainments.
Math teacher efficacy and students math self-efficacy

impact on the level of academic achievement and performance they may eventually achieve in learning mathematics (Chang, 2012; Kitsantas, Cheema, & Ware, 2011; Pajares & Miller, 1994; Pajares & Kranzler, 1995); that is, SMSE has been shown to predict students' mathematical achievement (SMA). In this study, the "Elementary Students Mathematics Self-Efficacy Instrument (ESMSEI) was employed to assess participating students' mathematics self-efficacy ratings; the instrument was developed and validated by the researcher (Chang, 2012) based on Bandura's (1977, 2006) theory and his guidelines. ESMSEI also consists of two cognitive constructs, "General Self-Efficacy—Related Mathematics (GSE-M)" and "Self-Efficacy for Mathematical Learning (SEML)".

Since teacher efficacy has a strong impact on student learning and achievement, do teacher efficacy beliefs have a direct influence on the development of students' self-efficacy in the classroom? In fact, several studies, domestically and internationally, have indicated that a teacher's efficacy beliefs and her/his students' self-efficacy are significantly correlated (Bandura, 1982; Janet et al., 1995; Shao, 2005; Liu & Zhou, 2007; Tang & He, 2006). However, little knowledge is available for the domain of mathematics learning, and even less so for elementary students. Further, empirical evidence has revealed that self-efficacy begins to decline in grade 7 or earlier (Urdan & Midegley, 2003); this is particularly evident in mathematics at the transition to middle school (Jacobs, et al., 2002). Of importance, fifth and sixth grade children are positioned right at the developmental transition period in which they confront dramatic psychological, physiological, and social changes. As students face new challenges during this fast-growing stage (Schunk & Meece, 2006), an understanding of the relationship between teacher efficacy and students' self-efficacy in the learning of mathematics may be particularly beneficial. Consequently, the first intention of this study is to assess the effect of mathematics teachers' MTE on the SMSE of their students, who are at the beginning stage of this transitional period (i.e. fifth-graders).

Empirical evidence has shown that teacher efficacy has a powerful impact on student learning and achievement (Cantrell, Young, & Moore, 2003; Gibson & Dembo, 1984; Goddard & Goddard, 2001; Tschannen-Moran & Woolfolk Hoy, 2001; Ross, 1998). Nevertheless, Goddard & Goddard (2001) argued that the effect of teacher efficacy on student achievement is indirect, where teacher efficacy is influential on teaching behaviors that, in turn, enhance students' learning achievement. Since less research has been done relative to elementary mathematics, Woolfolk Hoy and Davis (2006) proposed a framework to connect teacher efficacy and students' learning outcomes that hypothesizes the potential effects of teacher efficacy on students' beliefs (e.g. self-efficacy) and behaviors. Accordingly, it is worthwhile to examine the effect of mathematics teacher efficacy (MTE) on students' mathematics self-efficacy (SMSE). As verified in the current researcher's previous study (Chang, 2012), a student's mathematics self-efficacy is predictive of her/his mathematics achievement (SMA). In addition, teacher efficacy significantly influences students' learning. However, less empirical evidence exists to support the effect of teacher efficacy on students' mathematics achievement, especially for elementary mathematics in Taiwan. Therefore, besides assessing the effects of MTE on SMSE, it is also essential to assess the effects of MTE on students' mathematics achievement (SMA). Altogether, in this study, it is worthwhile to verify whether SMSE has a meditative impact on the effect of MTE on SMA. This effort will help to clarify the relationship among the three factors, and also should be useful for improving the quality of teaching and learning in mathematics.
Purposes and hypotheses

Based on the background and motivation stated above, the three purposes of this study are as follows: (a) to investigate the effects of teachers’ MTE on their students’ SMSE; (b) to examine the effects of teachers’ MTE on their students’ SMA; and (c) to assess the mediative impact of SMSE on the effect of MTE on SMA. Based on foregoing purposes, this study has three research hypotheses as follows:

H1: MTE has a significant effect on SMSE, and significantly predicts SMSE.
H2: MTE has a significant effect on SMA, and significantly predicts SMA.
H3: SMSE has a mediative impact on the effect of MTE on SMA.

