MODESTUM EURASIA Journal of Mathematics, Science and Technology Education, 2025, 21(5), em2630
ISSN:1305-8223 (online)
OPEN ACCESS Review Article https://doi.org/10.29333/ejmste/16307

AI artifacts in the mathematics didactical tetrahedron: A developed model

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Received 01 Feb 2025 ▪ Accepted 07 Apr 2025

Abstract
This paper studies the well-known model for didactic situations called socio-didactical tetrahedron in mathematics education. This study presents a developed model and considers a new vertex for artificial intelligence artifacts. Additionally, it emphasizes the mediator role of teachers in creating a favourable environment in student-centered mathematics education in the era of large language models. The idea of our proposed model comes from the differences between information technology artifacts and artificial intelligence artifacts.

Keywords: mathematics education, artificial intelligence, IT artifacts, AI artifacts, socio-didactical tetrahedron

INTRODUCTION

The initial idea of theorization of a didactical situation in mathematics education was based on the didactical triangle explained by Vygotsky (1997) (**Figure 1**). Considering the complexity of the teaching-learning process, this model reflects on the dynamics of the system and provides a platform to describe the relations between students, teachers, and mathematics.

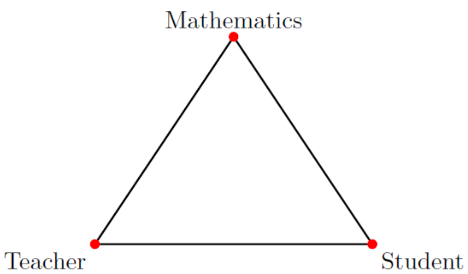

Figure 1. The didactic triangle (Vygotsky, 1997)

This model suggests that the teachers are representatives of sharing mathematical knowledge within an educational system. Indeed, in the traditional classroom, the teachers' explanations create a dynamic environment to deliver mathematical words and pictures as "statics". The triangle model explains dual relationships between mathematics, teachers, and students.

After emerging computers for educational purposes, Tall (1986) added a new dimension to the classical model and introduced a tetrahedron. Computers in David Tall's model are considered as software designed to provide "explicit" mathematics along with the process of calculations. Churchhouse et al. (1984) showed that a triple, namely, student-teacher-computer can be defined as an additional relationship in the tetrahedron model. Moreover, Rezat and Sträßer (2012) emphasized that the tools, both physical and non-physical ones which are used in the classroom can be named the so-called "artifacts" in the tetrahedron model. They proved that artifacts, including digital technologies, play a supportive role in the teaching and learning process of mathematics.

The vertex "Computer" was named as artifacts in Rezat and Sträßer's (2012) work (**Figure 2**). Then, they highlighted the teachers' ability to effectively utilize artifacts to structure the teaching and learning process in a modern manner. Indeed, their Socio-Didactical Tetrahedron model describes the relationships between different elements in didactical situations.

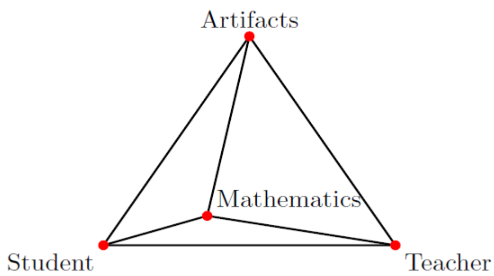

Figure 2. The Socio-Didactical Tetrahedron (Rezat & Sträßer, 2012)

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Contribution to literature

- This research reviews the well-known Socio-Didactical Tetrahedron model and employs an evidence-based method to recall its applications in mathematics education.
- This research highlights the key role of teachers as mediators in enhancing mathematics education, particularly in the context of the artificial intelligence era.
- This study proposes a revised version of the Socio-Didactical Tetrahedron, incorporation a new vertex to account for artificial intelligence artifacts, with a focus on student-centered educational environments in the 21st century.

