



Revisiting Early Research on Early Language and Number Names

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ABSTRACT

This article addresses the relationship between language and mathematical thinking by reconsidering early work on language and number names. The analysis examines theoretical assumptions, later empirical data, and critiques of those early studies. Researchers, practitioners, and curriculum designers in mathematics education working in multilingual settings need to develop an updated view of this early work on number names across languages, carefully considering what early research actually showed, how it has been critiqued, and how to theoretically frame claims about language and mathematical thinking. The analysis presented here suggests several ways to frame such an updated perspective, including work on linguistic relativity and ecological approaches to the relationship between language and mathematical thinking.

Keywords: cross-cultural research, language, mathematical thinking, multilingual classrooms

INTRODUCTION

Integrating language into research on mathematics learning is an important goal for both practical and theoretical reasons. Although this integration is important for all learners, it is crucial for improving mathematics learning and teaching for students who are bilingual, multilingual, or learning the language of instruction. This integration is also relevant to theory: since research in mathematics education uses language to provide one window into thinking, the role of language is central in theorizing about mathematical thinking and learning.

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State of the literature

- Early empirical studies explored how the structure of number names in different languages impacts mathematical thinking.
- These studies generated multiple claims about what phenomena differences in the structure of number names explain, from differences in strategies young learners use for base-ten problems to overall achievement in mathematics beyond early counting and operations with whole numbers.
- Recent work provides updated views of early research and includes contextual factors.

Contribution of this paper to the literature

- The paper reconsiders early work on language and number names.
- The paper provides an updated perspective of this research, using work on linguistic relativity and ecological approaches to frame the relationship between language and mathematical thinking.
- The analysis explores the theoretical assumptions for early studies, considers later empirical data, and examines critiques of early work.

Addressing the relationship between language and mathematics learning presents several challenges. The most significant challenge is that research examining language and mathematics learning must be grounded not only in current theoretical perspectives of mathematics cognition and learning, but also in current views of language. This article examines early work on the relationship between language and mathematical thinking, in particular, how differences in the structure of number names impact mathematical thinking, in particular about place value and, more generally, mathematics achievement.

This work on language and mathematical thinking has generated or influenced a constellation of claims regarding how differences in the structure of number names in different languages impact mathematical thinking. These range from the specific claim that differences in the structure of number names can predict or explain differences in the strategies very young children use for base-ten problems, to more general claims about early mathematical thinking, learning, or understanding, and, in the extreme, to explain differences in overall achievement in mathematics beyond early counting and operations with whole numbers.

This article will examine the theoretical assumptions for such claims and consider whether current empirical data supports or contradicts them. In closing, the paper suggests alternative approaches to mathematics education research and practice in multilingual or cross-cultural settings based, not only on empirical findings, but also theoretically grounded assumptions about the relationship between language and mathematical thinking.

In order to provide a theoretical grounding, the paper uses work by Lucy (1996, 1997) on linguistic relativity. The main section of the paper provides a critical review of early studies that examined the relationship between language and thought in the context of mathematics by focusing on number names (e.g., Miura, Chang, & Okamoto, 1988). This early work is often cited when summarizing what we know about the relationship between language and

mathematics learning. For example, it was cited in a review of the research literature on mathematics learning in early childhood published by the National Academies Press (Cross, Woods, & Schweingruber, 2009). It is difficult to summarize the complex relationship between language and mathematics learning in a few sentences (or even a few paragraphs). Any summary of a set of research studies with complex and sometimes contradictory results can be misunderstood, especially if interpreted through a reductionist theoretical lens.

This work and early results are not to be simply accepted as fact, but, instead, have been critiqued and considered in light of later research and competing claims, results, and theoretical perspectives. Researchers have also considered some of the nuances involved in reaching conclusions based on this research. For example, Ng and Rao (2010) provided a thorough review and critique specifically on the research relevant to Chinese number words and mathematics learning. It is crucial that researchers, practitioners, and curriculum designers in mathematics education working in multilingual settings develop an updated view of the early work on number names across languages, carefully considering what early research actually showed, how it has been critiqued, and how to theoretically frame claims about language and mathematical thinking. The analysis presented here suggests some ways to frame such an updated perspective.

