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Understanding the Conics through Augmented Reality

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ABSTRACT
This paper discusses the production of a digital environment to foster the learning of conics through augmented reality. The name conic refers to curves obtained by the intersection of a plane with a right circular conical surface. The environment gives students the opportunity to interact with the cone and the plane as virtual objects in real time and real place. They can perceive their own creation of the different curves: parabola, circle, ellipse and hyperbola. A dynamical animation to visualize the focus and directrix of the parabola is presented by introducing the Dandelin Spheres into the virtual environment. Besides the motivational element it offers, the purpose is to promote an active learning through the affordances of augmented reality technology. Anxiety towards mathematics could be tackled if students comprehend the mathematical knowledge. The integration of digital technology in the learning process helps this strengthening the development of spatial ability in students.
Keywords: Augmented reality, conics learning, Dandelin spheres, dynamical visualization, math anxiety, spatial ability.

INTRODUCTION

The conics are curves obtained by the intersection of a plane with a right circular conical surface. This way we identify the familiar circle, parabola, ellipse and hyperbola, curves that we studied in high school at Analytic Geometry. Nowadays we can easily access beautiful images of this spatial situation by Internet.

The teaching of conics has been engaged mostly with the performance of algebraic procedures in order to find some values related to those curves on the coordinate plane. A common mathematical task is to find the focus and directrix of a given equation representing a parabola, or vice versa, finding the equation of the parabola whose focus and directrix are given. We believe this way of dealing with mathematics content fits well with our common use of handwriting.

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State of the literature

- Math anxiety relates with math performance but also with teaching practices, and it is difficult to change this situation.
- Augmented Reality includes motivation with its use, and favors the development of spatial ability mainly introducing dynamical content.
- Conics, like the parabola, are obtained by the intersection of a plane with a cone; nevertheless, the teaching focuses on give the definition in terms of the focus and directrix, elements not present in the intersection.

Contribution of this paper to the literature

- The affordances of augmented reality allow us to interact with mathematics knowledge further than was possible before.
- The Dandelin's spheres are now available to our perception and we can visualize the focus and directrix of the parabola.
- Augmented reality is not constrained to the same mathematical knowledge in current curriculum, it allows new ways to understand mathematics actively.

Usually we ask students to express their mathematical skills by using formulas and producing a sequence of algebraic expressions whose organization reveals their mastery of the knowledge.

Nowadays the development of new technologies, like Augmented Reality (AR), could bring new ways to deal with mathematical knowledge. With AR we can emphasize geometrical aspects of the elements of the mathematics. Visual aspects of mathematical knowledge now have the opportunity to reveal its potential for our reasoning through the use of digital technology.

The aim of this paper is to discuss about the creation of a digital environment connecting our thoughts with a visual and tangible mathematics using AR. We look to foster the learning of conics through this digital technology by offering to the students the opportunity to interact with the cone and the plane as virtual objects, and perceiving their own production of the different conics in real time and place. A second use of this technology will allow us to present, as a kind of virtual movie, the way to conceive the focus and directrix of the parabola, recreating dynamically the mathematical proof for this, introducing the named Dandelin's Spheres into the virtual environment.

We begin the discussion by bringing up some rationale background to support our work, claiming for the transformation of the teaching of mathematics by means of the pedagogical integration of digital technologies into the learning process. We will discuss about the potential for education of AR, not just by bringing motivational elements during the learning process but mainly assessing the opportunity to stress the importance of having an external visualization in our 3D space for a visual and active perception when we learn mathematics.

We want to bring an enjoyable interaction with mathematics knowledge, looking to foster a positive attitude for its learning.

