

A refined framework for qualitative content analysis of mathematics textbooks

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Abstract

Within the realm of mathematics education, the examination of textbooks has emerged as a focal point of scholarly investigation. This endeavor necessitates the development of a comprehensive instrument capable of facilitating both qualitative and quantitative textbook analysis. To address this need, we draw upon a corpus of seminal studies, and supplement our analysis with local research reports that assess the quality of mathematics textbooks through a qualitative content analysis approach. Content analysis was utilized to examine the documents and identify the themes relevant to the analysis of mathematics textbooks. Our data analysis revealed the identification of six overarching themes for the systematic evaluation of mathematics textbooks: (1) language and communication, (2) content analysis, (3) pedagogical approach, (4) cultural sensitivity and inclusivity, (5) assessment and exercises, and (6) visual aids and presentation. Each of these themes is meticulously expounded upon, shedding light on their significance in shaping the intricacies of mathematics textbook authorship. This proposed analytical framework presents an invaluable resource for enhancing the quality of mathematics textbooks at the school level. The themes elucidated herein offer a structured approach to textbook assessment, ultimately fostering the advancement of mathematical education by improving the instructional materials that are integral to the teaching and learning process.

Keywords: mathematics textbooks, textbooks analysis, qualitative content analysis

INTRODUCTION

Formal educational systems across the globe are composed of three fundamental components: curriculum, pedagogy, and assessment and evaluation. Although there are variations in educational standards among different nations, the curriculum invariably operates on three distinct levels: the intended curriculum (curriculum objectives), the implemented curriculum (what is conveyed in the classroom), and the achieved curriculum (what students are able to demonstrate) (Hadar & Ruby, 2019; Houang & Schmidt, 2008). Textbooks, serving as instructional tools, are meticulously designed for the facilitation of the teaching and learning processes.

Mathematics textbooks, typically distributed in printed form or, more recently, electronically, serve as comprehensive resources containing explanations and

exercises for student engagement. These textbooks assume a central role in guiding the pedagogical activities of educators and stand as the primary instructional conduit for teachers and students (Van Steenbrugge et al., 2013). Indeed, textbooks can be perceived as integral components of the implemented curriculum (Houang & Schmidt, 2008), effectively bridging the gap between the intended curriculum and its actual enactment (Schmidt et al., 2001).

Numerous countries have formulated rigorous guidelines for their mathematics curricula, such as China, England, France, and Germany, while others have adopted more flexible frameworks, as observed in Finland, Scotland, Singapore, and the United States. In all cases, these national curriculum guidelines serve as reflections of the intended curriculum. In select nations, the intended curriculum is translated into official school textbooks that offer exemplars of the national

Contribution to the literature

- This study contributes to the literature by proposing a comprehensive analytical framework for the systematic evaluation of mathematics textbooks. By drawing upon a corpus of seminal studies and local research reports, the framework incorporates both qualitative and quantitative elements, facilitating a thorough examination of textbook content.
- Through content analysis, this research identifies six overarching themes crucial for evaluating mathematics textbooks: language and communication, content analysis, pedagogical approach, cultural sensitivity and inclusivity, assessment and exercises, and visual aids and presentation. These themes provide a structured approach to textbook assessment, enhancing the understanding of the intricacies involved in mathematics textbook authorship.
- By elucidating the significance of each theme, this study offers insights into improving the quality of mathematics textbooks at the school level. The structured approach provided by the proposed analytical framework enables curriculum developers to assess and enhance instructional materials, thereby fostering the advancement of mathematics education.

curriculum. These textbooks, in essence, manifest the ethos and values of a nation, serving as vehicles for expressing the educational aspirations of government officials (Reese, 2011). The selection of content within textbooks is instrumental in shaping the educational system, affording distinct learning opportunities at varying costs. The contents of textbooks determine the components and methodologies of learning, consequently constraining or expanding students' learning experiences based on their prescribed textbooks.

Textbooks, serving as hegemonic educational artifacts, act as intermediaries between the prescribed curriculum and educational practices (Tárraga-Mínguez et al., 2021). As primary sources, textbooks exert profound influence in shaping the methodologies through which mathematics is taught and absorbed. Research endeavors dedicated to the scrutiny of textbooks seek to enhance pedagogical methodologies and subsequently enhance students' academic progress. For instance, certain international studies involving textbook analysis are characterized as meta-analyses, endeavoring to unveil the learning opportunities embedded within textbooks, contextualized within specific frameworks and their correlative effects on academic achievement (Son & Diletti, 2017).

Fan et al. (2013) have categorized research inquiries concerning mathematics textbooks into four distinct domains: the role of textbooks, analysis and comparison of textbooks, the utility of textbooks, and miscellaneous domains. Notably, the literature pertaining to the role of textbooks in mathematics pedagogy engages with philosophical and non-empirical discourse concerning the significance of mathematics textbooks. Analyses of textbooks delve into the facets of interest to researchers in the realm of mathematics textbooks. Comparative studies scrutinize the congruencies and divergences between two or more mathematics textbooks. Investigations focusing on the utility of textbooks examine how mathematics textbooks mold teaching and

learning modalities. The fourth domain, referred to as "other domains" in Fan et al.'s (2013) categorization of textbook research, encompasses a wide spectrum of inquiries, including those pertaining to electronic textbooks and their impact on students' academic accomplishments.

The pivotal role of textbooks in the teaching and learning of mathematics has been widely acknowledged by scholars. By and large, it is concurred that textbooks hold a central position as the primary transmitters of curriculum content in contemporary education across various disciplines, with mathematics taking a particularly prominent role (Robitaille & Travers, 1992 as cited in Fan et al., 2013). Indeed, textbooks occupy a preeminent position as a foundational resource in the pedagogical process, notably within the field of mathematics. They serve as the principal conduits for conveying mathematical concepts and skills to students, concurrently providing indispensable guidance to educators. The quality of textbook content can exert both positive and negative influences on student learning, rendering the evaluation and enhancement of textbook content a valuable endeavor (Foster et al., 2022; Mahmood, 2011). Textbooks, in particular, possess the potential to structure a coherent sequence of ideas and information, instrumental in teaching, learning, and knowledge organization (Sosniak & Perlman, 1990). Textbooks can guide and mold the readers' comprehension, thought processes, and emotional engagement, affording them access to knowledge that enriches their personal development and augments their political agency. Fan and Kaeley (2000) have observed that teachers employ a variety of teaching strategies contingent on the specific textbooks in use and have deduced that textbooks wield substantial influence in pedagogy, transmitting pedagogical messages and configuring either motivating or discouraging instructional environments for educators through their incorporation of varied teaching strategies. Scholars' conceptualization of the intricate interplay between

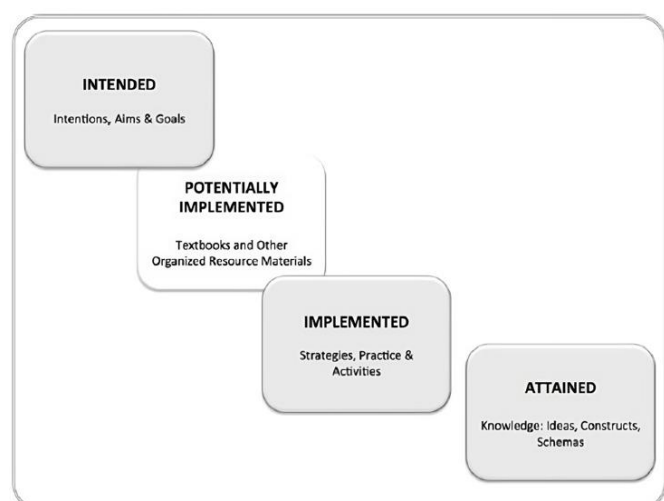


Figure 1. Textbooks–Potentially implementable curriculum (Houang, & Schmidt, 2008, p. 4)

textbooks and curriculum holds particular significance. Howson (1995) posits that textbooks stand one step closer to the classroom reality than the national curriculum itself, while Schmidt et al. (1997) argue that textbooks interweave the worlds of official educational intentions and the realities of classroom practice.

Role of Textbooks in Curriculum

A particularly noteworthy conceptualization, as developed by TIMSS (trends in international mathematics and science study), portrays textbooks as a potential implementable curriculum (Figure 1). Textbooks are meticulously designed to translate the abstract policies of the curriculum into practical operations that teachers and students can execute. They operate as intermediaries between the curriculum policy objectives established by curriculum designers and the educators who impart instruction in the classroom.

