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A teaching approach to quantum computing at the secondary school level

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Abstract

This paper presents a minimalist teaching-learning sequence for quantum computation in secondary education (5 × 60 minutes) grounded in the photon polarization approach. The framework has been tailored to the secondary school level education omitting complex numbers, matrices, and programming while relying on physics concepts with which students are already familiar from photon polarization. Employing a design-based research methodology, we identified the minimal set of concepts necessary for conceptual understanding: Dirac notation, the X and H quantum gates, and the quantum advantage demonstrated using the penny flip game as an example, which was tested on a real quantum computer. Students also explore the B92 cryptographic protocol and quantum entanglement in this course.

Keywords: quantum computing, quantum information science, quantum physics, polarization, photon, qubit

INTRODUCTION

In the second half of the 20th century, a new scientific era began: the second quantum revolution. As second-generation quantum technologies (e.g., quantum computing, quantum communication, quantum sensing, etc.) have become increasingly relevant to industry, the demand for a highly trained workforce with expertise in quantum physics has grown substantially (Fox et al., 2020; Greinert et al., 2023; Hughes et al., 2022; Merzel et al., 2024; Meyer et al., 2024; Venegas-Gomez, 2020). In Europe, efforts to develop such expertise have been integrated into the Quantum Flagship (2023) initiative, in which adapting knowledge for both secondary and higher education has played a central role (European Quantum Technology Education, 2024; QTIndu, 2023; Quantum Technology Education Project, 2020; Sherson & Goorney, 2023). The United Nations General Assembly declared 2025 as the international year of quantum science and technology, marking 100 years of quantum science (Goorney et al., 2025; UNESCO, 2024).

It is not necessary for students to master the entire underlying physical and mathematical apparatus in order to become familiar with the foundations of quantum computation. Indeed, a basic understanding of simple two-state quantum systems at the high school level is sufficient, given that a qubit is the state of such a two-state quantum system. In such a system, all states can be described by two basis states, a mathematical apparatus that is well within the scope of high school mathematics. In response to this need, numerous quantum technology educational resources have been developed in recent years, specifically targeting high school learners and beginning university students (Bernhardt, 2019; Billig, 2018; Bley et al., 2024; Bronner et al., 2009; Hughes et al., 2021; IBM, 2020; Kohnle et al., 2014, 2015; Microsoft, n. d.; Migdał et al., 2022; Müller & Greinert, 2024; University of St. Andrews, 2013). The present article introduces a new minimalist teaching-learning sequence on quantum computation that employs the photon-polarization approach, which is designed to be feasible even at the high school level.

RESEARCH BACKGROUND

Introducing Learners to Quantum Physics via Light Polarization: Brief Overview

Qubits can be physically realized in various ways, thus giving rise to multiple educational approaches for teaching quantum physics. Among the most widely used

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Contribution to the literature

- We present a novel minimal approach to quantum information science that employs the context of photon polarization.
- This approach avoids the use of complex numbers and matrices. The development process was informed by the cyclical principle of design-based research (DBR).
- This newly designed teaching-learning sequence was subjected to a pilot educational experiment in a real classroom setting. The experiment revealed certain difficulties in students learning.

are the electron-spin (Bernhardt, 2019; Feynman, 1965; Sakurai, 1985), the which-path (Bitzenbauer & Meyn, 2020; Marshman & Singh, 2017; Müller & Wiesner, 2002), the double-well (Faletič, 2020), and the photon-polarization approach (Michelini et al., 2000, 2004, 2022; Tóth et al., 2024b). Two-state approaches are not only effective for introducing quantum computing but also demonstrate superior performance over traditional methods in promoting conceptual understanding (see Bitzenbauer, 2021; Bitzenbauer et al., 2024).

Among the educational approaches that employ two-state systems, the photon-polarization approach is particularly significant because

(1) the underlying phenomenon (light polarization) can be readily explored and understood in school settings (Tóth et al., 2024a) and is also covered by several curricula,

(2) it has a decades-long tradition, providing access to well-established educational resources (Michelini et al., 2000, 2004, 2022; Tóth et al., 2024b), and

(3) it is especially effective in promoting students' conceptual understanding (Bitzenbauer et al., 2024; Michelini & Stefanel, 2008; Michelini et al., 2022; Montagnani et al., 2023; Tóth et al., 2024b; Zuccarini et al., 2024).

In the photon-polarization approach, students initially engage in thought experiments and computer simulations involving single-photon sources and polarizers. It is established that the transmission of photons through a polarizer is random by principle and that the outcomes of many repeated, identical experiments can only be described probabilistically. This probability is consistent with Malus law, with which students are already familiar. In this context, the $\cos^2 \vartheta$ term acquires a new interpretation at the single-photon level: it represents the probability p of photon transmission through a polarizer, with ϑ denoting the angle between the transmission axis of the polarizer and the photon's polarization direction.

Students develop an intuitive understanding that photon states can be represented by unit vectors in a plane, which is made concrete through the assignment of vectors to polarization directions. They recognize that the transmission of photons through a polarizer can be calculated as the square of a scalar product, with $p = \cos^2 \vartheta$ arising naturally: if the state vector $\boldsymbol{\psi}$ represents the initial state and $\boldsymbol{h}$ the transmitted state, then the probability of a state change is

$$p(\boldsymbol{\psi} \rightarrow \boldsymbol{h}) = (\boldsymbol{h} \cdot \boldsymbol{\psi})^2 = (|\boldsymbol{h}| \cdot |\boldsymbol{\psi}| \cos \vartheta)^2 = 1 \cdot 1 \cdot \cos^2 \vartheta \tag{1}$$

We emphasize that the photon polarization approach in secondary schools focuses exclusively on linear polarization, thus avoiding complex numbers (which would arise in the case of circular polarization). Therefore, the mathematical formalism is simplified, as shown in Eq. (1), because it does not require the adjoint of the state.

Building on their elementary knowledge of linear algebra, students also discover the superposition principle: not only the basis states corresponding to certain measurement outcomes (e.g., the horizontal polarization state $\boldsymbol{h}$ and the vertical polarization state $\boldsymbol{v}$) are possible, but also arbitrary linear combinations of these states (e.g., the diagonally polarized state),

$$\boldsymbol{\psi} = \psi_1 \boldsymbol{h} + \psi_2 \boldsymbol{v}. \tag{2}$$

In this representation, the squared coefficients (ψ_1^2 and ψ_2^2) correspond to the measurement probabilities associated with the respective states (the absolute value can be neglected because only linear polarization states are considered), restricting only real numbers.

The teaching-learning sequence also examines the features of photons emitted from a single-photon source onto a birefringent calcite crystal, where the behavior of the photons remains inherently random. In accordance with the superposition principle, photons are detected in the spatial locations of the ordinary and extraordinary beams, for example, in states $\boldsymbol{h}$ and $\boldsymbol{v}$ with probabilities ψ_1^2 and ψ_2^2, respectively. Birefringent calcite crystals allow students to observe that the behavior of photons cannot be described as continuous in space and time. This conclusion is based on the observation that the photonic state $\boldsymbol{\psi} = \psi_1 \boldsymbol{h} + \psi_2 \boldsymbol{v}$ does not correspond to a certain spatial location $\boldsymbol{h}$ and $\boldsymbol{v}$; rather, quantum states permit only statistical predictions. For more details on the photon polarization approach, see Michelini et al. (2000, 2004, 2022) and Tóth et al. (2024b).

