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A Comparative Analysis of Division in Elementary Mathematics Textbooks in Korea and Japan

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ABSTRACT

The purpose of the study is to examine how Korean and Japanese elementary mathematics textbooks series present division in terms of the perspective of making connections. For this purpose, units dealing with division of whole numbers, fractions, and decimal numbers were analyzed with foci on the meanings of division and procedures of division. Findings showed that, in the textbooks of both countries, the various meanings of division were consistently applied not only to whole numbers but also to fractions and decimal numbers. Moreover, the procedures of division were connected as numbers were expanded from whole numbers to fractions. Noticeable differences included: as for the organization of the contents of division, Korean textbooks presented division of fractions first, whereas Japanese textbooks dealt with division of decimal numbers first. Regarding the meanings of division, the Korean textbooks dealt mainly with partition and measurement division situations, while the Japanese counterparts were more inclusive with multiple situations, such as determination of a unit rate. This study is expected to provide information on how Korean and Japanese textbooks present division and to give implications for textbook developers and teachers to connect division meaningfully as students deal with whole numbers, fractions, and decimal numbers.

Keywords: comparative analysis, division, elementary mathematics textbooks, making connections

INTRODUCTION

The understanding of mathematical ideas involves making connections among such ideas (NCTM, 2000). As students are able to connect mathematical ideas to one another, they develop a deeper and more lasting understanding. The four basic operations, which are of great importance in elementary mathematics curricula, are interrelated to each other (Barnett-Clarke et al., 2010; Blanton et al., 2011; Carpenter et al., 1999). Addition and subtraction have an inverse relationship, as do multiplication and division. Moreover, as subtraction and division are defined in relation to addition and multiplication respectively, it is necessary for students to consider relations in order to fully understand the four operations.

The meanings of each operation remain constant and connected, even as numbers become more complicated (Barnett-Clarke et al., 2010). For example, division of whole numbers, 8÷2, can be interpreted as the idea of finding how many groups of 2 are in 8. As students move to division of fractions, this whole number understanding can be connected to similar examples, such as 8/15 ÷ 2/15, by asking how many groups of 2/15 are in 8/15. Likewise, most meanings are essentially the same although some need adaptation.

Many studies report that students have difficulty solving division problems, especially when fractions or decimal numbers are involved (Barnett-Clarke et al., 2010; Sinicrope et al., 2002). One possible reason for this difficulty is that students simply depend on memorized procedures without necessarily understanding the meanings behind such procedures (Gojak, 2013). They may be unable to connect division interpretations from whole numbers to rational numbers and rather deal with them separately.

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Contribution of this paper to the literature

- The study scrutinized and compared contents of division in the Korean and Japanese elementary mathematics textbooks from the perspective of connections.
- The results showed that the meanings and procedures of division were connected in the textbooks of both countries. However, notable differences were found, namely, in sequencing division of fractions and division of decimal numbers, and in employing multiple meanings for division of fractions.
- The study is expected to provide insights for international readers on directions of connecting division in mathematics textbooks.

However, few studies have focused on division in a comprehensive way. Given that textbooks are one of the main instructional materials (Stein et al., 2007), it is necessary to scrutinize how division is presented in textbooks from the perspective of connections. In addition, given that Korean and Japanese students have shown high achievement on recent TIMSS and PISA tests (Ku et al., 2016; Sang et al., 2016), these positive results may be attributed to the influence of the textbooks of each country on such performance. With this background, the study targeted the textbooks from Korea and Japan and investigated how division was represented. As such, this study is expected to suggest implications for how to connect division from whole numbers to fractions and decimal numbers in mathematics instructional materials.

RATIONALE AND RESEARCH QUESTIONS

Analysis of Textbooks

With the importance of textbooks and their strong influence on instruction and students' achievement (Alajmi, 2009; Remillard, 2005; Weiss et al., 2003), there have been numerous studies which investigated textbooks from various perspectives. Regardless of the topics and analytic foci of such studies, overall structures of textbooks tend to be examined in common (e. g., Alajmi, 2012; Charalambous et al., 2010; Son & Senk, 2010).

For example, Alajmi (2012) reviewed how fractions were addressed in mathematics textbooks in the USA, Japan, and Kuwait in terms of three aspects: the physical characteristics, the structure of the lessons, and the nature of the problems. Charalambous et al. (2010) compared the addition and subtraction of fractions in the textbooks from Cyprus, Ireland, and Taiwan. In their analytic framework, horizontal and vertical approaches were used: a horizontal approach means to examine background information and overall structures of textbooks, whereas a vertical approach indicates textbook analysis by topic-specific perspectives such as definitions, mathematical practices, and connections.