MATH ANXIETY, MATHEMATICS PERFORMANCE AND AR

Researchers in several countries have identified an educational phenomenon in students' attitude; it's named as anxiety towards mathematics. Its relation with low academic performance makes us aware about the importance of studying the impact of emotional aspects with the learning of mathematics. According to García-Santillán, Escalera-Chávez, Moreno-García & Santana-Villegas (2015), "math anxiety could be model by a five-factor structure including anxiety towards evaluation, understanding of mathematical problems and mathematical situations in real life". Our own experience in classroom makes us agree about the impact of those factors in students with lower or average mathematics achievement.

Now it is recognized that students´ attitude towards mathematics is one of the reasons determining their success or failure in courses. Math anxiety is a feeling of tension that interferes with the task of solving mathematical problems, blocking students´ reasoning, memory, understanding and appreciation of mathematics. Furner and Gonzalez-DeHass (2011) highlight the underlying causes of this situation related to the teachers´ instructional practices, and among them, the assessment process. But even when teachers could play an important role in reducing the levels of mathematics anxiety in their students, "researchers continue to emphasize the need to reform teacher education to promote a corresponding transformation in mathematics instruction" (p. 230).

Carey, Hill, Devine and Szücs (2016) reviewed the direction of the relationship between mathematics anxiety and mathematics performance. The authors present a mixture of evidences that support both causal directions and they suggest it indicates a bidirectional relationship. The unavoidable question about "the chicken or the egg?" makes us reflect that any effort done to break off with this cycle should be very welcomed. The level of importance for this is clear when we find in the cognitive literature that "Math anxiety leads to a global avoidance pattern—whenever possible, students avoid taking math classes and avoid situations in which math will be necessary, including career paths" (Ashcraft & Krause, 2007, p. 247).

According to the survey study performed by Jackson (2008), the majority of respondents think that it is acceptable to admit difficulties with mathematics and having negative perceptions of mathematics, including emotional and physical feelings. They relate the possible causes mainly with the learning of mathematics. There exists a lack of enjoyment with the teaching approaches that do not include interactive or creative or even relevant ways. The author argues that evidence suggests that the ability to do mathematics is strongly influenced by the attitude to learn mathematics and not due to cognitive skills.

Since 2010, AR technology has been mentioned year after year in the Horizon Reports of Higher Education; an electronic edition published by the New Media Consortium (NMC) organization. Augmented reality is one of the most promising emergent technologies having the potential to impact education (Johnson, Adams Becker, Cummins, Estrada, Freeman & Hall, 2016). Advancements due to mobile devices combined with pedagogical foundations lead to the overview on information about Mobile Augmented Reality (MAR) provided by Nincarean, Bilal Ali, Abdul Halim & Abdul Rahman, (2013). These authors state affordances of AR technology, like its potential on allowing users to view the learning content in 3D perspectives and supporting the inspecting of 3D objects. Also when it is used in a form of games, it could engage students and enhance collaboration. "It is proved that AR environments could boost students' motivation and interest, which in turn could help them to develop a better understanding in learning contents" (p. 659). Their analysis to the selected MAR studies lead them to give advice about focusing on pedagogical and learning theory since the educational value of AR does not depend only on its features.

The impact of AR use in motivation and achievement in high schools' students has been studied by Estapa & Nadolny (2015). For these authors, it is clear that technology in itself is highly motivating, despite that and even with the early stages of AR in the researchers' agenda, they stress as a prior goal for studies to focus on give evidence about the acquisition of knowledge. Their own experience with AR let them state that students gained experience with the topic of dimensional analysis; however, as they claim, further research is needed to indicate significant impact at both types of mathematical activity: conceptual and technical understanding.

For the purpose in this paper, we hold up the idea that the use of AR technology for the learning of mathematics represents an opportunity to create a learning environment offering a right feeling for interaction with mathematical knowledge, doing the mathematics understandable. This way we could fight against some of the causes of math anxiety and math performance at the same time. We want to take advantage of AR motivational aspects and, at the same time, advantages for the design of activities that make mathematics reachable to students. For us this is an important source of motivation. We should look further to strengthen a research agenda about the assessment process appropriate to the new ways for interaction with mathematics knowledge.

