Rise or Resist: Exploring Senior Secondary Students’ Reactions to Challenging Mathematics Tasks Incorporating Multiple Strategies

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Learning to solve more challenging mathematics problems using multiple strategies has been promoted by research and is a feature of numerous curricula. Yet teachers have been found to be reluctant to incorporate these tasks into their lessons predicting resistance from students. This study examined 87 senior secondary students’ reactions to a challenging quadratics task as part of their teachers’ participation in a design-based research project. It sought insights into students’ own perspectives on challenge and multiple strategies in mathematics teaching and learning, with the intent of understanding more about the cognitive, affective, and motivational aspects of this teaching approach. In responding to open-ended prompts, the students reported diverse but generally positive reactions related to their interest in the task and its relevance for their learning. Two thirds reported that they liked learning strategies from peers, and two thirds liked learning multiple strategies from their teacher. Implications for considering secondary students’ preferences for learning, engagement, and motivation in mathematics lessons are discussed.

Keywords: challenging mathematics tasks, multiple solutions, achievement goals, engagement; motivation, quadratic functions

INTRODUCTION

Being able to reason and think critically by evaluating different ways to solve a problem is an important skill in today’s world across many domains. In mathematics, learning multiple strategies for solving problems has been promoted by research and is incorporated in some high-performing international education...
systems. It is an important feature of challenging tasks, defined in this study as those tasks that “require students to process multiple pieces of information simultaneously and make connections between them, and for which there is more than one possible solution or solution method” (Sullivan et al., 2014). Many curriculum and education policy documents advocate the use of multiple-strategy tasks (e.g., Australian Curriculum Assessment and Reporting Authority, 2009; National Council of Teachers of Mathematics, 2000; Woodward et al., 2012). A number of theoretical models for mathematical problem solving and thinking incorporate them (e.g., Fraivillig, Murphy, & Fuson, 1999; Hufferd-Ackles, Fuson, & Sherin, 2004; Stein, Engle, Smith, & Hughes, 2008; Zazkis & Leikin, 2009). Silver and colleagues described their value for connective learning:

It is nearly axiomatic... that students should have experiences in which they solve problems in more than one way. Doing so has the potential advantage of providing students with access to a range of representations and solution strategies in a particular instance that can be useful in future problem-solving encounters. Moreover, different solutions can facilitate connection of the problem at hand to different elements of knowledge. (Silver, Ghousseini, Gosen, Charalambous, & Strawhun, 2005, p. 288)

Although studies on younger students learning multiple strategies are quite prevalent in the literature there is an increasing interest in how they are or might be used at secondary levels of schooling. The pervasiveness of teaching with multiple strategies in different education systems is not well understood (Lynch & Star, 2014). It is suspected that to utilise multiple strategies, suitable teaching approaches at secondary levels will differ from those at primary levels. There is not only the need to consider effective approaches for learning at secondary levels (Sullivan & Murnane, 2014) but also how to support their sustainable use by secondary teachers. Research has highlighted the constraints teachers can experience when trying to increase the task demand on their students by providing opportunities to solve challenging problems and explore different solution methods (Swan, 2007; Tzur, 2008).

This article discusses the findings of a sub-project within a larger project investigating the potential of challenging tasks for effective learning by examining teachers’ experiences of posing such tasks, strategies for helping students persist, the effect on students’ learning, and students’ responses to them. The sub-project

State of the literature

- Challenging mathematics tasks, which encourage students’ development of multiple strategy use, have been demonstrated in studies as effective for conceptual learning.
- Studies have found that secondary teachers are reluctant to use challenging tasks because they experience resistance from students or pressure to simplify the task; they are also reluctant to teach multiple strategies because high-achieving students are perceived as being bored by them and low-achieving students as being confused by them.
- Limited research exists on students’ perceptions of challenging tasks in relation to their motivation and engagement, since studies have focussed more on teachers’ views.

Contribution of this paper to the literature

- Novel and challenging tasks that secondary students viewed as relevant to their learning goals seemed to promote situational interest, engagement, and a positive affective response.
- High-achieving students who have experienced cognitive improvement on challenging tasks in an environment emphasising progress rather than performance in mathematics seemed spontaneously to evidence mastery goals rather than performance goals.
- Some high-achieving students expressed the desire to understand multiple strategies for solving one problem simultaneously, rather than learning about multiple strategies to choose one to adopt from among them. This finding is at odds with studies of teachers’ reasons for teaching multiple strategies.
focused on senior secondary students’ reactions to a challenging task after having experienced a number of them throughout the year, and examined their views on challenge in general and on learning multiple strategies. The central research questions for this study were:

1. What do senior secondary students focus on in describing their reactions to challenging tasks and their views on learning multiple strategies?
2. How do students’ perspectives relate to affective, cognitive, and motivational aspects of their mathematics learning?

The following section provides details on the context for the research and discusses theoretical perspectives from the literature, which provided a framework for the study.

RELATED RESEARCH AND CONTEXT

In the past, learning mathematics was viewed as an essentially cognitive process, but more recently the affective dimensions – emotions, attitudes, beliefs, and values – are being considered more seriously as important influences on learning (DeBellis & Goldin, 2006). Researchers examining intellectual functioning from different paradigms nonetheless advocate the integrating the cognitive domain with the affective and motivational domains (Dai & Sternberg, 2004). This study examined the students’ expressions of their views about challenging tasks and multiple strategies to see how they might relate to cognitive, affective, and motivational aspects of their learning. The following sections provide details on each of these three aspects.

Cognitive aspects of learning with challenging tasks incorporating multiple strategies

Involving students in wrestling with important mathematical ideas is considered an effective way of facilitating their conceptual understanding (Hiebert & Grouws, 2007). Therefore tasks that make higher cognitive demands on students elicit more opportunities for restructured and reconfiguring their connections between facts, ideas, and strategies (Stein & Lane, 1996). The types of challenging tasks explored in this project involve finding more than one solution or strategy, which encourages students to grapple with key ideas simultaneously and make connections between them. The literature highlights three approaches to multiple strategies, which are presented in Table 1. For ease of reading, each approach has been categorised (Types 1 – 3).

Table 1. Three approaches to teaching and learning with multiple strategies

<table>
<thead>
<tr>
<th>Type 1: Generated by learners</th>
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<tr>
<td>Individuals or groups are provided with the task and asked to solve the problem in more than one way. This supports students in inventing their own strategies based on their current conceptual and intuitive understanding of the mathematics involved. “The lesson then concludes with the whole class discussion and summary of various student-generated approaches… for the whole class to view and discuss” (Stein et al., 2008, p. 316).</td>
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<th>Type 2: Demonstrated by teachers</th>
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<td>Teachers introduce and explain different methods for solving the problem. They might use this approach for helping a diverse range of students find one way to learn the particular concept successfully (Lynch &amp; Star, 2014). A step beyond this is helping students develop logical thinking through comparing and evaluating the efficiency of different strategies to make connections between concepts (Rittle-Johnson &amp; Star, 2007).</td>
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<th>Type 3: Provided as worked examples</th>
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<td>Learners are provided with written examples of a type of problem, the solution steps for multiple strategies, and the final solutions. Teachers might aim to relieve learners of the struggle of having to find a solution on their own, to concentrate effectively on comparing the different strategies (Große &amp; Renkel, 2006).</td>
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There is much in the research literature about the Type 1 approach at primary levels of schooling and is a feature of reform mathematics teaching in the United States. Mathematics research on Japanese secondary mathematics classrooms has also highlighted that a key aspect of their problem-solving-oriented lessons is having students find and then present their solutions so that the class can discuss them (Fujii, 2014; Ohtani, 2014).