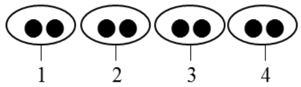
To reiterate, understanding the flow and structure of textbooks is necessary before delving into the detailed analysis tailored to the specific topic under study. This is especially true for this study because the topic of division is covered across multiple grade levels. Given this, first an exploration was conducted of how the contents of division are organized and connected in the Korean and Japanese textbooks. Then the meanings and the procedures of division were evaluated from a topic-specific perspective. Specifically, an examination was made of how the meanings and the procedures of division are presented and connected in the textbooks.

Meanings and Procedures of Division

Division is one of the main areas of school mathematics and is essential for students to succeed in elementary grades and their subsequent grades (Otto et al., 2011). In the Common Core State Standards for Mathematics (CCSSM), division is addressed in diverse domains such as 'Operations and Algebraic Thinking' and 'Number and Operations in Base Ten' (CCSSM, 2010). In this respect, understanding division is crucial in learning elementary mathematics.

While learning division, understanding what division means is of great importance. For whole-number division, division is usually interpreted as two different meanings: partition and measurement (Reys et al., 2014). In the partition context, also known as fair-sharing, a collection of objects is separated into a given number of equivalent groups and the quotient means the size of each group. In the measurement context, a collection of objects is separated into parts of a given size and the quotient means the number of equal-sized groups. **Table 1** summarizes partition and measurement division.

Table 1. Meanings of division: Partition and measurement

	Partition	Measurement
Whole	O (dividend)	O (dividend)
The size of each group	× *(quotient)*	O (divisor)
The number of groups	O (divisor)	× *(quotient)*
Ex. 8÷2=4		

Note 'O' refers to what is known in the problem, whereas '×' refers to what is unknown.

These two basic meanings of division are well applied in whole-number contexts, but adjustment of such meanings is necessary in fraction or decimal number contexts. In partition division, if a divisor is a fraction or a decimal number, the situation will not make sense. Rather, it must be modified as division of determination of a unit rate, which focuses on the size of a whole (Barnett-Clarke et al., 2010; Siebert, 2002). For example, $3 \div \frac{2}{5}$, a problem of division of fractions, can be considered as determination of a unit rate by asking a question, "If $\frac{2}{5}$ of a group gets 3, then how much does the whole group get?"

Other meanings such as 'inverse of multiplication' and 'inverse of a Cartesian product' may be considered (Sinicrope et al., 2002). The meaning of division as inverse of multiplication is usually approached by reversing the procedures of the original multiplication. Regarding the meaning of inverse of a Cartesian product, a prevalent example is that the total area and one dimension of a rectangular region are known, and thus quotient is the other dimension. **Figure 1** shows an example of the division of 8 ÷ 2, which is interpreted as determining the width of a rectangular array that has an area of 8 and a length of 2.

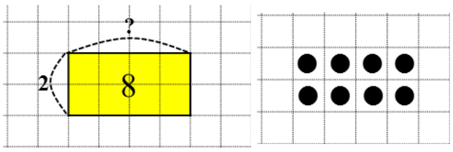


Figure 1. Models of the inverse of a Cartesian product

Up to now, the meanings of division are categorized as follows: (1) partition, (2) measurement, (3) unit rate, (4) inverse of multiplication, and (5) inverse of a Cartesian product. As each meaning needs different procedural reasoning process, it is important for students to understand what division means from various aspects.

The procedures of division are also connected as numbers are extended from whole numbers to fractions and decimal numbers. However, there are few studies which focused on connecting procedures of division of whole numbers, division of fractions, and division of decimal numbers. Rather, most of them tended to focus mainly on one aspect of division procedures. For instance, Van de Walle et al. (2008) recommended the instructional guidelines for improving computational competence for fractions. Among them, "connecting the meanings of fraction computation with whole number computation" (Van de Walle et al., 2008, p. 310) implies the need for connecting procedures of division. Li (2008) examined how the Chinese textbooks presented division of fractions compared to their counterparts in the United States. To be sure, connections of division of fractions with division of whole numbers were documented in part, but the main focus of Li's study was given to the conceptual procedures and the applications of division of fractions.

Many studies report that students experience difficulties in learning division (Barnett-Clarke et al., 2010; Sinicrope et al., 2002). For instance, Li and Kulm (2008) reported that students had difficulties in learning division of fractions and solved problems just by applying an "invert and multiply" algorithm without understanding its meaning. Regarding the division of decimal numbers, students usually focused on dividing vertically by moving a decimal point without knowing why it works (Sherman et al., 2015). Given that division procedures are connected even when the range of numbers is extended, these results indicate that students were not able to connect the procedures of division while learning division.

