

A Content Analysis Study about the Usage of History of Mathematics in Textbooks in Turkey

Mehmet Eren, Mehmet Bulut & Neslihan Bulut
Gazi University, TURKEY

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The present study aimed to investigate how history of mathematics was integrated to some mathematics textbooks in Turkey. On this account, four different textbooks with different grade levels were chosen. In total, 42 cases were detected and studied by three researchers. Results indicated that the usage of history of mathematics was materialized in six different manners: giving mathematics a human face, showing how concepts have developed, developing a multicultural approach, explaining the roles of mathematics in society, changing the perceptions of mathematics and providing opportunities for investigation. Among these manners, it is observed that giving mathematics a human face is the most frequent manner, while the least apparent one is providing opportunities for investigation.

Keywords: History of mathematics, mathematics education, mathematics textbooks.

THE HISTORY OF MATHEMATICS IN TEXTBOOKS IN TURKEY

$\forall n \in \mathbb{N}, ((n \geq 2) \wedge (n \text{ even})) \Rightarrow (\exists p, q \in \mathbb{P}, n = p + q)$ where \mathbb{N} is the set of natural numbers and \mathbb{P} is the set of prime numbers.

Imagine that in an 8th grade classroom, the topic is prime numbers. The teacher shares that, by putting what the above formula says in words, every even integer greater than 2 can be expressed as the sum of two primes. This is to some extent an effort that should be appreciated because it gives a chance for students to challenge their knowledge about prime numbers.

Imagine also that in another 8th grade classroom in which students are learning prime numbers. The teacher also wants to share the above statement with the students, yet in a different manner. Adding some mysterious parts, he or she starts to quote related part in the textbook in an excited voice: "In 1742, in a very rainy day, Cristian Goldbach was writing a letter to

Leonard Euler. In the letter, he was talking about a strange guess. That was every even integer greater than 2 can be expressed as the sum of two prime number. All the mathematicians worked on this strange guess could not prove it yet. Can you find 20 examples supporting this guess of Goldbach?" (Adapted from case 4071).

What do you think about the dissimilarity between these two scenarios? From what perspective, they are different from each other? We hope you will derive your answer for these questions as you go through the pages of this study.

The History of Mathematics in Literature

History of Mathematics to Give a Human Face to the Discipline

It is generally proposed by many researchers that using the history of mathematics gives mathematics a human face (Fauvel, 1991; Fried, 2001; Fenton, 2002; Russ, 1991). However, it can be useful to start by going beyond this definition with investigating the further interpretations of this perspective. Therefore, a broader viewpoint can be gained about this general idea. Firstly, it can be interpreted that mathematics is considered as "the part of the humanities where the focus is on the development of the individual rather than the acquisition of the expertise" (Fenton, 2002, p. 254).

Correspondence to: Mehmet Eren;
Gazi University, Gazi Faculty of Education, Primary
Education in Mathematics,
Ankara, Turkey
E-mail: mehmeteren@gazi.edu.tr
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State of the literature

- Giving mathematics a human face and showing how concepts have developed are two of the most chief logics behind the reason of using history of mathematics for both learning and teaching it.
- Two main strategy, addition and accommodation, are commonly defined so far for the integration of history of mathematics to mathematics teaching and learning.
- One of the foremost functions of textbooks is giving chances to its readers to internalize the presented information.

Contribution of this paper to the literature

- This paper draws a clear image about the integration strategy, which is on “addition” level, used for the integration of history of mathematics in some textbooks in Turkey.
- The results of this study shows that more emphasis should be given for “providing opportunities for investigation” part in the usage of history of mathematics.
- This study opens a new window for making textbooks more reader oriented by analysing them in terms of using history of mathematics.

