A Novel Method Based on Induced Aggregation Operator for Classroom Teaching Quality Evaluation with Probabilistic and Pythagorean Fuzzy Information

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ABSTRACT
The purpose of this study is to develop a novel method based on induced aggregation operator to evaluate classroom teaching quality with probabilistic and Pythagorean fuzzy (PF) information. Inspired by the induced ordered weighted averaging (IOWA) operator, a PF aggregation operator called the PF induced probabilistic ordered weighted average (PFIPOWA) operator is developed. This operator uses probabilities and order-induced variables in the same formulas to aggregate PF information. Some of key features and special cases of the PFIPOWA operator are also investigated. Finally, the practicality of the developed operator is tested by using realistic classroom teaching quality evaluation problems. Hopefully, the research of this paper is of great significance to the evaluation of classroom teaching quality problems.

Keywords: Pythagorean fuzzy set, induced aggregation operator, probabilistic information, classroom teaching quality evaluation, multi-attribute decision making

INTRODUCTION
The evaluation of classroom teaching quality (CTQ) is an important activity in higher education and is of crucial importance to ensure the improvement of teaching quality in colleges and universities, thus the continuous improvement of teaching is the central task for education. There is no doubt that the scientific and effective method of CTQ evaluation plays a significant role in stimulating teachers’ enthusiasm and enhance their teaching ability. Generally, teachers are assessed by professional people with respect to different attributes in the process of CTQ evaluation, which can be considered as a multi-attribute decision making (MADM) problem (Yu, 2013; Yu & Lai, 2011; Yu et al., 2009; Zhang et al., 2017b). The evaluation of CTQ is a very complicated decision process because of uncertain and fuzzy information involved. Pythagorean fuzzy set (PFS) (Yager, 2014), characterized by a membership degree ($\mu$) and a non-membership degree ($v$), has recently been introduced to deal with the complex uncertainty, which is satisfying the restrictions of $\mu^2 + v^2 \leq 1$. This extended constraint condition makes PFS stronger than traditional intuitionistic fuzzy set (IFS) (Atanassov, 1986) because it can describe imprecise and ambiguous information, whereas the latter cannot.

In the literature, Yager (2014) further developed a new and useful decision method to manage the MADM problems under PF environment after some aggregation operations of PFS were defined. Chen (2018) gave a remoteness index-based methods to measure distance between PF sets. Zhang and Xu (2014) extended the traditional TOPSIS method to handle the PF MADM problems. Zhang (2016) developed a novel ranking the PF number based on the closeness index. Peng and Yang (2015) introduced a superiority- inferiority ordering method to solve PF MADM. Peng and Yang (2016) presented a method based on Choquet integral for PF MADM problems. Zeng et al. (2016a) proposed a hybrid distance measure of the PFSs and studied its application in MADM problems. Zeng (2017) presented a PF MADM method based on probabilistic approach. Based on prospect and regret theory,
Contribution of this paper to the literature

- The developed new Pythagorean fuzzy aggregation operator is able to consider both the probabilistic information and complex attitudinal characters of decision makers.
- A new MADM model based on the developed operators are presented under Pythagorean fuzzy situation.
- A novel evaluation framework for Classroom teaching quality evaluation is given. This may be play a significant role in the improvement of classroom teaching quality evaluation.

Peng and Dai (2017) developed a PF stochastic decision making model. Wei (2017) developed some PF interaction aggregation operators and studied their application to MADM problem.

Considering that PFS is able to describe inaccurate and fuzzy information better than IFS and has been broadly applied in practical MADM problems, this paper will develop a new model to manage effectively PF MADM problems. It is based on the PFPIPOWA operator, which unifies the probabilistic information with the order-induced variables in the same expression to aggregate the PF information. Therefore, it can manage probabilistic information and represent complex attitudinal characters by using order-induced variables. The major advantage and particular cases of the proposed operator are investigated. Finally, the practicality of the developed operator is tested by using actual CTQ evaluation problems, which is similar to Shieh and Yu (2016).