Some research describes combinations of different approaches, for example eliciting students’ solution attempts before providing worked examples (e.g., Fraivillig, Murphy, & Fuson, 1999) or elaborating on students’ initial solutions and generalising them further with the class (Ohtani, 2014). In a review of studies on the use of multiple strategies for problem-solving, Woodward et al. (2012) concluded that all three types of approaches were recommended in the research literature for helping students learn.

One study considered two different approaches and their effect on students’ frequency and type of use of multiple strategies. Star and Rittle-Johnson (2008) evaluated 132 Year 6 students’ responses to prompts to discover strategies for themselves (Type 1 approach) and to direct teaching on multiple strategies (Type 2 approach) when solving linear equations. They found that prompts to solve the equation in two different ways led to greater use of multiple strategies whereas direct teaching led to the use of more efficient strategies. They asserted that direct instruction on multiple strategies after students had first experienced exploring different strategies for themselves provided “reliable gains in problem-solving flexibility” (p. 575).

Another study investigated the approach of providing multiple worked examples of solutions to task (Type 3 approach) to consider the effect on students’ procedural knowledge and conceptual knowledge. Rittle-Johnson and Star (2007) studied the use of multiple-solution examples with Year 7 students to solve linear equations in algebra. They found that comparing different solutions method side-by-side rather than simply learning each method sequentially produced greater gains in the students’ procedural knowledge and flexibility. They speculated that identifying similarities and differences was important, as well as the evaluation of each method’s efficiency. They cautioned that students would need sufficient prior knowledge to be able to do this effectively, which relates to concerns raised in another study about the cognitive load on students. Rittle-Johnson, Star, and Durkin (2009) found negative responses to and effects of the Type 3 approach for students with low or no prior knowledge of algebraic methods in problem solving. They speculated that the benefits of comparing alternative solution methods depend on sufficient prior knowledge, otherwise novices in a particular domain of mathematics could experience working memory overload and find the task demand too high. This could result in negative affect, and lowered engagement and motivation.

In another study, a schema-based instruction approach (provided by the teachers) aimed to help students interpret the problems, and compare and contrast multiple strategies (Type 3), with explicit attention paid to reducing the cognitive overload for low-achieving students. Jitendra et al. (2009) studied the effects of an intervention with 148 Year 7 students learning to solve ratio and proportion word problems (intervention and control groups randomly assigned). The intervention also included metacognitive strategies to help students with self-monitoring. The assessments – post-test and delayed post-test – demonstrated higher problem-solving performance for students who experienced the intervention. Yet both groups performed comparably on state standardised achievement testing. They speculated that a 10-day intervention was not long enough to encourage students to transfer their skills to other types of problems. They cautioned that it was difficult to establish which particular aspects of the intervention were responsible for the success or whether it was a combination of approaches.
Research with secondary teachers highlighted substantial reluctance to experiment with multiple-strategy tasks, even after professional learning (Leikin et al., 2006). From a group of 30 participants, all of whom indicated no prior experience with these types of tasks, only seven agreed to trial the tasks with their students. Of these teachers, only two agreed to implement the tasks and to also ask their students to complete post-task questionnaires about their perceptions. Discussions with the teachers revealed concerns about the inappropriateness of such tasks for low-performing students and about group-based learning involving students sharing solutions with each other. Leikin and Levav-Waynberg (2007) studied 12 secondary mathematics teachers and found that their lack of knowledge about and use of multiple-strategy tasks were related to their school curriculum, which for the most part did not include such tasks. Some teachers described the difficulty of having to respond to students’ alternative solutions (different to the one set out in the textbook). Leikin and Levav-Waynberg asserted that there is a gap between teachers’ theoretical valuing of multiple-strategy tasks for learning and their actual teaching practice – that they find it difficult to teach with multiple strategies and therefore do so rarely.

Another study examined middle and high school mathematics teachers’ perceptions of the advantages and disadvantages of using multiple strategies for teaching algebra (Lynch & Star, 2014). All of the teachers had reported using multiple strategies previously in their teaching; 90 out of 92 teachers indicated that they found the approach useful. A majority of the teachers described multiple strategies as helping a diverse range of students find at least one way to solve a problem that might eventually “stick” and therefore access the mathematics somehow. Lynch and Star compared this response to other studies on elementary teachers and noted that it was different in that elementary teachers were more likely to describe the developmental benefit of students learning to progress from intuitive (‘invented’) strategies to more efficient formal strategies. They speculated that either secondary teachers might have a “limited or incomplete view of the practice of teaching mathematics with multiple strategies” (p. 101) or that perhaps some approaches to multiple strategies in elementary grades such as strategy invention may not be appropriate at secondary levels. They suggested that the use of multiple strategies in elementary grades through inventing them might aim to build students’ intuitive knowledge, whereas their use at secondary levels might aim to help students “develop a more connected understanding of mathematics and to develop fluency” through knowing, comparing, and evaluating different strategies (p. 102).

**Students’ affective responses to challenging tasks and multiple strategies**

The interdependent nature of interactions between teacher and learner means that students make judgements about how they are taught and take an active role in determining the effects of teaching on their own learning. Student “interest and achievement mutually influence one another” (Shiefele & Csikszentmihalyi, 1995, p. 177) and teachers adjust their teaching to match their perceptions of students’ cognitive and affective reactions to their teaching and choice of tasks.

There is substantially more in the research literature on the perceptions of teachers about their students’ reactions than on students’ own expressions of their reactions. A few studies in the research literature focussed specifically on teachers’ perceptions of younger students’ reactions to learning with multiple-strategy tasks. Fennema, Carpenter, Levi, Jacobs, and Empson (1996) described elementary teachers’ perceptions that their students became eager to share their thinking and
were more enthusiastic about their learning when encouraged to solve problems using their own strategies to share them with the class (Type 1).

Research has highlighted that teachers believe students with higher ability resist opportunities to learn with challenging tasks because their “desire for immediate comprehension is very strong and may ultimately be debilitating” at more advanced levels of mathematics study (Davis, Hersh, & Marchisotto, 1995, p. 315). Studies on senior secondary mathematics students found that in response to being given challenging tasks involving “high-level cognitive processing or when the answers were not readily available”, “there was a tendency for students to resist task engagement or negotiate the task demands downwards” (Anthony, 1996, p. 42). Recent research with middle-school students also found that students exerted pressure on their teachers to over-explain the task or to provide simpler ones (Sullivan et al., 2009). In response to students’ resistance with challenging tasks, teachers are more likely to adapt their teaching by reducing the learning demands on them (Anthony, 1996; Swan, 2007; Tzur, 2008). This would then lead to more passive learning behaviours and teachers’ increased reluctance to incorporate cognitive demand in their lessons by using multiple strategies for learning.