Nevertheless, to some, this interpretation may not be clear in terms of its implication in the learning of mathematic in the classroom. An interesting, yet quite different perspective argued by Fauvel (1991) about its implication: Pupils gain comfort from knowing that they are not the only ones with problems. Also, this idea is corroborated by the study of Bakker and Gravemeijer (2006) which points that, thanks to the knowledge that it took many years for great mathematicians to figure out solutions, students may decrease their frustration of having troubles in mathematical problems. With the help of such comforts, students can deal with problems with a more enthusiastic character. Also, another interpretation can be learning that some important ideas in mathematics were simply achieved through human-specific actions such as exchanging letters or friendship minimizes the students’ complaints arising from their perceptions of mathematics as an unnatural discipline which is quite different than what they normally do as humans. To organize the argument and ground it on a more concrete base, the following example can be given: The general answer given by vast majority of the students about what Pythagoras’ Theorem means is that a squared plus b squared equals c squared. However, the idea that Pythagoras is an ancient Greek Mathematician whose ideas greatly reflecting the Greek’s desire for proof would be a great additive to Pythagoras’ Theorem

because of the arguments presented previously about the ‘giving mathematics a human face’ issue. In the following part, the role of history of mathematics in the development of concepts will be touched upon.

Case 4071

History Corner

In 1742, Christian Goldbach mentions about a strange inference in his letter to Leonard Euler. He says every integer greater than 2 can be expressed as the sum of two prime numbers.

It has not proven yet by mathematicians. Can you find 20 examples supporting this inference? (Özgin, 2007)

History of Mathematics in the Development of Concepts

In this section the role of history of mathematics is analyzed from the perspective of the development of mathematics. This issue is clearly formulized by Fauvel (1991) as drawing a clear picture of how the concepts developed foster students’ understandings. This idea is also highlighted by some researchers on the basis of evolutionary arguments which claim that “there can be no learning of mathematics without history” (Jankvist, 2009, p. 238). In addition, some researchers takes this issue one step further and discussed the connection between the difficulties mathematicians faced in the history and the problems of today’s learners (Thomaidis, & Tzanakis, 2007). Hence, it is emphasized that history of mathematics plays an indisputable role for shaping the framework of the mathematics as a discipline in the eyes of today’s teachers and learners.

To put these two arguments about why to integrate history of mathematics on a sound footing, it may be beneficial to highlight how they are stated in the literature. Jankvist (2009) gives a clear image for these arguments with the help of his term “indispensability of the whys” (p. 245). With this term, he analyses Fauvel’s Model (Fauvel, 1991), with the claim that showing how concepts have developed and giving mathematics a human face for motivational purposes are the predominant ‘whys’ behind the reason for the integration of history of mathematics. For that reason, the intent here is to help readers to form a general picture about the history of mathematics in the literature.

Succinctly to say, even though some studies are decomposed on the point of what the most dominant aim should be for the use of history of mathematics in mathematics education, the literature (e.g., Bakker & Gravemeijer, 2006; Furinghetti, 2007; Heiede, 1992; Jankvist, 2009; Miller, 2002; Weng Kin, 2008) mostly agrees on the notion declared by Augustus De Morgan, the first president of London Mathematics Society, in 11th January of 1865:

I say that no art or science is a liberal art or a liberal science unless it is studied in connection with the mind of man in past times. It is astonishing how strangely mathematicians talk of the Mathematics, because they do not know the history of their subject. By asserting what they conceive to be facts they distort its history in this manner. There is in the idea of everyone some particular sequence of propositions, which he has in his own mind, and he imagines that the sequence exists in history; that his own order is the historical order in which the propositions have successively been evolved. (As cited in Weng Kin, 2008).

The History of Mathematics in Mathematics Education

Besides giving mathematics a human face and showing how concepts have developed, the researchers study the role of history of mathematics in teaching and learning mathematics. At this point, two general trends seem to be apparent in the literature which are addition and accommodation as Fried puts in words (2001). In the former strategy, the history part is presented in an isolated manner in the main structure of the content. This addition strategy is mainly applied as presenting related history part in anecdotes, biographies or context-bounded problems. On the other side, the later strategy, accommodation, requires basing the flow of the content on its historical development. Thus, it entails broader change in the design of the curriculum as Katz (1993) argues:

By an historical approach to calculus, I do not mean simply giving the historical background for each separate topic or giving a biographical sketch of the developers of various ideas. I do mean the organization of the topics in essentially their historical order of development as well as the discussion of the historical motivations for the development of each of these topics, both those within mathematics and those from other scientific fields (p. 243).