Naturally, certain learning difficulties were also identified in the photon polarization approach: many students retained the classical way of thinking and found it particularly challenging to grasp and accept the absence of classical motion (Tóth et al., 2024b). Misconceptions regarding quantum state and measurement have also emerged, as indicated in the literature (Michelini & Stefanel, 2021; Michelini et al., 2022; Montagnani et al., 2023; Pospiech et al., 2021; Singh & Marshman, 2015; Tóth et al., 2024b). A primary benefit of the polarization-based approach is that its central element is not the lack of classical motion but polarization itself–a concept that is less burdened by classical interpretations. This approach appears to facilitate students' conceptual understanding, enabling them to comprehend numerous features of quantum physics, which may be particularly beneficial when introducing quantum computation.

Teaching-Learning Quantum Information Science in Secondary Schools

In recent years, books have been published that are aimed at high school students and focus on teaching quantum technology (Bernhardt, 2019; Billig, 2018; Hughes et al., 2021; Müller & Greinert, 2024). However, these books do not follow the minimalist approach that is espoused in this article (refer to our design principle later). For instance, Billig (2018) and Bernhardt (2019) immediately introduce column vectors and matrices, including concepts from infinite-dimensional systems, which exceeds the level of knowledge expected of students in public education. Bernhardt (2019) utilizes the Stern-Gerlach experiment in which electron spins are employed as qubits; however, it this experiment is not included in the standard physics curriculum in several countries (Stadermann et al., 2019). Hughes et al. (2021) introduce the superposition principle and its mathematical representation using the analogy of a coin toss. We believe this detour is unnecessary, given that some quantum physics concepts are already part of the general high school curriculum. Subsequently the Stern-Gerlach experiment is introduced, and the mathematical language employed is characterized by a substantial reliance on matrices. Müller and Greinert's (2024) book is designed for engineering university students; however, a substantial portion is also appropriate for secondary school students. Overall, the language is overly abstract, particularly due to the extensive use of complex numbers.

Quantum technology education literature also encompasses games (Piispanen et al., 2025), computer-based simulations and experiments (Bronner et al., 2009; Migdał et al., 2022; Solvang et al., 2025; University of St. Andrews, n. d.) and extended open-access learning materials that facilitate students toward a deeper understanding of the field (Hellstern et al., 2024; IBM, 2020; Microsoft, n. d.) and teaching experiments (Hu & Singh, 2024; Zuccarini et al., 2024). Escanez-Exposito et al. (2025) introduce quantum physics in a block-based programming format accessible to students, using classical physical analogies and minimal prior knowledge requirements. It extends well-established quantum learning material, but it includes classical analogies. However, the primary focus of our novel teaching-learning sequence is not on the programming aspects but rather on the application of a qubit. Similarly, Sun et al. (2024) have proposed an alternative approach that involves a reduction in the necessity for advanced mathematics. This approach entails a presentation of the subject starting from classical computing concepts, fostering students from IT to quantum information theory. Additionally, there are resources dedicated to certain topics, such as teaching quantum cryptography (see DeVore & Singh, 2020; Weissman et al., 2024). Zuccarini et al. (2024) underscore the physical implementation of quantum gates in a manner that is accessible and comprehensible to students within the context of photon polarization.

Despite the emergence of professional educational resources in recent years, there are still gaps in the literature regarding the combined appearance of the following three aspects:

1. It focuses solely on the minimal knowledge required for conceptual understanding and begins quantum computing from quantum physics rather than information theory. It is designed to be a comprehensive unit that spans multiple, interconnected quantum computing topics, providing a coherent and complete introductory journey rather than focusing on a single isolated concept.
2. It is built upon a research-validated and effective quantum physics educational approach: the photon polarization approach. This foundation is scientifically sound yet remains fully accessible and appropriate for the high school level.
3. It excludes mathematical formalisms not typically taught in high school, such as complex numbers and matrix algebra. This choice enables students to engage with fundamental quantum concepts without being encumbered by prerequisites in advanced mathematics, thereby ensuring the material's genuine applicability within the standard secondary school curriculum.

DEVELOPMENT OF A NEW TEACHING-LEARNING SEQUENCE ON QUANTUM COMPUTING AT THE SECONDARY SCHOOL LEVEL

Goal of the Teaching-Learning Sequence

The article presents a teaching-learning sequence that requires minimal prior knowledge and is based on proven-effective teaching approach, introducing quantum computing as an application of quantum physics. The sequence is designed to require a total of five hours, employs intuitive examples, and provides foundational knowledge necessary for the topic. As a

Table 1. The summary of the teaching-learning sequence (each lesson was 60 minutes long)

Lessons	Topic	Details
1	• Dirac notation • Quantum gates	Students are introduced to Dirac notation for quantum states, as well as X and H quantum gates.
2	• Quantum penny flip game • IBM quantum computer	A simple game provides students with an opportunity to experience the power of superposition from the perspective of quantum computation. They then ran this game on a real quantum computer, which can be programmed via the cloud using basic quantum circuit elements.
3	• Quantum cryptography	Students become familiar with the B92 quantum key distribution protocol.
4-5	• Quantum entanglement	The students explore single-photon entanglement along with its mathematical representation, where the spatial location and polarization state of photons are entangled. Through a program that simulates real experiments, students gain insights into photon pairs that are entangled in their polarization. The course content also encompasses the implementation of entanglement in quantum circuits.

result, students are introduced to a new discipline, quantum computing, and acquire the conventional notation, Dirac notation, which is imperative for comprehending the extant literature. They grasp key concepts, including qubits, quantum gates, and quantum entanglement. Moreover, they recognize the advantages of quantum computation over classical computation and gain hands-on experience with the noise inherent in real quantum computers and explore quantum cryptography. It is posited that this teaching-learning sequence can guide students toward future quantum engineering education, thereby enabling them to build on these conceptual foundations. We further posit that quantum computing can be reconciled with students' intuitive conceptions of nature because the probabilistic outcomes of measurements arise not from a lack of knowledge but from the fundamental superposition principle that goes beyond classical physics. This is of particular significance because students frequently hold two misconceptions:

(1) certain predictions are impossible and

(2) probabilistic descriptions stem from experimental contingency or lack of knowledge rather than the intrinsic dynamics of quantum mechanics (Bitzenbauer et al., 2024; Michelini et al., 2022; Montagnani et al., 2023; Singh & Marshman, 2015; Styer, 1996; Tóth et al., 2024b).

Within the domain of quantum computing, both misconceptions can be explicitly addressed. Indeed, the study of quantum computing can facilitate students' understanding of two fundamental principles:

(1) quantum states can be controlled and

(2) the inherently probabilistic nature arising from superposition states can be harnessed for societal benefit.

This highlights the constructive potential of the unavoidable probabilistic behavior. Therefore, we argue that our newly developed teaching-learning sequence not only represents a novel approach to promoting quantum computing and orienting students toward quantum engineering majors but also substantially improves students' understanding of quantum physics.

Description of the Design Process, Time Requirements, and Learning Prerequisites

The structure of the teaching-learning sequence is presented in **Table 1**. As demonstrated in **Table 1**, the underlying concepts can be introduced within five lessons, which fit within the limited timeframe available in high school education. However, we emphasize that students required two lessons for the phenomenon-based exploration of light polarization (Tóth et al., 2024a) and an additional six lessons for the photon-polarization approach to quantum physics (Michelini et al., 2000; Tóth et al., 2024b), resulting in a total of 13 lessons. Knowledge of light polarization and quantum physics can be integrated into the standard curricula, as these concepts are currently part of the standard school curriculum. This integration would facilitate the practical incorporation of our approach into real classroom settings.