The tetrahedron model has been widely used in designing strategies for appropriate teaching methods and integrating IT artifacts in the context of mathematics education. For instance, Albano et al. (2021) introduced an e-learning tetrahedron model and combined that with a model presented by Coppola et al. (2012) concerning teachers' attitudes. The combination of these models covered didactical situations by considering various elements in both face-to-face and distance education. Cao et al. (2021) analyzed teachers' perception of online teaching of mathematics in China. They suggested a need for rethinking relations between vertices in the tetrahedron model due to rapid growth of using information technology artifacts after Covid-19. Dasari et al. (2024) studied the Socio-Didactical Tetrahedron, emphasising instructors' role in avoiding negative side-effects of over-reliance on artificial intelligence tools in mathematics education. In a study conducted by Matić and Gracin (2015, 2016), the Socio-Didactical Tetrahedron was utilized to investigate the mediator role of teachers in the interaction of students and textbooks. A practical application of Vygotsky's interaction theory has been shown by Mota Lopes and Magalhães Netto (2021) in providing a system to improve teaching of Algebra in distance education. Also, the socio-didactical tetrahedron model has been used as a research methodology by taking the role of technologies into account of in mathematics education by Olive et al. (2010). The challenges faced by students using technology in their mathematical courses have been investigated employing the tetrahedron model (Hillmayr et al., 2020). Hjelmborg et al. (2020) analyzed the teacher-resources interaction and its impact on their teaching by employing using the Socio-Didactical Tetrahedron. Nowadays, the increasing presence of large language models and artificial intelligence in didactical situations puts these questions in front of us:

RQ1 How does/can AI emerge in the context of mathematics education redefine the relationship between vertices of the didactic triangle?

RQ2 Does AI define a new vertex and dimension to the latest model?

RQ3 Can large language models such as ChatGPT, Julius AI and Wolfram Alpha be considered in the category of IT artifacts?

Interaction Between Vertices in the Tetrahedron Model

Teacher - Technology

Technology can benefit mathematics teachers by engaging students in observing real-life applications of mathematics. It can play a supportive role in achieving learning outcomes and enhancing the visibility of abstract concepts. DeCoito and Richardson (2018) described how positive teachers' views on technologies can affect teaching, however, they provide a deeper insight into the use of technology as a part of the process of learning. In the case of teacher-technology relationship, some challenging factors have been identified, including the teachers' choice of appropriate technology to integrate into the class, assisting students in implementing technologies for their tasks, and availability of technologies in different languages.

Student - Technology

The capability of technology in helping students personalize the learning process and develop self-study skills has been proven. A review study based on 300 published papers was done by Ní Shé et al. (2023) to examine the impact of technology on students' mathematics performance in higher education. The findings of this study show that certain factors, such as students' skills, combined with precise guidelines of integrating technology into teaching, are crucial for achieving a successful contribution of technology in mathematics education. In another study, Hossein-Mohand et al. (2021) demonstrated the relationship between technology used for educational purposes and students' realization of its effectiveness in their learning. The relationship between students and mathematics has also been investigated in terms of the selection of digital resources by Pepin et al. (2024). They argued that the content of digital resources and the ways in which they are selected can influence students' beliefs and engagement in mathematics.

Technology - Mathematics

The use of technology in mathematics education is a mature area of research and has been well-developed. As one of the most comprehensive contributions to the investigation of employing information and communication technologies, Lagrange et al. (2003)

consider two directions: better mathematics learning through the analysis of students' knowledge and the improvements that technology induces in didactical situations. Additionally, Ghavifekr and Rosdy (2015) showed that the technology-mathematics relationship enhances the learning of mathematics. There are software and graphing calculators, such as GEOGEBRA, that provide an excellent platform for innovatively designing various topics.

Student - Mathematics

As a matter of fact, the positive attitudes of students toward mathematics play a significant role in overcoming difficulties in mathematical courses and achieving success. In this regard, Hwang and Son (2021) proposed a set of educational interventions, including those by teachers, parents, and administrators, to promote students' attitudes toward mathematics. Bonne (2016) examined several studies in New Zealand regarding the relationship between students' attitudes and their achievements in mathematics, with an emphasis on self-efficacy to illustrate this relationship. Moreover, Bonne (2016) argued that the role of teachers as interveners can enhance achievements, along with students' beliefs, which in turn can be interpreted as the teacher-student vertex in the socio-didactical tetrahedron.

