

# Pedagogical Demands in Mathematics and Mathematical Literacy: A Case of Mathematics and Mathematical Literacy Teachers and Facilitators

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Received 21 May 2017 • Revised 14 September 2017 • Accepted 29 September 2017

## ABSTRACT

The purpose of this article is to examine teachers' and facilitators' (subject advisors) views of the approaches to teaching mathematics and mathematical literacy (ML). Using Bernstein's (1996) constructs of recognition and realisation rules, I analysed data from interviews conducted with mathematics and ML teachers and facilitators. The analysis shows that some teaching strategies are associated with mathematics and others with ML. That is, teachers and facilitators refer to teaching strategies that are domain specific (mathematics and ML). I therefore ask what it means for teaching strategies to be domain specific, particularly in the context of mathematics and ML.

**Keywords:** curriculum and assessment policy statement, mathematics, mathematical literacy, pedagogical practices

## INTRODUCTION

This article emerges from an analysis of teachers' and facilitators' (subject advisors) views on mathematics and (ML) pedagogical practices in South Africa. In 2006, ML was introduced in the Further Education and Training (FET) band in South Africa as a learning area that goes hand in hand with mathematics in the FET band (grades 10-12). Learners who are taking mathematics as a subject are not allowed to take ML as well. Therefore, mathematics and ML are viewed by the Department of Basic Education as two distinct content areas in South Africa.

On one hand, since mathematics and ML have different goals and a different student audience, one may expect that the teaching approaches may be different. On the other hand, since teachers who teach ML have not received specific (official) training to teach ML and have been trained to teach mathematics but often teach both subjects, one may expect that the teaching approaches would not be very different.

In this article, teachers' and facilitators' views on mathematics and ML pedagogical practices are explored to gain a better understanding of the possible tension around whether the teaching approach for the two domains should be different or similar. In what follows, I first explain Bernstein's (1996) constructs of classification, framing, recognition and realisation rules that were used to analyse two main learning areas at the core of this article, mathematics and ML documents, to explore the "bias and the focus" in terms of the kind of teacher expected in mathematics and ML. Using Bernstein's constructs of recognition and realisation rules, I analyse data from interviews conducted with mathematics, ML teachers and facilitators.

## THEORETICAL FRAMEWORK

The key theoretical constructs informing this study come from the field of sociology of education, as described by Bernstein. According to sociologist Bernstein (1996), there are essentially two types of boundaries (classifications) in relation to curriculum structuring: weak and strong boundaries (classification). Bernstein (1982:59) refers to classification as:

### Contribution of this paper to the literature

- Teachers' and facilitators views on mathematics and ML pedagogical practices to gain a better understanding of this possible tension around whether the teaching approach for the two domains should be different or similar.
- If mathematics and mathematical literacy are, being considered "separate" subjects. Does it mean that they are inherently different discourses, and therefore requiring different identities of teachers? Does it mean that for one teacher to work productively with M and ML, the discourses in M and ML should not be consistent (not in conflict) with the identity(ies) of the teacher? When would it be important for these discourses to be consistent?

*The nature of differentiation between contents. Where classification is strong, contents are well insulated from each other by strong boundaries. Where classification is weak, there is a reduced insulation between contents, for the boundaries between contents are weak and blurred.*

The subject knowledge, such as ML for example, is organised through integration between mathematics and everyday context. Therefore, it could be viewed as a weakly classified subject. Everyday context has been used as a tool to weaken the boundary. The difference of structures of knowledge between mathematics and ML is underpinned by classification of knowledge and pedagogical practices.

Bernstein (2000) defines pedagogical practices as forms of communication (interactions) in the process of acquisition of pedagogical knowledge. He further argues that forms of communication entails forms of control, which controls the nature of the talk and kinds of space constructed for viability of pedagogical practice. Bernstein alludes to framing as a form of control that legitimises the selection of communication, sequencing of that selection, pacing and criteria used to control communication. Framing refers to the "form of the context in which knowledge is transmitted and received and refers to the specific pedagogical relationship between the teacher and the taught" (Bernstein, 1982:59).

A strong framing is a pedagogical practice where the transmitter has explicit control over the selection of subject matter, sequencing of the lesson plan or work schedule, pacing of subject matter, and criteria for assessment and classroom interactions. In a weak framing the "learner has apparent control over the modalities of communication" (Bernstein, 2000:13). Bernstein asserts that it is possible and practical to have strong framing over the selection of subject matter and weak framing over other aspects of modalities of communication. According to Bernstein, the concepts of classification and framing yield to concepts of recognition and realisation rules.

According to Bernstein (1996: 31), recognition rules "at the level of the acquirer", are the means by "which individuals are able to recognize the speciality of the context they are in". The recognition rule, essentially, enables appropriate realisation of putting things together. The realisation rule determines "how we put meanings together and how we make them public" (Bernstein, 1996:32). The realisation rule means that the acquirer (teacher) is able to produce a legitimate text in the required discourse. "Text" refers to anything that attracts evaluation; for example, the way one talks about the mathematics and ML pedagogical practices. Possession of the realisation is reflected in the ability to produce (act, speak or write) the expected (legitimate) text, in this context, of mathematics and ML pedagogical practices. The acquisition of the recognition and realisation rules for a specific practice, for example, teaching mathematics or ML, will depend on the evaluation rules of the pedagogic discourse, that is, the criteria of what is seen to be the "legitimate text" in mathematics or ML teaching practices. Therefore, different specialised consciousness - orientations to meaning - could be acquired, depending on the selection and organisation of knowledge contents of mathematics or ML and how they are made available to teachers (what is recognised as legitimate knowledge and practice).