THEORETICAL FRAMEWORK

Mathematics teacher efficacy

Based on Bandura’s (1977) social cognitive theory, perceived self-efficacy (SE) refers to individual “beliefs in one’s capability to organize and execute the courses of action required managing prospective situations” (Bandura, 1997, p. 2). Self-efficacy substantially influences individuals’ task choices, effort, persistence, and achievement (Bandura, 1997). As related to teachers, teacher efficacy (TE) is “a type of self-efficacy” (Ross, 1998, p. 50) and has been viewed as “self-efficacy beliefs directed toward a teaching context” (Knoblauch & Woolfolk Hoy, 2008, p.167). Based on Bandura’s (1977) theory, teacher efficacy is relevant to the effort teachers invest, the objectives they set, and their persistence and resilience in the face of problems and setbacks (Tschannen-Moran & Woolfolk Hoy, 2007). In addition to the effects of teachers’ decision-making actions in the classroom, teacher efficacy beliefs are influential in their thought processes, motivations, and affective and psychological states (Bandura, 1997). Bandura (1997) proposed four factors that would affect the development of teacher efficacy: mastery experiences, verbal persuasion, vicarious experiences, and physiological arousal. Further, Tschannen-Moran, Woolfolk Hoy, and Hoy (1998) proposed an integrated model of teacher efficacy emphasizing its cyclical nature (see Figure 1). That is, teacher efficacy, as an internal belief, is a powerful influence on a teacher’s external performance, which in turn cyclically adds new impetus to the future development of their efficacy beliefs.

Figure 1. The cyclical nature of teacher efficacy
Note: “Performance” was meant to indicate “teachers’ instructional behaviors in the classroom while teaching mathematics and their teaching effectiveness”.

In addition to the effect of teachers’ instructional performance, teacher efficacy has a strong relationship to student learning and achievement (Cantrell, Young, & Moore, 2003; Gibson & Dembo, 1984; Ross, 1998). While TE is defined as “a judgment about capabilities to influence student engagement and learning” (Woolfolk Hoy & Davis, 2006, p. 117), supporting empirical evidence drawn from studies over the past four decades has shown that teachers with a higher sense of efficacy for teaching may lead to their students’ positive learning outcomes, such as students’ self-efficacy, motivation, and academic achievement (Tschannen-Moran & Woolfolk Hoy, 2007). In fact, Ashton (1984) indicated that there were eight dimensions that distinguished teachers with high and low efficacy; they are: a sense of personal accomplishment, positive expectations for student behavior and achievement, personal responsibility for student learning, strategies for achieving objectives, positive affect, sense of control, sense of common teacher-student goals, and democratic decision-making. For instance, in the dimension of “positive affect”, mathematics teachers with a high sense of efficacy usually feel good about their own teaching and are more confident of having positive effects on their students’ mathematical learning. On the contrary, low efficacious teachers are typically frustrated with their own teaching, as well as often expressing discouragement and negative feelings about their instructional tasks with students. In relation to the aspect of “personal responsibility for student learning”, high-efficacious teachers demonstrate that “it is their responsibility to see that children learn, and when their students experience failure, they examine their own performance for ways they might have been more helpful” (Ashton, 1984, p. 29). Additionally, high efficacious teachers are willing to apply new instructional methods to better meet their students’ learning needs (Woolfolk Hoy & David, 2006); for example, they employ multiple or more effective instructional strategies to improve students’ mathematical problem-solving skills (Chang, 2010). However, teachers who have low efficacy tend to find excuses related to students’ ability, family background, motivation, or attitude.