Sullivan and Moranne (2014) sought junior secondary students’ own perceptions of challenging tasks and found mixed reactions, but all of the students reported preferring such tasks to textbook exercises or teachers writing questions on the board. Some students connected feeling less stressed about the task with knowing that there was more than one way to tackle it. Leikin, Levav-Waynberg, Gurevich, and Mednikov (2006) studied both Year 10 mathematics teachers’ perceptions and their students’ reactions to challenging multiple-solution tasks. The students were from only lower-level and intermediate-level classes. Initially the teachers (none of whom had used such tasks before) predicted that their students would dislike the activities because they would be confused by the different strategies and would not be able to explain their solutions to others. Yet the students subsequently reported positive reactions, related to appreciating being able to look for easier solutions, to think and understand the tasks in their own way, and expressing situational interest in the variety of the tasks. They found that some students who were in lower-level mathematics classes did express the view that the use of the tasks was confusing and unnecessary for them, indicating a preference for the usual way of being taught in a more teacher-directed way. The researchers expressed surprise that students in the intermediate level classes were more likely to prefer learning from receiving others’ strategy explanations whereas those in the lower-level class were more likely to prefer giving their strategies to others.

In studying 92 middle and high school mathematics teachers’ perceptions of using multiple strategies for teaching algebra, Lynch and Star (2014) found the teachers were more likely to pay attention to cognitive or pedagogical issues rather than the students’ affective reactions. Approximately 15% of the teachers made reference to the perception that multiple strategies got more students involved, increased their confidence, and reduced their boredom or frustration. Interestingly, a similar percentage described the opposite perception that multiple strategies bored high-ability students and de-motivated struggling students who were confused by trying to understand different methods. Nearly one third of teachers described experiencing resistance from students who preferred simply to learn one single method.

It appears that students’ affective reactions to tasks incorporating multiple strategies are diverse; there is more to understand about the factors influencing their reactions and how these might also relate to cognitive and motivational aspects of learning, such as the teaching approaches used, the relevance of the tasks, the set-up of the learning environment, and the students’ own dispositions, self-efficacy or ability levels. There were references to a student’s level of mathematics...
Reactions to challenging mathematics tasks

Motivational perspectives on learning

A student's disposition, goals and actions influence their participation in learning; their own curiosity and wish to learn may contribute to subsequent educational achievement (Cohen, Raudenbush, & Ball, 2003). Student motivation has been discussed extensively in the literature and is considered a key variable in students’ regulation of themselves – their decision to engage or not, how much effort to expend (Middleton & Toluk, 1999; Schoenfeld, 2007), their goal setting, and their choices in long-term academic pursuits (Pintrich & Schunk, 2002). There are different constructs for conceptualising student motivation. Motivation can be described as relating to both intrinsic and extrinsic factors – an individual's disposition and the influences on them from their environment (Middleton & Jansen, 2011).

One theoretical perspective describes the dichotomy of an individual's mastery or performance goal orientation to describe how an individual defines competence (Ames 1992; Pintrick & Schunk, 2002). Dweck (2007) related these to growth and fixed mindsets. A fixed mindset incorporates the belief that one’s qualities are carved in stone – that one has a fixed amount of intelligence, a certain personality, and certain moral character. A growth mindset views these qualities as able to be cultivated or improved through one’s effort, and although everyone differs “in their initial talents, attitudes, interests, or temperaments”, each person can grow through application and experience (Dweck, 2007, p. 7). A mastery goal orientation focuses on improving one’s own learning or progress and on task-based outcomes whereas a performance goal orientation focuses on comparing oneself with others, such as through test results or competitive situations. Hulleman, Schrager, Bodmann and Harackiewicz (2010) asserted that outcome (or achievement) goals, such as getting a good grade, ought to be classified separately from performance goals since they are neutral in terms of how competence is defined. They can be related to mastery (an "A" grade indicates mastering the content) or to performance (an "A" indicates outperforming others) orientations. Within the performance goal orientation Hulleman and colleagues further distinguished between appearance (demonstrating competence), normative (performing better than others), and evaluative, which is a hybrid of both appearance and normative components (demonstrating competence relative to others).

Another dichotomy considers the direction of motivation – approach and avoidance – to describe how competence is valenced: the inherent attraction (leading to approach) or aversion (leading to avoidance) of a situation or experience (Elliot, 1999). The resulting two-by-two goals framework using Elliot’s original conceptualisations and Hulleman’s and colleagues’ further distinctions is presented in Figure 1. It has been empirically supported by research substantially for 3 of the 4 motivation types but more recently also with mastery-avoidance (Jang & Liu, 2012).

In a meta-analysis of several studies Elliott (1999) noted that mastery goals were found to be linked to persistence, absorption during task engagement, challenge-related affect, self-regulated learning, deep processing, and intrinsic motivation. Examples of mastery-avoidance goals were found in which an individual strives to avoid losing or stagnating one's skills / abilities – of trying to avoid forgetting what one has already learned, and associated with a perfectionist need for achievement (Elliott & Murayama, 2008). Performance-approach goals were found to link to some positive consequences such as persistence, absorption during task engagement, challenge-related affect, and intrinsic motivation. But they were also linked to test
anxiety, shallow processing, and reluctance to seek help with tasks. Performance-avoidance goals were found to link to distraction, low absorption during task engagement, threat-related affect, less self-regulated learning, shallow processing, and anxiety about evaluation (Elliot, 1999). A study on student engagement in the middle years using the concept of Dweck’s mastery-performance goals dichotomy found that all of the higher-achieving students gave evidence of performance goals and that the students with mastery goals were neither in the low or high-achieving groups (Sullivan, Tobias, & McDonough, 2006).

In the literature, there remains ongoing debate about whether or not it is possible for an individual to hold both mastery and performance goal orientations at the same time (Brophy, 2005; Hulleman et al., 2010; Martin, 2013). Inconsistency with definitions and measuring across studies seems to have contributed to the lack of resolution and ongoing controversy (Hulleman et al., 2010). Brophy (2005) also raised the issue that most research using goal theory has involved measuring with experimental induction procedures or pre-defined Likert-scale surveys, which do not allow investigation into the degree to which students spontaneously generate these types of goals. He emphasised the need for research into students’ expressions in their own words and speculated that there would be less evidence of performance goals compared to results from normative surveys.

The study described in this article sought to explore students’ own expressions of their reactions to challenging tasks using an open-ended response questionnaire. It elicited their views about learning with multiple strategies to compare with their teachers’ perceptions, and used cognitive, affective, and motivational perspectives from the literature to understand more what matters to secondary students for their mathematics learning. The following section provides details of how the study was designed.