Teacher - Student

In the context of mathematics education, a positive teacher-student relationship can foster a dynamic learning environment, leading to the achievement of learning outcomes. Appiah et al. (2022) examined the relationship between teachers and students, as well as the role of self-efficacy, and highlighted the impact of teachers on students' attitudes toward mathematics. In a multidimensional study conducted by Flint et al. (2024), the teacher-student relationship was examined in the context of teachers' self-efficacy, focusing on teachers' beliefs, work engagement, and motivation.

Teacher - Mathematics

The relationship between teachers and mathematics has been described in various dimensions. In a pioneering study, Gonzalez Thompson (1984) demonstrated that teachers' conceptions of mathematics and their instructional behaviors are shaped by their thoughts and preferences regarding mathematics. A study conducted by Amirali and Halai (2021) has shown that the socio-cultural practices of teachers and their religious beliefs about the origin of mathematics can be utilized as a motivational approach for teaching mathematics in Pakistan. In some studies, the relationship between teachers and mathematics has been highlighted on a topic-by-topic basis. For example, Pournara (2013) investigated teachers' knowledge in the field of financial mathematics and showed that their knowledge on banking content will be effective in teaching compound interest. In fact, there is evidence showing that teachers' mathematical knowledge influences the quality of delivering concepts using appropriate instructional practices.

METHODOLOGY

The research methodology in our study was narrative summary (Green et al., 2006). We also took advantage of PRISMA to support our analytical study. We divided our analysis into two stages. In the first stage, we reviewed publications related to the theorization of didactical situations by the Socio-Didactical Tetrahedron in the context of mathematics education. To this end, we focused on indexed publications from 2012 to 2024. This time interval was set since the socio-didactical tetrahedron was first introduced in 2012 by Rezat and Sträßer as an extension of the didactical triangle. In the second stage, we concentrated on the role of AI in today's mathematics education. We focused on the differences between IT artifacts and AI artifacts to analyze the socio-didactical tetrahedron, resulting in the development of Rezat and Sträßer's model. We used the keywords "Human-AI interaction", "AI" and "Mathematics education" in the second stage of research.

By using PRISMA, the following steps were implemented:

1. We identified publications in Scopus, Google Scholar, IEEE and PubMed based on keywords "Mathematics education" and "Socio-Didactical Tetrahedron".
2. Having considered the titles and keywords, we identified some irrelevant documents in PubMed and Google Scholar, resulting in 265 documents of interest.
3. For eligibility, after applying the inclusion and exclusion criteria, we removed irrelevant documents which resulted in 89 documents.
4. The inclusion step was performed by reviewing abstracts, which resulted in a final list of 58 references.

Table 1 and **Figure 3** show the steps involved in the inclusion and exclusion of sources identified.

We optimized our work by careful reading of selected publications to determine the role of tetrahedron model in describing the relations between its vertices, as well as its applications. We then reviewed the most recent papers on artificial intelligence tools in mathematics education.

AI and Mathematics Education

The impact of AI varies across different fields, and its impact depends on the interactions between AI and

Table 1. Inclusion and exclusion criteria

Criteria	Inclusion	Criteria
Language	English	Language
Publication period	2012-2024	Publication period
Area of research	Mathematics education	Area of research
Type of publication	Theoretical and empirical papers, peer reviewed conference proceedings, systematics reviews, book chapters	Type of publication
Criteria	Inclusion	Criteria

