An Analysis of Singaporean versus Indonesian Textbooks Based on Trigonometry Content

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ABSTRACT
Organization for Economic Co-operation and Development (OECD) (2016) released the results of Programme for International Student Assessment (PISA) 2015 and reported that the students’ performance in mathematics of Singapore and Indonesia had significant differences. There is a strong relationship between textbooks used and mathematics performance of the students. If textbooks differ, students will get a different opportunity to learn and the opportunity to learn influences students’ achievement (performance). The purpose of this study was to analyze the trigonometry contents and cognitive demand levels in Singaporean and Indonesian mathematics textbooks. The data were primarily qualitative. Horizontal and vertical analyses were used in this study. The result showed that Singaporean textbooks put more emphasis on all the concepts of trigonometry on right-triangle and further trigonometry (sine/cosine rules) while Indonesian textbook provided more discussions on angle and its concepts, trigonometry on right-triangle, and graph function of trigonometry (which is a lot more difficult than sine/cosine rules). In addition, Singaporean textbook provided more mathematical questions requiring higher cognitive demand levels while, Indonesian textbook provided more questions requiring lower levels. The differences of textbooks contents and required cognitive demand level probably influenced students’ mathematics performance in the two countries. It is hoped that the results will inform curriculum designers and/or textbooks’ author(s) in Indonesia, Singapore, and other countries as they review and update the mathematics curriculum and/or mathematics textbooks.

Keywords: analysis of content, cognitive demand level, mathematics textbook, trigonometry

INTRODUCTION
Textbook is an important source for giving students an opportunity to learn (Fan, 2013; Schmidt, McKnight, Houang, Wang, Wiley, Cogan & Wolfe, 2001; Wijaya, van den Heuvel-Panhuizen, & Doorman, 2015) and a major enterprise in the teaching of mathematics (Weinberg & Wiesner, 2010). Recently, there are many studies comparing textbooks from different countries to find the strengths and weaknesses of textbooks and were considered to
State of the literature

- Mathematics textbook is an important source for giving students an opportunity to learn and a major enterprise in the teaching of mathematics.
- If textbooks differ, students will get different opportunity to learn; it products different students’ achievement (performance).
- Trigonometry is an inseparable part of mathematics, it takes some subjects of arithmetic and geometry as sources, and it is a product of algebraic techniques, geometrical realities and trigonometric relationships.

Contribution of this paper to the literature

- In early learning of trigonometry, Singaporean textbook covers all the concepts of trigonometry on a right triangle and further trigonometry (sine/cosine rules), while Indonesian textbook provided concepts of trigonometry on a right triangle and graph function of trigonometry which is a lot more difficult than sine/cosine rules.
- Singapore textbooks emphasized the high-level of cognitive demand while Indonesia textbook provided more problems requiring lower level of cognitive demand.
- The difference of textbooks design (mathematics topics and cognitive demand level) is a factor which influences student’s mathematics performance in Singapore and Indonesia.

Develop future textbooks (Yang & Lin, 2015; Charalambous, Delaney, Hsu, & Mesa, 2010; Son & Senk, 2010). In addition, textbooks influence what teachers teach, how they teach it, and what homework or activities they assign to students (Alajmi, 2009; Hirsch, Lappan, Reys, & Reys, 2005). Teachers’ decisions about the selection of content and teaching strategies are often directly influenced by the textbooks teachers use (Reys, Reys, & Chavez, 2004). Therefore, textbooks are considered to determine largely the degree of students’ opportunities to learn (Tornroos, 2005). This means that if textbooks differ, students will get a different opportunity to learn (Haggarty & Pepin, 2002). As a result, different student outcomes will appear which is confirmed by several studies that found a strong relationship between the textbook used and the mathematics performance of the students (Tornroos, 2005; Xin, 2007).

Organization for Economic Co-operation and Development [OECD] (2016) released the results of PISA 2015 and reported that the students’ performance in mathematics of Singapore and Indonesia had significant differences. Students’ performance in Singapore was ranked at the top performance while In Indonesia, the performance is always among the bottom place. Due to high scores in international mathematics exams lately, Singapore has attracted the attention of people in education all around the world. For instance, in the United States, Singaporean textbooks were used in some school districts as teachers and mathematicians like them because of their simple approach to problem solving (Hoven & Garelick, 2007).