SPATIAL VISUALIZATION AND AUGMENTED REALITY

As stated by Newcombe (2013), "ideally, learning science, mathematics and social studies ought to be intensely spatial activities" (p. 27). The role of spatial ability for the learning of STEM (Science, Technology, Engineering and Mathematics) has been a focus of research lately, and its improvement should be a relevant academic goal. This author suggests that we can "spatialize" the existing curriculum. We agree about the complex undertaking that this action represents, and the great amount of people involved making it happen. From policy makers, curriculum developers to administrators and teachers, they all need to know more about spatial ability to be convinced of this need to promote its development in formal education.

Over the last years Martín-Gutiérrez, Contero and Alcañiz (2015) have been working on several experiences creating fast courses in order to remedy the lack of spatial ability. Through an augmented book they provide students with a didactic set of varied exercises for spatial ability training. Their study shows a significant improve in spatial skills after performing the training with AR technology, which also provides students with an attractive content compared with paper books.

The experience of using 3D Visual technologies to train spatial skills in college has been addressed for several researchers. Among them Martín-Gutiérrez, García and Roca (2015) provide us with a promising insight because of the positive impact in the academic performance and the great satisfaction resulting from students using three different technologies for 3D visualization exercises developed by the authors for the Engineering Graphics course. This is another study that states the importance of strong spatial skills for the learning of STEM and the need for its development in students. As they state, the research performed by several authors connect academic success with good levels of spatial ability; their own research shows the benefit of using 3D visualization improving spatial ability, therefore it makes sense that 3D visualization is an ally for improving students' performance.

We find spatial ability characterized as the ability to construct and maintain high-quality internal spatial representations (spatial images) and to accurately transform these representations. Hegarty and Kriz (2008) research about learning with animations make us aware that animated diagrams and static diagrams, both can be thought of as external visualizations; those are external visual-spatial representations. On the other hand, spatial ability can be thought of as internal visualization ability. Thus, effects of spatial ability on learning from animations could be framed as the interplay between perception of external visualizations and internal visualization processes. The authors suggest the possibility for helping the lack of internal spatial visualization ability through watching an external visualization like an animation on a computer.

As researchers in mathematics education we believe that learning to think spatially is an important outcome that has a great opportunity to take place if we make an appropriate use of AR. For the learning of mathematics, it seems to be very useful to build animations with AR functioning as an external animation process in order to promote in students an internal visualization process. Mathematics deals with visual aspects that require the spatial visualization ability. Before the development of digital technologies we were limited to remain teaching algebraic processes, but nowadays this does not have to be the same; in our view, it shouldn't be the same. Visual aspects are available for students doing mathematics and this may bring the change we need in order to make math knowledge attainable. Regarding mathematics courses, it seems quite possible that we could drive a positive change if we succeed with "spatialize" curriculum through 3D visual technologies.

The focus on new visualization methods to improve the current teaching models can be seen necessary because our students in their ordinary life are constantly interacting with a lot of graphic information provided by videogames, Internet or 3D movies. For Martín-Gutiérrez et al (2015), AR is a promising technology "allowing a combination of real world elements such as text, images, video, or 3D models and animations" (p. 752). AR environments allows to see the real world with virtual objects merged, combining real and virtual, interacting in real time and allowing to visualize elements from different angles and perspectives. As these authors claim, they "have performed small virtual animations for viewing invisible abstract concepts in the real world (this is precisely one of the main strengths of the AR technology)" (p. 757).

In our case, mathematics is distinguished from other sciences because of the high level of abstractions that should be worked through their symbolic representations; among them we have algebraic and graphical representations. AR represents a great opportunity to teach mathematics in a visual way, and with this we mean the development of spatial ability when interacting with mathematical representations in a 3D environment.