Textbook analysis, in its broadest sense, encompasses two major dimensions: firstly, the scrutiny of individual textbooks or collections of textbooks, often focused on how specific topics are addressed or how a given topic or aspect is represented within the textbook; and secondly, the comparative analysis of textbooks originating from a single country or multiple countries, frequently concentrating on the identification of commonalities and disparities among these texts. The latter variant, referred to as comparative textbook analysis, necessitates an analysis of each individual textbook, forming the basis for this comparative assessment (Fan et al., 2013).

Impact of Textbooks on Mathematics Achievement

While it is acknowledged that various factors, such as students, teachers, and schools, influence mathematical achievement, Yeap (2005) has underscored the pivotal role textbooks play in the impressive mathematical achievement of Singaporean students in international

assessments, including PISA and TIMSS. The prominence of textbooks as primary resources for learning mathematics is reflected in their function as the primary source of tasks that form the foundation for meaningful conceptual learning, essential mathematical skill acquisition, and engagement in critical mathematical exercises. To such an extent, the International Congress of Mathematics Education has established a research group dedicated to this precise domain. Textbooks are not merely regarded as tools for implementing mathematical activities (Johnston-Wilder & Mason, 2004); they are also catalysts for promoting mathematical reasoning (Lithner, 2004) and fostering learning opportunities in mathematics (Sullivan et al., 2012). The tasks presented within textbooks can have a direct bearing on students' cognitive processes, influencing the organization of their thoughts, and either constraining or broadening their perspectives on a given subject (Sullivan et al., 2012). Solved examples and exercises are considered the quintessential components of textbook content, utilized both within the classroom and at home, and thus, have a substantial impact on students' mathematical understanding.

Numerous studies have undertaken comparisons of mathematics textbooks from different countries, highlighting the crucial role textbooks play in the engagement with mathematical content, provision of teaching resources, and the evaluation of curriculum reforms (Ball & Cohen, 1996; Erbas et al., 2012; Pepin, 2018). Baker et al. (2010) have demonstrated that, due to the lack of direct evidence regarding students' past learning experiences, math textbooks serve as historical evidence of curriculum objectives and the learning opportunities afforded to students. Other investigations have suggested that disparities in mathematics textbooks can significantly affect students' mathematical achievements (Fan et al., 2013; van den Ham & Heinze, 2018; Zhu & Fan, 2006). In particular, the quality of textbooks can shape how students learn mathematics (Fan et al., 2013; Organization for Economic Cooperation and Development [OECD], 2013; Stein et al., 2007).

Moreover, the analysis and comparison of textbooks can offer a glimpse into the expectations of students' learning experiences in a particular country, helping to ensure alignment with curriculum goals (Cai & Park, 2013; Li, 2000; Zhu & Fan, 2006). For example, Zhu and Fan (2006) scrutinized the representation of math problems in textbooks used in China and the United States, revealing that the United States textbooks contained a higher proportion of non-routine, non-traditional, and applied problems, whereas Chinese textbooks featured more challenging multi-step problems. Furthermore, Aljami (2012) analyzed elementary math textbooks from Japan, Kuwait, and the United States, focusing on how fractions were presented, and found that textbooks in these three countries emphasized standard algorithms as the primary

computational methods. Japanese textbooks employed linear models and linked fractions to measurement, American textbooks provided examples to aid students in understanding fraction patterns, and Kuwaiti textbooks utilized visual representations of area models to elucidate fraction concepts.

Recent studies, such as that conducted by Yang and Sianturi (2022), have analyzed algebra math problems in textbooks for elementary students in Finland, Indonesia, Malaysia, Singapore, and Taiwan. Their findings revealed significant differences in problem design based on representations, cognitive demand levels, contextual aspects, and problem-solving types. These variations underscore that students in these five countries encountered diverse challenges in solving mathematical problems, and they suggest that differences in math problems designed within textbooks can inform understanding of student performance. Finnish textbooks contained the highest number of problems among the five countries studied, leading researchers to posit that providing ample opportunities for engagement with numerous math problems may contribute to improved math performance in Finland. These findings establish a positive correlation between student opportunities for learning and academic progress in mathematics. Viewing textbooks as a potential vehicle for the implemented curriculum underscores their capacity to influence the provision of learning opportunities.

Consequently, such studies underscore the significance of improving the quality and variety of math problems within textbooks to align with students' diverse learning experiences and enhance their comprehension of different problem types. Moreover, educational stakeholders must prioritize teacher training to ensure a continued emphasis on the effective utilization of textbooks and the enhancement of pedagogical practices within their respective countries.

Hence, the analysis of textbooks offers an effective method for gaining deeper insights into the teaching and learning processes in diverse countries. However, the theoretical underpinning of textbook analysis is inherently challenging, given the wide scope and variety of these analyses (Fan et al., 2013). In essence, a comprehensive framework encompassing factors for analyzing mathematics textbooks, particularly within non-Euro countries, remains absent from the scholarly discourse. This lacuna prompts inquiries into the salient factors relevant to textbook analysis. Accordingly, the present study aims to elucidate the pivotal factors pertinent to the analysis of mathematics textbooks. Thus, the research endeavors to address the following research question:

What factors merit consideration in the systematic analysis of mathematics textbooks utilized in educational settings?

METHODOLOGY

This study is characterized by its descriptive nature and adopts a content analysis approach, a method commonly employed when researchers seek to synthesize and quantify information that has not been prearranged or organized (Frankel & Wallen, 2000). Specifically, directed content analysis was employed for qualitative content analysis within this research (Hsieh & Shannon, 2005).

A directed approach to content analysis is fundamentally concerned with affirming or extending a theoretical framework or theory, necessitating a more structured process than the conventional approach (Hickey & Kipping, 1996). The outset of this method involves the selection of key concepts or variables as initial coding categories, drawing upon existing theory or prior research. Subsequently, operational definitions are formulated for each category in accordance with the theoretical underpinning. If deemed necessary, subcategories may be identified through a more in-depth analysis of the category's type and scope. Outcomes of a directed content analysis offer both supportive and non-supportive evidence for the theory, which can be presented through illustrative code examples and descriptive substantiation (Hsieh & Shannon, 2005).

The present study draws upon a multifaceted dataset comprising evaluation reports on mathematics textbooks and curricula, in conjunction with internationally published articles germane to the analysis of mathematics textbooks. Specifically, articles delineating frameworks for the systematic analysis of mathematics textbooks were selected for qualitative content analysis.

In the contextual framework of this investigation, it is noteworthy that the ministry of education assumes the responsibility of publishing mathematics textbooks for the entire educational spectrum, owing to the strictly centralized nature of the educational system. These textbooks undergo meticulous evaluation by seasoned researchers within the field, with the resultant evaluation reports serving as primary source material for this study. The inclusion criteria encompassed evaluation reports spanning various educational levels, encompassing secondary school grades 3, 7, and 10, as well as middle school grade 6 and elementary school grades 1, 2, and 3. Additionally, two reports stemming from the evaluation of preschool curriculum and science education were incorporated, given their relevance to the broader discourse on textbook analysis.

Simultaneously, a systematic search of prominent databases such as Elsevier, ERIC, Google Scholar, and Springer repositories was undertaken to identify pertinent literature on mathematics textbook analysis, extending from 1999 to the present time. We, first, focused on meta-analyses, and then added individual studies. The inclusion criteria for articles entailed a focus

on textbook analysis, particularly within the domain of mathematics education. Such a comprehensive approach was adopted to elucidate the multifarious factors underpinning textbook analysis, thereby enriching the qualitative content analysis conducted in this research endeavor.

Qualitative content analysis is a systematic and structured research method deployed for the examination of textual data. This method encompasses the systematic identification, coding, and interpretation of patterns, themes, and meanings inherent in the content. MAXQDA 2022 was used for qualitative data analysis in this research. Researchers compile a set of codes or categories that encapsulate the concepts or themes manifest within the data. These codes are devised to capture the prominent concepts, emotional nuances, or phenomena inherent in the content under examination. Coding can be executed either manually or facilitated through dedicated software tools. Subsequently, the process involves the systematic categorization and formation of themes, which evolve through the analysis of coded segments to discern recurrent patterns, commonalities, and overarching themes inherent in the data. The identified themes undergo a thorough examination to elucidate their significance and implications within the context of the research question. Subsequently, the findings are synthesized and articulated within a comprehensive analysis report, featuring the themes supported by relevant quotes or examples derived from the data. Themes represent the key factors that are deemed essential for the analysis of mathematics textbooks. The analysis report encompasses interpretations, explanations, and insights gleaned from the rigorous content analysis process. It is imperative to underscore that qualitative content analysis is an adaptable and iterative process, with researchers frequently revisiting and refining the codes and themes as they attain deeper insights into the data.