On the other hand, Singapore and Indonesia have a national curriculum. Indonesia is a big country with many different cultures and varying students’ ability. As textbook indicates the intended curriculum, Indonesian government always develops textbooks based on students’ needs and ability in all areas (Ministry of Education & Culture of Indonesia, 2013).
Therefore, the Indonesian government always considers the appropriateness of textbooks design with students in those areas in mind. In conclusion, both countries can learn from each other to develop future textbooks related to trigonometry. In addition, the primary goal of Singapore’s school mathematics curriculum is mathematical problem solving (Ministry of Education of Singapore, 2006) while in Indonesia, mathematics curriculum emphasizes more scientific approaches for investigating phenomena, acquiring new knowledge, or correcting and integrating previous knowledge in learning mathematics (Ministry of Education of Indonesia, 2013).

Furthermore, we analyzed trigonometry topic because trigonometry is inseparable part of mathematics in high school. Trigonometry has long been a standard component of the secondary school curriculum in all countries, usually in the latter half of the secondary years (Kissane & Kemp, 2009). Trigonometry is a unit in which algebraic techniques, geometrical realities and trigonometric relations come together. However, it was found that students had difficulty in understanding some basic concepts of trigonometry and trigonometry is not interested for them (Akkoc, 2008).

Overall, this study reports the analysis of trigonometry presentation in Singaporean and Indonesian textbooks. This study presents content analysis and cognitive demand level of trigonometry topics in the textbooks of these two countries. It is hoped that the results will inform curriculum designers and/or textbooks’ author(s) in Indonesia, Singapore, and other countries as they review and update the mathematics curriculum and/or mathematics textbooks.

THEORETICAL BACKGROUND

The Role of Mathematics Textbooks

Textbooks have been shown to have a great impact on classroom work and to form the backbone of mathematics teaching (Kajander & Lovric, 2009; Tornroos, 2005). Textbooks have been used as a basic resource for teaching in many countries with the aim of facilitating both student understanding and teacher instruction. As they support teachers and instruction, textbooks constitute an integral element of mathematics education. Mathematics textbooks influence what topics are covered and how these topics are presented. Stein, Remillard, and Smith (2007) argued that, “what mathematical topics are covered in a given set of curriculum materials is of fundamental importance” (p. 327). How topics are presented in the text is important because it sets in motion “pedagogical approaches and different opportunities for students’ learning” (Stein et al., 2007, p. 327). Results of the TIMSS (Trends in International Mathematics and Science Study) showed that teachers use mathematics textbook as their main resource when selecting their teaching method (Mullis, Martin, Foy, & Arora, 2012). When a topic is not covered in the textbook, it is unlikely that it will be presented in the classroom (Alajmi & Reys, 2007). In addition, the mathematical questions (tasks, problems) in the textbooks that students engage in largely determine what mathematics students learn and how they learn it (Stein et al., 2007).
In addition, Singapore and Indonesia implement different approaches in learning mathematics concerning mathematics curriculum implementation. The primary goal of Singapore’s school mathematics curriculum is mathematical problem solving (Ministry of Education of Singapore, 2006) while in Indonesia, mathematics curriculum emphasizes more scientific approaches for investigating phenomena, acquiring new knowledge, or correcting and integrating previous knowledge in learning mathematics (Ministry of Education of Indonesia, 2013). The different approaches in mathematics curricula would influence the mathematics textbooks design in the two countries.

Textbooks analysis became crucial due to its role in teaching and learning, especially in international comparison (Fan, 2013). Previous studies analyzed textbooks from different countries to find the advantages and disadvantages of these textbooks, so the results could shed light to designing future textbooks (Yang & Lin, 2015; Fan, 2013; Charalambous et al., 2010; Son, 2012).

International Research Involving Singapore and Indonesia

Saglam and Alacaci (2012) compared quadratics units in Singaporean, Turkish and IBDP Mathematics Textbooks. They found that Singaporean textbook covered the biggest number of mathematics topics on quadratics units among the three mathematics textbooks. On the other hand, Siregar and Ziebarth (2015) compared mathematics textbooks from Indonesia, Singapore, and the US Common Core State Standards focusing statistics and probability. They found that Singaporean textbook requires students to master a higher cognitive demand level while Indonesian textbooks focused on students’ understanding the basic concepts of statistics and probability and provided more low cognitive demand levels. In addition, Fowler (2015) reported that Singaporean mathematics textbooks provided more questions requiring higher cognitive demand levels than US textbooks on linear functions topics. Singaporean textbook reflected simple features of text density and enriched use of visual elements, more number of mathematics topics, and an easier inner organization to follow (Erbas, Alacaci, & Bulut, 2012; Soh, 2008).