**RESEARCH DESIGN**

Much of the literature on challenging tasks and the use of multiple strategies has focussed on these issues from teachers’ perspectives. To consider specifically the voice of students (Creswell, 2007; Flutter & Rudduck, 2004; Clough & Nutbrown, 2007) this study was designed to elicit their views in an open-ended rather than normative approach. It sought to examine what students choose to focus on in describing their reactions to a particular challenging task and how their views on multiple strategies might relate to cognitive, affective, and motivational dimensions of learning. It aimed to “address both the pragmatic and highly theoretical issues

<table>
<thead>
<tr>
<th>Mastery-approach goal orientation</th>
<th>Performance-approach goal orientation</th>
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<tbody>
<tr>
<td><strong>Interest and curiosity:</strong> learning something interesting</td>
<td><strong>Appearance:</strong> demonstrating competence / ability</td>
</tr>
<tr>
<td><strong>Task:</strong> mastering a task</td>
<td><strong>Normative:</strong> performing better than other students</td>
</tr>
<tr>
<td><strong>Challenge:</strong> mastering a challenge</td>
<td><strong>Evalulative:</strong> Demonstrating my ability relative to others in the class</td>
</tr>
<tr>
<td><strong>Improvement or attainment:</strong> Learning as much as possible; improving my knowledge; understanding the content as thoroughly as possible; acquiring new skills</td>
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<thead>
<tr>
<th>Mastery-avoidance goal orientation</th>
<th>Performance-avoidance goal orientation</th>
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<tbody>
<tr>
<td><strong>Task:</strong> Avoiding forgetting what I have already learnt</td>
<td><strong>Appearance:</strong> avoiding looking incompetent / ‘dumb’</td>
</tr>
<tr>
<td><strong>Improvement or attainment:</strong> Avoiding losing my skills / abilities / knowledge; avoiding stagnation or lack of development</td>
<td><strong>Normative:</strong> Avoiding performing poorly in the class</td>
</tr>
<tr>
<td></td>
<td><strong>Evalulative:</strong> Avoiding demonstration of lack of ability relative to others (as judged by authority figure)</td>
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Figure. 1. Conceptualising four types of student goals using mastery-performance and approach-avoidance dichotomies (Elliot, 1999; 2008; Hulleman et al., 2010)
simultaneously” to achieve “reflexivity between theory and practice” (Cobb, 2000, p. 308). The study incorporated multiple sources of data (Creswell, 2007) to triangulate different views (Hesse-Biber, 2010) and seek plausible interpretations (Stake, 1995; Wolcott, 2009).

The study involved a cohort of Year 10 high-achieving students whose teachers were participating in a large design-based research project on challenging tasks. The students were from four mathematics classes and had begun attending a select-entry government school that year. Through a testing and interview process, their school had deemed them to be high achieving or to demonstrate an aptitude for mathematics. They were from a diverse range of socio-economic and ethnic backgrounds. Their teachers were invited to participate in the sub-study because of their students’ high ability and the research focus on this type of cohort’s reactions to challenging tasks incorporating multiple strategies. Previous research had highlighted teachers’ perceptions of high-achieving students’ reluctance to engage with challenging tasks, evidence of pervasive performance goal orientations, and teachers’ perceptions that such students react with boredom to learning multiple strategies. This study sought to explore this further but from the students’ perspectives.

**Data collection and analysis**

Towards the end the academic year, two of the six Year 10 mathematics teachers from the larger project agreed to a joint semi-structured interview to discuss their perceptions of their students’ reactions throughout the year to a number of challenging tasks (one hour’s duration – Appendix 2). They also raised issues they had encountered when implementing the tasks and responding in class to students. The teachers then suggested a topic for a subsequent challenging task, for which the researcher developed a selection of three quadratic tasks. The teachers chose a particular task and this was used in the sub-study (Appendix 1).

The researcher observed the implementation of the quadratic task across four classes (two large groups each with a pair of teachers in an open-plan area) and collected work samples of the students’ written responses to the task. The focus of the lesson observation and analysis of the work samples was on the perceived situational interest in the task, the perceived engagement of the students in tackling it, and their use of multiple strategies to answer the questions (to be reported elsewhere). The two teachers who participated in the earlier interview led the lesson with the four combined classes in two groups, with their co-teachers watching. They each started with a quadratic equation example for the students to sketch; they asked a pair of students to draw their solutions on the board and to explain a way of finding key points on the graph. The students were then asked to work in groups of three or four (their choice) and to record their solutions on the provided A3 sheets. The teachers led a concluding discussion, inviting students to explain their responses to the task, and making connections between the different strategies for answering the questions and the general formulae of quadratic graphs. The students were then invited to complete an anonymous individual open-ended reflection (Appendix 3) and 87 chose to participate (nearly 100% response rate). It was an important tenet of the research that the students were free to respond in any manner they chose without fear of reprisal from their teachers. After the lesson, the researcher and teacher participants debriefed.

The teachers’ interview responses and the students’ questionnaire responses were analysed using “descriptive and interpretive” approach (O’Toole & Beckett, 2010, p. 43). The teachers’ responses were transcribed from an audio recording and coded inductively using line-by-line coding with NVivo 10 qualitative analysis software (Creswell, 2007). The resultant coding framework (Miles & Huberman,
1994) is presented in Appendix 4. The students’ written responses were transcribed into Excel spreadsheets for inductive categorisation. The previously presented theoretical framework was used to examine which cognitive, affective, and motivational aspects of learning the students chose to focus on in describing their reactions to challenging tasks and views on learning multiple strategies. The cognitive and affective dimensions of learning described by the students, and evidence of goal-oriented language used in the students’ written reflections were examined to explore how high-achieving students view challenging tasks and multiple strategies in terms of their learning, engagement, and motivation.

RESULTS AND DISCUSSION

This section begins with the teachers’ perceptions of their students’ reactions to challenging tasks earlier in the year before encountering the quadratics task from this study. Four sub-sections then present results from the responses of the students to the reflective questionnaire which explored: their reactions to the quadratics task; their views on challenging tasks in general; their perspectives on learning strategies from a peer; and their views on learning multiple strategies from a teacher. A final sub-section discusses issues the teachers raised about incorporating challenging tasks into their mathematics program with high-achieving students.

The teachers’ perceptions of students’ previous reactions to challenging tasks

The two Year 10 mathematics teachers who were leading the mathematics program participated in a joint interview to share their experiences of using challenging tasks in the larger project. One was the Head of Mathematics (“Anne”) with several years of teaching experience and the other (“Barb”) had been teaching at the school for nearly three years and had experience of teaching senior secondary mathematics at another school. Barb explained that in her own earlier efforts to use challenging tasks at a previous school with her specialist mathematics class, the high-achieving students resisted tackling the tasks independently:

Every single lesson I would run from kid to kid to kid while they had their hands up. And there was no moving forward until I went and helped them. And I put in strategies like see three before me and that kind of stuff and would keep to that and go I know that you’ve only asked one person, go and ask two more.

She recognised the need for teachers to have strategies to manage the resistance of high-achieving students to challenging tasks:

I kind of tried to break them away from asking the teacher all the time, but I don’t think I had enough strategies to be able to do it consistently. And I think because I’d already recognised that as a problem and particularly a problem with kids who are good at maths – it’s that idea where they’re scared to try something and they might fail and they’re not used to failing and that’s a really uncomfortable area for them.