In order to assess the reliability between coders, Cohen's kappa was employed, yielding a coefficient of 0.62. This statistical measure, as defined by Krippendorff (2004), serves as a research technique for generating replicable and valid inferences from texts or other meaningful content to the contexts within which they are employed.

Significance of the Study

In light of our qualitative data analysis, which has unveiled six overarching themes for systematically evaluating mathematics textbooks, we find it imperative to elucidate how our framework differs from and extends beyond the prevailing frameworks in the field. These six key themes, namely language and communication, content analysis, pedagogical approach, cultural sensitivity and inclusivity, assessment and exercises, and visual aids and

presentation, not only serve as the backbone of our framework but also exemplify the distinctive character of our approach. Our framework differs from existing models (e.g., Aineamani & Naicker, 2014; Kalmus, 2004; Mahmood, 2009; Rahimah & Visnovska, 2021) by providing a comprehensive and holistic perspective that synthesizes these themes into a unified evaluation system, offering a more integrated and interrelated view of the textbook evaluation process. Furthermore, our framework adds an element of depth and nuance by introducing a nuanced layer of analysis within each of these thematic dimensions, enabling a richer and more fine-grained assessment of mathematics textbooks. This novel approach, which considers not only the presence of these thematic elements but also their interplay and synergy, significantly contributes to the literature by establishing a more rigorous and sophisticated framework for textbook evaluation. Consequently, our work highlights a crucial gap in the existing literature and underscores the importance of our framework in advancing the field of mathematics education.

FINDINGS

The findings section of this paper delves into a comprehensive examination of mathematics textbooks, revealing crucial insights into their diverse facets and their impact on mathematics education. These findings are derived from an in-depth analysis of existing literature, encompassing a range of themes that shed light on the complexities and nuances of this educational resource. In this section, we present key discoveries regarding cultural sensitivity, inclusivity, values, visual presentation, and the evolving landscape of mathematics textbooks, providing a deeper understanding of their role in shaping students' mathematical learning experiences.

Theme 1: Language & Communication

The analysis of the data has revealed that "language and communication" emerges as a prominent theme within this research. The categories that constitute this theme encompass various facets, including "gender-inclusive language," "inclusive language concerning ethnicities, race, and special needs," "accurate utilization of technical vocabulary," "incorporation of first-person pronouns," "clarity of language employed," and "appropriateness of vocabulary with respect to age and grade." **Figure 2** delineates several subthemes (categories) and corresponding codes pertaining to the "language and communication" theme.

Scholars such as Foster et al. (2022) and Huang et al. (2022) have underscored the need to assess the gender sensitivity of textbooks, as these texts serve as reflections of prevailing societal attitudes toward gender. Additionally, Howson (2013) has drawn attention to the

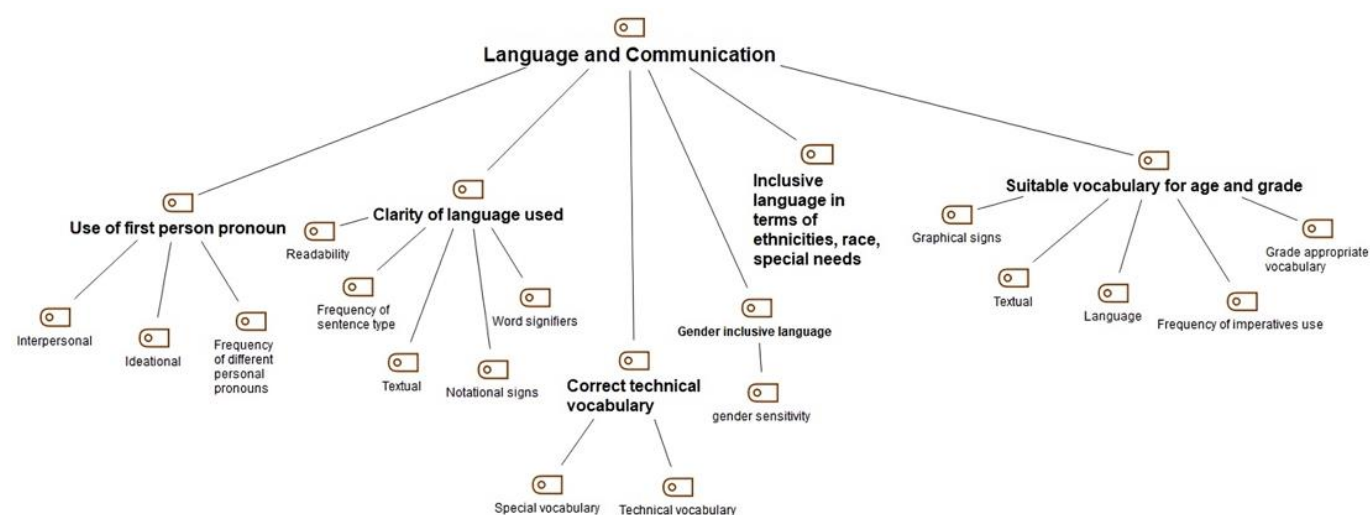


Figure 2. Language & communication theme codes (Source: Authors' own elaboration, using MAXQDA 2022)

imperative of gender, racial, and other social equilibriums in the context of educational materials.

Regarding the “correct technical terminology and vocabulary” usage, O’Keeffe and O’Donoghue (2015) contend that the frequent incorporation of technical language and symbols, accompanied by a judicious restraint in employing casual language, serves as evidence supporting the author’s identification with the mathematical community and their readiness to shoulder intellectual responsibility. It is pertinent to note that rare textbooks incorporate a glossary or a lexicon of pertinent terms. Moreover, the language characteristics of textbooks, as elucidated by O’Keeffe and O’Donoghue (2015), encompass the incorporation of special vocabulary, encompassing words commonly employed in daily life but imbued with distinct mathematical connotations, such as “match,” “set,” “group,” or “figure.” In a similar vein, technical vocabulary, featuring words unique to mathematics, for instance, “heptagon” or “multiple,” is expected to be utilized with precision.

Addressing the “use of first-person pronouns,” the incorporation of ‘you’ alongside the frequent deployment of ‘we’ signifies a profound and direct engagement with the reader, assuming their active participation in the learning process (O’Keeffe & O’Donoghue, 2015). While textbooks are not meant to emulate classroom dialogues, they may employ familiar language to effectively involve readers. Notably, the use of technical language in school mathematics textbooks holds substantial significance for student learning, as corroborated by numerous studies (O’Keeffe & O’Donoghue, 2015). The usage of first-person pronouns within textbooks can establish a formal relationship between the author and the reader; however, employing informal strategies can prove efficacious for problem-solving (Ho, 2002; Silver, 1990). Notably, while textbooks should not seek to replicate classroom conversations, they may employ accessible language to engage the reader. The inclusion of “I” can signify the author’s

involvement and authority, while “we” can encompass the reader (O’Keeffe & O’Donoghue, 2015). Additionally, the utilization of “you” can assert authority over the reader, directing their attention to the text (Huang et al., 2022).

The exploration of “second-person pronouns (you)” aligns with the framework endorsed by numerous experts worldwide for the analysis of textbooks (Huang et al., 2022, p. 10). Further codes, such as “interpersonal” and “identical,” were identified within the dataset, particularly in reference to the use of the present tense and the avoidance of passive sentence structures in textbooks (O’Keeffe & O’Donoghue, 2015).

Howson and Rogeers (2014) underscores the significance of clarity and accuracy in explanations found within mathematics textbooks. Explanations are meticulously structured to ensure accessibility to students who are grappling with novel concepts. The accuracy of these explanations holds paramount importance, as mathematical concepts are often interdependent, with any misinterpretations or errors significantly impeding a student’s ability to grasp and apply more advanced concepts. The readability of textual expressions within textbooks has also been a subject of local evaluation reports and international scholarly discourse. Readability, in this context, relates to the complexity of the language employed and the ease with which it can be comprehended through reading. This underscores the extent to which the linguistic complexity of a textbook is suitable (Huang et al., 2022).