To summarize, the meanings of division are multiple and need to be connected and adjusted as the ranges of numbers are extended from whole numbers to fractions and decimals. In a similar vein, the procedures of division must also be connected as the ranges of numbers are changed, because students do not realize such connections by themselves. In this respect, it is significant to investigate how the meanings and procedures of division are presented and connected in textbooks and to explore implications for the organization of textbooks in order to help students gain a deeper understanding of division as a coherent whole.

Research Questions

Given the rationale and literature review, the aims of this study are to compare how divisions of whole numbers, fractions, and decimal numbers are presented in the Korean and Japanese textbooks series and to explore instructional implications to connect division. Specifically, this study addresses the following research questions:

(1) How are the contents of division organized and connected in the Korean and Japanese textbooks?

(2) How are the meanings of division presented and connected in the Korean and Japanese textbooks?

(3) How are the procedures of divisions presented and connected in the Korean and Japanese textbooks?

METHODOLOGY

Mathematics Textbooks Selection

In order to investigate how division is presented in elementary mathematics textbooks series, textbooks in Korea and Japan from grades 3 to 6 were selected based on their similarities and differences. Both countries have a national curriculum respectively, and textbooks are aligned with the curriculum. In Korea, there is only one kind of mathematics textbooks series developed by the Ministry of Education. In Japan, however, private authors or publishers develop textbooks based on the curriculum guidelines. The series developed by Hitotsumatsu et al. (2011a~f) was chosen for analysis in this study. The Japanese series was originally translated into English, so only the Korean ones were translated into English by the authors as needed for this paper. In total, the following 11 mathematics textbooks in Korea and Japan were examined as shown in **Table 2**.

Table 2. Subjects of analysis

Country	Korea	Japan
Textbook	Mathematics textbook 3-1, 3-2, 4-1, 5-2, 6-1	Study with Your Friends MATHEMATICS for elementary school 3-1, 4-1, 4-2, 5-1, 5-2, 6-1
Publication	Ministry of Education (2016a~c, 2015a~b)	Gakkohtosho co., LTD. (2011a~f)

Data Collection and Analysis

All units dealing with division of whole numbers, division of fractions, and division of decimal numbers were analyzed. In every unit, main activities were included for analysis but the problems for individual practice or assessment were excluded from the study. These were excluded for the following reasons. First, most problems for individual practice are presented without situation. Also, problems for assessment tend to be aligned with the problems in the main activities. So there are few opportunities to find other meanings of division by including the problems for individual practice or assessment. **Table 3** shows the analytical framework used for the study.

Table 3. Framework used to analyze the contents, meanings, and procedures

Analytic foci	Focus questions	Aspects investigated
Contents of division	• When and how is division introduced and developed? • In which contents are the units composed?	• Overall structure • Development of the units
Meanings of division	• How are the meanings of division presented? • How are the meanings of division connected in fractions and decimal numbers?	• Types of meanings of division • Frequencies of meanings of division • Connections of the meanings of division to fractions and decimal numbers
Procedures of division	• What kinds of strategies are used? • Are the procedures of division interconnected with each other?	• Types of strategies • Connections of the procedures

As shown in **Table 3**, the collected data was analyzed in three aspects. Firstly, we examined how the contents of division were organized and sequenced in the Korean and Japanese textbooks. Specifically, we examined what contents were addressed and in which grade and semester they were presented. For example, in the Korean textbook series, whole-number division was introduced during the 1st semester of 3rd grade and addressed until the 1st semester of 4th grade. Next, we analyzed how the meanings of division were distributed in the textbooks, focusing on whether such meanings were presented in a way to facilitate the connection of the contexts to division of fractions and decimal numbers. To this end, we categorized problem contexts into six types of division meanings: Note that, in addition to five meanings described in the previous section, 'no situation' was added because it was frequently presented in every learning theme. Table 4 illustrates examples of the meanings of division from the Korean and Japanese textbooks.