The remainder of this article is organized as follows. Section 2 reviews some basic preparations. Section 3 presents the PFPIPOWA operator and studies some basic properties and special cases. Section 4 discusses the applicability of the PFPIPOWA operator with a MADM example concerning CTQ evaluation. Insection 5 we summarize the main results of the paper.

PRELIMINARIES

We will briefly review basic concepts of PFS in this section. Meanwhile, the Pythagorean fuzzy OWA (PFOWA), the IOWA and the probabilistic OWA operator are further presented.

Pythagorean Fuzzy Set

**Definition 1.** Let a set \( Y = \{y_1, y_2, \ldots, y_n\} \) be fixed, a PFS \( P \) is defined as:

\[
P = \{(y, P(y)) | y \in Y\}
\]

The numbers \( \mu_P(y) \) represents the membership degree, while \( \nu_P(y) \) non-membership degree of the element \( y \) to \( P \). \( 0 \leq (\mu_P(y))^2 + (\nu_P(y))^2 \leq 1 \), for all \( y \in Y \). \( \pi_P(y) = \sqrt{1 - (\mu_P(y))^2 - (\nu_P(y))^2} \) represents the degree of indeterminacy of \( y \) to \( P \). For convenience, we call the pair \( P(y) = (\mu_P(y), \nu_P(y)) \) a Pythagorean fuzzy number (PFN), denoted as \( \beta = (\mu_P, \nu_P) \), where \( \mu_P, \nu_P \in [0,1] \) and \( \mu_P^2 + \nu_P^2 \leq 1 \).

Consider any three Pythagorean fuzzy numbers (PFNs) \( \beta_1 = (\mu_{\beta_1}, \nu_{\beta_1}), \beta_2 = (\mu_{\beta_2}, \nu_{\beta_2}) \) and \( \beta_3 = (\mu_{\beta_3}, \nu_{\beta_3}) \), some operational rules are defined as follows (Yager, 2014):

1. \( \beta_1 \otimes \beta_2 = \left( \frac{\mu_{\beta_1}^2 + \nu_{\beta_1}^2 - \mu_{\beta_2}^2}{\mu_{\beta_1}}, \frac{\nu_{\beta_1}^2 + \nu_{\beta_2}^2 - \mu_{\beta_1}^2}{\nu_{\beta_1}}, \frac{\nu_{\beta_1}^2 + \nu_{\beta_2}^2}{\nu_{\beta_1}} \right) \).
2. \( \beta_1 \otimes \beta_2 = \left( \mu_{\beta_1} \cdot \mu_{\beta_2}, \frac{\nu_{\beta_1}^2 + \nu_{\beta_2}^2 - \nu_{\beta_1}^2}{\nu_{\beta_1}}, \frac{\nu_{\beta_1}^2 + \nu_{\beta_2}^2}{\nu_{\beta_1}} \right) \).
3. \( \lambda \beta = \left( 1 - (1 - \mu_{\beta_1})^\lambda, (\nu_{\beta_1})^\lambda \right), \quad \lambda > 0 \).
4. \( \beta^\lambda = \left( (\mu_{\beta})^\lambda, 1 - (1 - \nu_{\beta})^\lambda \right), \quad \lambda > 0 \).

**Definition 2** (Zhang and Xu, 2014). For two PFNs \( \beta_1 = (\mu_{\beta_1}, \nu_{\beta_1}) \) and \( \beta_2 = (\mu_{\beta_2}, \nu_{\beta_2}) \), \( \beta_1 \geq \beta_2 \) if and only if \( \mu_{\beta_1} \geq \mu_{\beta_2} \) and \( \nu_{\beta_1} \leq \nu_{\beta_2} \).

**Definition 3** (Zhang and Xu, 2014). For a PFN \( \beta = (\mu_{\beta}, \nu_{\beta}) \), \( S(\beta) = (\mu_{\beta})^2 - (\nu_{\beta})^2 \) and \( H(\beta) = (\mu_{\beta})^2 + (\nu_{\beta})^2 \) are named the score function and accuracy function of \( \beta \), respectively. For two PFNs \( \beta_1 = (\mu_{\beta_1}, \nu_{\beta_1}) \) and \( \beta_2 = (\mu_{\beta_2}, \nu_{\beta_2}) \), if \( S(\beta_1) > S(\beta_2) \), then \( \beta_1 > \beta_2 \); if \( S(\beta_1) = S(\beta_2) \), then

1. If \( H(\beta_1) < H(\beta_2) \), then \( \beta_1 < \beta_2 \);
2. If \( H(\beta_1) > H(\beta_2) \), then \( \beta_1 > \beta_2 \).
Pythagorean Fuzzy OWA Operator

To aggregate PFNs, based on the basic operational laws of PFNs, Zhang (2016) defined the PF ordered weighted averaging (PFOWA) operator as follows.