Furthermore, another facet examined in the analysis of mathematics textbooks pertains to word signifiers, encompassing general terms, mathematical terminology, technical vocabulary, specialized vocabulary, abbreviations, and letters (O’Halloran, 2005). The appropriateness of vocabulary with respect to the educational level is often assessed through metrics such as the Flesch-Kincaid grade level. It is rational to anticipate a balanced integration of both inclusive and

exclusive imperatives within mathematics textbooks, given that mathematicians are both thinkers and communicators, with the equilibrium between these facets deemed essential (O’Keeffe & O’Donoghue, 2015). The language employed within explanations typically leans toward the technical, although textbooks frequently include examples, diagrams, and other visual aids to facilitate student comprehension. These visual elements serve to illustrate mathematical principles and concepts. In sum, in alignment with extant literature, it can be posited that language profoundly contributes to the effectiveness of mathematics textbooks (O’Keeffe, 2013).

In the realm of language and communication within mathematics textbooks, it is crucial to acknowledge the multifaceted dimensions and nuances associated with the utilization of language. The literature underscores that gender-inclusive language serves as a fundamental component of this theme. This aligns with the imperative of assessing and addressing the gender sensitivity of textbooks, as they not only convey mathematical content but also reflect and perpetuate societal gender norms and attitudes (Howson, 2013; Huang et al., 2022).

The meticulous selection and employment of technical terminology and vocabulary are paramount, as they serve to convey the author’s expertise and credibility within the mathematical community. This resonates with O’Keeffe and O’Donoghue’s (2015) assertion that the proficient use of technical language is indicative of one’s affiliation with the mathematical discourse community. Notably, presence of a glossary or dictionary can further enhance the accessibility of mathematical content, making it more comprehensible to learners (O’Keeffe & O’Donoghue, 2015).

The clarity and accuracy of explanations represent another pivotal facet of this theme. The design of explanations is purposefully structured to ensure that they are intelligible to students who may be encountering the concepts for the first time. The paramount importance of accuracy stems from the cumulative nature of mathematical concepts, where misunderstandings at earlier stages can significantly impede later learning (Howson & Rogers, 2014). The readability of mathematical expressions in textbooks is contingent on the balance between linguistic complexity and comprehension ease. Huang et al. (2022) reinforce this dimension by highlighting the importance of linguistic appropriateness in mathematical content.

Furthermore, within the analysis of mathematics textbooks, particular attention is devoted to the selection and deployment of word signifiers. These encompass general terms, mathematical language, technical vocabulary, specialized terminology, abbreviations, and letters. The suitability of vocabulary in alignment with the educational level, as assessed through metrics such as the Flesch-Kincaid grade level, serves as a critical

criterion in this context. This underscores the importance of balancing inclusive and exclusive linguistic elements in mathematical communication (O’Keeffe & O’Donoghue, 2015).

The language and communication theme accentuates the intricate interplay between linguistic choices and mathematical pedagogy. It affirms that mathematical textbooks are not solely repositories of mathematical knowledge, but intricate documents shaped by linguistic decisions that influence how mathematical concepts are perceived and learned by students. It underscores the pivotal role that language plays in rendering mathematics accessible, comprehensible, and inclusive.

In sum, this theme underlines the complexity and multifaceted nature of language within mathematics textbooks, emphasizing the need for conscientious consideration of linguistic choices to enhance the effectiveness of mathematics education materials.

Theme 2: Content Analysis

In the course of the qualitative analysis, the paramount theme christened “content analysis” emerged as the focal point of this investigation, occupying a pivotal position within the purview of mathematics textbooks evaluation (see **Figure 3**). This theme comprises a spectrum of intricate subthemes or categories, each of which contributes to the comprehensive understanding of the theme’s overarching significance. These categories encompass “the structure of the textbook,” “motivational factors,” “relationship between images and argumentation,” “accuracy of content,” “technological integration,” “relevance of content to curriculum,” “suitability of content to the grade level,” “sequence of concept development,” “variety and range of illustrative examples,” “diversity of exercises,” “interdisciplinary connections,” and “availability of exercise solutions.”

A foundational facet of content analysis is the examination of the structural underpinnings of the mathematics textbook, which entails probing into elements such as the title, total page count, author and advisory committee profiles, publisher, year of publication, and the presence of accompanying materials like workbooks and teachers’ guides (Charalambous et al., 2010; Huang et al., 2022). Equally germane is the consideration of the organization of chapters and units within the textbook. This examination transcends mere quantification, delving into the alignment and coherence between lessons and the specific topics covered (Huang et al., 2022).

The incorporation of visual aids, including figures and tables, alongside textual content significantly contributes to the clarity and comprehensibility of mathematical concepts (Foster et al., 2022; Huang et al., 2022). The quantification of illustrations within a

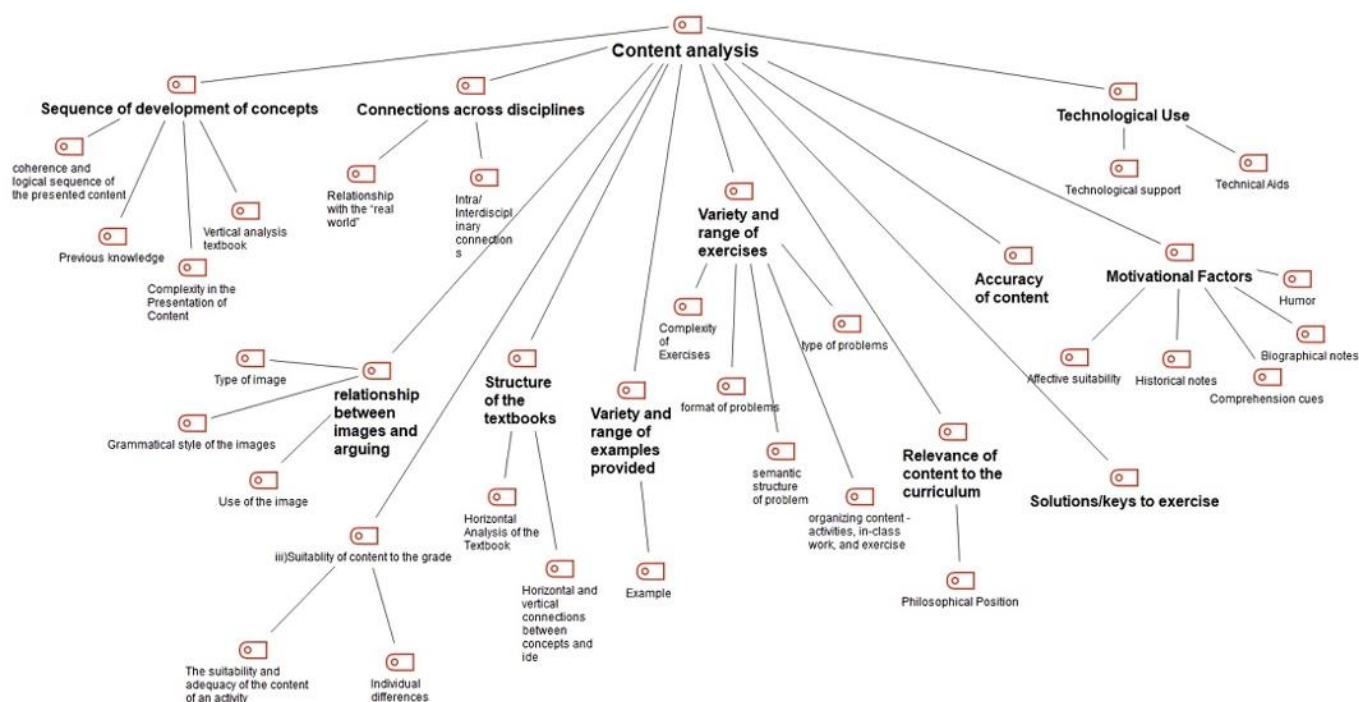


Figure 3. Content analysis theme codes (Source: Authors' own elaboration, using MAXQDA 2022)

textbook is an essential measure to gauge the degree of reliance on visual representations (Ainsworth, 2006).

Within the motivational factors category, historical notes, storytelling, and biographies of prominent mathematicians and scientists play a pivotal role in engendering students' interest and motivation for learning (Huang et al., 2022). This juncture interweaves with comprehension factors, where the appropriate utilization of colors, layout, and graphics is demonstrated to enhance students' grasp of the content (Berisha, 2015; O'Keeffe, 2013).