Table 4. Examples of meanings of division

Meanings of division	Korea	Japan
Partition	Let's share 8.2L of milk equally with 4 students. (2015a, p.124)	Let's share 12 candies equally amongst 4 children. (2011a, p.58)
Measurement	You cut a fabric of 1m by 1/4m to make a magic tie. How many magic ties can you make? (2015b. p.42)	There are 12 sweets. If one child receives 3 sweets, how many children can receive sweets? (2011a, p.64)
Determination of a unit rate	–	We used 5.76dL of paint to paint a 3.2m^2 wall. How many dL of paint will we use to paint a 1m^2 wall? (2011d, p.70)
Inverse of multiplication	You transform a magic stick of 0.46m into a tree of 3.68m. How many times taller is the tree than the stick? (2015b, p.72)	–
Inverse of a Cartesian product	–	There is a rectangular flower bed that is 2.3m long and has an area of 12m^2. How long is the width in meters? (2011d, p.69)
No situation	Write an expression into a vertical form and figure out a quotient: 48÷8. (2016a, p. 103)	Let's think about how to divide 96÷33 in vertical form. (2011b, p.83)

All the problems were coded by the authors and intra-class correlation coefficient between measurements taken by the authors was 0.974 indicating very high inter-rater reliability. After that, frequencies and percentages of meanings were calculated. Problems which were classified into different meanings by the authors were discussed until the authors agreed on a single classification.

Finally, we investigated what and how the procedures of division were addressed in the textbooks. For this, we examined what kinds of strategies were presented and in what sequences such strategies were arranged. In doing so, connections among division strategies of whole numbers, fractions, and decimal numbers were examined.

RESULTS

Contents of Division

Table 5 shows the organization of content regarding division in the textbooks. In both countries, division was introduced in 3rd grade with whole-numbers and it was dealt until the 6th grade.

Table 5. Organization of contents about division in the textbooks

Grade	Semester	Korea		Japan	
3	1	W	3. Division	6. Division	W
	2		2. Division		
4	1		2. Multiplication and Division	2. Division 5. Division by One-digit Numbers 7. Division by Two-digit Numbers	W
	2			16. Multiplication and Division of Decimal Numbers	D
5	1			5. Division of Decimal Numbers	
	2	F	3. Division of Fractions	10. Multiplication and Division of Fractions	F
		D	4. Division of Decimal Numbers		
6	1	F	2. Division of Fractions	4. Division of Fractions	
		D	3. Division of Decimal Numbers		

Note W: Whole numbers, F: Fractions, D: Decimal numbers

The division of whole numbers was addressed in partition situations, i. e., a total number of objects were fairly shared into 2 plates or 4 children respectively (see Figure 2). Here, discrete quantities such as Baduk stones or candies were used.

Korea
(MOE, 2016a, p. 95)

8 divided by 2 is 4. It is written as 8÷2=4 and read as "8 divided by 2 equals 4". Sentences like 8÷2=4 are called division and 4 is the quotient of when 8 is divided by 2.

Japan
(Hitotsumatsu et al., 2011a, p.64)

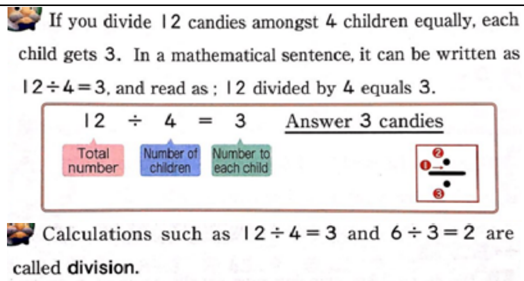
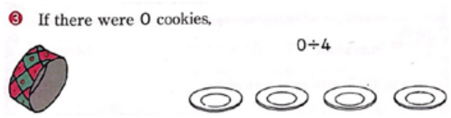


Figure 2. Introduction of the definition of division

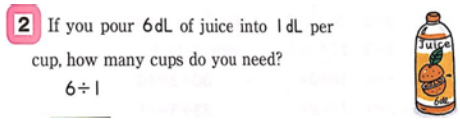
Most of the content regarding whole-number division was similar in both countries. The units dealing with the division of whole numbers were usually sequenced in terms of how many place-values the divisor had or whether or not the dividend was divisible in the set of whole numbers. Only the Japanese textbooks included division with 1 and 0 as divisor or dividend. **Figure 3** shows the situations in which the given dividend is 0 and when the divisor is 1 in the Japanese textbook. By dealing with these specific types of division within meaningful situations, students may understand why dividing 0 by any number yields 0 and dividing any number by 1 yields the same number.

When the dividend is 0



(2011a, p.67)

When the divisor is 1



(2011a, p.67)

Figure 3. Problem situations of division with 0 and 1 in the Japanese textbook

Another remarkable difference was the sequence of content regarding division. The Korean textbooks introduced the division of fractions earlier than the division of decimal numbers, but this order was reversed in the Japanese textbooks. Specifically, in the Korean textbooks for the 5th and the 6th graders, the unit dealing with division of decimal numbers is always followed by the unit dealing with division of fractions as shown in **Table 5**. This sequence of the units is to connect division of fractions with division of decimal numbers. In fact, in the unit dealing with the division of decimal numbers in the Korean textbooks, the main strategy is to change the given decimal number into the fraction, which leads to the division of fractions (see **Table 6**).