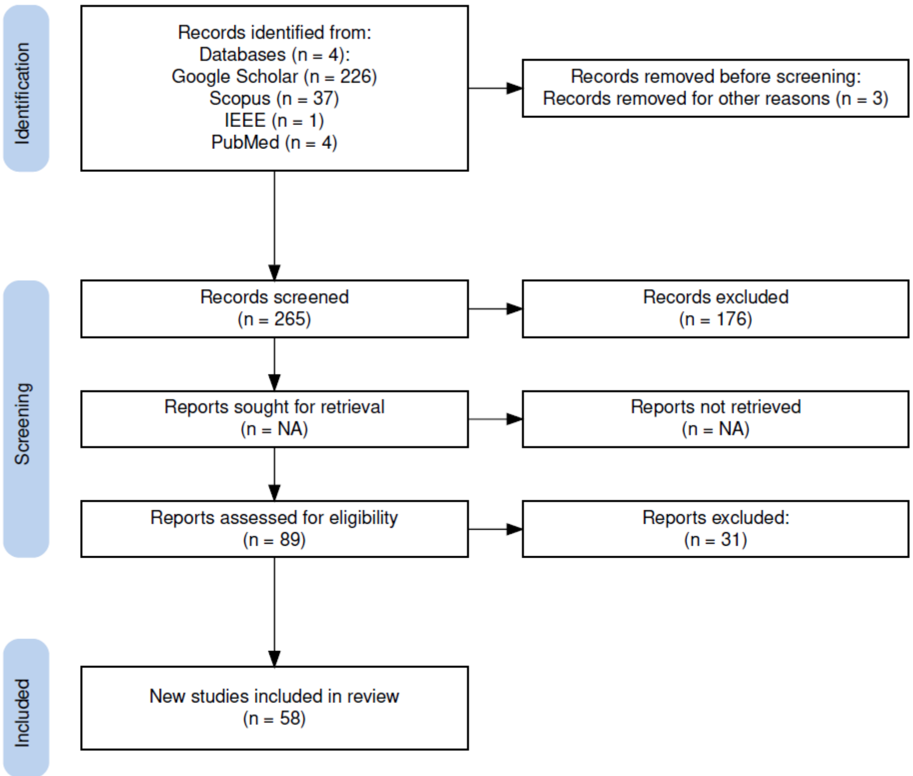

Figure 3. PRISMA flow diagram (Haddaway et al., 2022)

teachers and students, as discussed by Ansari et al. (2023). Large language models can enrich learning mathematical concepts and provide teachers with tools and methodologies to promote their instructional practice. AI tools also facilitate other aspects of the educational process, including grading (e.g., by Gradescope) and designing courses assessments (e.g., by Scantron). Interested readers can explore other examples of contributions of artificial intelligence in educational environments, such as task automation, getting students' feedback, and supporting students with special needs, as noted by Paliwal and Patel (2025). Although ChatGPT is one of the revolutionary AI tools being used in schools and colleges, various AI artifacts, such as Photomath, Socratic, and Mathway, are also used at different levels for the mathematics discipline. For instance, Socratic has the capacity to provide step-by-step solutions to algebraic questions, and Photomath gives graphical representations of mathematical problems. Among various AI tools, Mathway has been shown to cover a wide range of mathematical topics in detail and to provide additional support by suggesting related materials, such as videos, for its proposed solutions (Paliwal and Patel, 2025). Moreover, the combination of IT artifacts and AI artifacts has been observed to enhance the capacity of IT artifacts in delivering mathematical concepts precisely and accelerating the learning process. For instance, Funes et al. (2024) showed that the combining computer algebras systems, such as GeoGebra applets, with Gemini AI facilitates personalized learning.

The high potential of AI artifacts in assisting students necessitates a renewed focus on equity, inclusion and diversity as an issue in the wake of AI's emergence. It is worth noting that digital inclusion has always been a topic of discussion since the emergence of information technology artifacts in education, as Paliwal and Patel (2025) have discussed. Besides the auxiliary role of AI in educational environments, concerns have been raised about potential negative consequences of its overuse, including academic integrity and the development of critical thinking, as noted by Dasari et al. (2024) and Lo (2023). Apart from the above-mentioned cases, Li et al.

Table 2. Differences between IT artifacts and AI artifacts

Dimension	IT artifacts	AI artifacts
Neutrality	Neutral	Not neutral (depends on user)
Evolution	Evolve only through human intervention	Evolve without need for human intervention
Adaptation to users	Users adapt IT to their needs	AI adapts to user needs
Learning capabilities	No learning capabilities	Has learning capabilities
Adaptation to the environment	Cannot adapt itself to the environment	Can adapt itself to the environment
Perception	Shaped by the interest of the development team	Shaped by the interest of the user over time

Source: Bawack et al., 2019.