Definition 4. Let \( \beta_j = P (\mu_{\beta_j}, v_{\beta_j}) \) (\( j = 1, \ldots, n \)) be a set of PFNs, the PFOWA operator associated weighting \( W = (w_1, \ldots, w_n) \) with \( \sum_{j=1}^{n} w_j = 1 \) is defined by the following formulas:

\[
PFOWA(\beta_1, \beta_2, \ldots, \beta_n) = \sum_{j=1}^{n} w_j \gamma_j = P \left( \frac{1}{1 - \sum_{j=1}^{n} (1 - \mu_{\beta_j}) w_j}, \sum_{j=1}^{n} v_{\beta_j}^2 \right)
\]

where \( \gamma_j \) is the \( j \)th largest of the \( \beta_j \). Note that if \( \gamma_j \) and \( \beta_i \) have the same ordered position, then the PFOWA operator becomes the PF weighted averaging (PFWA).

The IOWA Operator

The IOWA operator (Yager & Filev, 1999) is a widely used operator in decision making problems. Until now, it has been studied and extended by thousands of publications in various kinds of journals and conferences (Merigó & Gil-Lafuente, 2013; Xia et al., 2011; Xian et al., 2016; Yu, 2014; Zeng et al., 2017; Zhang et al., 2014; Zhou & Chen, 2013).

Definition 5. An IOWA operator of dimension \( n \) is a mapping IOWA: \( R^n \times R^n \rightarrow R \) that has an associated weighting \( V = (v_1, \ldots, v_n) \) with \( \sum_{j=1}^{n} v_j = 1 \) such that:

\[
IOWA((u_{a_1}, a_{b_1}), (u_{a_2}, a_{b_2}), \ldots, (u_{a_n}, a_{b_n})) = \sum_{j=1}^{n} v_j a_j
\]

where \( (a_1, a_2, \ldots, a_n) \) is the reordered version of \( (b_1, b_2, \ldots, b_n) \) induced by \( (u_1, u_2, \ldots, u_n) \).

The Probabilistic OWA Operator

The probabilistic OWA (POWA) operator (Merigó, 2010, 2011a) is a new aggregation method that combines the main advantages of the probability and the OWA operator (Yager, 1988).

Definition 6. A POWA operator of dimension \( n \) is a mapping POWA: \( R^n \rightarrow R \) that has an associated weighting vector \( V \) with \( v_j \in [0,1] \) and \( \sum_{j=1}^{n} v_j = 1 \), such that

\[
POWA(b_1, b_2, \ldots, b_n) = \sum_{j=1}^{n} \hat{p}_j a_j
\]

where \( a_j \) is the \( j \)th largest of the \( b_i \), which has an associated probability \( p_i \) satisfying \( 0 \leq p_i \leq 1 \) and \( \sum_{i=1}^{n} p_i = 1 \), \( \hat{p}_j = \lambda v_j + (1 - \lambda)p_j \) with \( \beta \in [0,1] \) and \( p_j \) is the probability \( p_i \) ordered based on \( a_j \), that is, according to the \( j \)th largest of the \( b_i \). Especially, when \( \lambda = 0 \), we get the probabilistic average, and if \( \lambda = 1 \), the OWA operator.

THE PYTHAGOREAN FUZZY INDUCED PROBABILISTIC OWA OPERATOR

The PFIPOWA Operator

The PFIPOWA operator is a new aggregation model that integrates the IOWA operator and the POWA operator in the same formula. Therefore, the PFIPOWA is very relevant because it provides more flexibility to consider the importance of each concept has in the analysis. It is defined as follows.