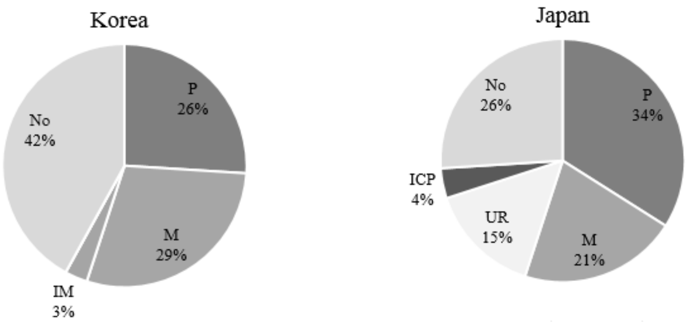
Table 6. Connections among division strategies in the Korean textbooks

Strategy	Example
• Change the given division into the division of fractions with the common denominator, and then deal with it as division of whole numbers	$14 \div 3.5 = \frac{140}{10} \div \frac{35}{10} = \square \div \square = \square$ (2015b, p.77)
• Change the given division into the division of fractions, and then follow the algorithm for the division of fractions	$91.44 \div 4 = \frac{\square}{100} \div 4 = \frac{\square}{100} \times \frac{1}{\square} = \frac{\square}{100}$ (2015a, p.119)

Note that both Korean and Japanese textbooks had two units on division of fractions and division of decimals respectively. The two units are differentiated according to the kinds of divisors. For instance, the unit 'Division of Fractions' appeared in both grade 5 and grade 6 in the Korean textbooks, the only difference being is the kinds of divisors. The textbook for grade 5 deals only with natural numbers as divisors in division of fractions, whereas the textbook for grade 6 deals with fractions as divisors. From the perspective of connections, the differentiation of the units across grades is intended to connect the division of fractions with the division of whole numbers.

Meanings of Division

Division problems from each textbook were analyzed in terms of the meanings of division. As shown in **Figure 4**, the situations were classified into partition (P), measurement (M), determination of a unit rate (UR), inverse of multiplication (IM), inverse of a Cartesian product (ICP), and no situation (No). In the Korean textbook series, the most frequent meaning of division was Measurement (29%) followed by Partition (26%), Inverse of Multiplication (3%). Note also that No situation was as much as 42%. In contrast, the Japanese textbooks included more varied situations such as UR and ICP. Here, the percentage of partition situation was the highest (34%), whereas No situation was 26%.



Note P: Partition, M: Measurement, IM: Inverse of Multiplication, UR: Unit Rate, ICP: Inverse of a Cartesian Product, No: No situation.

Figure 4. Distribution of division meanings in the Korean and Japanese textbooks

Table 7 displays detailed results by presenting the frequencies of division meanings according to the types of numbers. In total, the Japanese series included 95 problems and the Korean series only 76, indicating that more problems were in the Japanese textbooks. In fact, one learning theme in the Japanese textbooks included 3 to 5 word problems, whereas the Korean textbooks began with a word problem, moved to 1 or 2 activities to solve the given problem, and then ended with 1 or 2 problems without situations mainly for practice. This textbook structure explains why the Korean textbooks included a smaller quantity of problems than their counterparts in Japan.

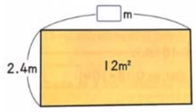
Table 7. Frequencies of meanings of division in the Korean and Japanese textbooks

	Korea							Japan						
	P	M	UR	IM	ICP	No	Total	P	M	UR	IM	ICP	No	Total
Whole Numbers	9	11	0	0	0	9	29	23	15	0	0	0	20	58
Fractions	2	6	0	0	0	20	28	4	3	8	0	3	3	21
Decimal Numbers	9	5	0	2	0	3	19	5	2	6	0	1	2	16
Total	20	22	0	2	0	32	76	32	20	14	0	4	25	95
(%)	(26)	(29)	(0)	(3)	(0)	(42)	(100)	(34)	(21)	(15)	(0)	(4)	(26)	(100)

As for the meanings of division, two countries revealed different results. In the Korean textbooks, the percentage of measurement division was higher than those of partition division both with whole numbers and with fractions. As for the division of decimal numbers, the percentage of partition division was higher than measurement division, because divisors were natural numbers so that partition situations could be more easily understood. In contrast, the percentages of partition division were higher in the Japanese textbooks than those of measurement division, regardless of the types of numbers.