A CONFLICT WITH THE TEACHING OF THE CONICS

In mathematical curriculum the conics are part of an Analytic Geometry course in high school. These curves are obtained by the intersection of a plane with a right circular conical surface. This is a simple geometric description for the curves familiar to us by the names of circle, parabola, ellipse and hyperbola. Today we can reach beautiful pictures of them with Internet access.

At Wolfram Web Resources, Weisstein (2016) relates the conics description with the study of the ancient Greek, long before their application to inverse square law orbits. Apollonius of Perga (262 BC – 190 BC), Greek geometer and astronomer wrote the ancient book entitled "On Conics". There he refers to his predecessor's work, like the one in Euclides' four Books on Conics. It is well recognized the legacy of the Greek and their influence in later scholars, like Kepler (1571 – 1630), who noticed that planetary orbits were ellipses, and Newton (1643 – 1727) whom, under the assumption that gravitational force goes as the inverse square of distance, was able to derive the shape of the orbits mathematically, using calculus.

Our claim with this work binds to that expressed by others: teaching about the conics, the fact that they are conics, which means, derived from the cone, is something named by the way. Immediately they are defined in the x-y coordinate plane by their equations, and tied to their focal properties (Bertrand, 2015).

Therefore, in one hand, the attribute of being conic is related to the intersection between a plane and the cone, perhaps shown in a 2D figure representing the 3D situation. On the other hand, the curves are defined as the locus of points (x, y) that satisfy certain conditions. As stated by Weisstein (2016), a more formally definition of a conic section sets the locus of a point P that moves in the plane containing a fixed point F (focus) and a fixed line d (directrix, not containing F) such that the ratio of the distance from P to F over the distance from P to d, is a constant. This constant is represented by e, and called eccentricity. Different values for the eccentricity determine the different conics.

In this paper we want to stand out the conflict in the teaching of conics with the absence of an explanation between this "being conic" and fulfilling the conditions of the formal definition. If a conic is a conic, where are the focus and directrix that seem imperative in order to establish its equation? We selected the parabola as the conic to illustrate our point. Its eccentricity is 1, so, its definition states that the distance from P to F equals the distance from P to d. For this situation, the plane intersects the cone with the same inclination angle that has the generatrix of the cone.

We are used to a common practice when teaching conics: once we give the students the definition of the parabola (for example), it is time to work algebraically with this definition in order to arrive to the general equation of the parabola. Students may be so struggled repeating the algebraic task this requires, thus, maybe they do not care or do not notice about the relation between the words 'conic' and 'parabola'. But, let's think for a while in this: why is a parabola a conic? We could face that question talking about the way that the parabola shows up: if we consider a cone, a three-dimensional (3D) figure in space, and we consider a plane in that 3D space, and we intersect the cone with the plane, tilt in the right way, finally there we will see the parabola in that intersection. If we remain with this inquiring attitude, we could ask about the place for the focus and the directrix (named in the definition) in such 3D figures intersecting in space. We believe that imaging the cone and the plane intersection in our mind could be a difficult task, but even conceive how the focus and directrix stand in that image, it is a problem that goes far beyond a simple spatial visualization. It really involves a rich presence of geometric figures that combine in 3D space, those that the great Belgian Mathematician Germinal Pierre Dandelin was able to visualize internally in his mind and demonstrated to the world in 1822, interestingly adding another 3D figure, the sphere.

THE CONICS AND AR POTENTIAL FOR AN ACTIVE INTERACTION

Standing in the middle of the so-called virtuality continuum for computer/human interactions, augmented reality is a real-world space in which virtual elements are inserted in real time (Kapp & Mcaleer, 2011). Diaz, Hincapié and Moreno (2015) agree about the advantages of AR use in education because it engages students and increases the understanding of some topics, especially if the spatial skill is involved. These authors classify the type of content deployed in AR as static or dynamic, and analyze how they affect the learning experience. The difference between types is in terms of the no variation or the variation of AR appearance during the interaction with the user. An example for static content is text, and an animation exemplifies a dynamic content. Even though their study is not a statistical hypothesis test, they observe a difference in the learning level of students, favoring the dynamic use of AR content.