Theoretical Framing

The first theoretical issue to examine is the term language. Many commentaries on the role of academic language in teaching practice focus on the structure of language and ignore the functions of language or reduce the meaning of the term language to single words (vocabulary or lexicon) and the proper use of grammar (e.g., Cavanagh, 2005). In contrast, work on the language of academic disciplines provides a more complex view of mathematical language (e.g., Pimm, 1987) as not only specialized vocabulary (new words and new meanings for familiar words) but also as extended discourse that includes syntax and organization (Crowhurst, 1994), considers the functions of the mathematics register (Halliday, 1978), and includes broader sociocultural constructs such as Discourse practices (Moschkovich, 2007).

Theoretical positions in the research literature in mathematics education range from asserting that “mathematics is a *universal* language,” to claiming that “mathematics is a language,” to describing how mathematical language is problematic for learners. Rather than joining in these arguments to consider whether mathematics is a language or reducing language to single words, I use a sociolinguistic framework to frame this essay. From this theoretical perspective, language is a socio-cultural-historical activity not something that can either be mathematical or not, universal or not. I use the phrase “the language of mathematics” not to mean a list of vocabulary words or grammar rules but the communicative competence (Hymes, 1972) necessary and sufficient for competent participation in the mathematical Discourse practices of a variety of communities. I sometimes use the term “language(s)” to remind us that there is no pure unadulterated language and that all language is hybrid.

The relationship between language and thought has been a long-standing object of study. One perspective can be summarized by the Sapir-Whorf hypothesis, the conjecture that one’s

thoughts are determined by the language one speaks. The strong version of this hypothesis would claim that all human thoughts and actions are constrained by language. The weaker (more accepted) version, sometimes referred to as linguistic relativity (Lucy, 1996, 1997), claims that language only somewhat shapes one's thinking. When reviewing and framing studies on linguistic relativity, Lucy distinguished between research projects interested in exploring the question of linguistic relativity and those that simply focus on providing accounts "for the noteworthy (often 'deficient') behavior at issue" (1996, p. 302). According to Lucy, if a study focuses on providing an account that addresses only noteworthy behavior, instead of considering more broadly the multiple and varied linguistic and cognitive phenomena of a community, that study does not explore issues of linguistic relativity. Also, if a study focuses only on deficient behavior, it should be suspect as providing evidence for linguistic relativity. This distinction seems fundamental for revisiting early work on number names.

According to Lucy, common defects of research that claims to address linguistic relativity "include" working within a single language, privileging the categories of one language or culture in comparative studies, dealing with a relatively marginal aspect of language (e.g., a small set of lexical items), and failing to provide direct evidence regarding individual cognition" (1997, p. 37). Instead, Lucy proposes that any study on the question of linguistic relativity should:

- 1) Distinguish between language and thought (examine outcome behavior that can be observed independently of language use),¹
- 2) Elaborate the mechanisms by which language influences thought, and
- 3) Explore the extent to which other contextual factors affect the influence and the operations of those mechanisms (1996, p. 306).

I use these criteria to examine the set of studies claiming evidence for the effect of number names on young children's mathematical thinking, because these studies raise more general issues related to linguistic relativity (Lucy, 1996, 1997) and they have been cited repeatedly. Below is one example taken from a volume produced by the National Research Council in the United States that summarizes research on mathematics learning in early childhood. Cross et al. (2009) cite work by Miura and others (Miura, 1987; Miura & Okamoto, 1989, 2003; Miura, Kim, Chang, & Okamoto, 1988; and Miura, Okamoto, Kim Steere, & Fayol, 1993) and conclude:

They have found that speakers of languages whose names are patterned after Chinese (including Korean and Japanese) are better able than speakers of English and other European languages to represent numbers using base-ten blocks and perform other place value tasks (p. 108).

Statements such as this can be misinterpreted, misunderstood, or used to reduce the complex relationship between language and mathematics learning.

Revisiting and Reframing Work on Language and Number Names

Early studies by Miura et al. have been used to support claims about how the structure of number names impact mathematical thinking, in particular about place value and, more generally, mathematics achievement. Miura et al. (1988) claimed to have provided evidence that the structure of number names in several languages (Korean, Chinese, and Japanese) makes it easier for young children to develop number concepts such as the base-ten structure. In these languages number names for the teens follow the structure of the base ten system: ten-one, ten-two, ten-three, and so on. In contrast, in English the names for 11-19 (and in Spanish for 11-15) do not correspond as transparently to the base-ten structure.