Allowing for the powerful effects of TE on their own teaching performance, Goddard & Goddard (2001) claimed that “the relationship between teacher efficacy and student achievement appears to be indirect, with teacher efficacy influencing numerous teacher behaviors that, in turn, promote student achievement” (p. 808). In fact, previous studies have revealed that teacher efficacy has a powerful impact on student learning and achievement (Cantrell, Young, & Moore, 2003; Gibson & Dembo, 1984; Ross, 1998). However, little knowledge has been attained for the subject domain of mathematics, even less at the elementary level. In addition, both theoretical underpinnings about TE and empirical studies support the assertion that students’ self-efficacy is influential to their own learning and achievement (Bandura, 1977; Schunk & Meece, 2006). Further, Woolfolk Hoy and Davis (2006) have proposed a framework to link teacher efficacy and student outcomes, which postulates that students’ beliefs (e.g. self-efficacy) and behaviors might be one of the two types of potential consequences of teacher efficacy. Consequently, within the context of mathematics teaching and learning, the relationship between teacher efficacy and students’ self-efficacy needs to be closely investigated.

**Students’ mathematics self-efficacy**

With reference to students, "self-efficacy" is regarded as one of eight influential factors for their learning performance (Bandura, 1977). Compared with students who question their own capabilities, those with higher level of self-efficacy beliefs about learning or academic performance are likely to set higher goals, work harder in schools, and persist longer while facing difficulties and setbacks, which in turns lead to higher academic achievement (Chang, 2012; Schunk & Meece, 2006). Pajares
and Miller (1994) indicated that students' self-efficacy had a direct effect on their academic achievement and performance. As Schunk (1991) argued, students' academic self-efficacy refers to their subjective beliefs of successfully achieving learning tasks given in classrooms at desired levels. Since assessing SE needs to be context and task specific (Zimmerman, 2000), empirical evidences revealed that students' self-efficacy and their academic achievement were positively correlated in diverse content domains (Lent, Lopez, & Bieschke, 1991; Pajares, 1996; Schunk & Meece, 2006).

In regard to mathematics, several studies found that students' mathematics self-efficacy (SMSE) could significantly predict their mathematical achievement and/or performance (e.g. mathematical problem-solving skills) in different levels of educational settings (e.g. secondary schools and colleges) (Kitsantas, Cheema, & Ware, 2001; Nasirian, Azar, Noruzy, & Dalvand, 2011; Pajares & Kranzler, 1995; Pajares & Miller, 1994). Even though there is robust evidence about the influence of SMSE on mathematical achievement or performance for adolescents, little knowledge has been obtained for elementary students. Because of this deficiency, Chang (2012) has conducted the present study to examine the effects of 1244 fifth-graders' mathematics self-efficacy on their mathematical achievement in school, as well as assessing the effects of gender, family socio-economic status, and parenting styles on SMSE. It was found that these fifth-graders' SMSE ratings (the whole scale and two sub-scales—GSE-M and SEML) could effectively predict their mathematical achievement. In brief, this finding indicated that, corresponding to the previous studies, SMSE has a significant impact on the gradation of attainment students might achieve while learning mathematics.

**RESEARCH DESIGN**

**Samples**

Fifty eight fifth-grade classrooms were included in the study. A total of 58 mathematics teachers and 1244 students participated in this study. They were selected, by using a stratified random sampling method (by school size), from elementary schools of two regions in southern Taiwan. There were two reasons for choosing these two regions as the target of this investigation: The two regions were diverse with respect to elementary students’ backgrounds (e.g. family socio-economic status) and where they live (e.g. urban and rural). Students in big cities typically obtain more resources and attention. These two regions had a balanced distribution of both rural and urban schools. Based on the purposes of this study, data were collected by administering the EMTEI to teachers and the MSEI to students, as well as collecting students' mathematics achievement scores from school.

**Data collection and analysis**

Tools used in this study in order to explore mathematics teachers’ efficacy beliefs were the “Elementary Mathematics Teacher Efficacy Instrument (EMTEI, Chinese version)” (Chang & Wu, 2004; Chang & Wu, 2009), adapted from Mathematics Teaching Efficacy Beliefs Instrument (MTEBI) for pre-service teachers (Enochs, Smith, & Huinker, 2000). The EMTEI consists of “Personal Mathematics Teaching Efficacy (PMTE, 13 items)” and “Mathematics Teaching Outcome Expectancy (MTOE, 8 items),” rated on a 5-point Likert scale; five items were written in a positive orientation and 16 items were written negatively (see Figure 2 for the construct and sample items). The EMTEI has respectable internal consistency of .77, .81, and .71.
for the whole scale, PMTE and MTOE subscales respectively; PMTE and MTOE accounted for 20.82% and 15.86% of variance, respectively (Chang & Wu, 2004).