Anne and Barb were asked about their Year 10 students’ responses to earlier challenging tasks that year. Anne said, “A lot of them are really uncomfortable for the first time; the really good kids who have been good at maths are really uncomfortable in that space.” They highlighted their efforts at the start of the year to explain to the students the purpose of giving them challenges:

We just explain and explain all the time with the Year 10s, “We don’t know where you’re at. You don’t want to be bored in maths, this is what you tell us, well here you go, here’s your chance, show us what you
At the time of the interview, the students were weeks away from the end of the academic year, and the teachers described the gradual shift in the students’ initial reluctance:

We’re just starting to see the results now; they’re just starting to come on board and they go to me, “Yeah yeah I know, you’ve got another problem that I don’t think I can do. I know, now I’m going to have a go.” (Anne)

They attributed this change to students experiencing “successes from what they’re doing”; “they understand why it’s being done like that” (Anne).

We’ve won the kids over in a way and their maths results may not necessarily have improved, but they’ve enjoyed being in there and they’re keen to come to maths. And they know that it’s a bit of a journey and they know they’ve got to work in a different way. (Anne)

**Students’ reactions to the quadratics task**

At the conclusion of the quadratics challenging task lesson, the students were asked how they found the task and their reasons. Their responses were categorised according to cognitive and affective aspects of learning. They are presented in Table 2.

**Table 2. Students’ descriptions of their reactions to the quadratics task (n = 87)**

<table>
<thead>
<tr>
<th>Type of Response</th>
<th>Percentage of Students (%)</th>
<th>Illustrative Examples</th>
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<tr>
<td><strong>Affective response: Emotion</strong></td>
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</table>
| - interesting / engaging | 27.6 | “interesting” because “it was interesting to look at the different forms and connect them together”  
“interesting” because “we got to able to explore different approaches to quadratics [sic]” |
| - enjoyable / fun | 5.7 | “fun” because “it was a group task”  
“enjoyable” because “it posed a challenge and it wasn’t over even when it was done, the teacher was very helpful too” |
| - confusing | 1.1 | “confusing” because “I haven’t done quadratics in a while and I’ve forgotten quite a bit” |
| **Affective response: General evaluation** | | |
| - okay / alright | 13.8 | “alright” because “I knew the concepts that we had to use but applying them was a challenge”  
“okay” because “I understood what was going on” |
| - good / cool / swell etc. | 8.0 | “good” because “it was a challenge but it was still doable”  
“swell” because “we had done similar work but not as an investigation” |
| **Cognitive response: Level of perceived challenge** | | |
| - easy / simple | 13.8 | “easy” because “I understand the concept of where x and y intercepts sit on a graph in an equation”  
“simple” because “we have generally covered this topic in class” |
| - difficult | 6.9 | “difficult” because “only understood certain aspects”  
“difficult” because “I haven’t done it in a while” |
| - average / moderate | 5.7 | “moderate” because “the task wasn’t a level which was unachievable”  
“moderately difficult yet achievable” because “I kept confusing concepts” |
| **Cognitive response: Effectiveness for learning** | | |
| - useful / beneficial / helpful | 10.3 | “beneficial” because “it helped to understand the turning point formula better and the discriminant in the quadratic formula”  
“useful” because “I had lost all memory on quadratics. Helped me remember” |
| - not useful | 3.4 | “useless” because “it took too much time and was unnecessary”  
“not effective” because “I already know this stuff” |
| Other | 2.3 | “supercalafragal…” because “for a loss of other words” |
| No response | 1.1 | |
Overall, over one half of the students described an affective response to the task, with most using positive words such as “interesting” or “engaging”. No students described the task as boring. Only one student indicated that they found the task “confusing” because they “[hadn’t] done quadratics in a while and [they’d] forgotten quite a bit”. Just over one quarter of the students described their perceived level of difficulty of the task. One student who described finding it “difficult” also explained that they “[kept] forgetting how it all works”. Another student who found the task “moderately difficult yet achievable” reported that they “kept confusing concepts”. For this cohort of high-achieving students who had previously experienced a number of challenging tasks, only a handful reported experiencing confusion or forgetting. Nearly 9% of the students reported that the task helped them revise or remember things they had already learned. Putwaine and Symes (2011) described senior secondary students as likely to develop a mastery-avoidance goal orientation in trying not to forget the content at higher levels of schooling. This may relate to assessment procedures such as examinations, which require longer-term retention of knowledge compared to shorter-term topic tests. These students’ comments about not wanting to forget and needing to revise may relate to this particular type of goal orientation.

For the 14% of students who reported finding the task easy or simple, most gave a reason related to already knowing the concepts, and a further 7% also commented on understanding the task. For this cohort, one fifth reported not experiencing the task as challenging. Another fifth, who mostly described the task as interesting or beneficial, specifically referred to some aspect of the task being novel for them – different to usual, something new to be learned, applying their knowledge in a new way, investigating, or exploring. It appears that the students’ evaluation of the task’s situational interest to ascertain if engagement was worthwhile (Middleton & Toluk, 1999) considered both its novelty (affective aspect) and its relevance to their learning (cognitive aspect). Middleton (2013) asserted that a number of studies have demonstrated that “the development of interest promotes more effective cognitive processing of academic content and engenders positive affect” (italics mine). This finding resonated with this present study’s results in that many students described their focus on considering their situational interest in the task from both cognitive and affective perspectives.

From the observation of the quadratics task lesson across the four classes, the researcher noticed that the rhythm of the lesson seemed familiar to the students. Their collaborative group work (sociocultural approach to learning), substantial time attempting the task, and the lack of any widespread resistance or disruption indicated that the expectations of the teachers for challenging tasks were being met by students. The concentration of most of the students appeared to confirm the teachers’ perceptions that the students had been “won over” by that stage in the year and were more likely to engage with a challenging task on their own now without premature teacher intervention. It was interesting to observe a continuation of the students’ engagement during the final whole-class discussion and their attention to the teachers’ drawing together of different strategies and formulae for quadratic functions.

**Students’ views on challenging tasks**

The results for students explaining their reasons for liking or not minding or disliking challenging tasks in general are presented in Table 3. Approximately 60% of this cohort indicated that they didn’t mind challenging tasks and a further 30% reported liking them. For those students who didn’t mind them, most of the reasons related to tasks being of value for their learning (cognitive aspect). For those who didn’t mind challenging tasks and gave an affect-related
Reactions to challenging mathematics tasks

reason, some referred specifically to their novelty: enjoying a change from the usual learning tasks. One wrote, "it’s a change from daily exercises" and another wrote, "they are not too mainstream". For those students who liked challenging tasks, half provided an affective reason related to being engaged, and the other half described a cognitive reason, related to their perceived usefulness of a task. Overall, nearly 7% of the students referred to challenging tasks as helpful for revision and 15% described learning something new or in a new way. Interestingly, nearly 14% described challenging tasks in terms of improving their thinking – "they make you think and aren’t just memorising equations", and they “make you think about maths in a different way". A few students described enjoying working together with peers on a challenging task.