The results of these early studies have been critiqued, are contested, and can be interpreted in multiple ways. Later work critiqued the methods used, showed that children can be influenced by the instructions provided by interviewers, and found that differences disappear when changing from oral to written modes. A study by Towse and Saxton (1998) concluded that children's representations of numbers can be heavily influenced by experimental conditions and that the influence of language on the cognitive representation of number was less direct than had been suggested in earlier studies. Another study (Brysbart, Fias, & Noel, 1998) examined differences between French and Dutch (in Dutch the order of tens and units is reversed). Although they found differences in naming latencies for the solutions to simple addition (two-digit plus one-digit numbers), these differences disappeared when participants were asked to type instead of say the answer.

How do these studies fare when using Lucy's framing to assess the defects and requirements for studies on linguistic relativity? Most studies worked across more than one language not within a single language, a common defect pointed out by Lucy. Whether any one study privileges the categories of one language or culture over the categories of other languages can be debated, and the conclusion depends on the nuances of how these categories are framed in each study. If a study uses nonverbal data to document mathematical thinking (or even to complement the analysis of verbal data), then it would fulfill Lucy's requirement that data "distinguish between language and thought and examine outcome behavior that can be observed independently of language use" (1996, p. 306). However, these studies clearly share two common defects described by Lucy, since they deal with a relatively marginal aspect of language (e.g., a small set of lexical items). Do the studies provide direct evidence regarding individual cognition? The critique provided by Towse and Saxton (1998) seems to address this defect, since one could argue that they fail to provide direct evidence regarding individual cognition. The results do not reflect individual cognition but instead individual cognition that is limited to mathematical thinking during a particular kind of interview context.

More recent studies provide further empirical evidence for the complex nature of the relationship between language and mathematical thinking, even for this particular small set of lexical items. Competencies in counting or arithmetic most likely consist of several components (Dowker, 1998), and some of these components are not linguistic, such as cardinality (Sarnecka & Carey, 2008) or using a number line for estimation (Muldoon, Simms, Towse, Menzies, & Yue, 2011). These competencies follow complex development paths (Dowker, 1998; Kimura, Wagner, & Barner, 2013; Muldoon et al., 2011; Sarnecka &

Carey, 2008). Even when findings suggest that a counting system can have some influence on arithmetic performance, the “effects tend to be limited to rather specific areas of arithmetic” (Dowker, Bala, & Lloyd, 2008, p. 536). Lastly, and perhaps most importantly for teaching, since studies have shown that cues provided by the interviewer can change those effects (Alsawaie, 2004; Towse & Saxton, 1998), any imagined or possible deficit for learning the base-ten system related to language structure can be addressed through instruction.

Miura (1987) also linked the differences in number names to students’ mathematics scores. This study of American and Japanese children residing in the United States found evidence of a “differential cognitive organization of number resulting from differences in primary-language characteristics.” These results may support the limited claim that the names for the numbers from 10 to 20 in three languages (Chinese, Korean, and Japanese) may make the base-ten system more transparent than the number names in non-Asian languages (English, Spanish).

However, the findings in these studies cannot be used as evidence for the much broader claim that Asian children’s advantage in mathematical reasoning originates in or is caused by those linguistic differences. A jump from a limited claim about early counting to broad causal arguments about the mathematics achievement of Asian children disregards the ecological nature of mathematical reasoning and learning. Extending these studies to more general and broader explanations for differences in mathematics achievement use simplistic views of both culture and language.

These results only support the limited claim that the names for the numbers from 10 to 20 in Asian languages (Chinese, Korean, and Japanese) make the base-ten system more transparent than the number names in non-Asian languages (English, Spanish), and this claim holds only in particular circumstances. The findings in these studies cannot be used as evidence for the broader claim that Mandarin or Japanese linguistic differences cause, hinder, or facilitate the learning of mathematical notions beyond early understanding of place value or mathematics achievement in general. We cannot make broad claims about the role of language with respect to children who, in addition to using different languages, may also have had different school experiences, different mathematics instruction, as well as different experiences outside of school, such as after school training in using the abacus. There are important contextual aspects to include that impact learning about number, such as the effect of schooling, tutoring, out of school experiences with numbers, cultural differences in beliefs about the importance of mathematics, and parental instruction on numbers. These cannot be reduced to one single and over simplified factor such as the names for numbers.

Claims that mathematics achievement or success can be attributed causally to early facility with the names for numbers in different languages are suspect for many reasons. In general, claims that language differences are the single explanation for cognitive differences are suspect, based on research on cross-cultural cognition that describes the complex relationships between language, cultural practices, and thinking (i.e., Cole, 1996a, 1996b; Gay & Cole, 1967; Glick, 1975). In terms of Lucy’s list of requirements, research would need to explore other contextual factors before making such claims.