Considering the integration of history of mathematics to the mathematics teaching and learning, it can be stated that there are two main components: teachers and students. From this perspective, apart from ‘addition and accommodation’ heading it seems that there is also a categorization for the use of history of mathematics. First, it can be taught as a separate subject. However, this idea is commonly related with the teacher education programs rather than presenting this to a student course level. This idea is reasonably supported by Thomas (2002) as “mathematical facts without understanding why they are the way they are, are impossible to learn” (p. 46). Thereby, it is useful for teachers to know history of mathematics to have a complete picture about the fact they are going to teach. Also, the proponents of ‘separate subject on teacher education level’ idea support their rationale by the studies showing that students’ ability to think in

historical terms is not developed enough to build a strong reasoning before the ages of fifteen (Partington, 1980).

The Textbooks’ Role

As it was mentioned previously, there are two components for the integration of history of mathematics: teacher and students. However, as equally important, the dialectical relationship between these components should be also structured efficiently to create a successful integration. By feeding the nexus between the teachers and the students, the textbooks can play a significant role to boost the effectiveness of the history of mathematics in mathematics learning and teaching. In addition, this nexus is vastly formulated in related literature by many researches (e.g., Fan, & Kaeley, 2000; Li, Chen, & Kulm, 2009; Li & Zhang, 2009; Stein, Remillard, & Smith, 2007; Törnross, 2005). Among these studies, the previously mentioned nexus is backed up by argument that textbooks, not only affect what to teach, but also affect the way to teach (Fan & Kaeley, 2000). Despite the growing research tendency to analyze the role of textbooks for effective learning environment, this study is unique in terms of analyzing mathematics textbooks from the perspective of usage of history of mathematics.

Within a theoretical framework, to understand the role of textbooks the reader-oriented textbook theory proposed by Weinberg and Wiesner (2011) is taken into consideration. This theory provides a more functional lens for this study because the core idea of this theory is that textbooks are not only a source of mathematical definitions, theorems or operations. Indeed, successful textbooks need to offer some effective ways to its readers to internalize the presented information (Erbaş, Alacacı & Bulut, 2012). Also, the internalization aspect of textbooks is clearly stressed by the study of Tyson & Woodward (1989) which postulates that “textbooks, of course, are the messengers, not the message” (p. 14). As a result, the purpose of this present study is to investigate how the history of mathematics is used in some mathematics textbooks in Turkey in the aim of creating opportunities for students to internalize the presented information in textbooks (Further information about these textbooks will be given under ‘Decision for textbooks’ section).

METHODOLOGY

The general structure of the present study is decided as a content analysis. “Content analysis is a research technique for making replicable and valid inferences from texts (or other meaningful matter) to the contexts of their use” (Krippendorff, 2004, p. 18). Therefore, in this study, it is planned to draw a picture and to make

Table 1. The Textbooks

Book code	Book Name	Publisher	Date	# Cases
1000	Mathematics Course Book 8 th Grade	Hayalgücü	2010	10
2000	Geometry 11 th Grade	Aydın	2011	9
3000	Mathematics Course Book 7 th Grade	Evrensel İletişim	2007	8
4000	Mathematics Course Book 6 th Grade	Özgün	2007	15
Total				42

inferences about the use of history of mathematics in textbooks. Cohen et al. (2007) succinctly describes the processes of a content analysis as “coding, categorizing, comparing, and concluding” (p. 476). In the following parts, firstly the information about the codes will be stated. Then, you can find information about the categorization of the initial codes. Under “Results” heading, the compare and contrast of these categories, and the history as a separate discipline will be stated. Lastly, the conclusion of findings and the reflection for further research will be discussed.

The Decision for Textbooks

The books intended to use the history of mathematics are selected for the study. It is underlined by the researchers that the data inevitably should provide functional evidence for testing the hypotheses or responding the research questions (White & Marsh, 2006). Hence, it would be meaningless to include the books that do not show any interest in the integration of history of mathematics. In this aim, 42 cases from four various books are investigated. The information about the textbooks can be seen in Table 1.

The Codes

The first step to start to analyze the data is to determine the parts to be classified. To many researchers, this step is defined as one of the most essential decisions for coding (e.g.; Cohen, Manion, & Marrison, 2007; Weber, 1990). For the present study, “syntactical unit”, as Rourke, Anderson, Garrison, and Archer put in words (2001, p. 16), such as a sentence, a word or a phrase is used to determine the codes. At the end of this process, 17 codes are determined (Table 2).