In assessing students' mathematics self-efficacy (SMSE), Zimmerman (2000) mentioned that items should be context and task specific. In order to gain the preferred predictive and conceivably interpretive effects, items should also be constructed in a multidimensional approach (Bandura, 2006; Pajares, 1996). Grounded in Bandura's (2006) guidelines and his multidimensional scales, the “Mathematics Self-Efficacy Instrument (MSEI, Chinese version)” was developed and validated in a previous study (Chang, 2012). Two subscales were included in this instrument (see Figure 3 for the construct and sample items): “General Self-Efficacy—Related Mathematics (GSE-M, 24 items)” and “Self-Efficacy for Mathematical Learning (SEML, 23 items),” rated on a 100-point scale. The GSE-M subscale is designed to assess elementary students' general SE, which is associated with their mathematical learning. In GSE-M scale, items are categorized into four aspects, i.e. enlisting social resources and parental support, academic achievement, self-regulated learning, and meeting others' expectations. Another subscale is designed to correspond to the practical learning context within and after school. Beginning from the higher grade levels in elementary schools, students are usually taught within test-oriented activities, which place greater emphasis on memorizing and calculating while learning mathematics. Therefore, within the SEML subscale, three types of items are included to contextually assess students' realistic learning circumstances, i.e. mathematics cognitive, strategy, and test preparation items. MSEI has high internal consistency of .96, .93, and .95 for the total scale, GSE-M, and SEML sub-scales respectively (Chang, 2012). In this study GSE-M and SEML accounted for 27.68% and 20.41% of variance, respectively. Both subscales were significantly correlated, r = .74, p < .001.

Students' mathematical achievement (SMA) in school was represented in terms of their overall mathematics scores at the fifth-grade level. SMA scores were collected at the end of the school year and then transformed into T scores for further analyses. Statistical analyses were applied to address the research objectives,
including descriptive analysis, analysis of variance (for H1 & H2), and regression analysis (for H3). Included below is a discussion of the findings, and recommendations for future study and further improvements of MTE, SMSE, and SMA.

FINDINGS

Descriptive reports

For teachers, the mean rating of all 58 fifth-grade mathematics teachers on MTE was 77.93 (SD=7.91), which meant that on average they had approximately 74% confidence in their own mathematics teaching capabilities. For students, the mean rating of all 124 fifth-graders on SMSE was 69.84 (SD=18.53), which meant that on average they had nearly 70% confidence in their own mathematics learning abilities. The characteristics of teacher and student samples are shown in Table 1 and Table 2 respectively.

The effects of fifth-grade teachers’ MTE on SMSE

In order to examine the effects of MTE on SMSE using ANOVA, all teachers’ MTE ratings were divided into three levels, i.e. "high (top 27% of ratings)", "middle", and "low (bottom 27% of ratings) MTE. Regarding the effect of MTE on SMSE, the results indicated statistically significant differences in fifth-graders’ SMSE ratings among the three levels of MTE, F (2, 1241) = 49.42, p< .001. The strength of the relationship between MTE and SMSE, as assessed by η², was strong, accounting for 7.4% of the variance for MTE. The post hoc comparison using the Scheffé test concluded that fifth-graders taught/led by teachers with high MTE (M=76.88)
scored significantly higher on SMSE than those taught/led by teachers with middle MTE (M=69.54) and low MTE (M=63.35). Additionally, fifth-graders taught/led by teachers with middle MTE also scored significantly higher on SMSE than those taught/led by teachers with low MTE.