In her description of recognition and realisation rules, Parker (2008:79), drawing from Bernstein, indicated that:

*It is through the evaluative rules (rules of recognition and realisation) that specific pedagogic knowledge and practices are constituted as legitimate in practice and orientations to meaning are acquired.*

Parker indicted that recognition rules are criteria (special relationship) for making distinctions, for distinguishing the speciality of a thing / a practice / a specialisation / a context; what makes it what it is. They are principles for recognising the "legitimate text", the voice to be acquired, and are determined by the classification principle at work (relation between different knowledge discourses and practices).

In this study, the official knowledge of both subjects (mathematics and ML), is packaged differently in terms of organisation of knowledge, which might demand different recognition and realisation rules of their pedagogical practices. Mathematics appears to be strongly classified (boundary between mathematics and everyday context is

strong) and representing the collection curriculum code. On the other hand, ML appears to be weakly classified (boundary between mathematics and everyday context is weak) and representing the integrated curriculum (Bernstein, 1996). By the fact that weaker classification changes the recognition and realisation rules “by means of which individuals are able to recognize the specialty of the context that they are in” (Bernstein, 1996:31), we might have different pedagogical practices in mathematics and ML.

Harley and Parker (1999) maintain that by making the context clear, strong classification orients individuals to what is expected and appropriate. If weak classification can cause ambiguity and confusion by making the recognition rules elusive, weakening of the classification could create a set of recognition rules unfamiliar to ML teachers. This suggests that for ML teachers to acquire the new recognition rules to teach ML, they need to have a strong balance of knowledge between mathematics content real-life context (Graven & Venkat, 2007; DoE, 2003a). Parker (2005) notes that the changes in the mathematics curriculum represent major shifts for prospective and practicing mathematics teachers whose mathematical identities were constructed under an ‘old’ curriculum education system. Teachers are required to implement these new ideals in their classroom practice. This means that they are required to develop new images of ‘good practice’ for mathematics teaching (recognition rules) and new pedagogic identities (forms of consciousness) that enable them to carry out these practices (realisation rules). In this study, most teachers who are teaching ML are not officially trained to teach this subject, but they are mathematics teachers. Thus, it would be interesting to see what recognition and realisation rules they reflect on.

Hence, this study explores kinds of mathematics and ML teachers’ recognition and realisation rules acquired and reflected in the teaching and learning of mathematics and ML. In the next section I draw from the official documents on mathematics and ML. These are two school curriculum documents that protrude symbolic images of what the state considers valuable knowledge and pedagogic practices, that is, what officially counts as legitimate mathematical and ML knowledge and teaching approaches that will enhance the principles of these curricula. In doing this, I will use some of the tools from Bernstein’s theory to illustrate the teaching practices implied in those documents. This will provide the background for the study I present in this article.

## MATHEMATICS AND MATHEMATICAL LITERACY

### Mathematics

Mathematics has been defined in the Curriculum and Assessment Policy Statement (CAPS) of the FET phase in the following terms:

*Mathematics is a language that makes use of symbols and notations for describing numerical, geometrical and graphical relationships. It is a human activity that involves observing, representing and investigating patterns and qualitative relationships in physical and social phenomena and between mathematical objects themselves. It helps to develop mental processes that enhance logical and critical thinking, accuracy and problem solving that will contribute in decision-making. Mathematical problem solving enables us to understand the world (physical, social, and economic) around us, and most of all, to teach us to think creatively (DBE, 2011, p. 8).*

The definition of mathematics seems to suggest that mathematics as a discourse has specialised language, images and symbolism different from that of everyday language. The practical implication of this definition is that teachers should use the correct mathematical language, including symbols and notations when teaching mathematics, which is key in enhancing learners’ conceptual understanding and procedural fluency. It further implies that mathematics should be taught in context, bringing real-life situations into the classroom to enhance problem-solving skills. However, it appears that in mathematics, everyday context is used as a vehicle to access mathematical knowledge. The fact that mathematics “involves observing, representing [and] investigating patterns [and] requires active participation of learners which enhance mental process required for logical and critical thinking and accuracy” (DBE, 2011:8) implies using teaching approaches that are mainly learner-centred (weak framing in Bernstein’s terms) and will enhance critical thinking. This means that teachers should be able to monitor and extend learners’ thinking and identify learners’ misconceptions (Simpson & Haltiwanger, 2016).

Moreover, one of the specific aims of mathematics, as captured in the curriculum, is to foster the use of real-life problems in the teaching and learning of mathematics in the classroom. It has been indicated that “real life problems should be incorporated into all sections whenever appropriate (weak classification in Bernstein’s term). Examples used should be realistic and not contrived. Contextual problems should include issues relating to health, social, economic, cultural, scientific, political and environmental issues whenever possible” (DBE, 2011:8). This suggests that in the teaching of mathematics, teachers should ensure that they draw from learners’ everyday experiences whenever appropriate.

In spite of this call, international and national literature seems to suggest that the teaching of mathematics still focuses on the application of rules, formulae and procedures without conceptual understanding (Hierbert, Carpenter, Fennema, Fosun, Human Murray, Olivier & Wearne 1996; Van de Walle, Karp, & Bay-Williams, 2015; Machaba, 2016a; Machaba & Makgakga, 2016). While mathematics teaching focuses on procedures and the application of rules, a contrast seems to be the opposite in ML. It appears that the relationship between mathematics, everyday context and pedagogical practices in mathematics seems to be different than that of ML. The pedagogical practices of ML are discussed next.

### **Mathematical Literacy**

In 2006, ML was introduced into the FET curriculum. Its introduction made a mathematically orientated subject, either mathematics or ML, compulsory for all FET learners. The Curriculum and Assessment Policy Statement (DBE, 2011:10) defines ML as:

*a subject that develops competencies that allow learners to make sense of, participate in and contribute to the twenty-first century world – a world characterized by numbers of different ways. Such competencies include the ability to reason, make decisions, solve problems, manage resources, interpret information, schedule events and use and apply technology.*

As noted in the definition, ML addresses the need to develop life-oriented competencies for a range of everyday situations which aim at developing “ways of acting in the world” (Steen 2001:6) to develop both a sense of the need to engage with real-world (weak classification in Bernstein’s term) issues and the mathematical tools with which to understand, analyse and critique these issues. Therefore, it is clear that ML aims to develop capabilities and willingness to use mathematical thinking to make sense of a wide range of life-related situations.