For situations in which the divisor was a fraction or a decimal number, both countries revealed different results. Since interpreting the division of fractions such as '$\frac{3}{4} \div \frac{2}{5}$' as partition situations does not make sense, alternative meanings needed to be included. In this case, the Japanese textbooks included situations of 'determination of a unit rate' and 'inverse of a Cartesian product'. However, in the Korean textbooks, measurement situations were used more frequently rather than adapting new situations. In fact, division as the inverse of multiplication was addressed only twice. These differences between two countries may have an influence on students in a distinct way. For instance, Japanese students are expected to have an opportunity to apply division to diverse situations, whereas Korean students are expected to have an opportunity to consistently connect the meanings of division from whole numbers to fractions. **Table 8** illustrates problem situations for all kinds of division meanings, as the numbers were expanded from whole numbers to fractions or decimal numbers.

Table 8. Examples of problem situations

		Whole Numbers	Fractions, Decimal Numbers
Korea	*Partition*	Equally sharing 20 oranges among 5 persons. (2016a, p.98)	Equally sharing sweet potatoes of 2.6kg between two persons. (2015a, p.114)
	Measurement	When dividing 12 bags of sweets into 2 for each child, how many children can receive the sweets? (2016a, p.96)	Making an invitation card of $\frac{1}{6}$ width with $\frac{5}{6}$-width paper. (2015b, p.44)
	Inverse of multiplication		When transforming a magic stick of 0.46m into a tree of 3.68m, how many times taller is the tree than the stick? (2015b, p.71)
Japan	*Partition*	Sharing 12 candies amongst 4 children. (2011a, p.58)	To make 6 ribbons from $\frac{8}{9}$m long tape, how many meters long will each ribbon be? (2011b, p.22)
	Measurement	Dividing 12 sweets by giving 3 per child, how many children can receive sweets? (2011a, p.59)	There is $1\frac{4}{5}$ L of milk. If you drink $\frac{3}{5}$dL each time, how many meals can you have? (2011f, p.47)
	Determination of a unit rate		We used $\frac{3}{4}$dL of blue paint for a $\frac{2}{5}m^2$ fence. How many m^2 did we cover with a 1dL of paint? (2011f, p.43)
	Inverse of a Cartesian product		 (2011d, p.69)

As mentioned earlier, the situations such as 'determination of a unit rate' or 'inverse of a Cartesian product' were not included in the Korean textbooks. However, the Korean workbooks included situations of inverse of a Cartesian product. Considered that workbooks in Korea were made for students' self-directed study at home for practice, it seems that they were intended to give an opportunity for students to apply division in diverse situations on their own. In contrast, as Japan has only textbook series without workbooks, it seems that multiple situations were included in the textbooks.

As the situation of 'determination of a unit rate' is related to the concept of rate, students need to learn what rate is to understand such a situation. In the Korean textbooks, the concept of rate is addressed immediately following division of decimal numbers in 6th grade. Because of the order of units, the division of fractions is not explained in the situation of unit rate. In contrast, in the Japanese textbooks, the concept of rate was addressed in 5th grade and the situations of 'determination of a unit rate' were introduced in 6th grade. This implies that it is important to understand the curriculum thoroughly for suggesting suitable content for textbooks.

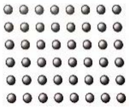
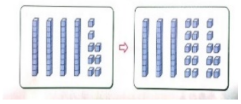
Procedures of Division

The procedures of division of whole numbers, fractions, and decimal numbers in the Korean and Japanese textbooks were analyzed by how they were structured and connected to one another. Before looking into the procedures of division, it is important to understand how a learning theme is composed in the textbooks. In the Korean textbooks, a problem situation was initially presented, along with 2 or 3 strategies, and followed by step-by-step questions. It ended with simple problems for students to practice their calculation skills. The Japanese textbooks are the same with Korean ones in that a problem situation was presented at first and 2 or 3 strategies followed. In the textbooks of both countries, the strategies presented earlier were more concrete and subsequent strategies more abstract. However, the difference was that the Japanese textbooks let students find the commonalities or principles of given strategies and led them to apply what they found to the subsequent problems.

The Korean Mathematics Textbook Series

Table 9 summarizes representative examples of division procedures presented in the Korean textbook series. Division of whole numbers, division of fractions, and division of decimal numbers were introduced in order. The kinds of numbers (i.e., whole numbers, fractions, and decimal numbers) are different, but the overall procedure of addressing division with such numbers is similar. Firstly, a problem situation is presented in a word problem. Two or three activities to solve the given problem follow. Here the first activity often includes using concrete materials or demonstrating visual representations for students to understand the problem situation. The second or third activity addresses various methods to solve the problem including the standard algorithm at the end.