To determine whether a mathematics teacher's efficacy belief could predict her/his students' mathematics self-efficacy, a simple regression analysis of MTE regressing on SMSE was conducted. The findings showed that MTE significantly predicted SMSE, $F(1, 1242) = 104.96$, $p < .001$, with 7.8% of SMSE variance explained by MTE. The standardized regression coefficients indicated that MTE ($B = .279$, $t = 10.25$, $p < .001$) had significant effects on SMSE. In brief, these findings indicated that fifth-graders who were taught/led by teachers with higher MTE were positively influenced on their SMSE. It means that a fifth-grade mathematics teachers with high MTE are potentially can help fifth-graders increase their SMSE in the classroom. Accordingly, H1 was supported in this study.

Further, to determine whether a teacher's efficacy ratings on the two subscales of MTE (i.e. PMTE and MTOE), together, could predict a student's mathematics self-efficacy, a simultaneous regression analysis of PMTE and MTOE regressing on SMSE was conducted. The findings showed that PMTE and MTOE, together, significantly predicted SMSE, $F(2, 1241) = 49.56$, $p < .001$, with 7.4% of SMSE variance explained by both PMTE and MTOE. The standardized regression coefficients indicated that PMTE ($B = .172$, $t = 5.32$, $p < .001$) had significant effects on SMSE, which were greater than the significant effects of MTOE ($B = .138$, $t = 4.28$, $p < .001$) on SMSE.

### The effects of fifth-grade teachers’ MTE on SMA

Regarding the effect of MTE on SMA, the results revealed statistically significant differences in fifth-graders' SMSE ratings among the three levels of MTE, $F(2, 1241) = 53.65$, $p < .001$. The strength of the relationship between MTE and SMA, as assessed by η², was quite strong, accounting for 8.0% of the variance for MTE. The

---

**Table 1. Characteristics of teacher samples (N=58)**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Category and Frequency (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gender</td>
<td>Male: 25 (43.1%) Female: 33 (56.9%)</td>
</tr>
<tr>
<td>Expertise</td>
<td>Math and/or science education related: 17 (29.3%) Non-math or science education related: 41 (70.7%)</td>
</tr>
<tr>
<td>Age</td>
<td>30 or less: 10 (17.2%) 31 to 40: 33 (56.9%) 41 &amp; up: 15 (25.9%)</td>
</tr>
<tr>
<td>Education</td>
<td>Doctorate: 16 (27.6%) Master: 37 (63.8%) University/College: 5 (8.6%)</td>
</tr>
<tr>
<td>Years of teaching Math</td>
<td>5 or less: 11 (19.0%) 6 to 10: 22 (37.9%) 11 to 20: 21 (36.2%) 21 &amp; more: 4 (6.9%)</td>
</tr>
</tbody>
</table>

**Note:** Analyses of these background variables were not included in this paper.

**Table 2. Characteristics of student samples (N=1244)**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Category and Frequency (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gender</td>
<td>Boy: 607 (48.8%) Girl: 637 (51.2%)</td>
</tr>
<tr>
<td>School Size</td>
<td>Large (25 classes &amp; up): 614 (49.4%) Middle (7–24 classes): 426 (34.2%) Small (6 classes &amp; less): 204 (16.4%)</td>
</tr>
<tr>
<td>SES</td>
<td>SES I: 47 (3.8%) SES II: 320 (25.7%) SES III: 625 (50.24%) SES IV: 244 (19.61%) SES V: 8 (0.6%)</td>
</tr>
<tr>
<td>Parenting Style</td>
<td>Authoritarian-Autocratic: 216 (17.4%) Authoritative-Reciprocal: 884 (71.1%) Indulgent-Permissive: 44 (3.5%) Indifferent-Uninvolved: 434 (34.1%)</td>
</tr>
</tbody>
</table>

**Note:** Only the briefest descriptions of background variables (e.g. gender, socio-economic status [SES], and parenting style) were provided; please refer to Chang (2012) for details.
post hoc comparison based on Scheffé concluded that fifth-graders taught/led by teachers with high MTE (M=53.16) scored significantly higher in SMA compared to those taught/led by teachers with middle (M=48.93) and low MTE (M=45.18), and fifth-graders taught/led by teachers with middle MTE scored significantly higher in SMA than did those taught/led by teachers with low MTE.