To categorize the codes, the cases analyzed again through the lens of initial codes. As a result of this iterative process, as White and Marsh (2006) puts in words ‘hermeneutic loop’ (p. 34), the main intent is to build a relationship among codes to see, behind the foreshadowing indicators, the actual conceptual

framework. In this concern, the model about the reasons for using history in mathematics education proposed by Fauvel (1991) is mainly benefited. In his models, he mentions 15 reasons why to use history in mathematics education. Among those reasons, six of them are selected to use as superordinate categories:

Giving mathematics a human face: pc, hs, coo.

Showing how concepts developed: concept, cumu, dynamic, proof, significance.

Developing a multicultural approach: iac, co.

Explaining the role of mathematics in society: need.

Changing the perceptions of mathematics: mdf, career, iad, ct.

Provides opportunities for investigations: encour, ref.

Lastly about the codes, the accuracy and consistency among coders is one of the core issues in the process of coding (e. g, Strijbos, Martens, Prins, & Jochems, 2006; Weber, 1990; White, & Marsh, 2006). To ensure this conceptual consistency, the researchers arranged three meetings. In first one, the codes were studied on five random cases from the books so that the meanings of the codes are clarified. In the case of conflict, the researchers discussed the case again until full agreement was reached. In the other two meetings, the coding of the whole cases was presented by three researchers. Thus, the conflicts about the cases were resolved in those meetings before the analysis was conducted.

RESULTS

Giving mathematics a human face

Case 3055

The Dignitaries of Mathematics -Blaise Pascal (1623-1662)

...When he was a child, he asked his father what geometry tries to explain. His father replied as “drawing shapes correctly and the relationship between the parts of those shapes.” With the inspiration of this answer, he started to construct geometry theories.

After a while, his father noticed his gift in geometry and gave him Eukleides’ “Elements” and Apollonius’ “Conicals”

In his free time after language courses, he wrote an article about Conicals when he was 16. Descart, at first glance could not believe that the author of this article was only 16 years old. In his 19, he invented a calculator for arithmetic operations. ... (İnci, 2007).

Table 2. The Explanation for Codes

Exemplary Case No.	Abbreviation Used for the Code	Codes	Brief Explanation for the Code
1054	pc	Person-Centered	There is a mathematician at the center of the case. The life of that mathematician, the discoveries and contributions are mentioned.
2004	hs	Human-Specific	Any action that reminds human-specific activity such as “he uses that word for the first time” or “he stands up to his father to get a career in mathematics.”
1234	coo	Cooperation	There is an active relationship between another scientist/person so they share their ideas about a problem.
3200	concept	Concept-centered	There is a mathematical concept at the center of the case such as a pyramid, probability or astronomy.
4178	cumu	Cumulative	Cumulative property of the discipline. A mathematician adds a new perspective to an existing phenomenon.
4176-7	dynamic	Dynamic Process	There can be a change in a concept. A mathematician criticizes an existing proposition so it can be replaced with the new idea.
4207	proof	Proof	Proof- based. A proposition is scientifically proved by a mathematician.
4071	sign	Significance	A statement which indicates the importance of the unit in the history of the concept. Example: “for the first time in the history” or “one of the most important discoveries in the history”
4116	co	Culture oriented image	Any model, idea, material or tool which is specific to a particular culture, nationality or region.
3102	iac	Interaction Among Cultures	The relationship or sharing among different cultures, nationalities or regions.
4142	need	Need	Any statement explains the role of the discipline in the real life situations. Example: using mathematics to construct buildings, canals, or to compute the area of lands.
2036	mdf	Multi-discipline focus	A mathematician is interested in different disciplines such as physics, philosophy, or medicine.
3055	career	Career Message	Any statement which intends to convey a message about the career in mathematics.
2155	iad	Interaction Among disciplines	The relationship between mathematics and different disciplines.
4033	ct	Critical thinking	Any question that provokes the critical thinking about the mathematical topic in the case.
1119	encour	Encouragement for further research	Any statement that encourages the reader to make further research.
1119	ref	Reference	The resources used for the case are stated. These can be a website, a book, or a journal.