Definition 7. A PFIPOWA operator of dimension \( n \) is a mapping PFIPOWA: \( \Omega^n \times R^n \rightarrow \Omega \) that has an associated weights \( V \) with \( v_j \in [0,1] \) and \( \sum_{j=1}^{n} v_j = 1 \) such that:

\[
PFIPOWA((u_{a_1}, a_{\beta_1}), (u_{a_2}, a_{\beta_2}), \ldots, (u_{a_n}, a_{\beta_n})) = \sum_{j=1}^{n} \hat{w}_j \gamma_j
\]

where \( \gamma_j \) is the recorded value of \( \beta_i \) induced by \( u \), each PF \( \beta_i \) has an associated probability \( p_i \) satisfying \( 0 \leq p_i \leq 1 \) and \( \sum_{i=1}^{n} p_i = 1 \), \( \hat{w}_j = \lambda v_j + (1 - \lambda)p_j \) with \( \beta \in [0,1] \) and \( p_j \) is the ordered value of \( p_i \) related to \( \gamma_j \), that is, based on the \( j \) the largest of the \( \beta_i \).

In the next example, we present a numerical example to show aggregation process of the PFIPOWA operator.

Example 1. Let \( \beta = (P(0.9,0.3), P(0.6,0.5), P(0.7,0.4), P(0.8,0.2)) \) be the aggregated arguments with the order-induced variables \( \theta = (7,4,1,9) \), the weights \( v_1 = v_2 = 0.2, v_3 = v_4 = 0.3 \), and the probabilistic weights vector be
\[ P = (0.2, 0.4, 0.1, 0.3). \] It is assumed that the probability information has an importance of 70%, while the degree of the weight vector is 30%. Then we should calculate the new weighting vector if we use the Eq. (5):

\[
\begin{align*}
\bar{w}_1 &= 0.3 \times 0.2 + 0.7 \times 0.1 = 0.13, \\
\bar{w}_2 &= 0.3 \times 0.2 + 0.7 \times 0.2 = 0.2, \\
\bar{w}_3 &= 0.3 \times 0.3 + 0.7 \times 0.3 = 0.3, \\
\bar{w}_4 &= 0.3 \times 0.3 + 0.7 \times 0.4 = 0.37.
\end{align*}
\]

And then, based on the PFIPOWA operator, we have:

\[
PFIPOWA((u_1, \beta_1), \ldots, (u_4, \beta_4)) = P(0.76, 0.37)
\]

If the weighting vector of the OWA and probabilities are not standardized, i.e., \( \bar{\nu} = \sum_{j=1}^{n} \bar{\nu}_j \neq 1 \), then the PFIPOWA operator should be formed as:

\[
PFIPOWA((u_1, \beta_1), (u_2, \beta_2), \ldots, (u_n, \beta_n)) = \frac{1}{P} \sum_{j=1}^{n} \bar{\nu}_j \gamma_j
\]  

(6)

**Main Properties of the PFIPOWA Operator**

The IFPIOWA operator has the similar properties as the IOWA and POWA operators, that is, it satisfies monotonicity, commutativity, idempotency and boundedness. Suppose \( f \) is the PFIPOWA operator, these properties can be expressed by the Theorem 1 to Theorem 4.

**Theorem 1** (Monotonicity). If \( \beta_j \geq \beta'_j \) for all \( j \), then:

\[
PFIPOWA((u_1, \beta_1), \ldots, (u_n, \beta_n)) \geq PFIPOWA((u_1, \beta'_1), \ldots, (u_n, \beta'_n))
\]  

(7)

**Theorem 2** (Commutativity). If \((u_1', \beta'_1), \ldots, (u_n', \beta'_n)\) is a permutation of the aggregated pair \((u_1, \beta_1), \ldots, (u_n, \beta_n)\), then

\[
PFIPOWA((u_1, \beta_1), \ldots, (u_n, \beta_n)) = PFIPOWA((u_1', \beta'_1), \ldots, (u_n', \beta'_n))
\]  

(8)

**Theorem 3** (Idempotency). If \( \beta_j = \beta \) for all \( j \), then

\[
f((u_1, \beta_1), \ldots, (u_n, \beta_n)) = \beta
\]  