To determine whether a mathematics teacher's efficacy belief could predict her/his students' mathematics achievement, a simple regression analysis of MTE regressing on SMA was also conducted. The findings showed that MTE significantly predicted SMA, F (1, 1242) = 132.24, p< .001, with 9.6% of SMA variance explained by MTE. The standardized regression coefficients indicated that MTE (B = .310, t = 11.50, p < .001) had significant effects on SMA. In short, these findings suggested that teachers with higher MTE positively influenced their fifth grade students' SMA. It indicates that a fifth-grade mathematics teacher with high MTE potentially can help fifth-graders increase their SMA in the classroom. Accordingly, H2 was supported in this study.

Moreover, to determine whether a teacher's efficacy ratings on the two subscales of MTE (i.e. PMTE and MTOE), together, could predict a student's mathematics achievement, a simultaneous regression analysis of PMTE and MTOE regressing on SMA was also conducted. The findings showed that PMTE and MTOE, together, significantly predicted SMSE, F (2, 1241) = 67.12, p< .001, with 9.8% of SMA variance explained by both PMTE and MTOE. The standardized regression coefficients indicated that the significant effects of PMTE (B = .220, t = 6.91, p< .001) on SMA were greater than the significant effects of MTOE (B = .133, t = 4.19, p < .001) on SMA.

The mediative impact of SMSE on the effect of MTE on SMA

To determine whether a mathematics teacher's efficacy belief and a student's mathematics self-efficacy could, together, predict a student's mathematics achievement, a multiple regression analysis of MTE and SMSE regressing on SMA was conducted. The findings showed that MTE and SMSE, together, significantly predicted SMA, F (2, 1241) = 210.06, p< .001, with 25.3% of SMA variance explained by both MTE and SMSE. The standardized regression coefficients indicated that SMSE (B = .412, t = 16.13, p< .001) produced significant effects on SMA, which were greater than the significant effects of MTE (B = .195, t = 7.64, p < .001) on SMA (See Figure 4).

As reported in 4.2 and 4.3, MTE had a significant effect on both SMSE (B =.279, p < .001) and SMA (B =.195, p < .001). Also, according to previous research (Chang, 2012), SMSE had a significant effect on SMA (B = .412, p < .001). The mediation

![Figure 4. Standardized regression coefficients of the mediative impact](image-url)
Math teacher efficacy and students' math self-efficacy
effect of MTE on SMA carried a standardized parameter estimate of .115 (p < .001). These findings showed that MTE significantly predicts SMSE, which in turn significantly predicts SMA. This indicates there is a mediation effect which is partial. In summary, this finding revealed that a fifth-graders who were taught by mathematics teachers with high MTE tended to have better mathematics achievement (SMA), with a partial support of her/his own higher mathematics self-efficacy (SMSE). Therefore, H3 was supported in this study.

CONCLUSION AND DISCUSSION

MTE significantly influence fifth-graders' SMSE and SMA

First, the findings of regression analyses, paralleling the results of ANOVA, indicated that MTE significantly predicted fifth-graders’ SMA. This finding of significant effects of a mathematics teacher's efficacy belief on her/his students' mathematical achievement in school corresponds to the findings of previous studies (Ashton & Webb, 1986; Rosenholtz, 1989); and the results were consistent with studies in different subject areas (e.g. Bandura, 1982; Janet et al., 1995; Shao, 2005; Tang & He, 2006). It is notable that MTE had substantial effects on SMSE as well, where the mediative impact of SMSE on the effect of MTE on SMA is partial. This result appears to show that the more efficacious a mathematics teacher the better her/his students’ mathematics self-efficacy, and that, in turn, promotes their mathematical achievement in school. In addition, it also was found that PMTE yielded superior effects on both SMSE and SMA than did MTOE. Since PMTE refers to teachers' beliefs in their ability to teach mathematics effectively, the importance of this critical influence on the development of students' mathematics self-efficacy deserves to be noted. As previous studies argued, a teacher's efficacy belief is influential on her/his teaching behaviors that, in turn, enhances students' achievement (Goddard & Goddard, 2001; Woolfolk Hoy & Davis, 2006).

