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On the Importance of Subject Matter in Mathematics Education: A Conversation with Erich Christian Wittmann

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Particularly in the second half of the 20th century, historical approaches became Erich Christian Wittmann is one of the primary founders of mathematics education research as an autonomous field of work and research in Germany. The interview reflects on his role in promoting mathematics education as a design science and its consequences as well as his view on current developments within the discipline and his activities.

Keywords: Erich Christian Wittmann, mathe 2000, mathematics education, structure-genetic analysis, subject matter, design science

INTRODUCTION

Professor em. Dr. Dr. h. c. Erich Christian Wittmann's contributions to the learning and teaching of mathematics have shaped German mathematics education in the last three decades. Even ten years after his retirement he is as active as ever and talking to him shows that his passion for mathematics and its teaching and learning is still vivid.

One of the most prominent achievements of Prof. Wittmann is the foundation of the project 'mathe 2000' in collaboration with his close colleague and friend Gerhard Müller. The key idea of this project is the holistic development of mathematics education from kindergarten to upper secondary level, taking both a theoretical and practical approach. The most important results so far are a handbook for teaching arithmetic (two volumes) and the textbook DAS ZAHLENBUCH (available from kindergarten to grade 4) (e.g. Wittmann & Müller 1990 & 1992; Wittmann & Müller 2012). Along with these materials, a genetic view of mathematics as well as its teaching and research has been established.

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Figure 1. Prof. Wittman while giving the talk at the symposium mathe 2000 in 2012

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E. Ch. Wittmann's work has greatly contributed to establishing the renown of Institute for Development and Research in Mathematics Education at the Technical University of Dortmund for its work in mathematics education in Germany. Similar to the project "mathe 2000" the Institute focuses on the processes of teaching and learning mathematics from kindergarten to university level. It shares the fundamental idea that mathematics education research has to take responsibility for the actual teaching and learning in schools, for instance by creating suitable learning environments.

For this interview we had the privilege to speak to a man who seems to know by heart most of the works of great thinkers in the broad field of education. John Dewey's works in particular are standing on a bookshelf right behind Professor Wittmann's armchair. To this day, Wittmann continues to concern himself with the discipline of mathematics education as a research domain. Thus, we took the chance to use the interview to give the reader a deeper insight into E. Ch. Wittmann's main contributions to the field and his opinion of the self-concept of mathematics education as a scientific discipline.

One of the most important principles followed by E. Ch. Wittmann is that mathematics itself lies at the core of all work in mathematics education. For this reason, we will use it as the connecting link and highlight his views on the importance of the subject matter from different perspectives.

The interview will address the following topics:

1)The importance of the subject matter in international traditions in mathematics education

2)Mathematics education as design science

3)The importance of the subject matter for research in mathematics education

4)The importance of the subject matter for teaching of mathematics

1959-1964	Study of Mathematics and Physics at the University of Erlangen
1964-1966	Teacher Training for the secondary level
1966-1969	Research Assistant at the Mathematical Institute, University of Erlangen
1967	PhD in Mathematics (Algebra)
1969 - 2004	Full Professor for mathematics education at the University of Dortmund
1987	Foundation of the developmental research project 'mathe 2000'
1998	Honorary doctorate awarded by the University of Kiel
2000	Plenary Lecture at ICME 9, Tokyo
2004	Professor Emeritus
2013	"Johannes-Kühnel"- Prize awarded by the German Association for the promotion of Mathematical and Scientific Teaching (MNU) for developmental research in primary mathematics

Box 1. Curriculum vitae of Erich Ch. Wittmann

Ganbare - The importance of the subject matter in international traditions in mathematics education

As E. Ch. Wittmann is interested in the international history and trends in mathematics education, he gladly agreed to give an interview for the journal EURASIA. For many years he has built strong connections to European and Asian researchers through collaborations as well as through lectures or school visits to get direct experience of how the different cultures handle the subject matter mathematics:

"The founding of the journal EURASIA has been a very good decision. I personally owe a lot to important inputs from the East. Already in my studies at the University of Erlangen I was lucky to use the textbook series by W.I. Smirnov, which stands for the admirable Russian style. Later on I gained a lot from the 'Encyclopaedia of Elementary Mathematics' written by eminent Russian mathematicians. Since my American colleague and friend Jerry Becker introduced me to Asian mathematics education in the mid-1970s I have learned a lot from Japanese and Chinese mathematics educators.