On the other hand, Indonesian curriculum has been implementing realistic mathematics education approach to school mathematics which is widely recognized as providing one of the best and most detailed elaborations of the problem-based approach to mathematics education (Hadi, 2002). Wijaya et al. (2015) argued that the lacking opportunity-to-learn in Indonesian mathematics textbooks may cause Indonesian students’ difficulties in solving tasks. They reported distribution of cognitive demand (reproduction, connection, and reflection) in Indonesian mathematics textbooks. There were 45% reproduction tasks requiring performing routine mathematical procedures, 53% connection tasks requiring linking different mathematical curriculum strands, and only 2% reflection tasks (which are considered as tasks with the highest level of cognitive demand).
Analysis of Textbooks contents and Cognitive Demand Level

In TIMSS, textbook analysis initially focused on investigating the content profiles of textbooks (Mullis, et al., 2012). Textbooks also were examined based on five measures (Valverde, Bianchi, Wolfe, Schmidt, & Houang, 2002). Those were the classroom activities proposed by the textbook; the amount of content covered in textbooks and the mode of presentation; the sequencing of content; physical characteristics of textbooks, such as the size of the book and the number of pages it has; the complexity of the demands for students are asked to perform.

Porter (2006) developed two-dimensional languages to describe the content of the mathematics curriculum. This two-dimensional language can be presented in a rectangular matrix with topics as rows and cognitive demands (sometimes called performance goals or performance expectations) as columns. Topics are content distinctions such as “trigonometric ratios” or “graph of trigonometric functions.” Cognitive demands distinguish memorizing; procedures without connections; procedures with connections; and doing mathematics. Our study utilizes methodology similar to that which Porter has described, in that we examined the content of textbooks in terms of topics (contents) and cognitive demand levels.

Cognitive demand is potentially required when solving mathematical questions (tasks/problems) (Charalambous et al., 2010). Therefore, it is crucial to examine what cognitive demand level required in the textbooks when solving problem (answering mathematical questions). When a teacher selects a task for use in a classroom setting, cognitive demand levels need to be considered to determine the extent to which a task is likely to afford an appropriate level of challenge students. Stein and Lane (1996) in their QUASAR [Quantitative Understanding: Amplifying Student Achievement and Reasoning] project found that the highest learning gains on a mathematics-performance assessment were related to the extent to which tasks were set up and implemented in ways that engaged students in high levels of cognitive thinking and reasoning. After analyzing the levels of cognitive demand of mathematical tasks, QUASAR project researchers noted that students “need opportunities on a regular basis to engage with tasks that lead to deeper, more generative understandings about the nature of mathematical concepts, processes, and relationships” (Stein, Smith, Henningsen, & Silver, 2000, p. 15).

Stein and Lane (1996) and Stein et al. (2000) defined four levels of cognitive demand: “memorization,” “procedures without connections,” “procedures with connections,” and “doing mathematics.” The first two are usually thought of as low cognitive demand levels, whereas the last two are considered as high cognitive demand levels. Memorization means that students reproduced previously learned facts (e.g., formula, definition, etc.) in problems; procedures without connections means that students used algorithmic or procedural knowledge without having a connection to the concepts or meanings; procedures with connections means that students are required to attend to the concepts or meanings when
using algorithmic or procedural knowledge; doing Mathematics means that students use complex, non-algorithmic thinking to solve problems.

Charalambous et al. (2010) analyzed textbooks in three categories, namely horizontal, vertical, and contextual. The horizontal analysis examines the general characteristics of textbooks, such as physical characteristics and the organization of the textbooks’ content. The vertical analysis addressed textbooks presentation and treatment of the content and an “environment for construction of knowledge”; including cognitive demand levels (Herbst, 1995, p. 3; Charalambous et al., 2010; Wijaya et al., 2015). The contextual analysis focused on the ways in which textbooks are used in instructional activities by either the instructors or the students (Mesa, 2007; Remillard, 2005; Rezat, 2006). Charalambous et al. (2010) argued that the first two categories (horizontal and vertical analysis) are appropriate criteria to analyze mathematics textbooks.

**Trigonometry Related Studies**

Trigonometry is an important part of mathematics. It takes some subjects of arithmetic and geometry as its source. In other words, it is a product of algebraic techniques, geometrical realities and trigonometric relationships (Niranjan, 2013). Trigonometry has applications in both pure and applied mathematics. Trigonometry is essential in many branches of science and technology. It is usually taught in secondary schools either as a separate course or as part of a pre-calculus course (Lial, Hornsby & Schneider, 2008). Throughout the time, there have been fundamental changes in the nature of trigonometry in the modern mathematics education; however, moving beyond the abstract mathematics to application has often been difficult for the students.