As mentioned previously, since teacher efficacy plays an important role in promoting students' learning achievement and their self-efficacy development in the classroom, teacher educators must devote extensive efforts to establish a positive and collaborative working and in-service learning environment that promotes mathematics and/or science teacher efficacy (e.g. Mavrikaki & Athanasiou, 2011; Siegle & McCach, 2007; Stevens, Harris, Aguirre-Muno, & Cobbs, 2009), as well as providing better teacher preparation programs for pre-service teachers’ efficacy development (e.g. Incikabi, 2013; Richardson & Liang, 2008; Sancar-Tokmak, 2013). In addition, the 58 mathematics teachers, on average, had nearly 74% confidence in their own mathematics teaching capabilities, about 20 of them were lower than 70%. This comparatively low efficacy and inadequate readiness in teaching elementary mathematics needs to be carefully acknowledged while discussing the future task of teacher professional development. Since teachers with high efficacy tend to put more effort in preparing and teaching, persist longer while facing students’ learning problems, and have more flexible selection of instructional activities, these enthusiastic actions combined with positive thoughts and adaptive expectations definitely will be beneficial for establishing a preferable learning environment, which in turn supports students' mathematical learning.

Fifth-graders' SMSE had an effect on their mathematical achievement

In this study, the 1244 fifth-graders had an average of 70% confidence in their own mathematics learning abilities. Findings of several studies (e.g. Kitsantas, Cheema, & Ware, 2011; Nasirian, Azar, Noruzi, & Dalvand, 2011; Ocak & Yama, 2013) revealed that students' mathematical self-efficacy had significant effects on them.
their mathematical achievement, which corresponded to Bandura's (1977) claim that "self-efficacy" was a powerful factor for students' learning performance. Additionally, the same phenomenon was evident in the researcher's previous study that demonstrated that the higher the SMSE score, the better the student's mathematical achievement (Chang, 2012), with the implication that "how to increase or maintain the status of their SMSE became more essential to help them be successful in learning mathematics in school both at this transitional period and in the future" (Chang, 2012, p. 524). It follows that providing an effective positive learning environment in this fast-growing and transitional stage will help to prevent possible decline of their SMSE (Jacobs, et al., 2002), which may also be helpful in promoting students' learning achievement in mathematics.

As shown in Figure 1, there are at least four sources that have an effect on teacher efficacy: mastery experiences, verbal persuasion, vicarious experiences, and physiological arousal (Woolfolk Hoy, & Hoy, 1998). In fact, active mastery experiences were verified as the most powerful source of efficacy information, with the most reliable evidence drawn from practical teaching performance accomplishments with students (Bandura, 1997; Tschannen-Moran & Woolfolk Hoy, 2007). Successful teaching attainments in mathematics "developed a positive and robust belief in a teacher's efficacy, which then contributed to the expectations of future proficient performance" (Chang, 2010, p. 274). These findings also are applicable for the development of students' mathematics self-efficacy. Sharing the same self-efficacious construct of the theoretical framework of social learning theory, students' successful learning attainments in mathematics are likely to be affected by their teacher's effective teaching performance, which would be partially reinforced by the teacher's efficacy belief attainments. Grounded in the framework of Woolfolk Hoy and Davis (2006) and the findings of this study, it is noteworthy that we can qualitatively examine the relationship between teachers' efficacy beliefs and students' outcomes (i.e. both self-efficacy and achievement) within the context of mathematics learning in a longitudinal design, which may be essential in exploring better practices for future mathematics teachers professional development.

ACKNOWLEDGEMENTS

This research was partially supported by NSC of Taiwan (NSC 98-2511-S-415-014-M; NSC 102-2511-S-415-003 MY2). The author is grateful to the editor, reviewers, and participants, as well as Prof. Dale Gentry for language editing.

REFERENCES