The teaching-learning sequence was developed according to the model of educational reconstruction (Duit et al., 2012). This paper is part of a DBR process involving development, evaluation, and refinement (Anderson & Shattuck, 2012). In this research, the first evaluation cycle is reported, with the main aim of constructing a secondary school teaching-learning sequence after a first trial.

Design Principles of the Teaching-Learning Sequence

DP1. Using the context of photon polarization

The teaching-learning sequence is structured around the photon-polarization approach, and this context is maintained throughout. Abstract and overly general reasoning can pose challenges for students, particularly in quantum physics. In contrast, the phenomenon of photon polarization provides an accessible, experimentally observable foundation. Prior to the quantum studies, we followed the suggestion of Tóth et al. (2024a) because light polarization can be understood phenomenologically without invoking wave optics. Leveraging this approach allows valuable instructional time to be saved while reducing the prerequisite knowledge required of students.

DP2. Dirac notation and vector representation

In the photon-polarization approach, the introduction of quantum states through vector representation is intuitive because polarization can be associated with a direction that is easily comprehensible to students and can be represented by a unit vector. A major advantage of this approach is that it utilizes two arrows to represent two orthogonal quantum states (e.g., horizontal and vertical polarization states), with a 90° separation between them. This is in contrast to approaches such as electron spin, where orthogonality corresponds to oppositely oriented spins. In quantum computing, the use of Dirac notation for quantum states (or qubits) is essential, as it is the prevailing notation in the field; without it, the literature becomes largely incomprehensible. Therefore, the vector representations of the states are expressed in Dirac notation.

DP3. Minimalistic approach to formalism

The introduction of concepts not covered in standard high school mathematics can imposes a significant cognitive and temporal burden on students. Therefore, one of our principles is to eschew mathematical knowledge that exceeds the level of high school. Consequently, complex numbers are avoided; therefore, only linearly polarized states are considered. The coordinate representation of quantum states and matrix operations is entirely omitted. Using linearly polarized states, quantum states are represented as vectors in a plane, which can be depicted on a unit circle, the state circle (Pospiech et al., 2021; Tóth & Tél, 2023). This provides a straightforward mathematical background that is also well known from high school mathematics. Consequently, the Hilbert space adopts a structure analogous to the familiar Cartesian coordinate system.

DP4. Nucleus-body-periphery distinction of a curriculum

In the design of the teaching-learning sequence, we adopted the innovative curricular approach of the discipline-culture framework (cf. Weissman et al., 2022). This framework classifies educational content into three categories: the nucleus (core features of a discipline), the body (applications and related experiments), and the periphery (border areas of the discipline). In this study, quantum physics is treated as a distinct discipline within physics, with a foundation that is structured around the photon-polarization approach. In this way, students discovered the most fundamental features of the topic, some of its basic laws, and a simplified version of its mathematical apparatus, which constituted the nucleus of learning. They also understood that, in the limit of very large numbers of photons (peripheral), classical physics is recovered, and light can be described using its classical representation. The presented quantum computing teaching-learning sequence complements the existing body of quantum studies, focusing on a specific application area (body). Care was taken not to include more advanced topics from classical computer science, thereby keeping students within the quantum physics context while still allowing them to explore connections with information theory (periphery).

DP5. Minimalistic approach to quantum computing

Owing to its cultural significance (The Nobel Committee for Physics, 2022), its positive results regarding the teachability of the topic in schools (Brang et al., 2024), and its role in quantum computing, quantum entanglement is included in the teaching-learning sequence. However, because the experimental components needed to generate entangled photon pairs are very expensive and the phenomenon itself is challenging to grasp, students are initially introduced to single-photon entanglement using inexpensive birefringent calcite crystals (Zuccarini & Michelini, 2023; Zuccarini et al., 2024). In this way, students acquire formalism in accordance with DP2 and subsequently deepen their understanding of photon-pair entanglement through an interactive screen experiment (Bronner et al., 2009).

Following DP3, a mere three quantum gates are incorporated into the teaching-learning sequence: the X gate (quantum NOT gate), the H gate (which can create a superposition), and the CNOT gate (sufficient for entanglement). These gates are sufficiently elementary for high school students to comprehend. We did not focus on the complex physical implementations of quantum gates, thereby saving time and cognitive resources. The physical realization of the quantum gates presented in the instructional sequence is well illustrated by Zuccarini et al. (2024) within the context of photon polarization, allowing seamless integration into classroom lessons.

In this study, we focused on the simplest instructional sequence that conveys the minimal knowledge necessary for conceptual understanding. We consider the quantum penny flip game and B92 protocol.

Pedagogical Trajectory in the Context of Teacher Training

The instruction of quantum computation requires substantial prerequisite knowledge from physics teachers. Quantum physics is a component of the Eötvös Loránd University teacher training program, and the photon polarization approach has been integrated into it (cf. Tóth, 2023, 2024). Students begin their studies with the study of photon polarization. In this phase, students learn the specific, photon-polarization-based educational material (Michelini et al., 2000, 2004, 2022, Tóth et al., 2024b), using only real numbers (i.e., considering only linear polarization states) and avoiding matrix representation. The only difference is that university students cover this material within a shorter timeframe than secondary school students. This is not only a pedagogical best practice that facilitates comprehension but also equips students with didactic skills. Subsequent to this, the description is generalized, while still avoiding complex numbers. The representation of the school-level material is extended by using column vectors, transposes, and matrices is introduced (Tóth, 2023).

Students then discover the complex description through circular polarization, as they can no longer describe circularly polarized states using only real numbers (Tóth, 2024). In this way, students become thoroughly familiar with the general description of two-state quantum systems. Thereafter, students are introduced to higher-dimensional systems through quantum entanglement via the entanglement of polarization. They were then introduced to the traditional wave mechanics approach and its application in atomic physics. The course concludes with didactics of quantum physics education. The teaching-learning sequence presented in this study was developed to ensure that students, based on their prior knowledge, are capable of teaching the described concepts, thus laying the groundwork for future quantum technology education.

TEACHING-LEARNING SEQUENCE

Lesson 1. Dirac Notation, Quantum Gates

Dirac notation

The bits 0 and 1 can be assigned to the horizontal ($\boldsymbol{h}$) and vertical ($\boldsymbol{v}$) polarization states, respectively. Students are introduced to Dirac notation using the standard conventions in quantum information theory: $|0\rangle = \boldsymbol{h}$, $|1\rangle = \boldsymbol{v}$. By full analogy with Eq. (1), not only the qubits $|0\rangle$, $|1\rangle$ are possible, but also their linear combination (in the classroom, with only real numbers ψ_1 and ψ_2 satisfying $\psi_1^2 + \psi_2^2 = 1$) is a qubit:

$$|\psi\rangle = \psi_1|0\rangle + \psi_2|1\rangle. \quad (3)$$

In the classroom, students encounter quantum measurements exclusively in the $\{|0\rangle, |1\rangle\}$ basis. As shown in **Figure 1**, every (linear polarization) state can be represented by a vector in a plane that lies on a unit circle. Owing to the choice of polarization context, the orthogonal states form a 90° angle, which is completely natural in the context of photon polarization (in contrast, for example, to electron spin). We note that this is not the standard Bloch sphere representation of quantum states; rather, it assumes a more simplified form.