(2024) raised several concerns, including the accuracy of AI artifacts and public's attitude toward AI. However, some concerns can be resolved over time by empowering AI tools and the mediation role of mathematics teachers.

From Socio-Didactical Tetrahedron to a Tetrahedron with a center

In this section, we propose a new model for the well-known Socio-Didactical Tetrahedron in mathematics education, particularly in the era of AI, and we hope that it will draw the attention of educational researchers to further develop these arguments. The motivation for our developed model stems primarily from the ongoing growth of human-technology interactions. Our proposed model can also be integrated into other theoretical frameworks, as the effectiveness of combining the theoretical frameworks has been suggested for evaluating the complex teaching-learning process in mathematics, as noted by Prediger et al. (2008). According to a comprehensive study recently conducted by Rezat and Geiger (2023), digitalization has led to fundamental changes in the field of education, particularly in the impact on student-teacher-mathematics interactions within the tetrahedron model, so that a remarkable update on these interactions is undeniable. Therefore, the whole Socio-Didactical Tetrahedron Model needs to be adapted to changes which are caused by the growth of digitalization and large language models.

Bawack et al. (2019) enumerate key differences between IT artifacts and AI artifacts (**Table 2**) which in turn justify consideration of a separate vertex for AI artifacts in our developed version of the socio-didactical tetrahedron model.

In the field of Mathematics Education, the roles of digital technologies, textbooks, and curriculum materials have been analyzed for teachers' use by Drijvers and Sinclair (2024) and Remillard and Heck (2014). For instance, Remillard and Heck (2014) showed that teachers' participation with curriculum materials is "highly interactive and multifaceted activity". Digital technologies and resources are shared by both teachers and students, as shown in the tetrahedron model. Sträßer (2009) used the term "artifact" as a concept to encompass textbooks, digital technologies, and other materials. The socio-didactical tetrahedron can be

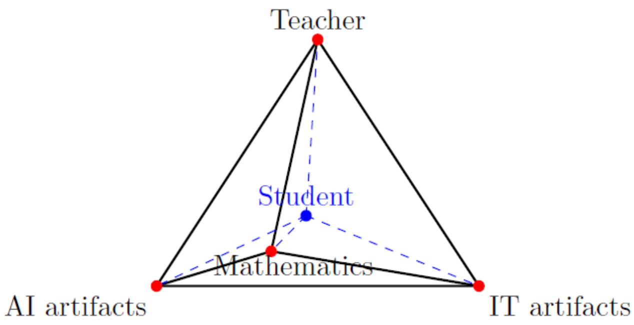

Figure 4. Developed socio-didactical tetrahedron (Source: Authors' own elaboration)

interpreted by the essential role of "artifacts," which is "mediated" by teachers in mathematics education. The precise definition of a mediatory role refers to "the intentional and systematic organization of the various artifacts available in a computerized learning environment by the teacher for a given mathematical situation, to guide students' instrumental genesis" (Drijvers & Trouche, 2008, p. 20). The term "AI artifacts" covers artificial intelligence tools and methodologies. The human-like capabilities of AI artifacts enable them to utilize IT artifacts like humans.

Our motivation to move from the known tetrahedron to a student-centered one stems from the role of AI in shaping the future of mathematics education and student-centered learning environments. Since we hold the view that AI-based applications are distinct from other digital technologies in terms of generativity, we propose a new vertex for this in our model (**Figure 4**).

In the developed model (**Figure 4**), the vertex representing students is located at the center, and a new vertex, "AI Artifacts," has been introduced as a separate vertex, distinct from the previous model (**Figure 2**). The era of large language models presents us with a more compelling reason to place the student's vertex at the center of the model, highlighting that students are surrounded by AI and IT artifacts, as well as their teachers. In this regard, Ginting et al. (2024) identified limitations in traditional classroom instruction and discussed how AI platforms can support student-centered learning by facilitating the personalization of learning for students. However, the instructors' role in supporting students to optimize material selection for complex tasks has been indicated. It is worth noting that students can directly learn with AI and use intelligence tutoring systems, which in turn promotes students' independent learning skills. In this regard, the pioneering research conducted by Kim et al. (2022) sheds a light on the potential role of educational AI in enabling students to become autonomous learners by externalizing their ideas, thereby enhancing their learning experiences and self-efficacy. For example, Julius AI, an AI math-solving tool, can help students recognize the possibility of having more than one solution to a math question, thereby fostering their creativity rather than simply writing the final answer.