Table 9. Procedures of division in the Korean mathematics textbooks

Whole Numbers	Fractions	Decimal Numbers	
48÷3 (2016b, p.54-55)	$\frac{4}{5}\div\frac{4}{15}$ / $\frac{2}{3}\div\frac{5}{7}$ (2015b, pp.48-49)	8.6÷5 (2015a, pp.122-123)	3.68÷0.46 (2015b, pp.72-73)
• Drawing (making 3 groups from 48) 	• Representing the problem in fraction bars and grouping by $\frac{4}{15}$ 	• Estimation	• Repeated subtractions Q. *How many times can you subtract 0.46 from 3.68 to make 0?*
↓	↓	↓	↓
• Base-ten blocks 	• Changing to division of whole numbers $\frac{4}{5}\div\frac{4}{15}=\frac{12}{15}\div\frac{4}{15}=\square\div\square=\square$ • Changing to multiplication of fractions $\frac{2}{3}\div\frac{5}{7}=\frac{\square}{\square}\times\frac{\square}{\square}$	• Changing to multiplication of fractions $8.6\div5=\frac{\square}{10}\div5=\frac{\square}{10}\times\frac{1}{5}$ $=\frac{\square}{50}=\frac{\square}{100}=\square$ • Comparing to division of whole numbers $860\div5=\square$ $8.6\div5=\square$	• Transformation of unit (m→cm) 3.68 m= $\square$ cm, 0.46 m= $\square$ cm ⇨ $3.68\div0.46=368\div\square=\square$ • Changing to division of fractions $3.68\div0.46=\frac{368}{100}\div\frac{46}{100}$ $=\square\div\square=\square$
↓		↓	↓
• Vertical form 16 3)48 3 18 18 0		• Vertical form 1.72 5)8.6 5 36 35 10 10 0	• Vertical form 8 0.46)3.68 368 0

From the perspective of connections, division of whole numbers was connected to division of fractions or division of decimal numbers. Knowing how to perform division of whole numbers serves as the foundation for understanding division of fractions and division of decimal numbers. For example, in the process of calculating division of fractions, division of whole numbers was used after reducing a common denominator and dividing between numerators.

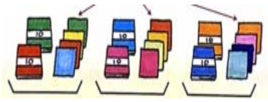
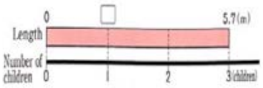
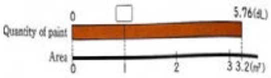
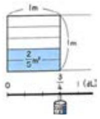
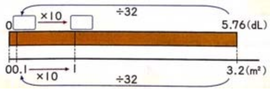
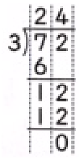
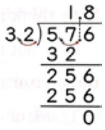
Division of fractions and division of decimal numbers interconnect with each other. Division of decimal numbers can be calculated by changing the given decimal numbers into fractions and simply applying the procedure for division of fractions. As each learning theme dealing with division of decimal numbers in the Korean textbooks includes at least one strategy of changing decimal numbers into factions, understanding how to perform the procedures and why the procedures work can lead to success in conducting division of decimal numbers.

The Japanese Mathematics Textbook Series

Table 10 shows representative cases of the procedures of division in the Japanese textbook series. Pictures, tables, or rectangles were frequently used to describe the given problem situations. After that, 2 or 3 approaches to solve the problems were displayed and their strategies were discussed. It ended by summarizing principles or methods of division, which were sometimes formalized algebraically.

Note that, unlike the Korean series, the Japanese series introduced division of whole numbers, division of decimal numbers, and then division of fractions in order. As division of decimal numbers preceded division of fractions, division of decimal numbers was not solved by changing the decimal numbers into fractions. Instead, 'the rules of division' were applied frequently from the beginning of introducing division of whole numbers: "*In division, the answers (quotients) are the same if the dividend and divisor are multiplied or divided by the same number.*" (Hitotsumatsu et al., 2011b, p. 23) This implies that division of whole numbers, fractions, and decimal numbers are

Table 10. Procedures of division in the Japanese mathematics textbooks

Whole Numbers	Decimal Numbers		Fractions
72÷3 (2011b, pp.44-45)	5.7÷3 (2011c, pp.63-64)	5.76÷3.2 (2011d, p.70)	$\frac{2}{5}\div\frac{3}{4}$ (2011f, pp.43-45)
• Representing problem situations 	• Representing problem situations 	• Representing problem situations 	• Representing problem situations 
↓	↓	↓	↓
• Applying the distributive property of division over addition 72÷3 < 60÷3=□, 12÷3=□ Total □	• Figuring out the length each of the children get by using 0.1 and dividing 10 • Using the rules of division	• Figuring out how much dL will be needed by using $0.1m^2$ and multiplying 10  • Applying the rules of division 5.76÷3.2=□ 10 times ↓ ↓ 10 times 57.6÷ 32 =□	• Regarding $\frac{3}{4}$ as an operator • Figuring out how many unit fractions are used by looking at the figure • Using the rules of division and changing fractions to whole numbers
↓	↓	↓	↓
• Vertical form 	• Vertical form	• Vertical form 	• Algebraic formalization $\frac{B}{A}\div\frac{D}{C}=\frac{B}{A}\times\frac{C}{D}$

interconnected by making use of the rules of division. In this way, division of fractions or division of decimal numbers can be calculated in the same manner as division of whole numbers.