Galadima and Yusha’u (2007) found that students has a low score in trigonometry caused by lacking understanding of the basic concepts, principles, terms and symbols involved. Yusha’u (2013) identified that trigonometry is a difficult topic that challenges students. Therefore, trigonometry is one of mathematical topics in textbook which is crucial to be analyzed of its design and development in textbooks; as textbook has strong relation with students’ mathematics performance (Tornroos, 2005; Xin, 2007).

**Fieldwork and Research Questions**

In this textbooks analysis, we followed the horizontal and vertical analyses developed by Charalambous et al. (2010) to analyze the two textbooks focusing on trigonometry contents and cognitive demand level required on trigonometry in Singaporean and Indonesian textbooks. The horizontal and vertical analysis is a complementary approach which is not only feasible but also worthwhile because it provides a means for better exploring what students learn, especially the opportunities to learn that students and teachers are afforded as they engage with the mathematics textbooks (Haggarty & Pepin, 2002). The horizontal analysis can provide information about the quantity of textbooks’ content including physical characteristics such as the size of the book, the number of pages it has, and the mathematics topics
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(Charalambous et al., 2010; Wijaya, van den Heuvel-Panhuizen & Doorman, 2015). On the other hand, for the vertical analysis, we focused on the cognitive demand levels.

Following horizontal and vertical analysis, previous studies have focused on the cognitive demand level of mathematical questions in the textbooks (Charalambous et al., 2010; Jones & Tarr, 2007; Porter, 2006). Charalambous et al. (2010) found that more than 85% questions in Cypriot and Irish textbooks require low cognitive demand; more than 70% of the questions in Taiwanese textbooks had high cognitive demands. Jones and Tarr (2007) studied the level of cognitive demand in probability tasks included in middle school textbooks from four recent eras of mathematics education (New Math, Back to Basics, Problem Solving, and Standards). The tasks were analyzed using the mathematical tasks framework (Stein et al., 2000). They found that the majority of mathematics tasks in the textbooks of the four recent eras of mathematics education required high cognitive demand level. Furthermore, this study expands on previous research by analyzing the trigonometry presentation in Singaporean and Indonesian textbooks. In particular, it focused on the trigonometry topics and cognitive demand level required on trigonometry. Therefore, the research questions are as following:

1. How is trigonometry covered in Singapore and Indonesia in terms of textbook contents including general structure, position and proportion of the trigonometry in the textbooks?
2. What cognitive demand level is required on trigonometry in Singaporean and Indonesian mathematics textbooks?

**METHOD**

The data of this study is primarily qualitative, as qualitative methods can enable a study to analyze issues deeply and comprehensively (Patton, 1990). According to the earlier study, this study followed the horizontal and vertical analysis to analyze the trigonometry contents and cognitive demand level in Singaporean and Indonesian textbooks (Charalambous et al., 2010; Jones & Tarr, 2007; Stein et al., 2000).

**Textbooks Selection**

**Singaporean textbooks**

Singapore’s mathematics textbooks are interesting for a few reasons. Singaporean students have been successful in international mathematics exams such as TIMSS and PISA (Mullis, Gonzalez, Gregory, Garden, O’Connor, Chrostowski, & Smith, 2000; Mullis, Martin, Gonzalez, & Chrostowski, 2004; Zhu & Fan, 2004). In addition, there is a widely-held view that the Singaporean textbooks are distinctive for setting high standards, containing both routine and non-routine problems, using a unique pedagogical approach (concrete pictorial abstract approach) to develop understanding of mathematics’ concepts, and being logically structured and focused on the essential skills of mathematics (Ahuja, 2005; Hoven & Garelick, 2007). Moreover, Singaporean textbooks are used in some school districts in the United States as teachers and mathematicians like their simple approach (Hoven & Garelick, 2007). Therefore,
Table 1. Selected textbooks from Singapore and Indonesia

<table>
<thead>
<tr>
<th>Country</th>
<th>Selected textbook</th>
</tr>
</thead>
</table>

The mathematics 6th edition 3rd series textbook for ninth grade in Singapore was selected in this study (Table 1).

**Indonesian textbook (2013 curriculum)**

This study selected *Matematika* [Mathematics] textbooks from Indonesia (Table 1). This textbook was published by ministry of education and culture of Indonesia. Safrudiananur (2015) found that this mathematics textbook is feasible to be used in teaching and learning in high schools because this textbook is good in terms of Indonesian teachers’ perceptions to the completeness and accuracy of the contents, the role in learning process, and as a teaching guide. Moreover, more than 60% schools in Indonesia have been using this textbook (Budiari, 2014).