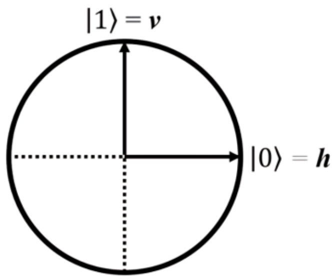

Figure 1. As posited by Tóth & Tél (2023), the geometrical representation of the Hilbert space (state circle) is consistent with secondary school mathematics

Students are also introduced to bra-vectors: the scalar product of state $|1\rangle$ with state $|0\rangle$ is written in Dirac notation as $\langle 0|1\rangle$. Since in the lessons every state was expressed as a superposition of $|0\rangle$ and $|1\rangle$, students were required to memorize the following relationships between the states: $\langle 0|0\rangle = \langle 1|1\rangle = 1$ and $\langle 0|1\rangle = \langle 1|0\rangle = 0$. These do not merely appear as mathematical rules but also carry a well-understood physical context: horizontally polarized photons are certain to pass through a polarizer with horizontal permitted direction, while certain photons are absorbed by a polaroid with vertical permitted direction.

Students then considered an experiment with a polaroid with horizontal permitted direction. The probability of transmission (see Eq. [1]) can be expressed as

$$p(|\psi\rangle \to |0\rangle) = \langle 0|\psi\rangle^2 = \langle 0|\psi\rangle \cdot \langle 0|\psi\rangle = \langle \psi|0\rangle \cdot \langle 0|\psi\rangle = \langle \psi|\hat{P}_0|\psi\rangle, \quad (4)$$

where $\hat{P}_0 = |0\rangle\langle 0|$ is a projector, and the “ket-bra” operation is the outer product. We note that the absolute value is neglected again because we omit complex numbers. Students recognize that the sequence of two states in the expression is not a scalar product. Rather, when an outer product acts on a state, that state is first scalar-multiplied by the bra-vector (yielding a number), and this result is then multiplied by the ket-vector of the projector, thereby producing a new vector.

Quantum gates

Building on students’ prior knowledge of quantum measurement and projectors, they are introduced to the fact that quantum states can be manipulated and that these transformations are carried out by quantum gates. Students are not expected to engage with the physical implementation of quantum gates; rather, their existence is assumed. The only requirement for a quantum gate is unitarity (Nielsen & Chuang, 2010): if a quantum logic gate is applied twice in succession, the result is as if nothing has occurred. Therefore, the inverse of a quantum gate (its reverse action) is the gate itself. Following the minimalist approach, students are introduced to only two single-photon gates: the X and H gates, which are presented in three representations - algebraically as a sum of outer products, geometrically

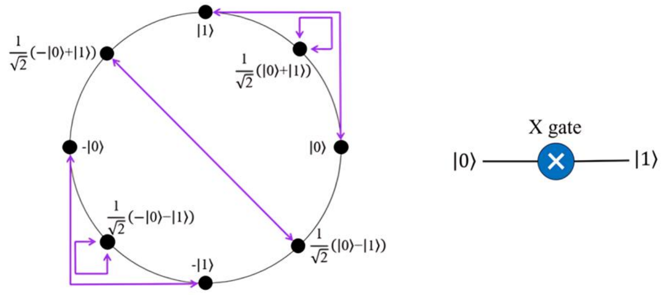

Figure 2. Effect of the X gate, indicated by blue arrows, on the state circle (left) & the quantum circuit symbol is a cross inside a blue circle (right) (by definition, the quantum circuit can also be read backward as $\hat{X}|1\rangle = |0\rangle$ (Source: Authors' own elaboration)

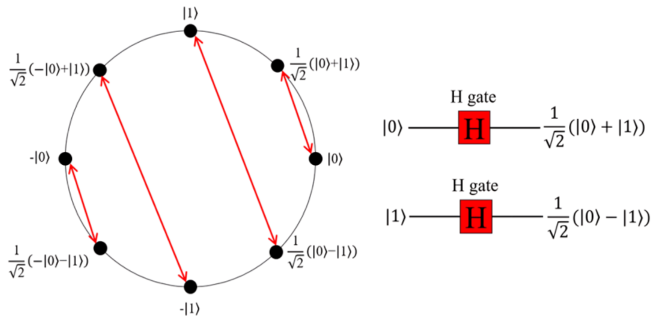

Figure 3. The effect of the H gate, which generates superposition states, is shown on the state circle (left) & its quantum circuit symbol is an "H" inside a red square (right) as demonstrated through two examples (Source: Authors' own elaboration)

on the state circle, and in the quantum circuit diagram notation used in programming quantum computers.

X gate: The X gate is the quantum analogue of the classical NOT gate: $|0\rangle \leftrightarrow |1\rangle$. The X gate also acts on superposition states, as illustrated on the state circle (see left-hand side of **Figure 2**), where the X gate reflects states across the +45° axis. The quantum circuit symbol for the X gate is a cross inside a blue circle, as shown on the right side of **Figure 2**. Its algebraic form is

$$\hat{X} = |0\rangle\langle 1| + |1\rangle\langle 0|. \tag{5}$$

Students verified that the X gate behaves as described above: $\hat{X}\,|0\rangle = (|0\rangle\langle 1| + |1\rangle\langle 0|)|0\rangle = |0\rangle\langle 1|0\rangle + |1\rangle\langle 0|0\rangle = |0\rangle \cdot 0 + |1\rangle \cdot 1 = |1\rangle$ and $\hat{X}\,|1\rangle = (|0\rangle\langle 1| + |1\rangle\langle 0|)|1\rangle = |0\rangle\langle 1|1\rangle + |1\rangle\langle 0|1\rangle = |0\rangle \cdot 1 + |1\rangle \cdot 0 = |0\rangle$.

Therefore, the action of the X gate on a general state $|\psi\rangle = \psi_1|0\rangle + \psi_2|1\rangle$ is $\hat{X}\left(\psi_1|0\rangle + \psi_2|1\rangle\right) = \hat{X}\,\psi_1\,|0\rangle + \hat{X}\,\psi_2\,|1\rangle = \psi_1\,\hat{X}\,|0\rangle + \psi_2\,\hat{X}\,|1\rangle = \psi_1\,|1\rangle + \psi_2\,|0\rangle$.

H gate: The H gate creates superposition states from the basis states $|0\rangle$ and $|1\rangle$, as shown on the state circle and in the two quantum circuit examples in **Figure 3**. Its algebraic form is

$$\hat{H} = (1/\sqrt{2})\big((|0\rangle + |1\rangle)\langle 0| + (|0\rangle - |1\rangle)\langle 1|\big). \tag{6}$$

Using Eq. (6), students can verify that the H gate generates a superposition state: $\hat{H}\,|0\rangle = (1/\sqrt{2})\big((|0\rangle + |1\rangle)\langle 0|0\rangle + (|0\rangle - |1\rangle)\langle 1|0\rangle\big) = (1/\sqrt{2})(|0\rangle + |1\rangle) = |D\rangle$ and $\hat{H}\,|1\rangle = (1/\sqrt{2})\,\big((|0\rangle + |1\rangle)\,\langle 0|1\rangle + (|0\rangle - |1\rangle)\,\langle 1|1\rangle\big) = (1/\sqrt{2})\,(|0\rangle - |1\rangle) = |A\rangle$

Lesson 2. Quantum Penny Flip

We adapted the idea of Müller and Greinert (2024) on teaching-learning the quantum penny flip game, which serves as an intuitive demonstration of the power of superposition. Students are first introduced to the classical version of the game, extending the approach of Müller and Greinert (2024).

Classical case

In the classical version, two players, Alice and Bob, play with coins. The outcome of the coin toss determines the winner: If the result is heads (bit 0), Alice wins; if it is tails (bit 1), Bob wins. The game proceeds as follows:

1. Alice places the coin in an arbitrary state (heads: 0 or tails: 1) inside a box that completely shields it from the outside.
2. Bob then reaches into the box and decides either to flip the coin or to leave it unchanged. He does not know the state of the coin before or after his action.
3. Alice next reaches into the box and likewise decides whether to flip the coin.
4. Finally, the box is opened, and the state of the coin reveals the winner of the round.