Through this model, we describe various relationships in didactical situations. In what follows, we add new relationships, namely student - AI artifacts, teacher - AI artifacts, mathematics - AI artifacts and IT artifacts - AI artifacts to the known socio-didactical tetrahedron.

Student - AI artifacts

Numerous studies have been conducted to investigate the relationship between students and AI artifacts. For example, Yilmaz et al. (2023) used the technology acceptance model to analyze students' attitude toward AI. They showed that students take advantage of using ChatGPT for learning and understanding of various topics. In another study, Raman et al. (2024) analyzed students' views on ChatGPT from the perspectives of gender and discipline, highlighting factors such as the accuracy of AI, which are faced by different disciplines. Additionally, Kim et al. (2024) investigated the interaction between students and AI, considering several key elements, including students' characteristics, attitudes toward AI, task performance, and emotional engagement. For instance, Kim et al. (2024) showed that students with a positive attitude toward AI tend to have a more positive emotional relationship with AI, whereas students with a negative attitude exhibit negative emotional reactions to AI.

Teacher – AI artifacts

Teachers' attitudes toward AI have been studied by Iqbal et al. (2022). Despite considering some positive effects of AI on motivating students and engaging them, teachers generally have serious concerns regarding the lack of learning. In another study conducted by Wardat et al. (2024), several challenges to applying AI in teaching were identified, including a lack of experience and knowledge. Moreover, Kim et al. (2022) described the interaction between teachers and AI, focusing on the role of teachers in monitoring student-AI interactions. They also highlighted the leading role of teachers in preparing strategic plans to address concerns about the potential negative effects of AI on cultivating critical thinking, creativity, analytical thinking and student motivation. We believe that the roles of teachers, as explained by Kim and his colleagues, can be seen as the so-called "mediator" role in the socio-didactical tetrahedron model.

Mathematics - AI artifacts

The relationship between mathematics and AI in a didactic situation can be described in terms of integrating AI tools in mathematics education. In this regard, a systematics review by Mohamed et al. (2022) shows the integration of AI in various algebraic or numeric approaches. Moreover, Van Vaerenbergh and Pérez-Suay (2022) have introduced four classes of AI systems for mathematics education that express the interaction between mathematics and AI artifacts. Based on this classification:

(1) Some AI systems convert real world problems into mathematical language;
(2) Some AI systems have reasoning capabilities;
(3) Some of them translate the AI-made reasons into understandable steps by humans; and
(4) Some of AI systems can model data generated by students.

Another interesting point in the paper is the introduction to AI-based calculators, which replace traditional ones and provide students with impressive features during their mathematical learning journey. An analytical study conducted by Bagno et al. (2024) provides some examples from linear Algebra, showing that ChatGPT makes errors in logical inference or complex problems, and it doesn't necessarily provide the best technique for a given question.

IT artifacts - AI artifacts

Digital learning platforms play a significant role in education in the post-COVID world. In fact, the pandemic accelerated the use of IT and educational online platforms more than ever. Moreover, the emergence of highly developed AI tools has opened a new avenue for mathematics education. Bawack et al.'s (2019) explained that AI artifacts become smarter through IT, and some features of AI artifacts, such as "adaptation to users," along with their learning and generative capabilities, set them apart from IT artifacts. There are two major IT artifacts used in mathematics education: Computer Algebra Systems and Dynamical Geometry Systems. The integration of AI artifacts into these tools is an ongoing fundamental project discussed by Botana and Recio (2024).