The countries in Eastern Europe and Asia have always given great importance to the teaching and learning of mathematics. This should be a model for Western countries, too. I wish that Germany would also understand that message instead of successively reducing the amount of mathematics in the school curriculum. In Asia the expectations on the students are obviously higher. At the same time, the confidence into students' capacity is also higher. The attitude is widespread that if you work hard for something then you will be able to achieve it. A lesson of Confucius' is that 'a mistake is only a mistake if you do not work on correcting it' (Confucius 2013, Chapter 15). Here mistakes are seen as natural elements of learning processes and as an indication that the student has to continue his or her study. In Japan the word 'ganbare!' encourages the learner to hold on and not to give up. In Western Countries mistakes are sometimes still seen as something that has to be avoided, and mistakes are interpreted as a personal failure. Often, in order to avoid mistakes, the wrong decision is made to reduce the level of difficulty, and that is not in the best interest of students."

In "The Science of the Artificial" Simon (1970) differentiates between two types of sciences: The well-established natural sciences like physics or chemistry and design sciences (also sciences of the artificial) like architecture, education, law or medicine. While the aim of the first type is to explain how natural things "are and work", design sciences are concerned with the design of artificial things. In his paper Simon discusses the role and the relation between these two sciences.
Wittmann (see for example 1995) contends that mathematics education also belongs to the category of design sciences. "The core of mathematics education concentrates on constructing 'artificial objects', namely teaching units, sets of coherent teaching units and curricula as well as the investigation of their possible effects in different educational 'ecologies'" (Wittmann 1995, 362-363). Wittmann has always fought for the acceptance of this notion of mathematics education as a design science and its primary aim of designing substantial teaching units (cf. box 4) and the strengthening of its position as an autonomous science.

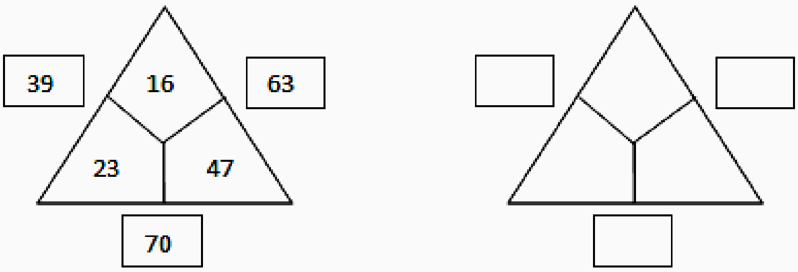
Box 2. Short information on Mathematics education as a design science

Substantial learning environments

Substantial learning environments are characterised as follows (Wittmann 1995, 365-366):

1. "They represent central objectives, contents and principles of mathematics teaching.
2. They provide rich sources for mathematical activities.
3. They are flexible and can easily be adapted to the conditions of a special classroom.
4. They involve mathematical, psychological and pedagogical aspects of teaching and learning in a holistic way, and therefore they offer a wide potential for empirical research."

An example for a typical substantial learning environment are arithmogons (Wittmann 2001, McIntosh & Quadling 1975) that appear in different mathe 2000 materials from first grade onwards.
The filled arithmogon in figure 3 displays the rules: Add the numbers of the two adjacent fields and note the sum in the box at this side.



Starting from simple addition and subtraction, Arithmogons offer different possibilities for rich mathematical activities, depending on the given numbers (c.f. Wittmann & Müller 2012). Here are only some examples of possible tasks.

- Can you fill the fields with numbers (in N) so that the numbers in the boxes are 30, 35 and 45 (10, 40 and 60)?

Can you find different possibilities (in N) so that the sum of the three boxes is 30 (25)? How many can you find?

Box 3. Substantial learning environments as one of the core concerns of mathematics education

The discussion about the correct understanding of mistakes is closely connected to a fundamental problem of education and teaching: How to deal with the apparent contradiction between demands imposed on the students from subjects on the one hand, and the right of the students to develop their personalities on the other hand?