For the 8% of students who indicated disliking such tasks, half of their reasons related to finding them too hard. It can be seen that even though the quadratics task was designed to be open-ended and to elicit multiple strategies, there were nonetheless students who found the task too easy and other students who found it too hard. Providing tasks that engage students within reach of their level of understanding and do not dishearten them is a challenge for teachers whatever the achievement level of their students.

### Students’ views on learning strategies from a peer

Table 4 presents the responses to students’ preferences for learning a strategy from a peer (Type 1 in Table 1). Comments that related specifically to the value of knowing multiple ways to solve a problem at the same time (rather than finding another way to use in future) were categorised separately to compare them to the later question on learning multiple strategies from the teacher (Types 2 and 3 in Table 1).

Nearly two thirds of the students reported liking to learn a strategy from a peer, with nearly 60% referring to a cognitive aspect: a perceived positive benefit to their learning. A further 30% indicated that they did not mind, again with the majority relating their reason to a positive learning outcome. Overall, nearly 30% of the students reported that they valued learning a new or a different way to solve a problem from a peer. Just over 10% indicated that they found a peer’s way of explaining easier to understand (than a teacher’s). One student wrote “I get to see their strategies and also helps me learn new things because they think and speak like teens". Another wrote, "I may not understand and they explain in layman's terms". It is possible that this may be a specific approach to learning multiple strategies, which is effective at secondary levels (rather than primary levels).
Approximately 7% of the students indicated a *dislike* for peers explaining strategies to them. A few students described a negative affective response such as feeling belittled or treated with condescension by their peers. This can relate to a performance-avoidance goal orientation, which is associated with concerns about appearance, looking incompetent or ‘dumb’ to others. A few students referred to a perceived negative cognitive effect on their learning, such as not being taught correctly or not having a clear explanation. A few students didn’t trust their peers’ knowledge, and another student wrote, “My strategy is the best”. Performance-approach goals focus on appearing to be more competent than others, but it is unclear here if the student’s comment is simply ‘tongue in cheek’ or what their reasons might be for thinking their strategy would be the best. No students referred to being bored by learning multiple strategies from a peer, a contradictory finding to those studies exploring teachers’ perceptions of high-achieving students.

**Students’ views on learning multiple strategies from their teacher**

The results for students’ preferences for learning more than one way to solve a problem from their teacher are presented in Table 5. Specific responses that related to wanting to be able to understand *multiple* strategies (rather than being able to choose one strategy from among those taught) were categorised separately.

Nearly 60% of the students indicated that they *liked* learning more than one strategy from the teacher and another 35% that they *did not mind*, with the vast majority giving cognitive reasons related to perceived effectiveness for their learning. Nearly 30% of the students referred to the value of being shown more than one way to solve a problem because they could choose a strategy for future use since it was *easier* or *quicker* or *more understandable* for them. Their reasons resonate with a recent study in which secondary mathematics teachers perceived the teaching of multiple strategies as a useful way of ‘covering all the bases’ to increase the likelihood of each student understanding one strategy that they could then competently use (Lynch & Star, 2014).

Slightly more students made a specific comment about a better or quicker strategy as such, but surprisingly, a similar proportion described wanting to know *more than one way* to tackle a problem *at a time*, for example:

| Table 4. Students’ descriptions of their views on learning strategies from a peer (n = 87) |
|-----------------------------------------------|-------------------------------|-------------------|-------------------|-------------------|
| Student Comments | Don’t Like (%) | Illustrative Example | Don’t Mind (%) | Illustrative Example | Like (%) | Illustrative Example |
| Affective response | | | | | | |
| Positive | - | | 3.4 | “it is interesting to know how other people think and learn from them” | 3.4 | “I like hearing other people’s input” |
| Negative | 2.3 | “I feel belittled” | - | - | - | |
| Cognitive response | | | | | | |
| Positive | - | | 12.6 | “I may need help” | 35.6 | “they often make more sense” |
| Positive & about different strategies | - | | 8.0 | “all methods are possible approaches and some might be easier methods” | 20.7 | “it shows me different ways to solve it” |
| Negative | 4.6 | “sometimes they are wrong” | 1.1 | “sometimes I don’t understand it” | - | |
| Other response | - | | 4.6 | “it doesn’t affect me” | 3.4 | “it means I get to show them the correct way” |
| TOTAL | 6.9 | 29.9 | 63.2 | | | |
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Table 5. Students’ descriptions of their views on learning strategies from the teacher (n = 87)

<table>
<thead>
<tr>
<th>Student Comments</th>
<th>Don’t Like (%)</th>
<th>Don’t Mind (%)</th>
<th>Like (%)</th>
<th>Illustrative Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Affective response</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Positive</td>
<td>-</td>
<td>-</td>
<td>1.1</td>
<td>“there are some ways which I don’t like doing, so an option is always fun”</td>
</tr>
<tr>
<td>Negative</td>
<td>3.4</td>
<td>1.1</td>
<td></td>
<td>“I get confused which one to use”</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>“sometimes at first it can be confusing”</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cognitive response</th>
<th>Positive</th>
<th>Positive &amp; about choosing a better / quicker strategy</th>
<th>Positive &amp; about learning multiple strategies</th>
<th>Negative</th>
<th>Other response</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>12.6</td>
<td>4.6</td>
<td>9.2</td>
<td>1.1</td>
<td>2.3</td>
</tr>
<tr>
<td></td>
<td>“it’s sometimes helpful”</td>
<td>“one way could be much easier than the other”</td>
<td>“in case one method isn’t working out I could use another, or I could use different methods to compare answers”</td>
<td>“I have not yet fully understood / remembered the first part”</td>
<td>“in tests we will probably have to show all ways to solve a problem”</td>
</tr>
<tr>
<td></td>
<td>17.2</td>
<td>24.1</td>
<td>16.1</td>
<td>2.3</td>
<td>4.6</td>
</tr>
<tr>
<td></td>
<td>“it improves the way of thinking”</td>
<td>“we can decipher a way to see which one is suited to us”</td>
<td>“it gives me options and I get to choose which method is suitable for certain questions”</td>
<td>“it is extended learning but can be unnecessary”</td>
<td>“I know the teacher is right and knows what they are talking about”</td>
</tr>
<tr>
<td>TOTAL</td>
<td>6.9</td>
<td>34.5</td>
<td>58.6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- “It gives me options and I get to choose which method is suitable for certain questions.”
- “I want to know more solutions.”
- “Different ways means more understanding of relationships between formulas.”
- “It allows students to use more than one approach.”
- “It is good to know more than one path.”

This reason for learning multiple strategies has not been evidenced in studies researching this approach from teachers’ perspectives. Elementary teachers refer to students inventing their own strategies and then learning to find a more efficient strategy (e.g., Stein et al., 2008). Secondary teachers refer to hoping students find one strategy amongst a selection that they can understand (e.g., Lynch & Star, 2014). It appears that for high-achieving students like these, knowing multiple strategies at the same time is a different motivation from those described by teachers.