Furthermore, the content analysis theme encompasses the scrutiny of the accuracy and authenticity of the mathematical content (Howson, 2013). The seamless integration of technology within the textbook fabric, ensuring the availability and adequacy of resources and temporal allocation, represents another layer of evaluation. The temporal allocation facet acquires pertinence in addressing issues related to the presentation of complex or challenging content, allocating sufficient time for in-depth comprehension (Castillo-Céspedes & Burgos Navarro, 2022).

A salient dimension underpinning content analysis is the transmission of social and cultural values through mathematics textbooks. This study hones in on six categories of values embedded in textbooks: collectivistic, individualistic, traditional, religious, ethnic, and social role values (Huang et al., 2022). The assessment of these values bears significant implications for the broader societal and cultural influences that textbooks may impart.

Inextricably linked to content analysis is the evaluation of the content's alignment with curriculum standards and its suitability for the intended grade level.

The potential for fostering interdisciplinary connections and weaving mathematical concepts into broader knowledge domains is a cherished theme in mathematics textbooks (Kristanto & Santoso, 2020). In this context, the narrative may be enriched by historical perspectives or excerpts from renowned mathematicians.

Moreover, the sequential development of mathematical concepts takes center stage, with meticulous consideration of prerequisites and attainability (Castillo-Céspedes & Burgos Navarro, 2022). This entails a judicious balance between the levels of cognitive demand and response types required of students (Charalambous et al., 2010). In addition, the complexity of content presentation, exemplified by the type of representations employed, plays a crucial role (Kochagina, 2021; Valverde et al., 2002).

The textual corpus of mathematics textbooks is further enriched by the diversity of worked examples and exercises, catering to both numerical and contextual problem-solving (van Zanten, 2020). Cognitively, exercises are scrutinized based on six distinct levels of cognitive requirement, ranging from procedural and conceptual knowledge to representation, reasoning, problem-solving, and problem-posing (Huang et al., 2022). Furthermore, the existence of solutions or answer keys to exercises, often found in teacher's guides, constitutes an integral dimension within the domain of mathematics textbook evaluation.

This theme underscores the intricate web of considerations surrounding the content analysis of mathematics textbooks, encompassing structural, motivational, and cultural dimensions, while also addressing pedagogical and cognitive aspects. The

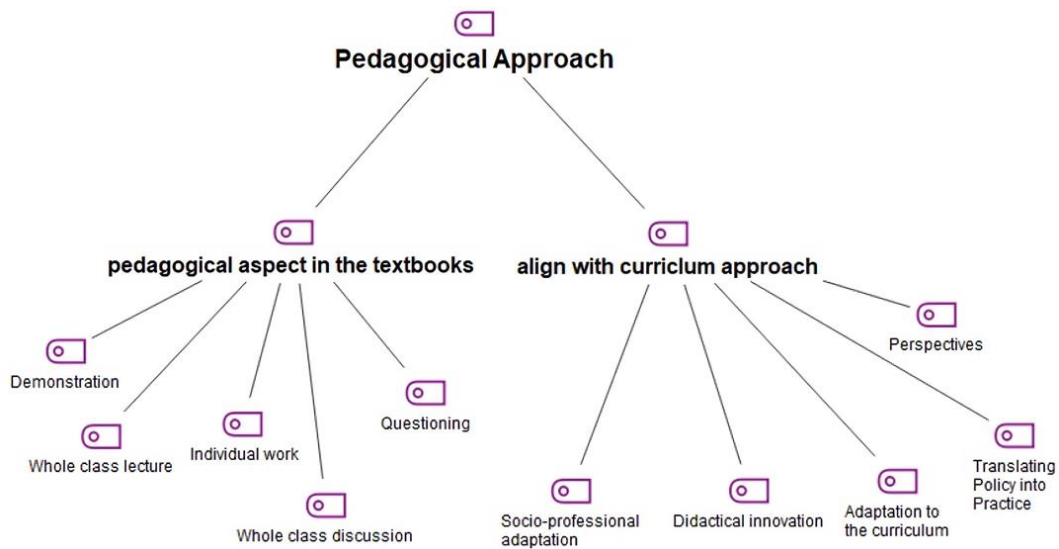


Figure 4. Pedagogical approach theme codes (Source: Authors’ own elaboration, using MAXQDA 2022)

theme encapsulates the pivotal role that content plays in the educational ecosystem and underscores the potential of mathematics textbooks to facilitate meaningful learning experiences.

Theme 3: Pedagogical Approach

Within the annals of this comprehensive analysis, the theme of “pedagogical approach” emerges as a cardinal point of emphasis, replete with constituent subthemes or categories, which engender a profound understanding of the role of pedagogy within mathematics textbooks (see Figure 4 for codes). This theme encompasses “pedagogical aspects in the textbooks” and “alignment with curriculum approach.”

A pivotal facet of pedagogical approach revolves around the various strategies and methods that textbooks employ to either extend or constrain the pedagogical milieu. These encompass “whole class lecture,” “demonstration,” “questioning,” “whole class discussion,” “individual work,” and “group work” (which, in turn, includes both cooperative and collaborative dimensions). Moreover, textbooks deliberate upon the integration of “online work” and “self-directed learning.” This multifaceted pedagogical approach integrates assessment strategies, encapsulating modes such as “investigation and problem-solving,” “drill and practice,” and “homework.” Each of these components is painstakingly dissected in the process of textbook analysis.

Significantly, the pedagogical approach theme dovetails seamlessly with the alignment with curriculum approach, ensuring a harmonious consonance with curricular guidelines (Castillo-Céspedes & Burgos Navarro, 2022). Textbooks, in this context, undertake the formidable task of translating policies into practical pedagogical implementation strategies, culminating in the realization of educational objectives (Valverde et al., 2002).

This theme underscores the intricate interplay between pedagogical strategies and the attainment of curricular goals, encapsulating the diverse array of teaching and learning models that textbooks must accommodate. Furthermore, the theme underscores the pivotal role of textbooks in offering a pedagogically robust foundation for mathematics education, which aligns with prescribed curriculum guidelines and educational policies. The pedagogical approach thus emerges as a critical determinant in the quality and effectiveness of mathematics textbooks.

Theme 4: Cultural Sensitivity & Inclusivity

An essential theme that has emerged from both local evaluation reports and extensive literature review is the issue of “cultural sensitivity and inclusivity” within mathematics textbooks (see Figure 5 for codes). This theme serves as a pivotal lens through which to examine the extent to which textbooks exhibit cultural awareness and foster inclusivity.

Within this theme, textbooks undergo rigorous scrutiny with respect to their portrayal of diverse cultures, including the names or contexts associated with problems presented. The manner in which textbooks represent men/boys, women/girls, and individuals with disabilities comes under particular scrutiny.

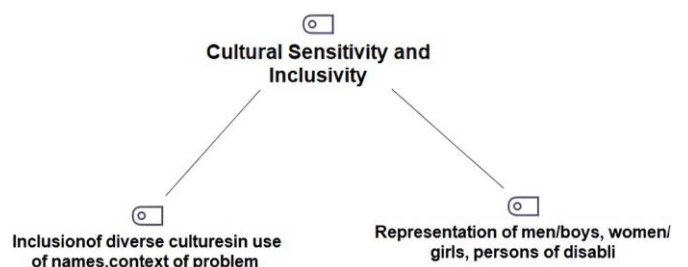


Figure 5. Cultural sensitivity & inclusivity theme codes (Source: Authors’ own elaboration, using MAXQDA 2022)

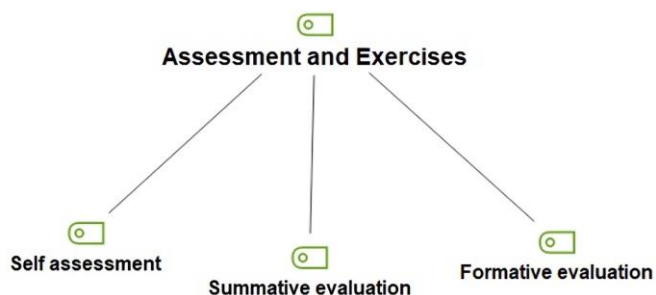


Figure 6. Assessment & exercised theme codes (Source: Authors’ own elaboration, using MAXQDA 2022)

This theme critically underscores the imperative of acknowledging cultural diversity and fostering an inclusive educational environment. It underscores the need for textbooks to transcend cultural biases and stereotypes and to provide equitable representation, thereby cultivating a learning environment that resonates with learners from diverse backgrounds. The manifestation of cultural sensitivity and inclusivity within mathematics textbooks constitutes an indispensable facet of their quality and impact on learners.