Here Wittmann refers to John Dewey, the great American educational philosopher, who - already at the beginning of the 20th century – warned of constructing an opposition between "the child" and "the curriculum". In Dewey's understanding, the two poles have to be understood as complementary: "Abandon the notion of subject-matter as something fixed and ready-made in itself, outside the child's experience; cease thinking of the child's experience as also something hard and fast; see it as something fluent, embryonic, vital; and we realize that the child and the curriculum are simply two limits which define a single process." (Dewey 1902, 11).

In order to develop fruitful relationships between students and subject matter, the traditional view of mathematics as a system of ready-made structures has to be replaced by a new view in which mathematics is seen as the science of beautiful and useful patterns that can be actively learned and applied. Wittmann considers it as a "lucky accident" that he became familiar with this view at the very beginning of his career:

"When I read W.W. Sawyer's 'A Prelude to Mathematics', published in 1955, the German translation of 'Notes on Primary Mathematics' (Wheeler 1967), Freudenthal's article 'What is axiomatics and what educational value can it have?' (Freudenthal 1963) and Polya's works not only my understanding of mathematics but also my understanding of teaching and learning mathematics was kind of re-set. I think this was the main reason why I moved from mathematics to mathematics education. I thought it was a most stimulating field of research to develop mathematics teaching along these lines. However, at that time I had no idea how to establish mathematics education as a scientific discipline accordingly. It turned out that progress on this task would take time and would not come along smoothly. However, in the early years I did not anticipate how immense the difficulties would be."

Mathematics Education as a Design Science

Much earlier than many other mathematics education researchers E. Ch. Wittmann pleaded for considering mathematics education as not a mere addition of different areas like mathematics, pedagogy, psychology or others, but as a scientific discipline on its own, with its own specific problems and aims (e.g. Wittmann 1974; Wittmann 1995). Through many years and up to the present day, Wittmann has always worked on defining this specific characterisation of mathematics education as a 'design science' with explicit reference to the Nobel Laureate Herbert A. Simon (cf. box 2).

From the design science perspective, the construction, the empirical research and the implementation of substantial learning environments are the main tasks of research in mathematics education (Wittmann 1984, Wittmann 2002). It is exactly these tasks that prove the autonomy of mathematics education (or 'didactics of mathematics') as a discipline on its own: "[…] mathematics education requires the crossing of boundaries between disciplines and depends on results and methods of considerably diverse fields, including mathematics, general didactics, pedagogy, sociology, psychology, history of science and others. Scientific knowledge about the teaching of mathematics, however, cannot be gained by simply combining results from these fields; rather it presupposes a specific didactical approach that integrates different aspects into a coherent and comprehensive picture of mathematics teaching and learning and then transposing it to practical use in a constructive way." (Wittmann, 1995, 356; emphasis in original version).

From his experience Wittmann knows that it is not easy to hold one's ground as a "designer" as this group forms only a minority in the international mainstream of mathematics education:

"In establishing mathematics education as a design science we are running into a real difficulty which, however, is typical for design sciences in general. It has been aptly formulated by Simon (1970, 55-58): 'Why would anyone in a university stoop to teach or learn about designing machines or planning market strategies when he could concern himself with solid-state physics? The answer has been clear: he usually wouldn't.' In the same way it is tempting for mathematics educators to digress into psychology, general education, sociology, history of science, mathematics."

The Importance of Mathematics for Developmental Research in Mathematics Education

Wittmann has always adopted a strong focus on mathematics, by emphasising that the specific aims described above should be addressed from a specifically mathematical perspective. It is this focus on mathematics as the core of mathematics education that raises his scepticism when looking at the current developments in mathematics education research:

"I think that mathematics education at the national and international level has developed an orientation that focuses too much on human sciences rather than on being a design science. The volume 'Mathematics Education as a Research Domain: A *Search for Identity' (Sierpinska & Kilpatrick 1997) pays testimony to this fact. The related disciplines are important, but the subject matter itself should not be reduced in favour of integrating them. In the past, mathematicians and mathematics educators used to cooperate closely. For example, mathematicians like Felix Klein and Hans Freudenthal took responsibility for the development of mathematics teaching. Now this connection seems to be more or less lost, at least in Germany."*