It is worth pointing out that, like online technologies, AI-based technologies interact with both teachers and students, thereby complicating the teaching and learning process. As teaching is not just about delivering a course, and there exist inevitable psychological and behavioral interactions within the process of teaching and learning, we believe that the key role of teachers in the future of learning and teaching mathematics remains unchanged. However, it must be adapted to transitions caused by AI. Our developed model was introduced by extending the known socio-didactical tetrahedron and addressing the following issue: Does the AI define another “Vertex”? The similarity between the cognitive activities of AI artifacts and human minds, such as reasoning, learning, and problem-solving distinguishes it from IT artifacts, as discussed by Rai et al. (2019). From a relational point of view, several pieces of evidence have been provided to show that there are fundamental differences between human relations with IT artifacts and AI artifacts (Farooq & Grudin, 2016). For instance, a consulting behavior of a software designed by Intuit for online tax preparation in the United States, is a good practical example of how capabilities of AI systems can transform AI artifacts and human relationships from a basic logical interaction to a dynamic one. The impact of development of AI systems on human interactions with computers and IT artifacts has been comprehensively investigated by Xu et al. (2023). Xu et al.’s comparative analysis between AI-based systems and other systems has characterized AI-based systems by their humanlike intelligence abilities, which gives a ground for the development of educational models focusing on AI artifacts. Indeed, the new vertex has been added to the socio-didactical tetrahedron based on the fundamental differences between IT and AI artifacts and the different ways human interact with them.

RESULTS

This study has described possible relationships between new pairs of vertices. The common role of both IT and AI artifacts is to enhance students’ mathematical learning abilities while also empowering teachers in their teaching. The constructive role of teachers in the developed model, particularly in the era of AI, extends beyond providing students with appropriate support, it also involves establishing a dialog-based environment in the classroom, which in turn leads to student-centered education, as shown in our model. This finding is also in line with recent studies by Baccaglini-Frank et al. (2024) and Huang and Sutherland (2022), which confirm that pedagogical strategies by teachers will play a crucial role in the interaction between students, IT and AI artifacts in the context of mathematics education. The importance of teachers’ mediating role in the new model is also in line with a study conducted by Dwivedi et al. (2023), which emphasizes teachers’ responsibilities in developing students’ critical thinking. As argued by Kim et al. (2022), teachers are in the first line of supporting their students in various aspects of interactions with AI, namely, cognitive, socio-emotional and artifact-mediated, so strategies for teaching will enhance creativity, imagination, and analytical skills of students.

AI artifacts share several features with digital technologies, but their generative capability sets them apart from other artifacts. The relationships between teachers, students, IT and AI artifacts reveal the new learning environment of mathematics and highlight teachers’ mediating role in the context of today’s mathematics education. In fact, teachers can take advantage of both IT and AI artifacts to prepare educational content that is compatible with students’ mathematical background and to reframe problems, Einarsson et al. (2024). Kasneci et al. (2023) highlighted the most challenging issue in the integrating AI-artifacts into educational settings, specifically the need for revision and moderation of AI outputs, which in turn underscores the essential role of mathematics teachers in the mathematical educational context. Indeed, the revised tetrahedron shows the importance of mediator role of mathematics teachers. Moreover, the generative capability of AI-artifacts can be considered in today’s’ mathematics education as a “co-mediator”.

FUTURE DIRECTIONS

Future research is required to explore methods for mitigating the risks associated with using AI in mathematics learning and enhancing its effectiveness. There is a need for research regarding assessing students’ learning and knowledge via AI platforms, which is a challenge lecturers encounter. Moreover, some comparative studies can analyze students’ performance in mathematics, considering the impact of AI across different subjects. Since large language models have become an integral part of mathematics education, further investigations might consider teachers’ experiences in using AI artifacts for mathematics education.

Author contributions: JM: writing – review & editing, methodology; **CZ:** conceptualization, analysis, writing – original draft, methodology. Both authors have agreed with the results and conclusions.

Acknowledgement: The authors would like to thank Modern College of Business and Science and Ministry of Higher Education, Research and Innovation in Sultanate of Oman for supporting this research.

Funding: The research leading to these results has received funding from the Research Council (TRC) of the Sultanate of Oman under the Block Funding Program. TRC Block Funding Agreement No [MoHERI/BFP/MCBS/2022].

Ethical statement: The authors stated that the study does not require any ethical approval. It is a review of available literature.

Declaration of interest: The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

Data sharing statement: Data supporting the findings and conclusions are available upon request from the corresponding author.

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