Internationally, other researchers have used concepts similar to ML. For example, in the United States of America (2001), Steen refers to quantitative literacy, which is aimed at developing students with a flexible range of quantitative skills to be applied in a diverse range of contexts. Steen differentiates between quantitative literacy and traditional mathematics as typical school mathematics problems, which involve simplified numbers and straightforward procedures but require sophisticated abstract concepts, while quantitative literacy involves mathematics that learners would use in their daily life experiences. In England, functional mathematics (QCA, 2005:2), which is similar to ML, is defined as:

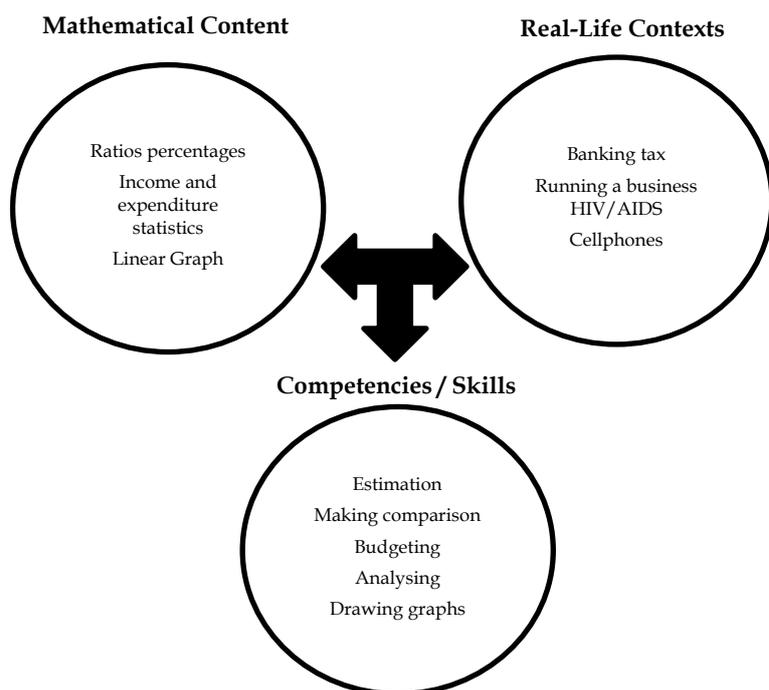
*Each individual has sufficient understanding of a range of mathematical concepts and is able to know how and when to use them. For example, they will have the confidence and capability to use maths to solve problems embedded in increasingly complex settings and to use a range of tools, including ICT as appropriate. In life and work, each individual will develop the analytical and reasoning skills to draw conclusions, justify how they are reached and identify errors or inconsistencies. They will also be able to validate and interpret results, to judge the limits of their validity and use them effectively and efficiently.*

Unlike in mathematics, these definitions of ML, quantitative literacy and functional mathematics seem to suggest the use of mathematical concepts to understand the context, and the use of the context to understand the mathematical concepts while developing skills such as reasoning, drawing conclusions and justifying as depicted in **Figure 1**. Furthermore, the relationship between everyday context and mathematics in ML has been foregrounded in five key elements of ML, which are: use of elementary mathematical content, use of authentic real-life contexts, solving of familiar and unfamiliar problems, use of decision-making and communication, use of integrated content and/or skills in solving problems. (DBE, 2011:9). The five key elements of ML are discussed in detail below:

### **Key Five Elements of Mathematical Literacy**

#### ***Mathematical literacy involves the use of elementary mathematical content***

ML is restricted to basics concepts and skills that are pertinent to making sense of mathematics and real-life context and scenarios (DBE, 2011). This implies that the teaching and learning of ML should not focus on abstract mathematical concepts. This further implies that classroom mathematical content should not be taught in the absence of context. In Bernstein’s terms, this implies a weak classification of ML.



**Figure 1.** Interplay between content, context and problem-solving skills in ML (DBE, 2011:9)

### *Mathematical literacy involves authentic real-life contexts*

This strand implies that in the teaching of ML, teachers should ensure that they draw from authentic situations (genuine and realistic context such as the usage of cellphone packages). It has been stated that “in exploring and solving real-world problems, it is essential that the contexts learners are exposed to in this subject are authentic (i.e. are drawn from genuine and realistic situations) and relevant, and relate to daily life, the workplace and the wider social, political and global environments” (DBE, 2011:8). This implies that teachers should draw on authentic real-life scenarios in the teaching of ML. In their study, Le Roux and Adler (2016) note that it is not easy for students to work with contextualised practical mathematical problems. It is not only a one-way movement from everyday context to mathematical knowledge with emphasis on vertical growth in mathematical ideas. It requires the to-and-fro movement between mathematics and everyday representations. The analysis thereof suggests that a contextualised mathematical problem can offer an innovative alternative to traditional representation of school mathematics. In the context of ML, it implies that the hybrid nature of ML text could offer opportunities for teachers and learners to work flexibly within and across text than what is valued in school mathematics.

### *Mathematical literacy involves solving familiar and unfamiliar problems*

Another feature that distinguishes ML is the fact that the everyday context from which teachers would be drawing, would not be the context they are familiar with every time. Sometimes teachers would draw from a context that is unfamiliar to learners with an aim of familiarising them with such a context. In the teaching of ML, teachers should ensure that mathematical content and real-life context dialectically work together. In other words, mathematical content provides learners with a means through which to explore context and the context add meaning to the mathematical content (DBE, 2011). According to Graven and Venkat (2007), in the learning of ML, the context should be explored to deepen mathematics understanding, to learn mathematics and to deepen understanding of that context. This involves selecting real contexts (possibly edited or adapted) that enable the dialectical relationship of content and context. Teaching needs discussion about contexts but this must be balanced with revisiting maths and the learning of new maths in new ways. In spite of all that ML could do, it remains a marginalised content area because it is not seen as promoting access to advanced mathematics and lucrative and attractive career courses such as engineering and accounting (DBE, 2011). Even learners who are taking ML remain marginalised.