Throughout the process, neither player discloses their decision.

During the lesson, the students quickly recognized the winning strategy for the players. If Bob does not flip the coin, Alice's chances of winning increase, because she knows the initial state she set in step 1. If this state is favorable to her, she will certainly win without further flips. To prevent this, Bob is forced to flip the coin. As a result, each player has a winning probability of 1/2.

Quantum case

In the quantum version, the coin is replaced with a qubit. The winner is determined by measuring the qubit in the {|0⟩, |1⟩} basis: If the outcome is |0⟩, Alice wins; if |1⟩, Bob wins. During the game, Alice understands and applies the laws of quantum physics, whereas Bob treats the qubit as a classical bit or coin. Neither party reveals their decision, and the game proceeds as follows:

1. Alice arbitrarily transforms the |0⟩ qubit state, corresponding to placing the coin in the box.
2. Bob performs an operation on the qubit. Since he treats it like a coin, he can only do one of two

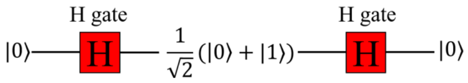

Figure 4. Quantum circuit for one scenario of the quantum coin-flipping game (by convention, qubits in quantum computing are initially in the $|0\rangle$ state; thus, in the first step, Alice's creation of the superposition state is implemented with the H gate & next, Bob does nothing, and Alice applies H gate again, resulting in the final $|0\rangle$ state) (Source: Authors' own elaboration)

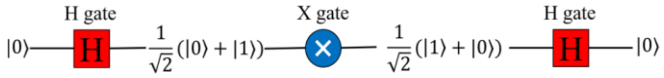

Figure 5. Quantum circuit diagram of the alternative scenario of the quantum coin toss game (Alice creates a superposition state using the H gate, then Bob "tosses the coin", and it "flips" (X gate), and finally Alice applies another H gate & the final state is always $|0\rangle$, so Alice always wins) (Source: Authors' own elaboration)

things: either do nothing or apply the quantum analogue of a coin flip. This "coin flip" consists of applying the X gate with a probability of 1/2.

3. Alice performs an operation on the qubit.
4. The qubit is measured, yielding the final state and determining the winner of the game.

The game is played in a classroom. Each player records their decisions in a notebook. To simulate the quantum "flip," students physically toss a coin: if heads, they do nothing; If tails, they record the application of an X gate. If Alice applies to the H gate twice, she will always win. The quantum circuit representation of this process is shown in **Figure 4** and **Figure 5**, respectively. In this game, Alice corresponds to quantum computation, and Bob represents classical computation. Alice's victory is enabled by the existence of superposition states.

Quantum penny flip using a real quantum computer

Students were introduced to a real quantum computer interface (IBM, 2016), and they tested the quantum penny flip game on a real quantum device. The quantum circuit executes on multiple real quantum computers online. Access to these machines is limited and subject to a queue that depends on the quality of the device. Students observed that on real quantum computers, the expected measurement outcomes are not always realized owing to factors such as environmental noise or nonideal experimental conditions. Imperfections in the experimental implementation prevent the perfect control of quantum states, resulting in measurement outcomes that may differ from the expected results. This phenomenon is referred to as noise.

Since higher-quality devices (with lower noise and more qubits) typically have longer queues, we used a

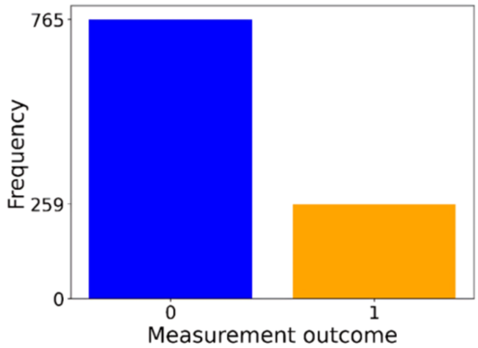

Figure 6. Results of 1024 repeated measurements on a noisy quantum computer for both cases (contrary to expectations (1024 measurements of $|0\rangle$), only 765 measurements yielded $|0\rangle$ and 259 measurements yielded $|1\rangle$ due to noise) (Source: Authors' own elaboration)

noisy machine with fewer qubits, allowing students to perform their first live quantum programming exercise during the lesson. Due to noise, Bob can occasionally win. **Figure 6** shows the results of 1024 repeated measurements conducted on a low-quality quantum computer. In this case, only 765 measurements resulted in Alice's victory, instead of the expected 1024. The quantum computer initially operated at higher error rates than students expected. To address this, we prepared in advance by selecting a higher-quality quantum computer several days before the lesson. This allowed students to observe the measurement outcomes with much lower noise after waiting in the queue. For example, following an approximately four-hour queue, 1023 measurements yielded $|0\rangle$ and only 1 measurement yielded $|1\rangle$.

Lesson 3. Quantum Cryptography (B92)

Students are introduced to a method for generating a secret key: the B92 quantum key distribution protocol. We demonstrated the protocol through a simulation of B92, available via the QuVis homepage (University of St. Andrews, n. d.). In this procedure, the 0 and 1 bits forming the secret key are determined based on the polarization states. Alice prepares photons using only two polarization states, keeping these polarizations a secret. The two chosen polarizations cannot be orthogonal; otherwise, a photon's polarization could be easily inferred using a polarizer aligned with one of the polarizations, thereby compromising the secrecy of the key.

Alice transmits photons either in the horizontal $|0\rangle$ or diagonal $|D\rangle = (1/\sqrt{2})(|0\rangle + |1\rangle)$ polarization, while Bob measures using polarizers with vertical or antidiagonal (orthogonal to diagonal) permitted directions. Photons polarized as $|0\rangle$ correspond to 0 bits, and photons polarized as $|D\rangle$ correspond to 1 bit (note that in this protocol, not qubit $|1\rangle$, but $|D\rangle$ that represents bit 1). Alice signals Bob that single photons have been sent but keeps their polarizations secret. Bob's goal is to infer the polarization of each transmitted photon. As a result, both parties maintain a list: Alice's list contains the indices of all signals and the corresponding bits for each transmitted photon, whereas Bob's list contains the indices but leaves some bits blank if he cannot infer the polarization or records a bit if he can deduce it. Bob indicates which photons were detected, allowing a shared secret key to be formed from the bits Bob recorded, while Alice discards the remaining bits.

The security of this protocol relies on quantum mechanics: An eavesdropper (Eve) cannot perfectly copy photons with unknown polarization states. Because the measurement outcomes are inherently probabilistic, Eve is unlikely to determine the polarization of each photon sent by Alice. Furthermore, the act of measurement affects the quantum state, so Eve's presence can be revealed if Bob's recorded bits deviate from Alice's.

Scenario without an eavesdropper

Bob randomly selects a polarizer with either a vertical or antidiagonal permitted direction with an equal probability of 1/2. If Bob detects a photon (i.e., the photon passes through his polarizer and triggers the detector), he can infer its original polarization. The students investigated the scenario in which Alice sends a diagonally polarized photon ($|D\rangle$) and Bob measures it with a polarizer with vertically permitted direction; detection occurs with a probability of 1/2. Across multiple trials, some $|D\rangle$ photons are detected. If Bob had used a 135° polarizer, the $|D\rangle$ photons would be absorbed, making detection impossible. Therefore, the students discovered that if Alice signals that a photon has been sent and Bob detects it, he can infer that the photon was in the $|D\rangle$ state and record a 1 bit.