Ng and Rao (2010) conducted an extensive review of the literature and critique of the research relevant to Chinese number words and mathematics learning that provides an

updated view of the research on this topic and includes other contextual factors. The review makes several important points. First, in particular for Chinese, NG and Rao (2010) report that Miller, Kelly, and Zhou (2005) “found that the influence of language on mathematics achievement was limited to the aspects of counting that involved number naming and the base-ten concepts” (Ng & Rao, p. 188). Second, “whereas number word systems and the application of more sophisticated strategies have a significant role in accounting for early cross-national differences in mathematics achievement, the relationship between number word systems and later achievement is more complex” (Ng & Rao, p. 186).

Multiple other factors need to be considered for later achievement including school curriculum, instructional methods, and strategies used in textbooks and some factors may vary across different Asian² language communities. For Korean children in second and third grades, other factors suggested by researchers included textbook presentation and the highly competitive educational system (Fuson & Kwon, 1992a, 1992b). Murata (2004) suggested that Japanese children’s learning of addition may be facilitated by the relevance of the mathematics curriculum. Stevenson and Stigler (1992) compared preschool through fifth grade children in the United States, Japan, and Taiwan and concluded that differences in achievement could have many possible reasons including children’s learning in and out of school, teacher training, lesson organization as well as parental beliefs, expectations, and practices.

Later studies of mathematics instruction, for example in Hong Kong and Korea (Leung & Park, 2002), of Chinese teachers’ beliefs and conceptions (Correa, Perry, Sims, Miller, & Fang, 2008; Ma, 1999), and of Japanese textbook representations (Murata, 2008) have documented the multiple and varied experiences that might influence achievement in those countries. One detail is that studies of curriculum and pedagogy have been conducted in early elementary settings, but not in pre-school. One word of caution is that “no studies have focused on cross-cultural differences in mathematics curriculum and pedagogy during the pre-school years, which makes it difficult to determine whether the number system or cultural support for learning of basic number concepts is more important in explaining early mathematics achievement” (Ng & Rao, 2010, p. 195).”

Lastly, there may be historical aspects to consider. For example, since achievement differences were not documented among older Chinese and American adults, these differences may not have existed 60 years ago and may reflect long term changes in the United States (Geary, Bow-Thomas, Liu, & Siegler, 1996). Therefore, the differences cannot be attributed solely to language. Another possibility is that mathematics instruction has changed in one or more of these countries over a longer time period than research has been able to consider. Overall, separating the influences of language from other factors is difficult and perhaps impossible.

Implications for Mathematics Education Research and Practice in Multilingual Settings

Having revisited and reframed this early work, in this section I consider the implications for mathematics education research and practice, especially in multilingual settings. One important implication is that curriculum and instruction for very young children can develop ways to compensate for the irregularities of English or other languages in naming numbers. Such

pedagogical strategies can help students overcome any early difficulties they might face. Another implication is the need to use theoretical frameworks from outside mathematics education, as exemplified by Lucy's requirements for work and claims regarding linguistic relativity. Rather than suggesting any one particular approach, in this section I describe how ecological perspectives are relevant for developing broader approaches to frame the study of language and mathematical thinking.

As alternatives to reductionist frameworks, researchers in the learning sciences have proposed using ecological frameworks for studying thinking, especially with students from non-dominant communities. These frameworks shift the focus to a) examine cognition across different situations, not only in school; b) document resources, as opposed to only obstacles, and c) consider learners' *repertoires of practice* (Gutiérrez & Rogoff, 2003), to avoid ascribing essential cultural practices to any one group or to individual traits. In particular, the construct *repertoires of practice* reminds us that individuals have access to a variety of practices. This perspective assumes that learners have access to multiple practices, that individuals develop, and communities change. Therefore, we should "neither attribute static qualities to cultural communities nor assume that each individual within such communities shares in similar ways those practices that have evolved over generations" (Lee, 2003).

Work on language and mathematical thinking cannot assume that language practices, communication styles, or home cultural practices are homogeneous throughout any community. For example, Gutierrez, Baquedano-Lopez, and Alvarez (2001) describe language practices as "hybrid" and based on more than one language, dialect, or practice. We cannot assume that *any* cultural group has "cultural uniformity or a set of harmonious and homogeneous shared practices" (González, 1995, p. 237). González decries perspectives that "have relegated notions of culture to observable surface markers of folklore, assuming that all members of a particular group share a normative, bounded, and integrated view of their culture" and suggests that "approaches to culture that take into account multiple perspectives can reorient educators to consider the everyday experiences of their student" (p. 237). Researchers should keep in mind that learners from any community can and do participate productively in a variety of roles, responsibilities, communication styles, and mathematical activity that includes hybrid linguistic and thinking practices.