The 7% of students who disliked learning more than one strategy referred to a sense of confusion or to being concerned about remembering them, which gives some evidence of mastery-avoidance goals. Only one student expressed learning more than one way to solve the problem as “unnecessary”. Earlier studies found this view amongst lower-achieving students; it appears that this may be different to the views of high-achieving students.

To compare the students’ views on learning strategies from a peer or from a teacher, a cross-tabulation of the previous two tables was made, and is presented in Table 6.

Even though a similar proportion of students (approximately 60%) indicated liking having a peer explain their strategy and having a teacher show more than one way to solve a problem, the cross-tabulation demonstrates that these are not the same students. Nearly 44% of the students indicated liking both, and every
combination of preference is represented with the exception of not minding peer explanations and disliking teacher demonstration. A Pearson product-moment coefficient (when the categories are ordered numerically – ‘don’t like’ as 1, ‘don’t mind’ as 2 and ‘like’ as 3) of 0.2 also adds to the finding that for this cohort, the students’ preferences for these two ways of learning were not correlated. It appears that these students have reasons for preferring a peer’s or a teacher’s strategies which do not necessarily overlap in both approaches.

The student who did not like *either* way explained that peers can be wrong and that they are confused when being shown multiple strategies by the teacher. Of the other two students who disliked peer explanations but did like their teacher showing multiple strategies, one felt belittled by peers but found the teacher helpful. The other wanted to solve problems individually, and liked to be given options by the teacher.

These results indicate overall that although there are variations in the students’ preferences for learning mathematics in ways that engage them, learning with challenging tasks and learning multiple strategies are viewed by them as engaging if they are interesting and also relevant to their learning. The reasons given by the students related to cognitive and affective aspects, and their use of language provided evidence more often of a mastery than a performance goal orientation.

**The teachers reflect on incorporating challenging tasks in their Year 10 program**

In addition to explaining to students the purpose of challenging tasks and the value of not always knowing how to do something, the teachers also found that they needed “to work out when is the best time to intervene” since “some of [the students] are so used to getting intervention straight away and they never get into that uncomfortable space” (Anne). They suggested that the students might over time ‘rise to the challenge’ if the teachers learned to hold back from modifying or over-explaining the task:

> If you give them a task like this, but then step in the minute that they struggle, then they get to learn that they don’t need to struggle and so they’re like, “If we just sit back for another two minutes they’re going to show us how to do it on the board.” So we made it really clear I think through the other tasks that we’ve done that we’re not just going to show them on the board and if it takes longer than we thought, then it takes longer than we thought. (Barb)

They addressed the issue of teachers having to “[let] go of the control” when using challenging tasks – “I don’t know what’s going to happen in this lesson and that freaks me out” (Barb). Their strategy for managing teacher anxiety was to timetable some of the lessons together with the other Year 10 classes and to communicate with each other during the task to discuss what was unfolding and decide collaboratively how and when to respond.

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**Table 6. Cross-tabulation of students’ views on learning strategies from a peer and from the teacher (n = 87)**

<table>
<thead>
<tr>
<th>Having the teacher show us more than one way to solve a problem</th>
<th>DON’T LIKE</th>
<th>DON’T MIND</th>
<th>LIKE</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Don’t like a peer explain their strategy for solving a problem</td>
<td>1%</td>
<td>3%</td>
<td>2%</td>
<td>7%</td>
</tr>
<tr>
<td>Don’t mind having a peer explain their strategy for solving a problem</td>
<td>-</td>
<td>17%</td>
<td>13%</td>
<td>30%</td>
</tr>
<tr>
<td>Like having a peer explain their strategy for solving a problem</td>
<td>6%</td>
<td>14%</td>
<td>44%</td>
<td>63%</td>
</tr>
<tr>
<td>Total</td>
<td>7%</td>
<td>34%</td>
<td>59%</td>
<td>100%</td>
</tr>
</tbody>
</table>
The teachers also highlighted the need for teachers to have a strong knowledge of the mathematics, particularly to be able to manage multiple strategies:

Sometimes you can't follow what the kids are doing. You've got to actually be able to engage with every different kid's strategy and make sure that they're mathematically on the right track. And that can be very, very hard to do right. (Anne)

They thought that the way to develop their knowledge was to plan tasks together to anticipate different strategies – "If teachers have some sort of structure around it and they know what they're looking for, then it might be a little bit less daunting, so it's all in the planning of that" (Anne).

**IMPLICATIONS AND CONCLUSION**

Previous research has highlighted that teachers may be reluctant to use challenging tasks because of their perception that students would resist (Anthony, 1996). They reported believing that learning more than one way to solve a problem bores high-achieving students and that less capable students become confused. Theoretical perspectives connect students' situational interest in a task with their decision to engage or not (Middleton & Toluk, 1999). An interesting task is more likely to promote effective cognitive processing and positive affect (Middleton, 2013). Research on the motivation of students has found that high-achieving students are likely to hold performance goals, related to appearing or performing better than others (Sullivan, et al. 2006), rather than mastery goals, related to mastering a challenge or learning as much as possible. Senior secondary students have shown an increase in mastery-avoidance goals, related to trying not to forget what has been learnt (Putwain & Symes, 2011).

This study focused on the perceptions of high-achieving Year 10 students after their experience of a year-long teaching intervention incorporating challenging tasks as part of a design-based research project. Data were sought (anonymously) on: the students' views of a particular task immediately after the lesson; their views on challenging tasks in general; and their preferences for learning multiple strategies from peers and from the teacher. This was compared to the perceptions of their teachers.

Having experienced a number of challenging tasks throughout the year, the majority of students in the study expressed a positive response to them, giving reasons that related to their affect – their interest or engagement or enjoyment – or to a cognitive aspect: their perception of benefit to their learning. Their responses resonate with different theoretical perspectives on student motivation. In terms of an adaptive theory of motivation (Middleton & Toluk, 1999), the students had prior experience of challenging tasks and evaluated the particular quadratics task to ascertain its situational interest. The findings suggest that this cohort considered the novelty of the task and its relevance to their learning; assessing the task as both interesting and relevant meant that they were more likely to engage with it than resist. A focus on learning something interesting and mastering a challenge are related to a mastery goal orientation. Teachers may be able to increase the likelihood of students like these engaging with challenging tasks by choosing unusual problems that are also relevant to the content students believe they need in a particular course of study.

In their responses, the students overwhelmingly made reference to aspects showing a mastery goal orientation rather than performance. Their cognitive reasons for liking a challenge or multiple strategies related to learning something new or interesting, rather than to achieving high grades or improving performance. Unlike previous research, these high-achieving students described wanting to learn new concepts, to understand the content, to think actively, and to be challenged.
There were very few spontaneous references to performance goals, which Brophy (2005) speculated would be the case when not using pre-set Likert-scale survey instruments for data collection. Yet these students were approaching the last two years of schooling in which competitive performance on external examinations play a significant role in determining future eligibility for university entrance. It is therefore surprising that the students did not reference this in their responses. There was some mention of not wanting to forget and appreciating the value relevant challenging tasks for revision purposes, which resonates with research on the likely development of mastery-avoidance goals by senior secondary students (Putwain & Symes, 2011).