According to Bernstein (2000), there would be weak classification and framing, which results in recognition and realisation rules being less vague. Authenticity of context and progression of maths must be balanced. Authenticity

**Table 1.** Comparing mathematics and ML

Topics	Mathematics	Mathematical Literacy
Pedagogical Practices	<p>Prescribed algorithms and a focus on symbolic manipulation deprived of meaning</p> <p>Teaching needs discussion about contexts, but this must be balanced with revisiting maths and the learning new maths in new ways</p> <p>Whenever appropriate, teachers should ensure that they draw from learners' everyday experiences</p> <p>Everyday context is used as a vehicle to access the learning of mathematics</p> <p>Teaching approaches that are mainly teacher centred</p> <p>Teaching should involve observing, representing and investigating patterns and require active participation of learners, which enhance mental process required for logical and critical thinking and accuracy</p> <p>Teaching that monitor and extend learners' thinking, and identify learners' misconceptions</p> <p>Prescribed algorithms and a focus on symbolic manipulation deprived of meaning</p>	<p>Teachers integrate content and/or skills in solving problems</p> <p>In the teaching of ML, teachers should provide learners with opportunities to develop and practice decision-making and communication skills</p> <p>Exploring of a context to deepen the understanding of mathematics and to learn math and to deepen understanding of that context</p> <p>ML should not be taught in the absence of every day context</p> <p>Teachers should ensure that they draw from authentic (genuine and realistic) contexts such as using cellphones packages</p> <p>Teachers draw from contexts that are familiar and unfamiliar to learners</p> <p>Teachers integrate content and/or skills in solving problems</p> <p>In the teaching of ML, teachers should provide learners with opportunities to develop and practice decision-making and communication skills</p> <p>Exploring of a context so as to deepen the understanding of mathematics and to learn math and deepen understanding of that context</p> <p>ML should not be taught in the absence of everyday context</p> <p>Teachers should ensure that they draw from authentic (genuine and realistic context such as the usage of cellphones packages</p> <p>Teachers draw from the context which is familiar and unfamiliar to learners</p> <p>Teachers should use elementary mathematical content</p>

of context and progression of maths can be experienced as a problem. The interplay between content, context and solving problems has been explained in [Figure 1](#).

### *Mathematical literacy involves decision-making and communication*

This strand states that “In the teaching of Mathematical Literacy, teachers should provide learners with opportunities to develop and practice decision-making and communication skills” (DBE, 2011:9). For example, learners may be provided with a cellphone advertisement that appeared in the newspaper. They should then use mathematical tools to explore the options, interpret the findings and make a decision about which contract is the most cost-effective option in certain circumstances and communicate the decision with an awareness of non-mathematical considerations. In ML, the mathematical calculations are meaningless without decision-making and appropriate communication.

### *Mathematical literacy involves the use of integrated content and/or skills in solving problems*

In the teaching of ML, teachers should ensure that they integrate content and skills, which are organised and categorised according to topics in solving problems. Teachers have to ensure that in teaching ML, content and skills have to be integrated into any real-life problems, which are not necessarily structured according to topics. Drawing from the literature and the official documents on mathematics and ML, I provide a summary of the pedagogical practices for mathematics and ML (see [Table 1](#)).

## **Comparing Mathematics and ML**

[Table 1](#) seems to suggest that in the teaching of mathematics, and procedures, teachers should ensure that concepts in mathematics should be developed first, although the focus is on symbolic manipulations which yield to algorithms. Furthermore, the teaching of mathematics, if appropriate, should be connected to learners' everyday context. However, the usage of the everyday context in the teaching of mathematics should be used as a vehicle to access mathematical knowledge. On the other hand, in ML the usage of context is to deepen the understanding of the context itself and mathematical knowledge.

## Research Design and Methodology

The bigger research study on which this article is based, involved four secondary schools from contrasting backgrounds in Gauteng, South Africa. The research took on a qualitative approach, adopting a multiple case study approach. The four schools were located in socially divergent sites – two former model C schools that served middle- to upper-class well-resourced schools, where parents typically were high-income professionals. The other two schools were public township schools serving children from predominantly poor backgrounds, less affluent schools offering both mathematics and ML, where parents were typically low-income earners. However, for the purpose of this article, I will report on the data of only four mathematics and ML teachers from two schools (two mathematics teachers and two ML teachers) and two mathematics and ML facilitators (subject advisors), one mathematics facilitator and one ML facilitator from one district in Gauteng.

Therefore, the research design for this study is best characterised as an explorative, multiple case study design (Leedy, 2001) in a sense that it looks at mathematics and ML teachers' and facilitators' views of mathematics and ML pedagogical practices. McMillan and Schumacher (2001) and Opie (2004) argue that qualitative research uses a case study design in which the data analysis focuses on one phenomenon, which the researcher selects to understand in depth, regardless of the number of sites or the participants of the study. The selection of participants in this qualitative case study followed a purposive sampling design (Newman, 1994). Therefore, the participants in this study were chosen purposefully to illuminate the key theoretical constructs concerning their views of the pedagogical practices of these two discourses.