Theme 5: Assessment & Exercises

The comprehensive analysis of mathematics textbooks unearths a distinct theme, namely “assessment and exercises,” which encompasses the multifaceted landscape of evaluating students’ comprehension, knowledge, and progress (see **Figure 6** for codes). While elements of this theme were addressed in the content analysis, the recognition of its standalone significance necessitates a comprehensive exploration. This theme is further segmented into three distinct subthemes: “self-assessment,” “summative assessment,” and “formative evaluation.”

Within the realm of this theme, the intricacies of assessing students’ responses, including the nuanced approach of filling in the blanks, are expounded upon.

Mazana et al. (2020) provides insightful perspectives on assessing students’ responses in the context of completing sentences with appropriate content. This analysis prompts recommendations regarding the optimal number of blanks to be included in textbooks to facilitate an effective assessment process.

The examination of this theme highlights the pivotal role that mathematics textbooks play in implementing structured evaluation mechanisms to systematically monitor students’ cognitive progress. Assessment, in this context, is not viewed merely as an end but rather as a process intrinsically intertwined with teaching and learning. It serves as a dynamic tool for providing essential feedback to students, thereby fostering their growth. Mathematics textbooks incorporate a diverse array of evaluation techniques, encompassing problem-solving tasks and practical exercises. These techniques span the spectrum from self-assessment to co-evaluation and hetero-evaluation activities, aligning closely with the overarching learning objectives. Consequently, the assessment and exercises theme reflects the textbook’s role in not only imparting mathematical knowledge but also in the holistic cultivation of learners’ skills and understanding, bridging the divide between pedagogy and evaluation.

Theme 6: Visual Aids & Presentation

The realm of “visual aids and presentation” within mathematics textbooks represents a dimension of critical evaluation, illuminated by insights derived from local report evaluations (see **Figure 7** for codes). It is imperative to recognize the significance of the visual aspects and presentation styles that underpin mathematics textbooks, particularly in the context of “mathematics textbook analysis.”

Mathematics textbooks span the globe in a multitude of dimensions, including size, length, and structural features. There exists a diverse array of chapters and units within these textbooks, each adopting distinct

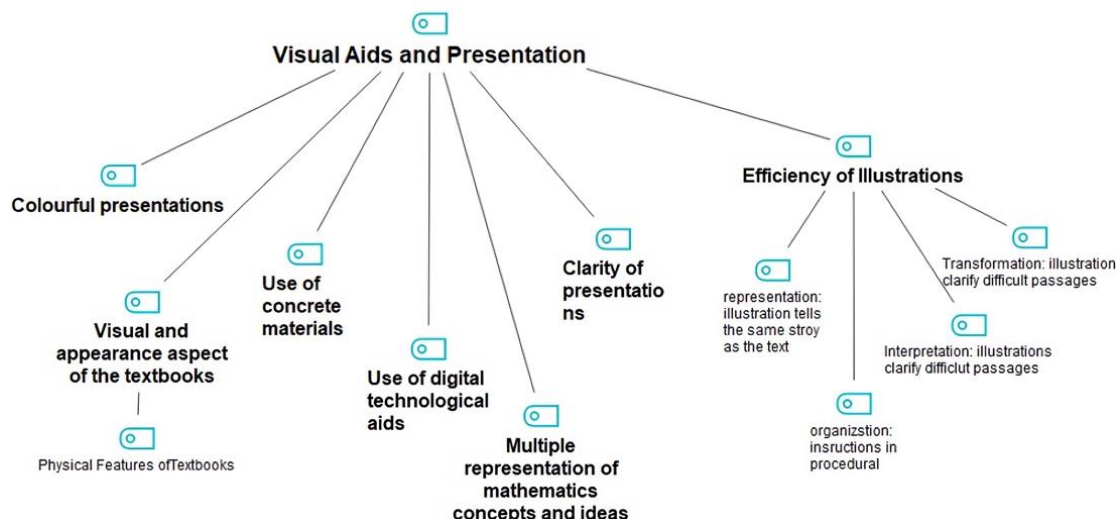


Figure 7. Visual aids & presentation theme codes (Source: Authors’ own elaboration, using MAXQDA 2022)

layouts and sequencing. Moreover, these variations extend to the complexity and presentation methods employed in explaining mathematical and scientific concepts (Valverde et al., 2002).

Valverde et al. (2002) have provided discerning observations regarding the use of visual aids, encompassing photographs, illustrations, tables, and graphs, across different textbooks. This analysis serves as a powerful lens through which to gauge the degree of reliance on visual elements, supplementing written content. The presence and deployment of visual aids within mathematics textbooks contribute significantly to the presentation and comprehension of specific mathematical and scientific topics, notably in areas such as geometry or forces and motion.

This theme underscores the intrinsic link between visual aids and effective pedagogical presentation, further emphasizing the influential role these elements play in enhancing the overall learning experience within the context of mathematics education.

DISCUSSION

The multifaceted nature of textbook analysis encompasses a spectrum of dimensions, each lending valuable insights to the evaluative process. Notably, within the realm of mathematics education, a range of critical aspects serves as focal points for in-depth examination, thus affording a more nuanced understanding of the field.

One established framework in this discourse, as articulated by Fan et al. (2013), delineates five distinctive aspects of textbook analysis. These include a comprehensive scrutiny of

- (1) mathematics content and topics;
- (2) cognition and pedagogy;
- (3) gender, ethnicity, equity, culture, and value;
- (4) comparative analysis across different textbooks; and
- (5) the methodological nuances inherent to the analysis process.

Such an analytical framework serves as a structured guide for researchers, aiding in the systematic assessment of mathematics textbooks.

Pepin and Haggarty (2001) have contributed to this framework by classifying textbook analysis into four primary domains. These domains encapsulate the

- (1) mathematical intentions,
- (2) pedagogical intentions,
- (3) sociological contexts, and
- (4) cultural traditions enshrined within the textbooks.

This sophisticated classification scheme underscores the multifaceted nature of mathematics textbooks, which are far more than repositories of mathematical concepts

but also carriers of sociocultural contexts, educational intentions, and broader cultural traditions.

Language & Communication

The paramount role of language and communication in mathematics education cannot be overstated. Indeed, the research of Joseph (2012) brings into focus the pivotal influence of communication and academic vocabulary on students' capacity to develop mathematical thinking. The choice of language employed in prompts, for instance, can profoundly impact students' comprehension and responses—a critical consideration for constructing well-informed responses in the context of mathematics. O'Connell et al. (2005) echo the importance of language comprehension, emphasizing that a firm grasp of mathematical terminology and symbols is vital for effective communication and content understanding.

The labyrinth of mathematical vocabulary, comprising both terms and symbols, poses a multifaceted challenge for students. As such, mathematical vocabulary warrants multifaceted approaches to enhance comprehension. National Council of Teachers of Mathematics (NCTM, 2000) underscores the significance of mathematical vocabulary, urging students to employ it with precision and clarity in mathematical discourse.

The interplay of language and mathematics education has drawn the attention of scholars like Haggarty and Pepin (2002) and Herbel-Eisenmann (2007), particularly concerning middle-grade learners. Nonetheless, there is a dearth of research concerning the alignment of elementary-grade textbooks with process standards, the positioning of writing prompts within elementary mathematics textbooks, and the nuances of language use in these prompts.

Academic vocabulary, an integral component of mathematical language, can be further dissected into domain-specific and general academic terms. The work of Baumann and Graves (2010) extends the vocabulary spectrum to encompass literary vocabulary, meta-language, terms describing processes, and symbols.

Constructed response items stand as open-ended questions that necessitate a written response, diverging from the typical closed-ended questions like multiple-choice, true/false, or matching questions. The National Assessment of Educational Progress (NAEP) Glossary of Terms (National Center for Education Statistics, 2010) offers a precise definition for constructed response items, delineating them as non-multiple-choice items that demand written or oral responses.

The realm of mathematical literacy and education extends its lexicon, often employing the terms "prompt" and "writing task" interchangeably, exemplified by scholars such as Murphy et al. (2011) and Yancey (2004).

Within the purview of mathematical textbook analysis, language plays a crucial role. Newall (1990) has identified various features of language within mathematical textbooks, encompassing discourse types, coordinators, and semantic structures. Such linguistic elements provide the foundation for textbook language analysis, shedding light on critical aspects of communication.