An external environment is seen as influencing an individual's adoption of goals independent of other intrinsic motivational features of that individual (Elliot, 1999). It is suspected that the school's explicit culture of valuing progress above performance, and the teachers' use of strategies to encourage task persistence, may have played a role in influencing these students' mastery-oriented responses, which were different from expected. Contexts promoting the possibility of success are more likely to activate approach goals whereas those promoting threat are more likely to activate avoidance goals. He cautioned, however, that goals emerging from environmental effects are likely to be weaker and less stable than those related to individual dispositions. Middleton et al. (2013) found that students display different motivational patterns with different teachers. Dweck (2007) asserted that students pick up on what teachers value and act on their messages. Middleton (2013) asserted that teachers play a role in changing a student's motivational pattern by inducing them to “buy in”. In this study, the school environment and the teachers' use of strategies for encouraging persistence on challenging tasks resonate with these perspectives on motivation. Yet one limitation of the study was that the students' motivational goals were not examined before the implementation of the teaching intervention to see if and how they might be influenced by environmental factors related to the teachers' strategies and the choices of tasks. It would also be valuable to explore the goals of these students in other subject areas and longitudinally to see how they might vary across different disciplines and over time after having experienced this environment and a mastery-oriented approach to learning in Year 10. Although a majority of the student responses in this study highlighted their interest in learning effectively (approach goals), there were still references to not forgetting or to needing revision (avoidance goals), which seems to indicate that assessment pressures may still be perceived by some individuals as threatening even in this environment. A cohort of younger or older students may exhibit different proportions of approach and avoidance goals. And of course, individual dispositions and other factors cannot be discounted.

Previous research found that teachers perceived that their students pressured them to over-explain or simplify a challenging task (e.g., Anthony, 1996; Sullivan, et al. 2009). In this study, and after the students had experienced a number of challenging tasks, this exertion of pressure was not noticeable during the quadratics task lesson (as perceived by the researcher). The teachers in their interview had highlighted their awareness of this issue and their explicit use of teaching approaches to resist the urge to modify the task or give in to student pressure. They explained to students the purpose of a challenging task for their learning and then deliberately provided time for the students to grapple with the task without stepping in. Some of the students did, however, express dissatisfaction afterwards in their reflections that they found the task too hard or too easy for them. Finding novel and relevant tasks to suit a range of levels of understanding, even for groups streamed according to achievement level, remains an ongoing issue for teachers.

With regard to learning more than one way to solve a problem, nearly two thirds of the students liked having a peer explain their strategy, and two thirds liked being...
shown more than one way by the teacher. Approximately 44% liked both of these approaches. Some students liked learning strategies from a peer because their language was easier to understand. This approach to multiple strategies may be one that is specifically useful at secondary levels. Students’ reasons related overwhelmingly to the cognitive aspect of perceived benefit to their learning rather than to their affect, unlike their views on challenging tasks. For those students who specifically referred to learning more than one way to solve a problem, similar proportions of students described wanting to find one strategy that they found easier / quicker / understandable, or wanting to know multiple ways at the same time. This latter preference is at odds with the findings of recent research on secondary teachers’ pedagogy (Lynch & Star, 2014). The teachers were more likely to show different ways of solving a problem for the express purpose of making sure each student could find one strategy that worked for them, rather than to promote understanding of multiple strategies at the same time. It seems that there is more to understand about if and how secondary teachers might incorporate multiple strategies in their teaching to help students learn, through comparing them and assessing their efficiency in different situations, rather than simply hoping that one strategy might at least ‘stick’. The students in this study showed that they valued knowing multiple strategies at the same time.

The findings of other studies – that teachers perceive less capable students as being confused by learning more than one way to solve a problem – could not be investigated in this study given its focus on high-achieving students. Yet a handful of students did mention their confusion with both challenging tasks and with learning multiple strategies. There is more to understand about the approaches to using multiple strategies at secondary levels and with different cohorts of students. Overall this particular cohort of high-achieving Year 10 students, with many evidencing mastery goals, viewed challenging tasks and the opportunity to learn multiple strategies as engaging their interest affectively and effective for their learning cognitively.

REFERENCES


Reactions to challenging mathematics tasks


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APPENDIX 1

Can you find some equations of parabolas that:
   a) Cut across the x-axis twice?
   b) Cut across the x-axis once?
   c) Don’t cross the x-axis at all?

What do you notice about each of the different groups of parabolas you have found?

Extra challenge:
Can you use your previous answers to find the equations of horizontal lines that cut across the parabola \( y = x^2 - 2x \) once, twice, or not at all?

Appendix 2 – Sample questions from teachers’ interview schedule

What year levels do you teach this year? Streamed?
How long have you been teaching at the school? In general?
What types of challenging tasks have you trialled with your class? Who wrote the task?
What was important to you in the design of the task? What did you want it to achieve in terms of the students’ learning? engagement?
Can you describe how you set up the lesson? How was this different to your usual lessons?
What did you notice about the lesson? The students’ different responses? Did anything surprise or puzzle you?
What was it about the task that some students found engaging / disengaging? Challenging? Uncomfortable?
Did you experience any “push-back” from anyone subsequent to the lesson? The students? Parents?
What do you think might make it difficult for teachers to use this sort of teaching?
Do you think the students learn better?
For what topics in Year 10 do you think the use of challenging tasks would be effective?
Have you been able to compare notes with the other teachers about their experiences?
How do you currently feel about using challenging mathematical tasks with this year level?
How would you change your approach to using challenging mathematical tasks in future?
What do you plan to do from here?

Appendix 3 – Student post-task reflective questionnaire

1. I found this task ____________________ because ____________________

2. I (choose option) like / don’t mind / don’t like doing challenging maths problems like this one because ____________________

3. I am most engaged in my maths learning when I get to ____________________

4. I (choose option) like / don’t mind / don’t like having a peer explain their strategy for solving a problem to me because ____________________

5. I (choose option) like / don’t mind / don’t like having the teacher show us more than one way to solve a problem because ____________________

Additional comments?

Appendix 4 – Teachers’ interview coding hierarchy

<table>
<thead>
<tr>
<th>Code</th>
<th>Number of references</th>
</tr>
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<tbody>
<tr>
<td>School mathematics program for Year 10</td>
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<tr>
<td>Previous challenging tasks trialled</td>
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<td>Learning intentions</td>
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<td>Mathematics content</td>
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<td>Students’ responses</td>
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<td>Teachers’ responses</td>
<td>2</td>
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<tr>
<td>Teaching strategies used</td>
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<td>Issues identified</td>
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<td>Kickback</td>
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<td>Knowing when to intervene</td>
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<td>Lack of control</td>
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<td>Students’ attitudes, mindset</td>
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<td>Students’ learning</td>
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<td>Teachers’ attitude, mindset</td>
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<td>Traditional teaching approaches</td>
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<td>Suggestions</td>
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