In addition, a diagnostic survey conducted by ministry of education and culture of Indonesia found that many mathematics teachers in Indonesia are still using mathematics textbooks published by Indonesian government (Hadi, 2012). Moreover, this textbook is provided online access for teachers and students without any payment. Therefore, this textbook is largely used in Indonesia.

**Method of Data Coding and Analysis**

**Analysis of Textbook Contents**

In this section, we used horizontal analysis (Caharalambous et al., 2010) to analyze textbook content including general structure of the textbooks, position of trigonometry and proportion of trigonometry in the two textbooks. Therefore, a table (Table 2) consists of mathematical units in the two textbooks was created to compare the position of trigonometry in textbook. In addition, another table (Table 3) consists of mathematics topics, total number of pages and number of pages allocated to trigonometry and the percentage of pages on the trigonometry units over the whole textbooks—was constructed in order to see the design of trigonometry in the two textbooks. Then, the percentage of pages allocated to trigonometry (proportion of trigonometry) was computed by calculating the percentage of the pages allocated to the units over the total number of pages in each textbook. Moreover, internal units of trigonometry were recorded and compared in the two textbooks. The internal contents of trigonometry indicated the learning objectives of trigonometry in both textbooks.
Cognitive Demand Level

The cognitive demand level of trigonometry tasks were analyzed based on Stein et al’s (2000) four cognitive demand levels: “memorization,” “procedures without connections,” “procedures with connections,” and “doing mathematics” (Authors, 2015; Charalambous et al., 2010; Stein et al., 2000). The memorization and procedures without connections are usually thought of as low-level cognitive demand, whereas the procedures with connections and doing mathematics are considered as a high-level cognitive demand.

Memorization means that students reproduced previously learned facts (e.g., formula, definition, etc.) in problems (Figure 1). This task has no connection to the concept or meaning that underlies the facts, rules, formulas, or definitions being learned or reproduced.

![Figure 1](image1.png)

**Figure 1.** Example of memorization question

Procedure with connections means that students are required to make connection with the concepts or meanings when using procedures (more than merely application of procedures). This task requires some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with conceptual ideas that underlie the procedures to complete the task successfully and that develop understanding (Figure 2).

![Figure 2](image2.png)

**Figure 2.** Example of procedures without connection question

Problem 8.2

A kid wants to determine an angle by trigonometric ratios. He was given a ratio as following: $\sin \alpha = 1/2$. His work is to find $\alpha$!

Translated from Indonesian textbooks (Sinaga et al., 2013, p. 377)
Doing mathematics means that students use complex, non-algorithmic thinking to solve problems (Figure 4). When solving this task, it is required to explore and understand the nature of mathematical concepts, processes, or relationships. This task requires considerable cognitive effort and may involve some level of anxiety for the student because of the unpredictable nature of the solution process required.

Procedure

We recorded the general structure of the textbooks including all mathematics topics and the internal units (subtopics) of trigonometry presented in the selected Singaporean and Indonesian textbooks. Then, we discussed the position and proportion of trigonometry in each textbook. Furthermore, we examined and coded all the trigonometry questions presented in the selected textbooks according to the four cognitive demand levels. For the overall accuracy of the coding, the inter-reliability of coding was checked between the researchers and/or with other external scholars, especially for a small number of seemingly equivocal cases, though most were quite straightforward. In addition, an independent coder was invited to code the trigonometry questions to meet the final decision of disagreement between the first two coders (the coding result by the independent coder was compared to that obtained by the researchers). There are 429 trigonometry questions in the two textbooks. According to the intra-class correlation coefficient (ICC) on absolute agreement, the reliability between the different coders was found to range from 0.85 to 1.00 among the four classifications, with an average being 0.92. Overall, the reliability of this study is high.
In this section, we reported mathematics topics in the two textbooks to attend the comparison of the topics design. In addition, we found that Singaporean textbook introduced trigonometry earlier than Indonesian textbook. Singaporean textbook started to teach trigonometry officially at 9th (ninth) grade junior high school level whereas Indonesia started at 10th (tenth) grade senior high school level. There were some differences of the mathematics topics structure in the two textbooks (Table 2). There were some more difficult mathematical topics in Indonesian textbook, such as quadratic function, geometry, and limit of function. Overall, the following were reported: the general structure of the two textbooks, the position and proportion of the trigonometry topics within the totality of the textbooks.