According to McMillan and Schumacher (2001), purposive sampling, in contrast to probabilistic sampling, is selecting rich information to study in detail when one wants to understand something about cases without desiring to generalise all such cases. Purposive sampling is done to increase the utility of the information obtained from small samples. Thus, two mathematics teachers from four schools (MTS1 and MTS2), two ML teachers (MLTS1 and MLTS2) and two facilitators from mathematics (MF) and from ML (MLF) were purposefully chosen for an interview. These participants were chosen because they are specialists in mathematics and ML. An in-depth interview of an hour was conducted with each of them.

Data was collected through interviews, using a tape recorder. Many qualitative researchers such as Cohen, Manion and Morrison (2007) emphasise that data analysis needs to be an ongoing process. In other words, data analysis starts with the first data that the researcher collects and continues throughout the data collection period and during the process of writing up the research itself. During the data collection process in this study, I kept my research journal in which I recorded my insights, hunches, speculations and reflections on a range of events during the data collection period.

After data collection, I had to transcribe all six hours of tape recording of interviews with mathematics and ML teachers and facilitators, nearly 10 pages of transcripts were analysed. As advised by Cohen et al. (2007), I ensured that I transcribed phrases and sentences as they were to keep the flavour of the original data, not because they were often more illuminative and direct than my own words. It was important to be faithful to the exact words used. Bowe et al. (1992) in Cohen et al. (2007) reported that they used much verbatim data in their research, as it is done in this research, not because those whom they interviewed were powerful people, but because they felt that justice had to be done to the exact words used by the respondents.

After ensuring that the data was sorted and typed, I read and reread it to familiarise myself with it. Although the recorded interviews were transcribed, I listened to them several times, writing down any impression I had in my reflective journal as I went through the data. In the process of coding, I read carefully through my transcribed data, line by line, dividing it into meaningful analytical units. When I located meaningful segments I coded them. Maree (2007:105) notes "Coding is marking the segments of the data with symbols, descriptive words or unique identifying names". I made a three-column typology, with the transcript in the left, the coding in the centre and comments in line with some theoretical orientations on the left.

Grounded theory (inductive theory) and deductive approaches were used in this data analysis. Although I was open to find the emerging codes in the process, those codes could be related to existing theoretical orientations. The recognition and realisation rules were used to analyse teachers' views on how they teach mathematics and ML.

In the next section, findings in terms of teachers' and facilitators views' about mathematics and ML pedagogical practices are discussed.

## FINDINGS AND DISCUSSIONS

### Pedagogical Practices and Mathematical Knowledge in Mathematics and ML

The issue discussed in this section concerns pedagogical practices related to the teaching of mathematics and ML. It was important to note that the pedagogy relates to mathematical rules, for example those linked to the

teaching of fractions. ML is also associated with reasoning and problem-solving strategies while mathematics is seen as a discipline that deals primarily with the application of rules. This difference suggests an issue needing further discussion, such as that strategy for teaching may be domain specific. These issues are discussed below

### Mathematics for Direct Teaching

When asked about how to teach mathematics, MLTS2 said:

*... to be a good mathematics teacher, one needs to be able to explain maths concepts, dedicated and able to control learners' work and give feedback, able to transfer your understanding of maths concepts to learners. Let learners do lots of maths practice.*

This extract implies that mathematics is taught by giving learners a “lot of practice”. Teachers have to transfer their understanding of mathematics concepts to learners. The fact that mathematics teachers are expected to “transfer their understanding of math concepts to learners” suggests that mathematics requires direct teaching (Selling, 2016) as opposed to a learner-centred approach (Brodie, 2008) as implied in the mathematics curriculum. This is in contrast with reform instructional practices which encourage learners to construct mathematical knowledge by themselves through investigations and discovery of knowledge that enhance conceptual understanding, as opposed to encouraging learners to do “a lot of practice”, which enhances procedural understanding (Hiebert et al., 1996). The transferring of knowledge (Boaler & Staples, 2008) and doing much math practice are not encouraged by most researchers (Boaler & Sengupta-Irving, 2016) in mathematics education and the South African curriculum (DBE, 2011). It appears that, according to Bernstein, both classification and framing are expressed as being strong. It appears that teachers have more control than learners over the way in which knowledge is selected, sequenced and evaluated in the classroom.

Similarly, when asked to describe and explain what makes a teacher a good mathematics or ML teacher, MF and MLF said the following:

*In mathematics as an educator, give a lot of activities for learners to practice, supervising them, giving them extra work, extra lessons on Saturday and tell learners that mathematics is simple (MF).*

MLF said:

*... is the teacher who let learners to practice a lot, ... mathematics does not need one to talk but to do a lot of practice; you talk a few minutes by providing examples and giving learners work to solve. You give them a similar problem of the same concept, but not different problems at a time.*

MLTS2, MF and MLF viewed mathematics as a subject that needs much practice, but not to be talked about much. This means that, according to MLF and MF and MTS2, mathematics is associated with a “lot of practice”, transfer of knowledge and less talk in class.

The fact that mathematics teachers are expected to “transfer their understanding of math concepts to learners” suggests that mathematics requires direct teaching (Selling, 2016) as opposed to a learner-centred approach (Brodie, 2008) as implied in the South African mathematics curriculum. This is in contrast with reform instructional practices, which encourage learners to construct mathematical knowledge by themselves through investigations and discovery of knowledge which enhance conceptual understanding, as opposed to encouraging learners to do a much practice which enhances procedural understanding (Hiebert et al., 1996). This suggests that mathematics teachers and facilitators have not acquired and developed new images of ‘good practice’ for mathematics teaching (recognition rules) and new pedagogic identities (forms of consciousness) that enable them to carry out these practices (realisation rules). Teachers and facilitators are still reflecting “old” recognition and realisation rules for teaching mathematics.