Mathematical textbooks unfold a spectrum of challenges by virtue of their language choices. Word length, for instance, can exert a considerable influence on student learning, with longer words often containing a wealth of information and complexity. Thus, the ability to comprehend mathematical terminology and its meanings stands as a formidable hurdle for students.

O’Keeffe (2013) offers valuable insights into the development of mathematical thinking, highlighting three distinct stages: active, figurative, and abstract. This analysis suggests that active engagement and the incorporation of physical objects, wherever feasible, are integral to effective learning and teaching. Furthermore, the visual component holds a distinct advantage, as the visual center in the human brain surpasses the audio center by a significant margin, rendering visual aids particularly potent in conveying mathematical concepts.

The language choices within mathematical textbooks have the capacity to frame the reader either as a “thinker” or a “scribbler” (Herbel-Eisenmann, 2007). When evaluating the interpersonal function within a mathematical text, linguistic forms such as imperatives, personal pronouns, and modality come into play. Imperatives, denoting commands or directives, instruct the reader and dictate their involvement in the mathematical discourse. This choice of imperatives may either induct the reader into the mathematical community or establish their role as a thinker actively shaping the mathematical narrative.

Two key pronouns bear significance in constructing the interpersonal function: first-person pronouns “I” and “we,” and the second-person pronoun “you.” The first-person pronouns manifest the author’s personal involvement in the discourse and may convey either authoritative information or foster reader involvement. “You,” the second-person pronoun, addresses the reader directly, guiding their attention to specific ideas or concepts.

Modality, expressed through modal auxiliary verbs, adverbs, or adjectives, reflects the level of certainty or authority attributed to a statement (Kress & Bezemer, 2009). This dimension aids in elucidating the degree of conviction or uncertainty within a mathematical discourse, with “hedges” serving as linguistic indicators of moments of uncertainty.

In the broader framework of textbook analysis, these linguistic elements are leveraged to deconstruct the construction of roles and relationships between the

author and the reader (Herbel-Eisenmann & Wagner, 2007). This intricate web of linguistic features shapes the interaction between the reader and the mathematical discourse, significantly impacting the reader’s comprehension and engagement with the text.

Diverse Reading Styles

Mathematics textbooks yield to a diverse spectrum of reading styles, each delineating the relationship between the reader and the text. These styles encompass close reading with strong connections, close reading with partial connections, scanning, skimming, and avoidance. Each style is imbued with unique characteristics, structured to suit the reader’s prior knowledge, experiences, beliefs, and capabilities.

Readers of mathematics textbooks invariably inject individual meaning into the text, which may or may not align precisely with the author’s intent. This aspect underscores the richness of mathematics textbooks, offering a multitude of interpretations and layers of meaning as they are absorbed by readers. This interpretive flexibility creates dynamic opportunities for varied meanings and interpretations, a fact that deepens the educational experience within mathematics.

Content Analysis

Mathematics textbooks serve as a central focus in educational research, with a particular emphasis on content analysis. The systematic analysis of mathematics textbooks, as asserted by Rezat and Strässer (2015), provides a robust foundation for research endeavors. This in-depth scrutiny encompasses a multitude of research topics, expanding from coherence, clarity, and accuracy of mathematical explanations to the breadth and quality of exercises. The following attributes, as outlined by Howson (2013), encapsulate key considerations for textbook reviewers:

1. Mathematical coherence.
2. Clarity and precision in explanations.
3. Range, quantity, and quality of exercises.
4. Integration with real-life scenarios and other curricular subjects.
5. Gender, racial, and social inclusivity.
6. Appropriate language and language development.
7. Evidence of research integration and professional experience.
8. Accommodation for diverse student abilities.
9. Visual attractiveness.
10. Signs of originality in content.
11. Comprehensive teachers’ guides surpassing mere answer keys.

The structural organization of content within mathematics textbooks tends to follow a consistent pattern. A typical structure involves the presentation of fundamental knowledge on a topic, succeeded by exercises and individual tasks that are not systematically graded in terms of difficulty (Berisha et al., 2021).

Past research endeavors on mathematics textbooks have explored various facets. For instance, studies have delved into the coverage of content topics in textbooks, the curricular treatment of mathematical concepts, and the nature of exercise problems (Cai et al., 2002; Chang et al., 2016; Fuson et al., 1988; Schmidt et al., 1997; Westbury, 2008; Zhu & Fan, 2006). Moreover, four key points in the development of mathematics textbooks significantly contribute to students' learning capabilities. These encompass discussions of content, the creation of student work plans or schedules, the presentation and exposure of content, and the interconnection and linkage of content aspects.

A fundamental categorization of mathematics concepts falls under five broad branches: Numbers and operations, measurements, geometry, algebra and arithmetic, and data analysis and probability. The classification of mathematical knowledge and content aligns with the curriculum, reflecting clear boundaries that segregate mathematical knowledge from other subjects on the curriculum (Gene, 2018). This discrete classification underscores the "mathematical purity" of textbook content, segregating it from other subject matter in the educational sphere.

The primary school mathematics textbook typically adheres to a structured framework, comprising chapters that correspond to specific teaching units. Each chapter encompasses two activities, which are carried out in class, and two applications, usually solved exercises tackled either in the classroom or as homework assignments. Furthermore, a section titled "Questions for self-assessment and discussion" incorporates evaluative questions. Workbooks, usually comprising four volumes, complement the textbooks and include exercises and extension activities aligned with the textbook chapters.

Secondary school mathematics textbooks structure their teaching units with two or three activities, some requiring formal exercises and others demanding a more investigative approach from students. These activities culminate in generalizations with the presentation of rules, accompanied by examples of solved exercises (Gene, 2018).

Pedagogical Approach

The role of mathematics textbooks extends beyond being mere instructional materials; they play a pivotal role in shaping the pedagogical landscape in classrooms (Choy et al., 2020). A significant strand of textbook analysis focuses on pedagogical and didactic aspects,

delving into topics such as modeling, differentiation, and heuristic approaches (Bråting et al., 2019; Rezat & Strässer, 2015).

The pedagogical content knowledge, a construct elucidated by Shulman (1986), represents the bridge connecting content and pedagogy, playing a central role in effective mathematics education. Researchers have undertaken studies exploring pedagogical features within mathematics textbooks, curricular treatment of mathematical content, and exercise problems, highlighting the critical role that textbooks play in conveying pedagogical messages and influencing the curricular environment in which teachers operate (Fan & Kaeley, 2000).

Textbooks, within the educational framework, serve as tools for guiding teachers, shaping their pedagogical approaches, and facilitating the teaching-learning process to align with the developmental capacities of students (Gene, 2018). The dependence on mathematics textbooks, as uncovered by Fan et al. (2013), is more pronounced compared to textbooks in other subjects, as they are often the chief resource upon which teachers base their instruction (Grouws et al., 2004).

The manner in which pedagogical instructions are embodied in textbooks, both in their organization and content, cannot be overlooked. This is evidenced in the structural arrangement and the content presentation, with content sequence and instructional expectations being pivotal factors in defining the pedagogical approach that a textbook promotes (Valverde et al., 2002). As such, it is imperative to consider how textbooks frame the presentation of content formats and the performance expectations they establish. These factors reflect broader teaching approaches and the conceptualization of mathematical concepts.

In summary, mathematics textbooks, aside from serving as repositories of mathematical content, exert profound influence on pedagogical strategies. Their structured organization, content layout, and instructional intentions collectively define the pedagogical approach in mathematics classrooms. These findings underscore the pivotal role that mathematics textbooks play in shaping the educational landscape and the pedagogical decisions made by teachers.

Cultural Sensitivity & Inclusivity

The values underpinning mathematics education, as discussed by Seah et al. (2016), represent the confluence of general educational values and mathematical values. These values guide the learning and teaching processes within mathematics education (Cao et al., 2006). They manifest in the curriculum, teachers' pedagogical approaches, and classroom activities (Tan-Sisman & Kirez, 2018). Students, in turn, develop their learning paths and strategies influenced by these values (Cao et al., 2006).

Seah et al.'s (2016) mathematical education classification of values, which consists of five value pairs—formalistic view-activist view, operational understanding-relational understanding, relevance-theoretical knowledge, accessibility-special, and evaluating-reasoning—plays a pivotal role in understanding the ethical and philosophical foundations of mathematics education (Bakirci & Biber, 2022).