DISCUSSION

Given the importance of connections among mathematical ideas, a promising result in this study was that the meanings of division and the procedures of division were connected from whole numbers to fractions or decimal numbers in the elementary mathematics textbooks of both countries.

First of all, regarding the meanings of division, although some meanings need to be adjusted as division extends from whole numbers to fractions, both partition and measurement division were consistently addressed across the division of whole numbers, the division of fractions, and the division of decimal numbers. Considering that it is important for students to experience mathematics in a context (NCTM, 2000), this result implies that the textbooks are expected to give opportunities for students to recognize the diverse contexts of division and to come up with expressions of division regardless of what types of numbers are included.

Note that there is a difference between the Korean and Japanese textbooks in presenting problem situations related to division. In the Korean series, two typical meanings (i.e., partition and measurement division) were consistently employed across almost all of division situations. However, the Japanese series included other kinds of meanings such as determination of a unit rate or inverse of Cartesian products when division was expanded into factions. On one hand, from the perspective of connections, the Korean textbooks are expected to give students opportunities to connect the essential meanings of division in a coherent way as the ranges of numbers are extended. On the other hand, Korean students may experience difficulties in gaining a conceptual understanding of division of fractions, because some meanings of whole-number division such as partition are not sufficient for interpreting division of fractions (Sinicrope et al., 2002). In this case, it may be informative to consider other interpretations for the division of fractions, as found in the Japanese textbook series.

Another prominent result was that procedures of division from whole numbers to fractions or decimal numbers were connected in different ways in Korea and Japan. In the Korean series, procedures of whole-number division were used in calculating division of fractions and division of decimal numbers. The processes of fraction division were applied when figuring out quotients of division of decimal numbers by changing the decimal numbers into fractions. In the Japanese series, however, the rules of division for whole numbers were used frequently. Unlike the Korean textbooks, the Japanese textbooks did not introduce a method of changing decimals into fractions when dealing with the division of decimal numbers. For instance, 320÷1.6, the Japanese textbooks solved it by multiplying 10 to both 320 and 1.6 and making 3200÷16. In contrast, the Korean textbooks changed the decimal into a fraction and treating the division such as dividing between numerators: $\frac{3200}{10}\div\frac{16}{10}$=3200÷16 for an instance.

These differences may result from the sequence of division. In the Korean textbook series, division was introduced in the order of whole numbers, fractions, and decimal numbers. In the Japanese textbooks, however, division of decimal numbers was presented earlier than division of fractions. Procedures of fraction division were not utilized in solving division of decimal numbers, and rather was emphasized changing decimal numbers into whole numbers by making use of rules of division. More fundamentally, these differences may come from different perspectives of how a decimal number is defined in each textbook series. The Korean textbooks defined decimal numbers as another name of fractions (MOE, 2016a) and they constantly emphasized connections between decimal numbers and fractions. Their counterparts in Japan, however, defined decimal numbers with units smaller than natural numbers without relating to fractions.

These results raise the issue of how to deal with division of decimal numbers: Focus on connections with division of whole numbers or division of fractions? On one hand, according to Stevin (1540-1620) who defined decimal numbers, operations of decimal numbers can be calculated using only decimal numbers, while fractions may be employed when justifying the results of such operations (Sanford, 1921). On the other hand, decimal numbers have usually been treated as another name for fractions, even calling them as 'decimal fractions'. Furthermore, fractions have been utilized whenever calculating decimal numbers (Resnick et al., 1989; Van de Walle et al., 2008). From the perspective of connections, the Korean textbooks are expected to focus on connections between decimal numbers with fractions, whereas the Japanese counterparts pay attention to the connections between decimal numbers with whole numbers. These imply that different conceptions or emphases of a mathematical topic may lead to different construction of textbooks.

In conclusion, this study is expected to provide information on how division from whole numbers to fractions and decimal numbers are addressed in the Korean and Japanese textbook series. The study also gives implications for how to connect division meaningfully in mathematics instructional materials.

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