### Mathematical Literacy for Problem Solving and Mathematics for Procedures

On the other hand, MLTS1 felt that mathematics has to be taught in rules, procedurally using steps, while ML is taught by means of a problem-solving approach. MLTS1 said:

*..... but if a mathematics teacher steps into a ML class starts to teach ML learners like he/she is teaching mathematics learners “step 1, step 2”, is not going to work. In mathematics problems are given, steps followed application of rules – never get into the problem solving. In ML I try to give the real-life problem. I try not to give facts, first I say this is the problem, try to figure it out. If they can't I refer them to the textbooks, if they can't still give them some nuggets on how to get to the problem*

*and what the problem is all about. So, in ML I try to let them try on their own if they cannot still get the answer I get into my projector, and show them how they can do it.*

MLF added:

*A lot of teachers take mathematics, change it to basic and give it to learners when they teach. The exam is also presented in a problematic way, it is step 1, step 2 which a lot of teachers do. So learners are not in a problem solving approach, but they are taught steps, procedures rules, and how to get to the answer.*

MF2 identifies ML with a problem-solving approach and mathematics with a procedural approach. He considers it problematic to use a procedural approach to ML.

The remark confirms that MLTS1 and MLF viewed mathematics and ML as subjects that have to be taught differently. He says, "in mathematics, problems are given, and steps [are] followed". Mathematics is taught / presented procedurally, step by step, following rules, with the main objective being to arrive at the answer, whereas ML is taught using a problem-solving approach and real-life problems are given. A "nugget" approach seems to characterise ML. To help learners get into the problem, they are "give(n) some nuggets on how to get to the problem and what the problem is all about". Problem solving is an approach or a mathematical teaching practice that has been encouraged in the teaching of mathematics in mathematics education worldwide (Le Roux & Adler, 2015; 2016; Selling 2016; Jacobs & Emson, 2015). Van de Walle et al. (2016) maintain that when students solve mathematical problems, they are not applying mathematics, but they are solving problems to learn new mathematics. This suggests that solving problems is not only a goal of learning mathematics, but it is also a major means of solving problems. Problem solving is an "integral part of all mathematics learning and so it should not be an isolated part of the mathematics program" (NCTM, 2000:52). Surprisingly, these teachers regard problem solving as a ML practice but not a mathematics practice.

The use of rules and procedures (step by step as indicated by MLTS1 without understanding in mathematics) has been discouraged as it enhances instrumental understanding. According to Van de Walle (2007), procedural knowledge of mathematics is knowledge of rules and procedures that one uses when carrying out routine mathematical tasks and includes the symbolism that is used to represent mathematics. This is the knowledge produced by a lack of connections of mathematical ideas. If mathematical ideas are seen as isolated from each other, the knowledge produced is referred as "procedural". According to Skemp (1976), this knowledge produces instrumental understanding. Boaler (1997) refers to this kind of knowledge as "inert" knowledge; that is, knowledge that cannot be used to a new situation.

It appears that time is also a key aspect that distinguishes how the two subjects are approached. MTS2 said:

*When I teach ML, I facilitate learning more than when I teach mathematics. I give the problem to learners, let them work on their own, go around and check what and how they are doing the problem. When I start teaching mathematics, I had these wonderful ideas that I am going to change it, but when I get into the class it is a different situation all together. For example, since the schools reopen for the past two weeks, we have been working everyday but still we are one week behind. So, in mathematics the syllabus is so huge I cannot facilitate learning, whereas in ML we have a lot of time, so I can facilitate learning. And so, in mathematics you are basically forced to give learners rules, content and let them work at home so that you cover the syllabus.*

MTS2 wants to facilitate learning even in a mathematics class. However, she cannot do this because of time constraints. On the other hand, MTS2 says that "in ML we have a lot of time so I can facilitate learning". It might be suggested that while time allows the teachers to facilitate learning in ML, the lack of time in mathematics forces a situation in which learners facilitate their own learning in mathematics. One might even suggest that time frustration is so irritating that the teacher is forced to facilitate homework instead!

When asked to elaborate why she is not facilitating learning in her mathematics class, MTS2 said:

*In mathematics, we have only 35 minutes to teach the sin rule and cos rule. If I give them these to do on their own, they would take more time. Yes, it would be more meaningful in the end if they had time to do it. The problem is I have only 35 minutes to get them to understand the proof, to do examples, and the next day I have to do the cos and sin, the next day area rule so there is no day wherein I can sit back and say 'ok let try, this one our own'. In addition, if I take longer time to explain the concept, we have also a standard test which we have to write, a particular amount of work to be covered in a certain period, so even if I relate it to other concepts as you are saying it won't work. Unless if the department could give us a year to do our own planning, to develop your own learning programme, work schedule.*

*So, we have a work schedule from the department which they always come and say today is the 15th, are you on track and this is what makes things difficult.*

The issue of time in ML is also consistent with what Graven and Venkat (2007) found in their classroom observations with ML teachers. They observed that the common use of real situations seemed to open opportunities for communication. Learners commented that group work and discussion were much more common in ML than in mathematics. It was reported that tasks were often covered over a week or two. They noted that the structure and the pace of work in ML were perceived as being more responsive to the understandings of learners (Graven & Venkat, 2007:75).

Therefore, it is clear that the issue of time is the key pedagogical practice to be employed in a mathematics and ML class. However, according to MLTS2, the fact that a more learner-centred, problem-solving approach is practiced in an ML class does not necessarily mean this approach cannot be applied in a mathematics class. There are external factors such as time and workload coverage which inhibit this approach from being effectively practiced in a mathematics class (strong framing). Hence, MLTS2 is forced to give learners rules and to teach in a procedural way so that she would be able to cover the syllabus; however, in an ML class where she has more time and the curriculum is not so loaded, it allows her to facilitate learning more (weak framing).