As it was mentioned in the history of codes, this intention ‘giving mathematics a human face’ has three dimensions in the study. In Table 3, the frequencies and percentages are presented for this heading. It is apparent

that the books analyzed for this study usually stays at the level of just introducing the related historical material by putting a character at the center of it or by reflecting an active relationship. Although it is rare

(about 8%), there is also an intention to take the human character one step further by stating an active relationship with other people. Specifically talking, this active relationship is, defined as ‘coo’ code, accomplished by letters between mathematicians (case 4071) and their responses to each other by writing (case 3055; case1234).

Case 1234

...In 1650s, the most popular gamblers in France think that mathematics can help them. The mathematicians like Pascal, Fermat, D’alembert, and De Moivre took on the problem so the probability took form. The definition of probability is formulized in the correspondence between Pascal and Fermat in 1654...(Güler, & Yücelyigit, 2010)

Showing how concepts developed

In the distribution of this code, the most dominant application is using the stress for the historical importance as it was denoted by ‘sign’ code (Table 4). Also, it is interesting that there is an exact correspondence with the argument proposed by Ernest (1984) that the history of mathematics also aims to challenge the myth of mathematics as perfectly finished body of knowledge, in the light of the difference between the frequencies of “dynamism” and “proof”. Thereby, the high frequency of ‘dynamic’ code and the low frequency of ‘proof’ code can be interpreted as an effort to create a conflict with the myth of mathematics as a dead discipline.

Case 4017

History Corner

Georg Cantor (1845-1918)

Cantor decides to pursue a career in mathematics despite his father. He gets his PhD degree when he is 22 years old. Even though he really wants to be a professor, he fails because the subjects he works are not that popular among the mathematicians of that time. ... (Göğün, 2007).

Changing the perceptions of mathematics

Different from giving the frequencies and perceptions, in this part, some findings about four codes will be mentioned. The ‘career’ code has the potential to offer an analysis. The frequency of this code is four, that’s to say in the four different parts of the cases readers are given the inspiring message to follow a career in mathematics. However, in the two cases (case3055, case4017) it is given as the resistance to the obstacles put by their fathers not to follow a career in mathematics. On the other side, there is also a positive example in one case (case 4049) which talks about that Nightingale Florence is very supported by her father who believes that women should definitely be educated. This particular example is also valuable because among all 42 cases, Nightingale Florence is the only woman who deals with the mathematics. In consequence, this concept, changing the perception of mathematics, can highly be questioned for its effectiveness about comprehensiveness in terms of the audience.

Case 4049

History Corner

Nightingale Florence (1820-1910)

The women at their ages do not usually prefer to follow a career in a university or higher education. However, thanks to her father-who thinks that every woman must have education, she and her sister effectively get a mathematics education. Besides nursery, she uses statistics to make reforms in health. She collects data during war and tries to record the data in a systematic way. She used those data, by demonstrating them in graphs, to improve the conditions in the hospitals. This was a very new method for statistics for that time. (Göğün, 2007)

Also, ‘ct’ code can be investigated separately. In the general picture for this concept, this code is used alone, except for one case (case2104), with a view to changing

Table 3. Giving Mathematics a Human Face

Codes	Frequency	Percentage
P.C	21	46 %
H.S.	21	46 %
Coo.	3	8 %
Total	45	

Table 4. Showing How Concepts Developed

Codes	Frequency	Percentage
Concept	8	22 %
Cumu	3	8 %
Dynamic	10	28%
Proof	2	6 %
Sign	13	36 %
Total	36	

the perception of mathematics. This may be due to the fact that since it is aimed to create a different perspective about the content, it would be confusing to use another code for the same issue, thanks to these critical questions. In the exceptional case, together with ‘ct’ code, ‘iad’ code is used by stressing the relationship between the classification method of living things and the classification in mathematics. Nevertheless, the general tendency to use this ‘ct’ code alone also can be read as it may be supportive to use another code because, for example ‘iad’ code, it may give a different view to the students to approach the critical question.