Students practiced this procedure in groups of three, taking on the roles of Alice, Bob, and a referee. The student playing Alice determined the bits of the secret key by tossing a coin, recorded them privately, and shared them only with the referee. Similarly, the student playing Bob randomly selected the orientations of the polarizers by coin tosses and reported them to the referee. The referee then simulated the probabilistic outcomes using additional coin tosses and informed Bob which bits he could infer correctly and which he could not.

Then, the students discussed that, because Bob chooses the polarizer randomly and detection occurs with a probability of 1/2 for the correct choice, he can infer the polarization of photons with a probability of 1/4. Alice needs to know which photons were detected, so Bob communicates the indices of detected photons via a public channel. Only photons whose polarization Bob confidently determines are used in the shared key, ensuring that no bits are disclosed to others.

Scenario with an eavesdropper

Students, together with the teacher, first investigated the B92 simulation available on the University of St. Andrews (2013) website. Eve's goal is to infer as many photon polarizations as possible while remaining undetected. If she knows Bob's polarizer orientations, she can attempt to measure Alice's photons in a similar manner. If successful, she sends identical photons to Bob to conceal her presence. If Bob detects these photons, he records the bits that are already known to Eve. If Eve fails to detect a photon, she must still send a photon to Bob and guess its polarization. Bob may detect incorrect guesses.

To detect Eve, Alice and Bob perform a subset of bit-checking: Bob randomly selects bits he knows and sends their values and indices to Alice over a public channel. These bits are not included in the secret key. If all the checked bits match, the communication is considered secure, and the remaining shared bits form the key. Any discrepancy reveals Eve's interference, and the key is discarded.

Students playfully experimented with this scenario, forming groups of four, with the fourth member acting as a referee (similar to the case without eavesdropping). Some students observed that Eve's strategy involves sending a photon aligned with her polarizer if she fails to detect the original photon. Errors arise when Alice's transmitted photons are absorbed and originally had a polarization at 45° relative to Eve's polarizer. Since the polarizer orientation is chosen with a probability of 1/2, and the chance of sending a photon in the wrong polarization is also 1/2, the probability that Eve sends a photon incorrectly is 1/4. Bob detects these incorrect photons with a probability of 1/4, resulting in an expected 1/16 fraction of all photons yielding incorrect bits. Therefore, among the bits Bob records, 1/4 are expected to be erroneous, allowing Alice and Bob to detect Eve's presence.

In summary, students explored that the security of the protocol relies on the quantum mechanical principle that an eavesdropper cannot perfectly replicate unknown photon polarizations. Due to the probabilistic nature of quantum measurement, Eve is unlikely to obtain full information about Alice's photons, and her presence can be detected because Bob's recorded bits will differ from Alice's.

Lesson 4-5. Quantum Entanglement

Single-photon entanglement

Students, together with the teacher, investigate an experiment that appeared in previous quantum physics studies: A single photon is emitted onto a birefringent crystal calcite. The students are able to express the photonic state (2) in Dirac notation as: $|\psi\rangle = \psi_1|0\rangle + \psi_2|1\rangle$, where the $|0\rangle$ state corresponds to the ordinary beam and the $|1\rangle$ state corresponds to the extraordinary beam. Previously, students considered the superposition state in terms of polarization, where the $|0\rangle$ and $|1\rangle$ states represented two orthogonal polarization states. We emphasized to students that these states can also reflect certain spatial modes, which can be explicitly indicated as $|\psi\rangle = \psi_1|0\rangle_{spat} + \psi_2|1\rangle_{spat}$. Since there is a correlation between spatial mode and polarization, the state can equivalently be written as

$$|\psi\rangle = \psi_1|0\rangle_{pol}|0\rangle_{spat} + \psi_2|1\rangle_{pol}|1\rangle_{spat}. \tag{7}$$

As indicated by the subscripts, the first ket refers to the polarization state, and the second ket immediately following it refers to the spatial mode. The pairs $|0\rangle_{pol}|0\rangle_{spat}$ and $|1\rangle_{pol}|1\rangle_{spat}$ do not denote scalar products but rather new states (tensor product). These joint states must be considered together, reflecting the fact that knowledge of the spatial mode also determines the polarization, since the corresponding properties are realized simultaneously. The joint states $|0\rangle_{pol}|0\rangle_{spat}$ and $|1\rangle_{pol}|1\rangle_{spat}$ correspond to mutually exclusive events and are therefore orthogonal. The polarization and spatial-mode degrees of freedom are entangled: It is not possible to separate the photon's state into independent polarization and spatial-mode states, because these properties are not independent and are always measured together, even if only one degree of freedom is accessed.

If we agree that the first ket corresponds to polarization, the subscripts in Eq. (7) can be omitted, yielding the more compact expression

$$|\psi\rangle = \psi_1|0\rangle|0\rangle + \psi_2|1\rangle|1\rangle = \psi_1|00\rangle + \psi_2|11\rangle. \tag{8}$$

In Eq. (8), the first number in the ket represents the polarization state, and the second number denotes the spatial-mode state: $|0\rangle_{pol}|0\rangle_{spat} = |0\rangle|0\rangle = |00\rangle$ and $|1\rangle_{pol}|1\rangle_{spat} = |1\rangle|1\rangle = |11\rangle$.

Photon-pair entanglement

In the following experiments, instead of using birefringent calcite crystals, nonlinear beta-barium borate (BBO) crystals (Dehlinger & Mitchell, 2002a, 2002b) were employed within the interactive screen experiment of Bronner et al. (2009). Students are introduced to the experimental details. When a laser beam of a specific frequency illuminates these nonlinear crystals, the photons in the beam can be absorbed by the crystals. The excess energy gained by the crystals is then released via spontaneous emission of indistinguishable photon pairs. Energy is conserved; therefore, the emitted photons each have half the energy (and, therefore twice the wavelength) of the absorbed photons. For example, an absorbed violet photon (405 nm) produces an emitted photon pair in the near-infrared (810 nm). Due to momentum conservation, the two photons of each pair lie in the same plane as the incident laser beam and form equal angles with its propagation axis.

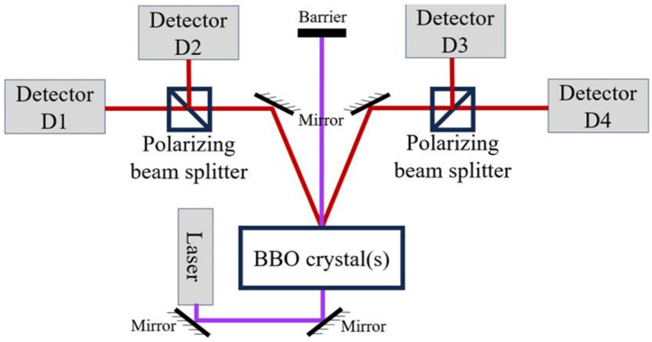

Figure 7. Schematic representation of the experiment. A laser beam impinges on the BBO crystal, and an obstacle intercepts the transmitted beam (the crystals occasionally emit photon pairs (red lines) & polarization beam splitters direct photons such that detectors D1 and D4 register vertical polarization, whereas D2 and D3 register horizontal polarization) (Source: Authors' own elaboration)