By necessity, researchers in mathematics education who address issues of language in multilingual settings have used work from fields outside of mathematics education to inform research on the relationship between language and mathematics thinking and learning. Work outside of mathematics education contributes theoretical frameworks for studying discourse in general, methodologies (e.g., Gee, 1996), concepts such as registers (Halliday, 1978) and Discourses (Gee, 1996), and empirical work on classroom discourse (e.g., Cazden, 1986; Mehan, 1979). This work has provided crucial concepts necessary for studying the role of language in doing and learning mathematics.

In considering what work might be relevant to research and practice on language and mathematics learning, it is important to distinguish between psycholinguistics and sociolinguistics, because these two perspectives differ in how they conceptualize language. While sociolinguistics stresses the social nature of language and its use in varying contexts, psycholinguistic studies have been limited to an individual view of performance in

experimental settings. From a sociolinguistic perspective, psycholinguistic experiments provide a different view of how people use language. As Hakuta and McLaughlin (1996) explain:

The speaker's competence is multifaceted: How a person uses language will depend on what is understood to be appropriate in a given social setting, and as such, linguistic knowledge is situated not in the individual psyche but in a group's collective linguistic norms. (p. 608)

Research in mathematics education provides theoretical frameworks for integrating language into research on mathematics learning without privileging formal mathematical discourse. As an example, Brenner (1994) provides useful distinctions among different kinds of communication in mathematics classrooms and describes three components of a "Communication Framework for Mathematics."

Communication About Mathematics entails the need for individuals to describe problem solving processes and their own thoughts about these processes. . . . Communication In Mathematics means using the language and symbols of mathematical conventions . . . Communication With Mathematics refers to the uses of mathematics which empower students by enabling them to deal with meaningful problems. (p. 241)

In general, work on language and mathematical thinking needs to avoid using deficit models of learners and their communities (Moschkovich, 2002). Many deficit models stem from assumptions about learners and their communities based on race, ethnicity, socio-economic status, and other characteristics assumed to be related in simple, and typically negative ways to thinking and learning in general. Deficit models are so pervasive and insidious that we may sometimes fail to recognize them. Focusing on the mathematical activity that occurs in a community, in homes, or outside of school is an important way to avoid using deficiency models.³ If research does not examine the mathematical activity in local communities, then it may seem that these learners do not engage in mathematical activity outside of school thus further contributing to seeing them as deficient. It is crucial to uncover the mathematical thinking learners use, both in and out of school settings. In order to uncover the mathematical meanings learners construct rather than the mistakes they make, researchers need frameworks for recognizing the mathematical meanings that learners are constructing in and with language.

A situated and sociocultural perspective (Moschkovich, 2002) is one framework for shifting the focus from looking for deficits to identifying the mathematical discourse practices evident in student contributions (e.g., Moschkovich, 1999). This framework assumes that mathematical Discourse is complex, grounded in practices, and connected to mathematical concepts. Discourses occur in the context of practices and practices are tied to communities. Discourse practices are constituted by actions, meanings for utterances, focus of attention, and goals; these actions, meanings, focus, and goals are embedded in practices. Instead of focusing on deficits, research using this perspective has documented how bilingual students communicate "about mathematics" using hybrid resources, a combination of everyday and

formal language as well as using gestures, objects, tables, graphs, and symbols (Moschkovich, 2015).

Since mathematical discourse is multimodal and multi-semiotic (O'Halloran, 1999), in order to document all the possible resources that learners use to think mathematically, research focused on language and mathematical thinking should include more than language and use a multimodal and multi-semiotic perspective of mathematical activity. Instructional practice needs to include opportunities for mathematical thinking that include multiple modes of communication, sign systems, and types of inscriptions.

FOOTNOTES

1. For examples, see Boroditsky (2000, 2001) and Boroditsky & Gaby (2010).
2. I use the term "Asian" although it is misused to refer to one language, population, or cultural category, when, in fact, there are multiple differences among "Asian" languages and cultural practices.
3. See, for examples, work on "funds of knowledge" in Latino/a working-class communities in the United States (Civil, 2002; González, Andrade, Civil, & Moll, 2001; Moll, Amanti, Neff, & Gonzalez, 1992).

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