(9)

**Theorem 4** (Boundedness). The PFIPOWA operator is bounded by the max and min values, i.e.,

\[
\min \{\beta_j\} \leq f((u_1, \beta_1), \ldots, (u_n, \beta_n)) \leq \max \{\beta_j\}
\]  

(10)

Note that the proofs of these theorems are straightforward and thus omitted for sake of brevity. Moreover, Given the PFIPOWA operator relies on probabilistic property, one can prove that it is a semi boundary condition:

\[
\lambda \min \{\beta_i\} + (1 - \lambda) \times \sum_{i=1}^{n} p_i \beta_i \leq f((u_1, \beta_1), \ldots, (u_n, \beta_n)) \leq \max \{\beta_i\} + (1 - \lambda) \times \sum_{i=1}^{n} p_i \beta_i
\]  

(11)

**Families of PFIPOWA Operator**

A series of particular aggregation operators can be obtained by analyzing the coefficient \( \lambda \), the weights \( V \) and the order-induced value \( U \) in the PFIPOWA operator. Some interesting special cases (among others) can be identified:

**Remark 1.** Basically, if \( \lambda = 0 \), i.e., the relative importance of the WA approach to zero, we get the Pythagorean fuzzy probabilistic aggregation (PFPA). Conversely, if \( \lambda = 1 \), PF the induced OWA (PFIOWA) operator. Furthermore, if \( \gamma = 1 \) and the ordering of order-inducing variables coincides with the input arguments, the PFOWA operator is obtained.

**Remark 2.** Another group of important cases are the maximum PFPA (Max-PFPA), the minimum PFPA (Min-PFPA) and the step-PFIPOWA.

- The Max-PFPA is found when \( v = (1, 0, \ldots, 0) \).
- The minimum PFPA is found when \( v = (0, \ldots, 0, 1) \).
- The general step-PFIPOWA is formed when \( v_k = 1 \) and \( v_j = 0 \), for all \( j \neq k \).

**Remark 3.** We assign \( v_{(n+1)/2} = 1 \) when \( n \) is odd, and \( v_j = 0 \) for all others, then we get the median-PFIPOWA. If \( n \) is even, then we assign \( v_{n/2} = v_{(n/2)+1} = 0.5 \).

**Remark 4.** Numerous other types can be analyzed in accordance with methods widely used in the OWA-based literature (Aggarwal, 2015; Merigó, 2011b; Xian et al., 2016; Zeng et al., 2016b; Zhang et al., 2017a).
A further extensions of the PFPIOWA operator can be studied by employing the generalized means (Merigó & Gil-Lafuente, 2013; Zeng et al., 2016b) which is very useful for representing a complete picture when we want to consider more choices. The result is generalized PFPIOWA (GPFPIOWA) operator:

**Definition 8.** A GPFPIOWA operator of dimension \( n \) is a mapping GPFPIOWA: \( \Omega^n \times R^n \rightarrow \Omega \) that has an associated weighs \( V \) with \( v_j \in [0,1] \) and \( \sum_{j=1}^{n} v_j = 1 \) such that:

\[
\text{GPFPIOWA}(u_1, \beta_1; u_2, \beta_2; \ldots, u_n, \beta_n) = \left( \sum_{j=1}^{n} \bar{w}_j v_j \right)^{1/t}
\]

(12)

where \( \bar{w}_j = \lambda v_j + (1 - \lambda) p_j \) with \( \lambda \in [0,1] \), \( t \) is parameter that satisfies \( t \in (-\infty, +\infty) - \{0\} \). Generally, we can see the families of GPFPIOWA by studying the coefficient \( \lambda \), the weight \( V \), the order-induced value \( U \) and the parameter \( t \). More specially,

- If \( t = 1 \), then the GPFPIOWA reduces to the PFPIOWA operator, thus all PFPIOWA’s particular cases can be seen as GPFPIOWA’s special cases.
- If \( t = 2 \), we get the PF induced probabilistic quadratic ordered weighted averaging (PFIPQOWA) operator
- If \( t = -1 \), the PF induced probabilistic harmonic ordered weighted averaging (PFIPHOWA) operator is obtained.
- Etc.