In Singaporean textbook, trigonometry is the tenth and eleventh of thirteen chapters in 9th (ninth) grade after linear graphs, congruent and similar triangles, area and volume of similar figures and solids; and coming before mensuration (arc length, sector area, radian measure) and geometrical properties of circles. On the other hand, in Indonesian textbook, trigonometry is the eight of twelve chapters in 10th (ninth) grade coming after quadratic equations and functions; and coming before geometry (points, lines, planes and angles). Both textbooks have the same vision to provide learning of geometry after trigonometry. Nonetheless, there were huge differences on providing mathematical topics before trigonometry as listed in Table 2 (Indonesian textbook provided more difficult mathematical topics combined together with trigonometry in the textbooks).

In terms of trigonometry proportion, Singaporean textbook was composed of 402 pages and 72 of them were devoted to the trigonometry, which means trigonometry covers 17.91%
Table 3. Proportion of trigonometry in Singaporean and Indonesian mathematics textbooks

<table>
<thead>
<tr>
<th>Textbook</th>
<th>Number of Chapters</th>
<th>Total number of pages</th>
<th>Number of pages allocated to trigonometry</th>
<th>Proportion of trigonometry over whole textbook</th>
</tr>
</thead>
<tbody>
<tr>
<td>Indonesia</td>
<td>12</td>
<td>391</td>
<td>35</td>
<td>8.95%</td>
</tr>
<tr>
<td>Singapore</td>
<td>13</td>
<td>402</td>
<td>72</td>
<td>17.91%</td>
</tr>
</tbody>
</table>

of the whole textbook (Table 3). Trigonometry was the main topic included in Singaporean textbook considering the number of pages in the textbook and it was presented in two main chapters (trigonometrical ratios and further trigonometry).

On the other hand, In Indonesian textbook, trigonometry were given in the 35 of the 391 pages; covering 8.95% of the textbook (Table 3) and it was only given in one chapter. Trigonometry was not the main topic in Indonesian textbook; the Indonesian curriculum put more emphasis on exponents and logarithms in the selected textbook (10th grade). In addition, trigonometry is also provided in mathematics textbooks of eleventh and twelfth grade level (this probably influenced the amount of trigonometry topics in the selected textbook from Indonesia).

**Internal Content of Trigonometry**

This section discusses the topics included in trigonometry in the two textbooks. There are some different topics of trigonometry covered in textbooks of these two countries (Table 4). Indonesian textbook discussed angles and angle measurement before learning trigonometry while Singaporean textbook didn’t provided a discussion about angle and concepts in the selected textbooks. Moreover, Indonesian textbook provided graphs of trigonometric functions whereas the Singaporean textbook did not introduce graph function of trigonometry (the main topic in Singaporean textbook was about trigonometry on right-triangle).

We found a difference of trigonometry design in the two textbooks. Singaporean textbooks put more emphasis on all the concepts of trigonometry on right-triangle and further trigonometry (sine/cosine rules). Meanwhile, Indonesian textbook provided more discussion on angle and its concepts, and graph function of trigonometry (which is a lot more difficult than sine/cosine rules) (Table 4). In addition, Singaporean textbook provided more different mathematics topics (subtopics) on trigonometry than Indonesian textbooks (Table 4).

**Cognitive Demand Level**

In this section, we reported the summary of the findings including the distribution of trigonometry questions in Singaporean and Indonesian textbooks in terms of their cognitive demand level. We found that there was a greater number of trigonometry question in the Singaporean textbook compared to Indonesian textbook (there are 316 trigonometry questions in Singaporean textbooks while, there are 113 questions in Indonesian textbook). It showed that Singaporean textbook put more emphasis on mathematical problem solving by providing
more mathematics questions (Ministry of Education of Singapore, 2006). In terms of trigonometry questions distribution, there were 68.99% of the trigonometry questions in Singaporean textbook and 39.82% of the trigonometry questions in Indonesia textbook requiring higher cognitive demand levels (Table 5).

On the other hand, the trigonometry questions coded as lower cognitive demand level were 31.01% in Singaporean textbook and 60.18% in Indonesian textbook. It showed that trigonometry questions in Singaporean textbook were dominated by requiring higher level of cognitive demand, while the Indonesian textbook provided more problems requiring lower cognitive demand levels.

Typically, the most common level of cognitive demand required by trigonometry questions was procedures with connections in Singaporean textbook and procedures without connections in Indonesian textbook. In addition, the trigonometry design in the two textbooks adopted the exposition-examples-exercises model (Love and Pimm, 1996) and therefore the exercises of the textbook for the relevant topic formed the bulk of the practice tasks.