Case 2014

... The first scientific classification in the history is made by Aristoteles in his book “Organon”. After his classification of living things, the similar classifications also made in mathematics, e.g. the classifications of numbers as even, odd, or prime numbers. Recently, a similar classification is made in geometry. Think about what is needed to be taken into consideration in a scientific classification and try to figure out a way to classify rectangular (Aydm, Camus, & Biberoglu, 2011).

Case 3161

Nile in Egypt is one of the most important rivers in the world. Nile is an indisputable source in such a desert. That is why, for centuries Nile serves as the main source for many civilizations. As it is today, the agriculture was developed near the Nile in ancient Egypt. However, in winter, when Nile floods, it sweeps away the signs farmers put to distinguish their lands from others. So, they figure out a solution for that.

What kind of solution you think those farmers might find? If you were one of them, what would you do? (Inci, 2007).

Explaining the role of mathematics in society

Since there is only ‘need’ code under this superordinate heading, it is not possible to talk about a distribution. However, it may be something valuable to explore how this code is used with other codes. In general, it can be said that the role of mathematics is explained around ‘mathematics as a human activity’ or ‘the development of concepts’. Thus, it can be inferred that need is highly integrated with practical purposes. Specifically talking, one case (case3161) uses this need code serves as context for problem solving. This also reveals the role of history in mathematics education as creating a meaningful context for the problems (Fried, 2001). Lastly, in one case (case4193), this code is used alone without any support from other concepts. In such a case, it is not difficult to propose that the rationale about the claim for need is weak in the eyes of students.

Case 4193

Area calculation is firstly used in Egypt to calculate the area Nile floods over, to calculate the land taxes, and to construct pyramids; in Babylon, to construct houses and canals (Göğün, 2007).

Developing a multicultural approach

One of the reasons for the integration of the history of mathematics is to show that mathematics is multicultural in its origins rather than being a product of Europe as many believe (Ernest, 1988). This is also the case for the message given in the books analyzed in this study. However, it is usually on the level of stating country names or some civilizations such as Babylon, Mayas or Mesopotamia. At the interaction among cultures level, the interaction among The Greek, The Babylon, The European and The Arabic are commonly stressed. From these sets of interactions, it can be derived that the familiarization of the target group to these cultures, to a certain extent, is taken into consideration. Otherwise, it would be very expected to present the Indians and Chinese whose works are keeping a considerable place in the scope of the history of mathematics.

Providing opportunities for investigations

This is one of the most crucial phases of the story the cases tell because it is possibly this part where it will be decided whether to go beyond what is written in the case. If it can make the reader go beyond, then the transfer issue of learning can be drastically encouraged. Furthermore, with a broader lens, through providing opportunities for further investigations, the readers may be helped to gain the tendency for taking initiative for their learning. Hence, this tendency for research will undoubtedly characterize their learning in other disciplines as well. As a result, even though it seems quite vital in the story, this is, unfortunately the part where the voice of cases is the hoarsest form with only one case by both encouragement for further research and a reference (case1119).

Case 1119

Pythagoras is thought to be born in B.C. 596 in Samos Island. He starts his education in Samos Island, and then he moves to Egypt and Babylon. Finally, he moves to Croton in Italy and he opens his famous school there. There, he makes reforms in mathematics, physics, astronomy, philosophy, and music.

Pythagorous’ theorem is one of most significant discoveries in mathematics, especially if it is considered that it is 2600 years ago. (Güler, & Yüceliyiğit, 2011)

The history as a separate discipline

Up to present, you may question the role of history discipline to increase the efficiency of mathematics teaching and learning. The same concern is vocalized by some researchers in the argument that history has its own problems as a separate discipline (Fenton, 2002). Therefore, this issue is also analyzed in the cases from the perspective of how the pure history is incorporated.

For this analysis, three types of integration are detected in the cases. First one is giving the exact dates for an event or a person such as “1889-1978”. In the second type, an approximate date is given in such forms of “in 1900’s”. Lastly, without giving any dates, the historical sense is given in the phrases like “the first time in the history” or “ancient times”. According to that classification, it is found that exact date is used 10 times; approximate date is used 15 times; historical sense is used 17 times. The result that ‘exact date’ type has the lowest percentage can be interpreted as the evidence of the effort to offset the problems of the history as a separate discipline in the eyes of the students whose attitudes towards history is relatively lower compared to their peers.