**CLASSROOM TEACHING QUALITY EVALUATION WITH THE PFPIOWA OPERATOR**

Next, a numerical example concerning CTQ evaluation (adapted Zhang et al., 2017b) is given to illustrate the use of the PFPIOWA in a MADM problem. Assume a university desires to enhance classroom teaching quality by way of teaching match. Several professor and students are invited to evaluate five teachers \( \{A_1, A_2, A_3, A_4, A_5\} \) from the following five attributes: teaching attitude \( (C_1) \), teaching ability \( (C_2) \), teaching content \( (C_3) \), teaching method \( (C_4) \) and teaching effect \( (C_5) \).

Due to the uncertainty associated with the analysis of the phenomenon, the evaluation values of various alternatives with respect to attribute given by the professor are represented by PFNs, showed in Table 1.

In this problem, the order-inducing variables is assumed \( U = (10, 12, 8, 9, 7) \), which presents complex attitudinal character in the decision process. We assume that \( \lambda = 0.4, P = (0.3, 0.3, 0.1, 0.2, 0.1) \) and \( V = (0.1, 0.2, 0.2, 0.2, 0.3) \). By exploiting the above information, the PFPIOWA operator can be used to aggregate the evaluated values and to select the best desired alternative. The results are shown in Table 2.

From the Table 2, we can get the optimal ranking order of these five companies: \( A_5 > A_1 > A_2 > A_4 > A_3 \), obviously the most desirable alternative is \( A_5 \). Moreover, it is interesting to examine the validation of results by using some special cases of the PFPIOWA and the GPFPIOWA operators. In this case, we will consider the Max-PFPA, Min-PFPA, PFWA, PFOWA, PFIOWA, PFIPOWQA and the PFIPOWHA. The ranking of results by the different cases are shown in Table 3.
As we can see, the ranking of the alternatives may be different depending on the particular cases used. Therefore, this approach is quite flexible because it enables the decision maker(s) to have more options to select aggregation schemes. Thus, the decision maker will choose the one that is most suitable for his or her beliefs or interests.

CONCLUSIONS

In this paper, a new PF aggregation operator that uses the principal features of the probability, the order-induced variables and uncertain information in form of PFNs is developed. Some of its main properties are analyzed. In addition, various particular cases of the PFIPOWA operator including the PFWA and the PFOWA operator are investigated. Moreover, the application of the new approach to MADM problem concerning CTQ evaluation is presented. We have seen that the PFIPOWA is very capable because we can assess the progress of decision making progress taking into account the probabilities and the attitudinal character of decision makers. Thus, this method is very flexible because by assigning different parameter values to the operator, more opportunities can be given to choose a particular situation. Therefore, it enriches the existing method of aggregating Pythagorean fuzzy information.

In future studies, we hope to further expand this operator by using distance measures and other new characteristics. The application of the presented model may be explored in other areas, such as engineering, economics and material recognition.

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REFERENCES


Table 3. Ranking of the potential teachers

<table>
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<th>Special Cases</th>
<th>Ordering</th>
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<tr>
<td>Max-PFPA</td>
<td>$A_1 &gt; A_2 &gt; A_3 &gt; A_4 &gt; A_5$</td>
</tr>
<tr>
<td>Min-PFPA</td>
<td>$A_5 &gt; A_2 &gt; A_1 &gt; A_4 &gt; A_3$</td>
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<tr>
<td>PFWA</td>
<td>$A_1 &gt; A_2 &gt; A_3 &gt; A_4 &gt; A_5$</td>
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<tr>
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<td>$A_1 &gt; A_2 &gt; A_3 &gt; A_4 &gt; A_5$</td>
</tr>
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<td>$A_1 &gt; A_2 &gt; A_3 &gt; A_4 &gt; A_5$</td>
</tr>
<tr>
<td>PFIPOWQA</td>
<td>$A_1 &gt; A_2 &gt; A_3 &gt; A_4 &gt; A_5$</td>
</tr>
<tr>
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<td>$A_1 &gt; A_2 &gt; A_3 &gt; A_4 &gt; A_5$</td>
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Table 3. Ranking of the potential teachers

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