It is not possible to have a real cone in our space and intersect it with a real plane. But if we use AR, those two objects could stand virtually in our real space giving the opportunity to move them and perceive the intersection that is taking place in front of us. Having the marks that will allow us to interact physically with the virtual cone and plane, we then can move those objects in such a way that our own thinking arouses trying to obtain different conic curves. The intention of having a parabola as the intersection guides our interaction with the virtual scene and benefits the engagement with our thoughts and learning process.

If we adopt the terms static-dynamic, even when the cone and the plane represent a static content, the action performed with them by the user promotes a personification. This is a kind of dynamic action that AR technology is bringing to mathematics education and that allows a radically different perception of the interaction with mathematical content.

Figure 1. a, b. Intersecting the virtual cone with the plane

In **Figure 1** (a, b) we show a mark displaying the virtual cone over a real surface in real time. The student is interacting with the cone through the use of an additional mark that displays a yellow plane pane. In order to obtain the different conics, the student interacts freely seeking to find the largest number of different curves generated by his actions and classified as circle, parabola, ellipse or hyperbola.

When we discuss about the design with educational features, we take care of advances in mathematics education, especially with focus on digital technologies affordances. This perspective is strengthened with the proposal of Moreno-Armella and Hegedus (2009) considering the impact of co-action. As we understand, the term co-action refers to the opportunity that digital artifacts offer to the user in order to promote a cognitive relation. It is a mutual reaction between the user and the artifact, fostering an intentional interaction. This way we are betting to offer students a tangible interaction with mathematics enhanced through AR that give rise to some intentional actions, related with a mathematical thinking. In next **Figure 2** (a, b) we show the student using a different mark, around his fist, which displays the cone in both directions. The natural interaction is intended to reproduce the curves obtained before, when the cone was standing in the table, but now the plane pane is fixed on the horizontal surface of the table.

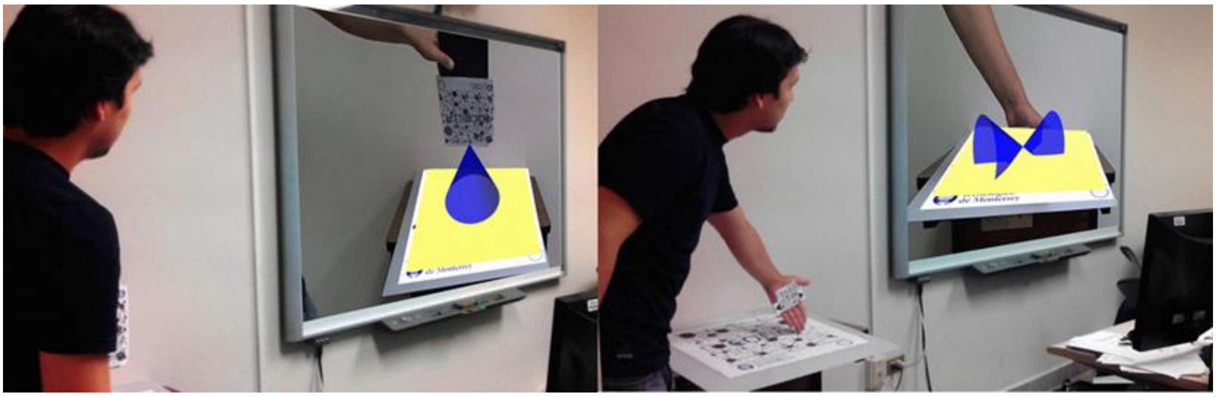

Figure 2. a, b. intersecting the virtual plane with the cone

Technologies have affordances; this is a concise way to understand the potential of AR regarding other technologies. Norman (1993) retakes the term affordance from the Ecological Psychology (Gibson, 1977) and applies it into several design areas, among them in Human Computer Interaction (HCI). With Figures 1 and 2 we highlight the affordances of AR for the learning of mathematics; the conics are conceived right there, through the active interaction that this technology affords. The user is invited to use his body motion to perceive in a natural way the production of the conics as the intersection of the cone with a plane.