CONCLUSION & DISCUSSION

As mentioned in the introduction, there are two trends in the integration of history of mathematics: Addition and accommodation (Fried, 2001). By looking what the results recommend, it is not hard to conclude that the cases in these books are greatly in the form of addition strategy. As well, Jankvist makes a similar categorization in his study with headings “the illumination, the modules and the history based” (2009). In his categorization (2009), he makes an analogy that “one way to think of these smaller supplements in the illumination approaches is to think them as “spices” added to the mathematics education casserole” (p. 246). Consequently, this can be perceived as an expected result since the later strategy, accommodation, or the modules or the history based approach, requires more than just adding some cases serving various aims discussed so far. Furthermore, a meaningful analysis for the accommodation strategy can be conducted at the curriculum level, which is beyond the scope of this study. In its small scale, this study shows that giving mathematics a human face is the most leading reason behind the integration of history of mathematics to textbooks while showing that the least concern is given to the encouragement for further research. Concisely, in the light of Jankvist’s term (2009, p. 246), the history of mathematics in the cases of this study is a spice added to the mathematics education casserole, not a brand new cuisine for mathematics education. In the next paragraph, a few words will be said about the heading “providing opportunities for investigation.”

The result of this study supports the literature that showing how concepts have developed and humanizing mathematics are the chief reasons for the integration of history of mathematics (Jankvist, 2009). On the other hand, this study, in the related body of research, opens a new window to demonstrate “providing opportunities for investigation”. In the Fauvel’s model (1991), even though it is one of the items of “why to integrate

technology”, in a very weak argument it is stated. However, this study underlines the importance of this section in the light of internalization of the presented information in the textbooks as reader-oriented theory mentions. In the process of internalization, this section is vital to make learners take initiative to go further than the given case for history of mathematics. In this study, there is only one case meeting the criteria of this heading (case1119). In this case, the internalization process is getting stronger by stating a source name and a question that triggers students to make further investigation. This case can be a model for further attempts to include history of mathematics to the textbooks. However, further research need to be conducted to solely analyze this process of internalization.

Besides the fact that the teachers and students can greatly affect the ultimate outcome of the integration by their attitudinal stance on the issue of history of mathematics, this study assumes the ideal atmosphere in the classroom in terms of teachers’ and students’ stance. The other shareholders, namely students and teachers, need to be further studied. Combining what those studies will propose and the results of this study, the most ideal use of history of mathematics in mathematics education can be constructed. Therefore, the textbook designers can use the findings of this study to publish more reader-oriented textbooks by making their product more comprehensive in terms of history of mathematics.

Let’s go back to the scenarios in the beginning of this paper. Do you think your initial answers changed after reading this paper?

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Appendix: The codes for 42 cases (The first digit of case number is the code for the books and the last three are the page numbers the codes belong)

Case No	PC	Human	Coo	Concept	Cumu	Dynamic	Proof	Sign	Co	Iac	Need	Mdf	Car-Eer	Iad	Ct	Encour	Ref
1013		1															1
1024									1						1		
1054	1	1							1					1		1	
1091-2	1	1				1			1								
1119	1	1						1	1	1		1				1	1
1127						1			1						1		
1152				1					1						1		
1177				1					1								
1194	1	1												1			
1234		1	1	1				1			1				1		
2004	1	1												1			
2005	1	1			1					1						1	
2036									1		1						
2046				1													
2075									1								
2104		1						1						1	1		
2115				1										1			
2148	1																
2214	1										1						
3055	1	1	1					1			1	1					
3101	1	1						1	1			1					
3102	1	1						1	1	1	1						
3161									1		1				1		
3172				1				1	1		1			1			
3187	1	1				1		1	1	1		1					
3200				1							1						
3217	1	1				1		1	1	1	1	1					
4017	1	1				1		1					1				
4033	1	1													1		
4049	1	1						1			1	1	1				
4071	1	1	1			1		1							1		
4082									1								
4116						1			1	1							
4132	1	1															
4142	1								1		1		1				
4143						1									1		
4150	1								1			1					
4176-7				1	1	1											
4178	1	1			1			1				1					
4193									1		1						
4200		1					1										
4207						1	1										