### Table 4. Comparison of trigonometry (internal contents) in the two textbooks

<table>
<thead>
<tr>
<th>Trigonometry Topics</th>
<th>Indonesian Textbook</th>
<th>Singaporean Textbook</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trigonometrical ratios</td>
<td>✓</td>
<td>-</td>
</tr>
<tr>
<td>Measuring angle (degree and rad)</td>
<td>✓</td>
<td>-</td>
</tr>
<tr>
<td>Basic concept of an angle</td>
<td>✓</td>
<td>-</td>
</tr>
<tr>
<td>Trigonometrical ratios</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Values of trigonometrical ratio</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Use of calculator</td>
<td>-</td>
<td>✓</td>
</tr>
<tr>
<td>Solving right-angled triangles using trigonometrical ratios</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Finding the value of an angle with trigonometrical ratios</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Graph function of trigonometry</td>
<td>✓</td>
<td>-</td>
</tr>
<tr>
<td>Applications of trigonometry</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

### Table 5. Distribution of trigonometry questions with respect to cognitive demand levels

<table>
<thead>
<tr>
<th>Cognitive Demand Level</th>
<th>Singapore</th>
<th>Indonesia</th>
</tr>
</thead>
<tbody>
<tr>
<td>Memorization</td>
<td>3.80%</td>
<td>10.61%</td>
</tr>
<tr>
<td>Procedure without connection</td>
<td>27.21%</td>
<td>49.56%</td>
</tr>
<tr>
<td>Procedure with connection</td>
<td>52.53%</td>
<td>30.09%</td>
</tr>
<tr>
<td>Doing mathematics</td>
<td>16.46%</td>
<td>9.73%</td>
</tr>
</tbody>
</table>
DISCUSSION and CONCLUSION

The purpose of this study was to analyze the trigonometry topics and cognitive demand levels in Singaporean and Indonesian textbooks. The analysis of textbooks focusing on trigonometry revealed some similarities and differences in terms of the general structure, position and proportion of trigonometry along with the grade levels these concepts had been taught. The positions of trigonometry in the two textbooks gave indications about the connection among the represented topics (Table 2). Trigonometry was at the tenth and eleventh of thirteen chapters in 9th (ninth) grade after linear graphs, congruent and similar triangles, area and volume of similar figures and solids; and coming before mensuration (arc length, sector area, and radian measure) and geometrical properties of circles. On the other hand, in Indonesian textbook, trigonometry is the eight of twelve chapters in 10th (ninth) grade coming after quadratic equations and functions; and coming before geometry (points, lines, planes and angles), limit of a function, statistics, and probability.

Considering the topics presented with trigonometry in the two textbooks, it showed that trigonometry takes some subjects of arithmetic and geometry as its source. In other words, it is a product of algebraic techniques, geometrical realities and trigonometric relationships (Niranjan, 2013). For instance, in Singaporean textbook, the trigonometry chapter was given after quadratic equations and congruence of similar triangles. In this way, students were expected to make connections with their prior knowledge about quadratic equations. For example, we can find that quadratic equation was used in solving the problems of trigonometry in the textbooks (Pythagorean formula).

Singaporean textbook provided topics related to arc length, sector area, radian measure and geometrical properties of circles which require trigonometry concepts, while Indonesian textbook didn’t provide these topics. On the other hand, Indonesian textbook introduced trigonometry after quadratic equations and functions, and before geometry (points, lines, planes and angles). Indonesian textbook provided questions and explanation of trigonometry related to prior chapters such as functions (linked to graph function of trigonometry) and quadratics (linked to Pythagorean Theorem). In terms of internal contents of trigonometry, Singaporean textbook covered all the concepts of trigonometry on a right triangle and further trigonometry (sine/cosine rules) while, Indonesian textbook provided all the concepts of trigonometry on a right triangle and plus the trigonometric functions which is a lot more difficult than sine/cosine rules.

In conclusion, Singaporean textbooks put more emphasis on all the concepts of trigonometry on right-triangle and further trigonometry (sine/cosine rules). Meanwhile, Indonesian textbook provided more discussion on angle and its concepts, trigonometry on right-triangle, and graph function of trigonometry (which is a lot more difficult than sine/cosine rules) (Table 4). In addition, Singaporean textbook provided more different mathematics topics (subtopics) on trigonometry than Indonesian textbooks (Table 4). It
showed that Singaporean textbook is consistent to provide more mathematics subtopics in a unit (chapter) in the textbook (Saglam & Alacaci, 2012; Erbas, Alacaci, & Bulut, 2012).