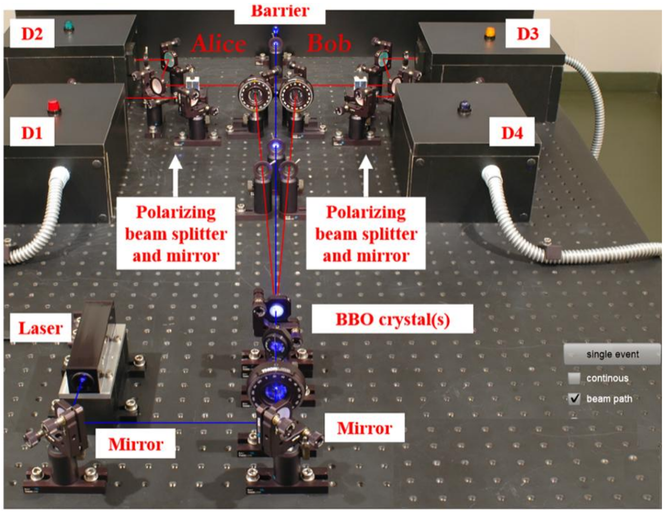

Figure 8. Screenshot from Bronner et al.'s (2009) interactive screen experiment, which replicates the real experiment and allows students to interactively explore photon pair behavior

Polarization measurements can be performed using 50/50 polarization beam splitters (or birefringent calcite crystals), because in this configuration, the photons' spatial modes and polarization become entangled. Signals from detectors placed at different locations then provide polarization information. The transmitted beam has vertical polarization, while the reflected beam has horizontal polarization. Consequently, photons polarized at 45° incident on the beam splitter have a probability of 1/2 of being detected in either the transmitted or reflected path. Detectors placed after the beam splitters provide information about polarization. **Figure 7** shows a schematic diagram of the experiment. The Bronner et al. (2009) interactive screen experiment also demonstrates this behavior: detectors signal for each photon of a pair simultaneously. **Figure 8** shows the realization of **Figure 7** in the interactive screen experiment.

Students investigated the behavior of a single photon pair generated by the deposited crystals. They explored that:

- Each photon has a probability of 1/2 of being detected by either D1 or D2 (first photon) and a probability of 1/2 of being detected by D3 or D4 (second photon).
- If D1 detects one photon, the other photon is always detected by D4.
- If D2 detects one photon, the other photon is always detected by D3.

From these detector signals, students concluded that both photons of a pair are always found with the same polarization; therefore, their polarization states must be entangled. Together with the students, we represented the state of a photon pair as a two-photon quantum system:

$$|\psi\rangle = \psi_1|0\rangle_1|0\rangle_2 + \psi_2|1\rangle_1|1\rangle_2, \tag{9}$$

where $\psi_1 = \psi_2 = 1/\sqrt{2}$. Here, each ket refers to polarization, and the subscript 1 and subscript 2 identify the individual photons. The coefficients $\psi_1 = \psi_2 = 1/\sqrt{2}$ correspond to a Bell state, reflecting that a measurement yields either two horizontally or two vertically polarized photons, each with a probability of $(1/\sqrt{2})^2 = 1/2$. In this entangled state (9), the two photons are indistinguishable, and their indices can be swapped or omitted: $|\psi\rangle = \psi_1|0\rangle|0\rangle + \psi_2|1\rangle|1\rangle$ (the photons are bosons). This indicates that the system can only be meaningfully described as a whole (holism). As before, the notation can be further compacted:

$$|\psi\rangle = \psi_1|00\rangle + \psi_2|11\rangle. \tag{10}$$

Upon measurement, the system collapses instantaneously into either the $|00\rangle$ or $|11\rangle$ state. This phenomenon is referred to as "nonlocality", because entangled photons exhibit correlated behavior despite being spatially separated. The entangled state $|\psi\rangle$ is a special superposition with the following features:

(1) its eigenstates are composed of multiple states, and

(2) the entangled state cannot be factorized into two separate states; the system is only meaningful as a whole.

Before measurement, the photon pair should not be imagined as two separate, spatially isolated photons, as doing so would conceptually fragment the quantum system. Entanglement requires functional and mathematical reasoning rather than purely visual intuition, which students have come to understand.

For this reason, students see that the state in Eq. (10) cannot be decomposed, illustrating feature (B). We observed that students often initially propose that if $|\psi\rangle$ were separable, each photon would be in the state $|D\rangle$. In that case, the joint state would be

$$|D\rangle_1|D\rangle_2 = [(1/\sqrt{2})|0\rangle_1 + (1/\sqrt{2})|1\rangle_1]\,[(1/\sqrt{2})|0\rangle_2 + (1/\sqrt{2})|1\rangle_2], \tag{11}$$

which expands to

$$(1/2)|0\rangle_1|0\rangle_2 + (1/2)|0\rangle_1|1\rangle_2 + (1/2)|1\rangle_1|0\rangle_2 + (1/2)|1\rangle_1|1\rangle_2 = (1/2)(|00\rangle+|01\rangle+|10\rangle+|11\rangle). \tag{12}$$

This state represents independent polarizations for the two photons, without entanglement. Therefore, the entangled state in Eq. (9) and Eq. (10) does not correspond to the separable state in Eq. (11) and Eq. (12).

Entanglement as a quantum circuit

A less commonly discussed classical gate is the controlled-NOT (CNOT) gate, which takes two input bits and produces two output bits. One of the input bits is called the control bit, and the other is the target bit. If the control bit is 0, the CNOT gate has no effect; if the control bit is 1, it acts on the target bit as a NOT gate.

The quantum CNOT gate operates similarly on two input qubits and produces two output qubits. Analogously, if the control qubit is $|0\rangle$, the gate has no effect; if the control qubit is $|1\rangle$, it acts on the target qubit via the quantum mechanical analog of the NOT gate, that is, the X gate. Its algebraic definition is given by:

$$\widehat{CNOT} = |00\rangle\langle 00| + |01\rangle\langle 01| + |11\rangle\langle 10| + |10\rangle\langle 11|. \tag{13}$$

Our students examined the action of the gate on the four fundamental qubit pairs (the first qubit is the control, and the second is the target qubit). The states correspond to mutually exclusive events and are therefore orthogonal, with scalar products equal to zero, as indicated in red:

$$\widehat{CNOT}|00\rangle = |00\rangle\langle 00|00\rangle + |01\rangle\langle 01|00\rangle + |11\rangle\langle 10|00\rangle + |10\rangle\langle 11|00\rangle = |00\rangle. \tag{14}$$

$$\widehat{CNOT}|01\rangle = |00\rangle\langle 00|01\rangle + |01\rangle\langle 01|01\rangle + |11\rangle\langle 10|01\rangle + |10\rangle\langle 11|01\rangle = |01\rangle. \tag{15}$$

$$\widehat{CNOT}|10\rangle = |00\rangle\langle 00|10\rangle + |01\rangle\langle 01|10\rangle + |11\rangle\langle 10|10\rangle + |10\rangle\langle 11|10\rangle = |11\rangle. \tag{16}$$

$$\widehat{CNOT}|11\rangle = |00\rangle\langle 00|11\rangle + |01\rangle\langle 01|11\rangle + |11\rangle\langle 10|11\rangle + |10\rangle\langle 11|11\rangle = |10\rangle. \tag{17}$$

The quantum CNOT gate only changes the two qubits in the cases of (16-17) because the control qubit is $|1\rangle$.