CONE, PLANE, PARABOLA, FOCUS, DIRECTRIX AND DANDELIN'S SPHERE

Alsina (2010) provide us of key information for making a connection between the conic section method of the Greek, and the focus-directrix method used commonly in Analytic Geometry. It is about a Theorem preceded by a Lemma about triangles whose proofs are due to Adolphe Quetelet (1796-1894) and Germinal-Pierre Dandelin (1794-1847). The fruit of their work is explaining how to derive the focal-directrix properties of the conic sections from their definition as sections of the cone. We are not thinking on giving this proof to students, which elegance we could appreciate as mathematicians. Instead, our plan is to produce an AR animation introducing the Dandelin Spheres necessary for each conic section. We want to allow the visualization of the existence of those fixed points and straight lines, known as foci and directrices. This way, the connection between "being conic" and the definition of these curves will be set up finally.

Even when the resourceful method combines visual elements that are hard to perceive, we are confident in AR affordances to offer a dynamic visualization for obtaining the focus and directrix of the parabola. We are working the animation in 3D of this mathematical result, which could bring the connection between the parabola as a conic, and its elements (focus and directrix) determining the behavior of each parabola's point in accordance with the definition.

According to Kühl, Scheiter, Gerjets and Gemballa (2011), looking to research literature on learning with static or dynamic visualizations, it seems unclear if there exist an advantage for dynamic compared with static visualizations. “To account for this inconsistency it has been recommended to consider when and why dynamic and static visualizations might be best suited” (p. 178).

The AR animation we are creating involves necessarily the use of a dynamic visualization, where Dandelin’s spheres will be introduced as new 3D objects that add to the cone and the plane in order to situate the focus and directrix of the parabola. A static visualization of this in 2D images, printed or displayed digitally, could be compared with that in our **Figure 3** (d); there the sphere is already placed tangent to the plane and the cone. Our aim is to arrive to that situation in a dynamic way, by producing an animation projected in AR, as we describe next.

First, we will present the virtual cone displayed in AR and a fixed plane intersecting it with the same tilt that the cone has. The parabola is identified with the intersection, as seen in **Figure 3** (a). After that, a small sphere will fall into de cone and once jammed at the bottom, it will start to grow until reaching the plane. While growing, the sphere keeps tangent to the cone in a circle of latitude (a parallel), as seen in **Figure 3** (b, c).

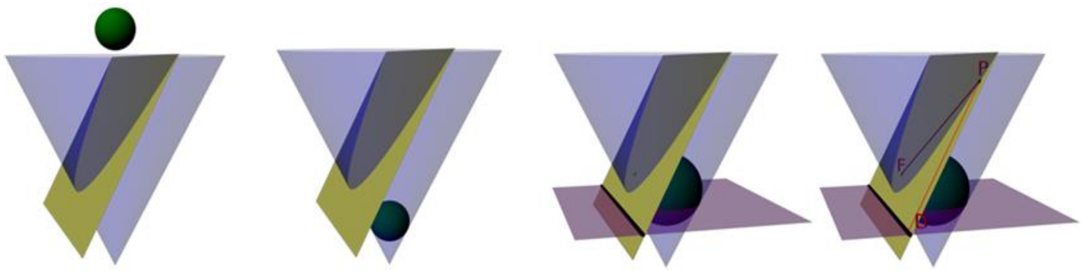

Figure 3. a, b, c, d. The sphere falls and grows to stay tangent to the plane

There will be a time when the sphere gets tangent to the plane also; the tangency point is the Focus. A horizontal plane enters in the animation passing through the circle of tangency in the sphere. This plane will also intersect the other plane; the one with the parabola. That intersection determines the directrix of the parabola (see **Figure 3** d). Focus and directrix are now an important piece for the visualization and it is time now to probe the parabola’s property given in the classic definition form Analytic Geometry.