It is evident from the above remarks that transfer of knowledge, much practice and procedures and pedagogical practices are associated with mathematics, while problem-solving, making sense of the problem, the use of real-life context and learner-centred practices are associated with ML. The big question to ask is: "Are pedagogical practices supposed to be different in mathematics and ML?" Furthermore, the concept of time in Bernstein's terms is explicit and punctuated in different activities in M than in ML. Again, data seems to suggest that by virtue of learner centredness, problem-solving practices in ML, knowledge transfers and much practice in mathematics (sequencing and pacing of activities and teacher learner interaction (learner centredness)) is explicit compared to in mathematics class. According to the literature as well as the mathematics and ML curriculum, pedagogical practices which teachers have associated with ML should also be in mathematics. Therefore, it appears that, since both mathematics and ML teachers teach mathematics and ML, they are unable to recognise the speciality of the context of mathematics pedagogical practice. Although the study could not observe their practice in practice, the researcher presumed that since they were unable to recognise the mathematics pedagogical practices, they might fail to put mathematical practice as captured in the literature and the curriculum into practice. In Bernstein's terms, it appears that in ML, the fact that learners could discuss and interact with each other in groups, the framing between teachers and learners was weak. However, learners do not have control over the sequencing, pacing, selection and evaluation of the knowledge transmitted.

### **Mathematics for Rules and Mathematical Literacy for Reasoning**

When asked how he could teach task 1, MTS1 said:

*I think the best way of teaching this kind of problem, is the way in which ML learners will approach it; by comparing the RHS and LHS. I think the interpretation of the problem is very important. I think algebraically is another way of solving this problem, but I think learners must understand what the problem means before they use algebra.*

It is clear from the above remarks that MTS1 acknowledges the importance of emphasising the concept of "equal sign" when working with equation problems. He thinks this strategy of emphasising the RHS=LHS should also be taught in a mathematics class, not only in an ML class. This is because it encourages both mathematics and ML learners to reason. According to MTS1, the differences and similarities between mathematics and ML are not only about the idea of the relationship between content and context, but it is also about the idea of reasoning. MTS1 said "I think it also involves thinking and reasoning".

In relation to the practice in mathematics classes, MTS2 said:

*In maths, learners do not make sense of their answers like in ML. For them, if they solved a problem and got the answer, they have achieved their objective. Their aim in this context is to get the answer; making sense of that answer is not what they do.*

It is clear that as far as MTS4 is concerned, mathematics learners do not need to bother with checking if their solution strategies make sense. If it is the case that the main objective of mathematics is to get the answer, while ML learners make sense of their solution strategies, it is suggested that even when teachers are teaching these subjects, they teach them differently in this way.

When asked to comment on strategies learners would use to solve task 4, MTS1 said:

*From my experience, as I taught ML before, learners should do this practically; they should cut and paste to understand the problem and the concept of volume. I think in this problem the formula to be used is  $V = \pi r^2 h$ . A learner needs to understand what this formula comes from before they use it. They should be able to view the height of a cylinder, radius. They should know where it starts, how to come?*

From the above extract, it appears that MTS1 suggests that learners, in particular those of ML have to be hands-on when solving this kind of problem. To solve this problem, ML learners need to understand the concepts embedded in it. They need to understand the problem and the concept of volume, and where it comes from, before using it.

When asked about mathematics learners, he said:

*In mathematics, when we teach this we don't teach it practically, we often teach them procedurally, get the values of variables and substitute them in a formula without emphasising the concept as I see now from this interview that teaching concepts is very important (MTS1, August 2011).*

When asked to comment on task 4, MLTS2 said the following:

*For mathematics learners, I would give them the formula to apply, as they are used to. And the ML learners would do with them practically. For example, I would take a cylinder cut it to have two circles and a rectangle and show them how the formula of volume of a cylinder is derived. I would show them that  $A = L \times B$  can give you the volume of everything. ML is trial and error, so if you let them study the formula and do it practically with them they, would remember it and it would make sense to them.*

The above remark shows that MTS1 suggests that mathematics learners would approach task 4 procedurally because they are often taught procedurally, "to get the values of variables and substitute without the concept". In other words, for task 4, according to MTS1, ML learners would approach the task with understanding, while mathematics learners would be more procedural because of how they have been taught.

When MF was asked to comment on task 1, we end up focusing on the discussion of addition of fraction since she was talking about LCD. I gave her an example, to add  $\frac{1}{2} + \frac{3}{4}$  and she said the following:

*As long as learners learn about the fraction, they know that there is always a numerator and denominator, so a good mathematics teacher will actually say 'Let's look at the LCD, you multiply each fraction by the LCD.'*

Therefore, according to MF, a mathematics teacher would use an LCD maths rule strategy to add  $\frac{1}{2} + \frac{3}{4}$ . When asked what other strategy could be used to add the above fractions, she said: "No, I don't have one, there is no any other strategy". I asked her to read  $\frac{1}{2} + \frac{3}{4}$ . And she read, "one half + three quarters". So, I said, "What is one half bread plus three quarters of bread". (Through asking this question, I was drawing from her household knowledge). She said, "One loaf of bread and a quarter loaf of bread, which is  $1\frac{1}{4}$ ". I asked whether it would need an LCD. It appears that MF started to make sense of the problem, when I drew from everyday knowledge. The other strategy used with her was a decomposition strategy, that is, to break  $\frac{3}{4}$  into one-halves or one-fourth. Although MF is subject advisor working with teachers, training teachers in her district could not realise that  $\frac{1}{2} + \frac{3}{4}$  could be solved conceptually without applying procedures (Herbert, et al., 1996). In fact, after spending much time trying to come up with alternative strategies, MF could not believe that those strategies could be used in mathematics class, although they were ML strategies. She said:

*ML deal[s] with the context, like, for example, the one we were dealing with now of  $\frac{1}{2} + \frac{3}{4}$ , to say how many quarters there are in  $\frac{3}{4}$  and  $\frac{1}{2}$ . I think this is an ML method, because it is the context, this is how the ML people teach.*

MF considers a way of reasoning which regards  $\frac{1}{2} + \frac{3}{4}$  as asking "How many quarters are there in  $\frac{3}{4}$  and  $\frac{1}{2}$ ", which is an approach for ML, but not for mathematics. It appears that MF characterises mathematics approaches as consisting of rules that must be followed. At some stage she said, "In mathematics we have rules and those rules I am telling you they must be followed to the letter. Mathematics is about rules".