The students investigated an important case in which the control qubit was in the superposition state $|D\rangle$ and the target qubit was $|0\rangle$. Since the control qubit was neither $|0\rangle$ nor $|1\rangle$, the effect on the target qubit was not straightforward. We expressed the initial system in the two-qubit form: $|D\rangle|0\rangle = (1/\sqrt{2})(|0\rangle + |1\rangle)|0\rangle = (1/\sqrt{2})(|0\rangle|0\rangle + |1\rangle|0\rangle) = (1/\sqrt{2})(|00\rangle + |10\rangle)$. The action of the CNOT gate can be calculated using Eq. (12) and Eq. (14):

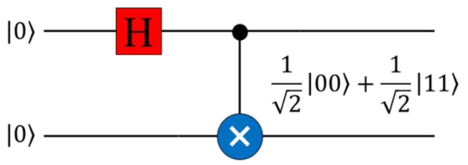

Figure 9. The bell quantum circuit in our notation (the solid black dot represents the control qubit, and the blue circle with a cross (X gate) acts on the target qubit & the line connecting them denotes the CNOT gate in the quantum circuit) (Source: Authors' own elaboration)

$$\widehat{\mathrm{CNOT}}(1/\sqrt{2})(|00\rangle + |10\rangle) = (1/\sqrt{2})\big(\widehat{\mathrm{CNOT}}|00\rangle + \widehat{\mathrm{CNOT}}|10\rangle\big) = (1/\sqrt{2})(|00\rangle + |11\rangle). \tag{18}$$

The calculation shows that if the control qubit is $|0\rangle$, the target qubit remains $|0\rangle$, and if the control qubit is $|1\rangle$, the target qubit becomes $|1\rangle$. Since the control qubit is in a superposition of $|0\rangle$ and $|1\rangle$, the target qubit also becomes a superposition and is entangled with the control qubit. The (18) Bell state describes an entangled photon pair, so the CNOT gate, when applied to an appropriate input state, acts as the quantum circuit element responsible for entanglement.

By convention, the initial qubits in quantum computers are $|0\rangle$; therefore, an H gate must be applied to the control qubit to generate entanglement. The combination of these two operations is called a Bell quantum circuit. **Figure 9** shows a quantum circuit that allows students to experiment with entangled qubits on quantum computers.

CLASSROOM EXPERIENCES

In this paper, we presented a novel teaching-learning sequence in the context of a pilot experiment conducted within an extracurricular high school study group. Six students aged between 17 and 18 years participated in five sessions, each of which lasted 60 minutes. The participants were recruited from a group of high school students who were interested in the topic and volunteered for the study. In the ensuing discussion, we will summarize the classroom experiences that were gathered during the initial pilot study, with a particular focus on the learning difficulties encountered by the students.

Learning Difficulties in the Context of Logic Gates

We found that three students encountered difficulties due to their inability to recall the fundamental principles of classic logic gates, a component that is not a central element of the IT and mathematics curriculum in Hungary. Consequently, a preliminary discussion was deemed necessary. The alignment of classical informatics knowledge with extracurricular courses is a significant future objective.

Five students held the conviction that the eigenstates of an operator are identical to the states resulting from the operator acting on a state. Consequently, some students erroneously assumed that an H gate could be realized by a polarizer with diagonally permitted direction of 45°. This learning difficulty arises because horizontally polarized photons in the initial $|0\rangle$ state passing through such a polarizer are transformed into the $|D\rangle = (1/\sqrt{2})(|0\rangle + |1\rangle)$ state. However, in quantum mechanics, the effect of an operator as an operation and the eigenvalues and eigenstates of an operator representing a physical quantity are entirely distinct concepts - an issue also identified among university students in the publication (Sing & Marshman, 2015).

A further challenge emerged from the perspective of four students, who held the conviction that quantum gates were non-reversible. This misconception originated from photon polarization experiments, wherein photons are frequently absorbed or annihilated, thereby rendering the transformations irreversible. The issue was identified in the treatment of quantum gates as black boxes; in this respect, the paper by Zuccarini et al. (2024) can be integrated into the teaching-learning sequence.

Learning Difficulties in the Context of Quantum Cryptography

It was evident that a conceptual revision of the notion of a secret key was necessary for some students, as the concept of classical encryption was not entirely clear. We observed that all students tended to use term "detection" and "quantum measurement" synonymously, a finding that is consistent with the observations reported Tóth et al. (2024b). This has given rise to certain difficulties in the B92 protocol, in which the absorption of photons must also be considered as a measurement outcome. Consequently, it was imperative to engage the students in a comprehensive discussion on the meaning of quantum measurement: when the quantum system interacts with a measuring device or its environment, information about the system is released to the outside world. It is noteworthy that interaction with a polarizer is regarded as a quantum measurement, whereas interaction with birefringent calcite crystals or beam-splitter cubes is not. In such cases, only the detector signal can be considered a quantum measurement. We found that this dialogue was sufficient to clarify the students' reasoning. The potential benefits of employing calcite crystals in future encryption applications in lieu of polarizers merit exploration, given the observation that detection and measurement occur concurrently at these points.

Learning Difficulties in the Context of Entanglement

Three students required assistance in rewriting the state of two non-entangled photons from two two-dimensional states into a four-dimensional state (and vice versa). Given the novelty of these operations, the lessons was supplemented with practical exercises to facilitate student comprehension. We also observed that some participants experienced difficulty distinguishing between the states $(1/\sqrt{2})(|0\rangle+|1\rangle)(1/\sqrt{2})(|0\rangle+|1\rangle)$ and $(1/\sqrt{2})(|00\rangle+|11\rangle)$. When expressing photon-pair states in four dimensions, some students mistakenly assumed that they were always entangled, which is not always the case.

Three students found it difficult to accept that the state of an entangled photon pair cannot be decomposed into the states of the individual photons. Therefore, for these students, we provided a derivation (cf. Bernhardt, 2019) showing that a state representing an arbitrary entanglement cannot be decomposed into a product of two individual photon states. We also emphasized that the components of an entangled photon pair are indistinguishable.

We also observed that the conceptualization of entanglement of photon pairs posed a significant challenge for students, as this phenomenon defies classical intuition. However, we discovered that students who has already demonstrated an understanding of the nonlocality of a single photon exhibited comprehension of the concept of photon-pair entanglement. A significant benefit of the polarization-based teaching approach is its capacity to promote the development of a quantum mechanical way of thinking (Bitzenbauer et al., 2024), thereby enhancing the comprehension of entanglement, which is crucial for quantum computing, in comparison to traditional teaching approaches.

CONCLUSIONS

In this study, we presented a new, minimalist teaching-learning sequence to quantum computing based on photon polarization. We found that students were strongly interested in topics at the forefront of science that are frequently highlighted in the media. One educational advantage of quantum computing is its ability to clearly demonstrate the possibilities inherent in superposition and entanglement.

Future Research

The teaching-learning sequence outlined in this article enable more in-depth analysis of students' mental models and conceptual understanding in future studies. For instance, students are instructed on quantum entanglement via a novel pedagogical approach that incorporates the use of a real quantum computer. However, a study was conducted that investigated students' conceptions about quantum entanglement (Brang et al., 2024), but through a completely different approach. Consequently, an investigation into students' conceptions about entanglement in our approach has the potential to contribute to existing literature. As Hennig et al. (2024) demonstrated, an acceptance survey can facilitate a more profound exploration of learning difficulties experienced during the teaching-learning sequence. Furthermore, educational experiments on teaching the BB84 quantum key distribution protocol have been conducted (DeVore & Singh, 2020; Weissman et al., 2024); however, the impact of teaching the B92 protocol described in this article on students' thinking has not yet been investigated. A comparison of the educational effectiveness of the BB84 and B92 protocols may prove to be an interesting avenue for future research. Another intriguing educational experiment could involve examining the challenges associated with utilizing diverse online programmable quantum computers within the context of a technological acceptance survey, similar to Dandl et al. (2024), who conducted a study in a different context.

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AI statement: The authors stated that Generative AI tools (e.g., ChatGPT by OpenAI) were used to check the English language clarity of the manuscript only. No content generation was performed by AI.

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