In **Figure 4** (a) we turn a bit the cone in order to show a more familiar view of the parabola, the focus (point F) and the directrix (line with point D). As said before, the definition establishes that, given any point P in the parabola, the distance from P to the focus equals the distance form P to the directrix; that means PF=PD, having in D a right angle. To prove that equality, the geometry should give the evidence, and there, the sphere makes its part.

You can follow the animation through **Figure 4** (b, c, d) where it is a key step to trace a line from P to the cone apex, remaining in the cone. The point E in that line is the one on the circle of latitude that is tangent to the cone and the sphere. The segment PE remains there, and then, from P again, a line is drawn down to the horizontal plane, determining point P´ in such a way that PP´ is perpendicular to the horizontal plane. Finally we trace segments DP´and EP´.

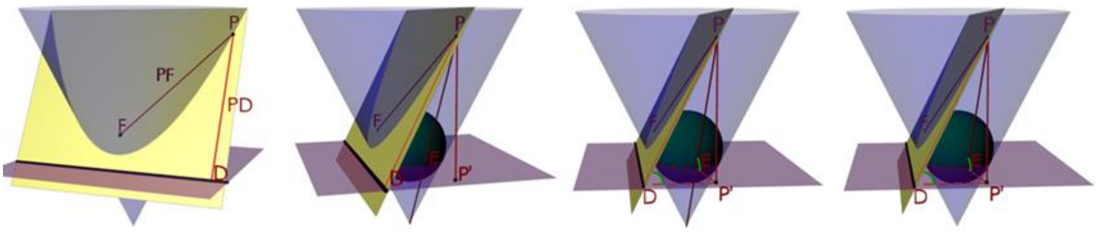

Figure 4, a, b, c, d. Taking any point P in the parabola we must prove PF=PD

At this moment, triangle PDP´ and triangle PEP´ are congruent because they have the same angle at D and E being that the cone and the plane producing the parabola have the same tilt. The triangles have also right angles at P´, so the third angle in each triangle must be equal also, due to the fact that they must sum 180 degrees. Finally, as the triangles share side PP´, the triangles congruence criterion Angle-Side-Angle guarantees they are congruent, so, the corresponding sides, PE and PD are equal. We have established that PE=PD, and looking to **Figure 4** (d) we can see that now we should get PF in the action. If we can prove that PE=PF, by transitivity we will have PF=PD.

Next **Figure 5** (a) shows lines PF and PE tangent to the sphere, this is a visualization that AR provides in an easy way, it makes us remember the tangency of PF because this segment is in the plane producing the parabola and F is the point of tangency with the sphere. It also reminds us of PE embedded in the cone and traced originally to arrive to the cone apex, thus, it has also point E of tangency with the sphere. AR allows the visualization of this situation: having two segments tangent to a sphere, as in **Figure 5** (a) we can draw a circle, as part of the sphere, where the segments are tangent. **Figure 5** (b, c, d) show from different visual perspectives the situation. It is the classic geometric fact of having two tangent segments to a circle traced from an exterior point (P), thus, those segments are equal to each other (PE=PF). This is a case of congruent triangles also, easy to visualize if we draw an extra segment from P to the center of that circle located at the sphere.