MF sees a clear demarcation in teaching approaches between mathematics and ML. Mathematics is about rules and ML is about reasoning, making sense of the problem according to MF. The idea that mathematics and ML have

approaches that are inherently different, raises important questions about what actually happens in mathematics and ML classrooms, and in training workshops for mathematics and ML. In addition, there is a need to explore the role of rules in both mathematics and ML, and how these incorporate aspects of reasoning and conceptual understanding of mathematics.

MF associated ML more with reasoning and mathematics with rules. When distinguishing between mathematics and ML, she said “ML deals with reasoning a lot and M with the application of rules”. This seems to suggest that teaching and learning strategies are domain specific. For example, in relation to fractions, MF1 viewed the use of the LCD as a strategy suitable for mathematics and the splitting (decompressing) strategy suitable for ML. Although ML strategy appears to be making more sense because they emphasise understanding of the concepts and making sense of the problem through drawing from everyday context, but its strategies are still devalued when compared to mathematics algorithm, procedural strategies that do not make sense (Civil, 2002; Civil, 2016). In Bernstein’s terms, it appears that mathematics teachers’ recognition and realisation rules (orientations to meanings based on how they have been socialised and inducted into the mathematics and ML discourses) seem to be reflecting the old apartheid curriculum of recognising and teaching mathematics as rules and procedures without understanding. It appears that teachers and facilitators have not yet institutionalised the bias and focus of the required mathematical official knowledge. That is, the “legitimate” text – what is accepted as good mathematics teaching practice.

The issue of associating strategies to a specific domain of knowledge in this study raises a number of questions: “Which strategies can be said to be exclusively for mathematics, which ones can be said to be for ML and which ones for both?”, “What does it mean for a strategy to be for mathematics and another for ML?”, “What are teachers’ perceptions of who knows and who does not know mathematics?” “Do they believe in a preferred way to do mathematics?” “Who determines which ways are the preferred ones?”, “As subject advisors, what should we be doing to prepare teachers to not only encourage learners to do home mathematics, but to do so in an authentic way, one that really values the cultural nature of mathematical knowledge” (Civil, 2016). These questions need to be discussed, because, according to these teachers and facilitators, there are strategies that appear to be linked to ways of working that are for mathematics and others for ML.

## CONCLUSION AND RECOMMENDATION

In this article, I explored teachers’ and facilitators’ views on mathematics and ML pedagogical practices to gain a better understanding of the possible tension around whether the teaching approach for the two domains should be different or similar. Drawing from Bernstein’s (1996) constructs of classification, framing, recognition and realisation rule, I analysed two main learning areas which were at the core of this article, mathematics and ML documents. I provided an analysis of teachers’ and facilitator’s views using the construct of recognition and realisation rules.

The data discussed reveals that learners’ and teachers’ strategies are domain specific. In other words, there are teaching strategies that are associated with mathematics and others that are associated specifically with ML. MLTS4 argued that because of time constraints in mathematics class, she is “forced to give learners rules and content and let them work at home so that you cover the syllabus”. This suggests that in a mathematics class, she gives learners rules and procedures. MTS1 concurred by saying: “That is how I have been taught to teach in this way, of allowing learners to follow the procedures, even if there is a quicker way of finding the answer”. The mathematics facilitator, MF1, supported this by saying “ML deals with reasoning a lot and mathematics with the application of rules”.

This implies that in Bernstein’s terms, the pedagogical practices (evaluative criteria) in mathematics are viewed as mathematics for rules and application of procedures. Bernstein refers to this type of teaching as visible pedagogy which is associated with a performance-based curriculum. On the other hand, pedagogical practices (evaluative criteria) for ML indicates that ML is for reasoning and problem solving. Bernstein refers to this type of teaching as invisible pedagogy which is associated with a competence-based curriculum. Thus, the evaluative criteria as being indicated by teachers and facilitators are inconsistency with those that are espoused in the mathematics curriculum document, which seems to suggest that mathematics should be learner centred. This implies that teachers and facilitators have not yet developed the required “legitimate” text, practices, criteria (recognition and realisation rules) for these specialised forms of consciousness in mathematics. Their reflections of the recognition and realisation rules still reflect unreformed pedagogical practices. One would argue that the recognition and realisation rules reflected in ML by teachers and facilitators seem to be the required “legitimate” text as advocated for mathematics by researchers in mathematics education.

The question to ask is what does it mean for a strategy to be for mathematics and a strategy to be for ML strategy? The idea of associating strategies with a certain learning area has an implication on the learning and teaching of mathematics and ML.

For learning, it suggests that ML and mathematics learners participate in different discourse practices. ML learners are expected to act, think and believe differently from mathematics learners because they are from different communities of practice and therefore their participation in their community of practices would be different.

For teaching, it raises critical questions linked to the issue of mathematics and ML being considered “separate” subjects. Does it mean that they are inherently different discourses and therefore require different identities of teachers? Does it mean that for one teacher to work productively with mathematics and ML, the discourses in mathematics and ML should not be consistent (not in conflict) with the identity(ies) of the teacher? When would it be important for these discourses to be consistent? It is suggested that it would be in the case where the same teacher is asked to teach both mathematics and ML.

I suggest that the differences between the two subjects (mathematics and ML) should not be inconsistent (in conflict). If they were in conflict, what does it mean for the teacher who is teaching both? He would have split identities, which can make the task of his teaching very difficult. Again, for the teacher who is teaching both subjects, the two subjects should not have discourses that are in conflict, otherwise it would require ways of behaving which are different from the identity of the teacher.

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