Values, as fundamental aspects of human behavior, have a profound impact on individuals' thoughts, emotions, and actions, serving as guides in differentiating right from wrong and influencing attitudes and preferences (Ata & Yaman, 2021; Dede, 2015; Hozum & Yildiz, 2023). The definitions of values vary across perspectives, encompassing beliefs, attitudes, experiences, preferences, and behaviors (Bishop et al., 2000; Dede, 2015; Grootenboer & Marshman, 2016). Grootenboer and Marshman (2016) highlight the interconnectedness of affective domain variables, including belief, value, attitude, and emotion, which often lead to diverse definitions of values. Hence, values are inherently personal and subject to individual beliefs (Seah & Bishop, 2000).

Values, in the context of society, serve as guiding principles for individuals' behavior within the cultural framework of their society (Tan et al., 1997). They can be understood as personal choices made based on the perceived importance of an idea or action and their alignment with societal goals (Dede, 2015). Values are instrumental in simplifying social life by promoting cohesion among people (Aydin & Akyol-Gurler, 2012).

It is imperative to recognize that values are not static; they evolve over time and are influenced by culture (Aydin & Akyol-Gurler, 2012; Chin & Lin, 2001). The societal preservation of these values necessitates their transmission to subsequent generations, which is primarily achieved through systematic education (Bishop et al., 1999; Kurtdede Fidan & Ulu, 2021).

In recent years, the discussion of values has gained prominence in mathematics textbooks, as evident in a comprehensive analysis of the literature (Cetin et al., 2021; Dede, 2015; Karaca & Uzunkol, 2019; Kilcan, 2020; Ozkaya & Duru, 2020; Sahin, 2021; Sahin & Basgul, 2018; Sayin et al., 2019). While some studies have focused on mathematical values introduced by Bishop et al. (1999) in middle and high school textbooks (Dede, 2015), others have explored the inclusion of moral values such as love, respect, responsibility, cooperation, tolerance, benevolence, universalism, and kindness in elementary and middle school mathematics textbooks (Sahin & Basgul, 2018).

Research efforts, such as the study by Cetin et al. (2021) analyzing ninth-grade mathematics textbooks, reveal the presence of values such as benevolence and patience, with less frequent mention of values like self-

control, honesty, and love. Similarly, Sayin et al. (2019) found that fifth-grade textbooks often emphasize values such as self-control, justice, benevolence, and responsibility, while values like friendship, honesty, patriotism, respect, and love are mentioned less frequently.

Visual Aids & Presentation

The physical features of mathematics textbooks have garnered attention, with comprehensive analysis conducted across 40 countries worldwide (Valverde et al., 2002). The selection and presentation of mathematical concepts play a pivotal role in student engagement and success in mathematics, although further research on a larger scale is needed to establish a comprehensive understanding of these findings (Macintyre & Hamilton, 2010).

Notably, mathematics textbooks have evolved to incorporate vibrant visuals and color to aid in explaining mathematical concepts, a shift recognized in various educational settings (Usiskin, 2018). Moreover, research has explored the different modes of representation employed in mathematics textbooks, including symbolic, geometric, and algebraic representations (Johansson, 2006; Usiskin, 2018). The five fundamental aspects in the presentation of mathematics, namely symbolization, deduction, modeling, algorithms, and representations, have a profound impact on mathematics learning across all grade levels (Usiskin, 2018).

Representations in mathematics textbooks serve as a means to provide students with multiple perspectives on the same mathematical concepts. These representations encompass geometric depictions of numerical relationships, concrete models using physical objects, graphs of functions, and statistical data visualizations, among others. These diverse representations help students transition between symbolic and pictorial modes, fostering a deeper understanding of mathematical concepts.

The inclusion of concrete teaching materials in mathematics textbooks is a prevalent practice, with a wide array of specialized materials incorporated in the activity and examples sections. These materials range from algebra tiles and counting pieces to unit cubes, base ten blocks, and fraction sets. However, it is noteworthy that their inclusion in unit assessment sections is comparatively less frequent, potentially indicating an area for improvement (Yazlik, 2022).

The choice of fonts in mathematics textbooks can have significant implications, particularly in early grades, where the characteristics of non-English alphabets, such as Persian, become relevant. These considerations highlight the significance of fonts in mathematics textbooks, and their impact has sparked scholarly interest (Chan et al., 2023). Research by Chan

et al. (2023) investigates the influence of font type on students' mathematical performance, revealing that Times New Roman font might affect students' accuracy on certain mathematical tasks, underscoring the importance of font selection in instructional materials.

Despite these insights, it is crucial to recognize the limitations of such studies, notably the variations in educational systems across different regions. Future research might consider applying this framework to assess mathematics textbooks on an international scale, following established criteria such as TIMSS or PIRLS. Additionally, further exploration of the digital format of textbooks is warranted to gain a more comprehensive understanding of its impact on mathematics education.

In summary, the discussion of values within mathematics textbooks and the influence of visual aids and presentation on mathematical learning are areas of ongoing research, reflecting the dynamic nature of mathematics education and the ever-evolving needs of students and educators.

CONCLUSIONS

In conclusion, this paper has delved into critical aspects of mathematics education, shedding light on the multifaceted dimensions of mathematics textbooks, and their role in shaping the learning experiences of students. The examination of mathematics textbooks, which serve as primary instructional resources in classrooms, has uncovered several essential themes and considerations that influence the field of mathematics education.

One salient theme explored in this paper is the intrinsic connection between cultural sensitivity, inclusivity, and values in mathematics education. The values that underpin mathematics education, such as those outlined in Seah's classification, play a pivotal role in shaping both the curriculum and the students' learning paths. Moreover, the integration of cultural and moral values within mathematics textbooks has become increasingly prominent, reflecting a growing awareness of the need for inclusivity in educational materials. The recognition of values as personal choices influenced by societal and cultural contexts is a testament to the dynamic and evolving nature of mathematics education.

It is prudent to draw the readers' attention to the predominant Euro-Western orientation characterizing the majority of textbook analyses. This Euro-Western-centric perspective has historically prevailed in the academic discourse, often overshadowing the critical nuances associated with cultural sensitivities within mathematics textbooks. Particularly, aspects concerning the portrayal of women, the use of gender-inclusive language, and other facets of inclusivity exhibit noteworthy variances across diverse cultural contexts, thereby conveying profound and distinct messages to the learners. This academic observation underscores the

necessity of acknowledging and addressing this inherent bias in existing literature when scrutinizing mathematics textbooks, emphasizing the need for a more comprehensive and culturally informed approach to textbook analysis.

The second major theme centers on the visual presentation of mathematical concepts within textbooks. The physical features of textbooks, including the use of color, various representations, and concrete teaching materials, have a profound impact on students' engagement and success in mathematics. The importance of fonts in mathematics textbooks, especially in the early grades and across diverse linguistic contexts, underscores the need for careful consideration when designing instructional materials.

It is important to acknowledge that mathematics textbooks are evolving in response to changing educational needs and technological advancements. The shift toward digital platforms and the ever-expanding array of learning resources have added complexity to the design and presentation of mathematical content.

However, it is vital to recognize the limitations in the existing body of research, particularly the differences in educational systems across regions. For this reason, future studies should consider applying the analytical framework discussed in this paper to evaluate mathematics textbooks on an international scale, potentially through established criteria like TIMSS or PIRLS. Additionally, research into the impact of digital formats on mathematics textbooks is a pressing need, as technology continues to transform the landscape of education.

In summary, mathematics textbooks play a pivotal role in the pedagogical landscape, serving as more than just repositories of mathematical knowledge. They are vehicles for cultural sensitivity, inclusivity, and the transmission of values in mathematics education. The careful consideration of their visual presentation and the adoption of innovative teaching materials can significantly impact students' understanding and engagement with mathematics. The stakeholders within the educational landscape, including curriculum developers, mathematics textbook authors, educators, and mathematics teachers, stand to derive significant benefits from the elucidated themes extracted in this study. By embracing these insights, stakeholders can effectively inform future endeavors in curriculum development, textbook authorship, pedagogical strategies, and instructional practices. Moreover, this research heralds new avenues for scholarly inquiry into textbooks, whether as subjects of investigation or as integral components within broader educational analyses. As such, it not only enriches the discourse surrounding textbooks but also contributes to the advancement of educational research methodologies and the enhancement of teaching and learning practices.

The exploration of these themes underscores the ever-evolving nature of mathematics education and the need for continuous research and adaptation to meet the diverse and dynamic needs of learners in the 21st century.

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