Wittmann is concerned that current research projects are running the risk of shifting the focus away from mathematics towards other related disciplines and to rely too much on empirical studies as they are common in psychology. He has a clear preference for "natural theories of teaching and learning" that are inherent in well-understood subject matter and include even empirical information. In his terminology, the common empirical research in mathematics education should be considered as "empirical research of the second kind", whereas "empirical research of the first kind" in his sense (cf. Wittmann 2013) focuses on empirical evidence implicit in designing substantial learning environments. The main sources for empirical research of this first kind are (elementary) mathematics, history of mathematics, mathematical textbooks, existing knowledge of learning processes, the curriculum and teaching experience. Wittmann's position, however, is not entirely opposed to the current research:

"I am not against empirical research of the second kind. But it requires research of the first kind to identify what is worth to be investigated empirically."

To further strengthen the role of mathematics within the research in mathematics education, Wittmann is currently working on formulating the research method underlying the design of learning environments. He calls this method "structure-genetic didactical analysis" (cf. Wittmann 2012 and 2013). "We have to analyse the reasons why the subject of mathematics has been eclipsed. I think one of the problems was that the mathematically grounded researchers lacked a language, which helped them to express how they work scientifically. I myself have been struggling a long time in trying to describe and to defend what I am doing. Now I have elaborated this approach in some detail and called it a structure-genetic didactical analysis. I hope that my work will provide an instrument for mathematics educators who work on a mathematically well-founded basis."

This method combines a profound analysis of mathematical contents and processes with the design of substantial learning environments. It aims at shaping mathematical landscapes, "mathscapes" in Yuri Manin's terminology (cf. Manin 1992), in which students at given levels can become active and through their activities learn mathematics.

"The experiences in the developmental research of the project 'mathe 2000' show that it was a fundamental mistake to conclude from the deficiencies of former didactical analyses that content-oriented analyses are no longer relevant as a research method in mathematics education. As soon as didactical analyses are not reduced to the subject in its final form, but also involve the processes that occur in working on mathematical contents in an authentic mathematical practice, the picture changes dramatically. For such structure-genetic didactical analyses, as I like to call them, mathematics is still of major importance, but in its process character as it appears in mathematical activity, when students work alone or in collaboration with others." (Wittmann 2012, 273 translated by the interviewers).

"The structure-genetic didactical analysis is close to the children": It also takes into account the prerequisites of students as well as a long-term perspective on further mathematical ideas that build upon the currently addressed one. In accordance with Dewey (1904), it makes use of the structures, theoretical and methodical aspects that are inherent in the mathematics itself.

"I think the best way to explain this is by illustrating it with an example. Let's say as a researcher you have a certain problem, for example you want to design a learning environment for the exercise of the algorithm of multiplication. I do not know how to solve this problem empirically straight away. I cannot go into the classroom and ask the children or the teacher to tell me how the algorithm should be practiced or which exercises should be used. So I look at elementary mathematics and try to find out which mathematical situations or ideas in the mathematical landscape around multiplication might be useful in practicing the algorithm so that children can in addition gain mathematical insights. Then I try to find out which prerequisites the students would need and to what extent these have been introduced earlier in the curriculum. Furthermore I have to look ahead and to evaluate what the learning environment means for future learning."

Wittmann highly recommends Wheeler (1967), Freudenthal (1983) or Winter (1987) as paradigmatic examples for structure-genetic didactical analyses. In addition we would like to mention also some of his own works, e.g. Wittmann & Müller 1984, 1990 and 1992.

The Importance of Mathematics for Teaching

The ideas mentioned above put emphasis on the relations between research and practice. This relationship has always been an important factor for E. Ch. Wittmann's engagement in teacher education. He has been working hard for over 35 years to introduce prospective and in-service teachers to mathematics that matters for teaching. This task has never been easy as, at the beginning of their study, many prospective primary teachers are interested in everything else but mathematics.

"Whenever I give a teacher training course for our textbook Zahlenbuch (Wittmann & Müller 2012) in Switzerland, I use to illustrate it with the following metaphor: The teachers should imagine that they are guides who accompany their children through a mathematical landscape. The most important thing is that the teachers themselves know this landscape very well from their own hiking tours, that they know what can be experienced and discovered and are aware of the critical spots. The tour should be comfortable, and the teacher should not feel obliged to guide every single student to the very top of the mountain. Some students might dwell a bit or wait until other children come back from some digression. But basically, all students are participating in the hiking according to their best possibilities. So teacher training is a kind of mountain guide training."