Cognitive demand is potentially required when solving mathematical question (task, problem) (Charalambous et al., 2010). In addition to literature, the findings on cognitive demand levels showed that Singaporean textbook contained questions requiring higher cognitive demand levels, whereas Indonesian textbook provided more questions requiring lower cognitive demand levels. Therefore, Singaporean textbook adhered to the recommendations of Stein et al. (2000) that students at each grade level should experience more mathematical questions requiring higher cognitive demand levels and have opportunities to “engage with tasks that lead to deeper, more generative understandings regarding the nature of mathematical processes, concepts, and relationships” (p. 15). Singaporean textbook is consistent to provide more questions requiring higher cognitive demand levels (Fowler, 2015; Siregar & Ziebarth, 2015).

On the other hand, Indonesian textbook provided more questions requiring lower cognitive demand level. This finding similar with the earlier studies (Siregar & Ziebarth, 2015; Wijaya et al., 2015) that Indonesian mathematics textbooks provided more questions requiring lower cognitive demand level. Textbooks containing tasks that predominately require lower levels of cognitive demand may not support student learning because students are rarely asked to grapple with difficult situations. Therefore, it is recommended to provide more questions in textbooks requiring higher cognitive demand levels which can improve students’ mathematics ability and achievement (Silver & Stein, 1996) in Indonesia. When completing tasks requiring higher cognitive demands level, students are engaged in a productive struggle that challenges them to make connections to concepts and to other relevant knowledge (Van De Walle, Karp & Bay-Williams, 2012).

Furthermore, the findings might add to earlier studies that showed a positive relation between textbooks design (providing opportunity to learn) in textbooks and student achievement. For example, Tornroos (2005) found a high correlation between student achievement in a test and the amount of textbook content related to the test items. Also, Xin (2007) revealed that the algorithmic strategy used by students to solve word problem tasks was the strategy suggested in the textbooks. In conclusion, the difference of textbooks design (mathematics topics and cognitive demand level) is a factor which influences student’s mathematics performance in Singapore and Indonesia. It is hoped that the results will inform curriculum designers and/or textbooks’ author(s) in Indonesia, Singapore, and other countries as they review and update the mathematics curriculum and/or mathematics textbooks.

IMPLICATION, LIMITATION, AND FUTURE RESEARCH

The trigonometry contents found in the two textbooks would give references to designing opportunity to learn trigonometry in the future mathematics textbook. On the other hand, information of cognitive demand levels would be considered to provide more mathematical questions (tasks, problems) requiring higher cognitive demand level. This study
is relevant for mathematics education policy, especially for developing and adopting textbook. Teachers may consider for implementing mathematical questions that require higher cognitive demand levels. Teachers can modify their daily instruction according to their students’ needs by raising or lowering the cognitive demand of the questions in the textbooks they are required to use.

There are some limitations in this study. First, we only analyzed one series of textbooks in each country. Although these selected textbooks were representative, it does not imply all the results found in this study are similar in other textbooks in both countries. Second, this study does not analyze how teachers use these textbooks. Although textbooks play a significant role in mathematics classes, we cannot assume that all teachers teach trigonometry in exactly the same ways. Textbook is one of the factors that influenced students’ mathematics learning and students’ achievement (performance). Researchers or policymakers should treat our results very carefully and not overwhelmingly use these results.

Finally, it is important to point out that textbook development itself is not a purpose, but a process that aims to produce high-quality textbooks. Textbook development closely relates to textbook studies that examine the quality of textbooks and its impact on teaching and learning mathematics. By taking this stance, efforts to improve textbook development can be informed and facilitated by the ever-growing research interests in examining and documenting teachers’ use of textbooks and use of textbooks in students’ learning (e.g., Li, Chen & Kulm, 2009; Stein et al., 2007). Further efforts are thus needed in studying textbook development, and connecting textbook development research with research on textbook use and its impact on teaching and learning mathematics. Moreover, further research is needed to investigate questions such as: (1) How do teachers in Singapore and Indonesia implement their textbooks in classrooms? (2) How do mathematics textbooks in Singapore and Indonesia affect middle-grade teachers’ teaching and students’ learning? (3) How do the cultures of Singapore and Indonesia reflect on their textbooks? Research has provided some information about how cultural differences may influence the textbooks’ design (Fan, 1999; Leung, 2001). Future research can further investigate this issue to reveal the major differences in mathematics textbooks between Singapore and Indonesia.

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