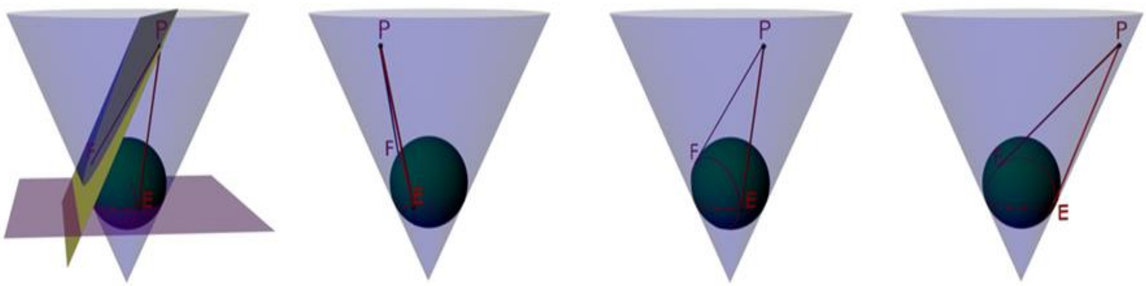

Figure 5. a, b, c, d.

This ends up the proof of PF=PD, which means that, given any fixed point P in the parabola (obtained by the intersection of the cone and a plane with the same tilt) the distance from P to the Focus F, equals the distance form P to the directrix (Focus and directrix were determined with the corresponding Dandelin Sphere).

The animation we described above includes some text that helps following the proof. The equal segments stand there available to our eyes, and the geometrical facts we used have the opportunity to be perceived actively. This is something that requires a continuous technical effort when we are conceiving and creating the environment. It is important to say that the design and development of this dynamic animation is a technological product that results from a PhD dissertation in the Program of Innovation in Education. Its use with students is actually another important element in the methodology that is carried out with the corresponding dissertation.

DISCUSSION AND CONCLUSION

As Bujak et al. (2013) comment, "disappearing are the days in which learning is an individual process limited to the confines of the classroom" (p. 541). These authors propose a framework for understanding AR learning. From a physical perspective, they argue about the affordances of a natural interaction with content; and from a cognitive perspective, the spatiotemporal alignment of information scaffolds the students' symbolic understanding. Besides that, they find the creation of collaborative learning and personal relevance as earnings from a contextual perspective. With their contribution, we have enough reasons to believe in the improvement of learning experiences through AR technology. The environment we presented in this paper looks for the scaffolding of mathematics symbolic nature in the students' mind. We take advantage of the affordances of AR to present important features of mathematical thinking related to the development of spatial ability in students through a natural interaction with mathematical content.

The present work aims to give an example of the creation of an AR environment in mathematics education which offers to the students important features of mathematical thinking that were reserved just for brilliant minds before, like the one of Germinal-Pierre Dandelin, capable of internal visualizations in the absence of external displays. The technology of AR allows us now to deepen in a mathematical thinking that binds to the "magic" this technology offers to our eyes. The pedagogical product we presented here should be considered as an external dynamical visualization fostering the development of internal visualization ability, thereby supporting the role of spatial ability for the learning of STEM education.

It always results interesting to reflect in how technology changes our manners. In the past we used to check information in books, and now we are used to find information in Internet. Seeking to know about Dandelin's work, we found Bertrand (2015) sharing what she names as an experience of effective revelation finding the "ice cream cone proof" in Calculus, Volume 1 from Tom Apostol (1961). There, she says, using the Dandelin's Spheres in the case of the ellipse, it was produced what could be recognized as a proof by picture that a cone cut obliquely by a plane results in an ellipse as defined by its focal property.

The work we are sharing here seeks to bring students the opportunity of that kind of "effective revelations" through the interaction with AR technology. We believe that nowadays AR reveals a new level of visual perception, promoting active interaction for understanding. Our aim is to deepen in the design and development of educational products supporting a different way to interact with mathematical content.

Our life goes by daily in a three-dimension space where our perception takes place over time. With the presence of AR we could enhance that perception in order to bring to the students new ways to deal with mathematics. We have now the opportunity to promote visual and gestural environments in our three-dimension space where mathematics could be "seen and touched" in real time and place, through the lenses of AR. We feel confident that this new approach to the conics should bring a positive attitude towards the learning of mathematics, releasing the anxiety that could have been built with past experiences where the name "conic" was not related with the teaching of this important curves.

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