This metaphor helps to point out why Wittmann sees the subject mathematics itself as the fundamental element of the teacher training at university level: "*It is the landscape where you can get lost if you don´t know it well enough. You need to have this knowledge first, before being able to guide others."* In other terms, a profound content specific knowledge of mathematics has been identified as a prerequisite for developing a pedagogical content knowledge.

This approach is not only appropriate for good students, but especially for those that have difficulties with the subject as long as the base of teaching is mathematics itself: *"The most important thing in teaching is to understand mathematical structures as teaching aids that facilitate* *learning. Slow learners in particular take profit from a sound mathematical frame. If you look at mathematics as a difficult and strange subject, you would tend to make it understandable for the children by using means from other disciplines such as pedagogy or psychology. However, the best support for learning comes from mathematics itself. The subject is not the born enemy of the learner but a helpful friend. And teachers that know the subject very well have the best means to help their students".*

Wittmann contends that prospective teachers should be introduced to subject-matter and theory first and to practical education only afterwards. He quotes from Dewey (Dewey 1904, 21-22): "Now the body of knowledge which constitutes the subject-matter of the student-teacher must, by the nature of the case, be organized subject-matter. It is not a miscellaneous heap of separate scraps. [It has] been selected and arranged with reference to controlling intellectual principles. There is, therefore, method in subject-matter itself – method indeed of the highest order which the human mind has yet evolved, scientific method. It cannot be too strongly emphasized that this scientific method is the method of mind itself. The classifications, interpretations, explanations, and the generalizations which make subject-matter a branch of study do not lie externally in facts apart from mind. They reflect the attitudes and workings of mind in its endeavor to bring the raw material of experience to the point where it at once satisfies and stimulates the needs of active thought." When learning mathematics properly, prospective teachers acquire a kind of implicit didactics which in subsequent didactical courses can be made explicit.

Despite his preference for mathematics in teacher education, Wittmann is well aware that a solid knowledge of mathematics is not the only thing that a good teacher must possess. "*I have to admit that there was a time when I thought that the mastery of elementary mathematics was necessary and sufficient for being a good teacher. However, I have stepped back from this position completely as there are many counterexamples in both directions. There are teachers who know mathematics very well, who, however, lack the feeling for different levels and for processes. There are also teachers, whose knowledge of mathematicians is somewhat limited, but who nevertheless are curious and like to learn together with the children. What counts in the last is that a teacher is able and willing to look at mathematics as a developing organism. Then she or he is in a position to take sides with students. I know many talented primary school teachers that I would not call 'elementary mathematicians'. Nevertheless they have a good intuition for the mathematics on the level of their students and are able to use substantial learning environments in a very reasonable way."*

This raises the question of necessary prerequisites for good mathematics teachers. *"The most important trait of a good teacher is an active relationship with mathematics and the willingness to investigate mathematical problems at the level of teaching, including harder ones where the solution requires some effort. To start with good feelings, to travel confidently, to communicate with others in the solution process, and to not be worried when the solution does not come easily, that's what matters. I have found this attitude more often in primary teachers than in mathematics teachers of the upper secondary level who understand themselves as 'mathematicians', and are sometimes not able to 'reach' the level of the students. My experience in teacher education shows that structure-genetic didactical analyses are a very powerful tool to develop an active attitude towards mathematics."*

Figure 2. Interviewers with C. Wittmann

Summarising the interview we noticed that during our talk we did not only speak about a lot of E. Ch. Wittmann's different ideas and achievements in mathematics education(figure 2, but we also got to know many of the future wishes and plans, that he has for upcoming materials, teacher training congresses and the development of the discipline. That finally brings us back to the question, when he wants to start enjoying his retirement, but fortunately E. Ch. Wittmann could not think of stopping his passionate work yet. "*I often remember the ones that have fought for mathematics education to be the way it is today. How then could I leave them alone*?" But with a wink he also reveals that he nevertheless places big hope on the next generation, who will continue his work).

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