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Alternative conceptions emerging in pre-university students while making mathematical connections in derivative and integral tasks

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Abstract

Understanding students' alternative conceptions is important, as such conceptions can prevent them from making mathematical connections, thereby hindering their conceptual development. For this reason, this research aimed to identify the alternative conceptions that emerge when a group of pre-university students engage with derivative and integral tasks presented in algebraic, graphical, and application problems contexts. Alternative conceptions were defined as conceptions inconsistent with those accepted by the mathematical community. Twenty-five students from a Mexican public institution participated. Data were collected through task-based interviews and analyzed using thematic analysis. Nine alternative conceptions were identified. The most frequent included: the integral of the derivative of a polynomial function is obtained by finding the derivative and the integral separately and, the instantaneous velocity of an object is calculated using the formula $v = d/t$. These findings highlight the importance of explicitly addressing such conceptions in mathematics instruction to promote deeper conceptual learning.

Keywords: alternative conceptions, derivative and integral, pre-university, thematic analysis, mathematical connections

INTRODUCTION

The derivative and the integral are two central concepts of calculus: they are necessary to understand the fundamental theorem of calculus (FTC) and are very important for later courses in mathematics and useful to solve problems in different contexts. However, the research focused on the understanding of calculus showed that some earlier concepts, as rate of change, limit, tangent and function, cause problems to the students, even in university level (Denbel, 2014; Dolores & García-García, 2017; Rodríguez-Nieto & Font, 2025). A great part of their knowledge of limits, continuity and differentiation lies in isolated facts and procedures; they have a deficient conceptual understanding of the relationships between these concepts (Bezuidenhout, 2001). On the other hand, one of the goals of teaching calculus is the achievement of conceptual understanding (Bezuidenhout, 2001), but to accomplish that, a student first needs to be able to make mathematical connections (García-García, 2024; García-García & Dolores-Flores, 2018, 2021a; Hiebert & Carpenter, 1992; Noss et al., 1997; Silver et al., 2009; Stekete & Scher, 2016).

For its part, Muzangwa and Chifamba (2012) admit that superficial knowledge of the concepts of calculus affects the understanding of a great number of disciplines where this knowledge is used, including mathematics itself. This superficial knowledge could also facilitate the emergence and persistence of mistaken–understood as errors that may be procedural or momentary–and alternative conceptions, which reflect deeper, stable mental models that conflict with accepted mathematical meanings, in students. In this sense, differentiation and integration are recognized as some of the concepts that cause alternative conceptions (Kaplan et al., 2015); therefore, it is important to identify these conceptions to find their origin and causes of emergence.

Chhabra and Baveja (2012) argued that it is important to explore the conceptions of the students to identify

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Contribution to the literature

- The study reports alternative conceptions in a population that has been little studied concerning a concept that is highly relevant in school mathematics.
- Most of the alternative conceptions identified have not been previously reported in the existing literature.
- The data collection method allowed for in-depth exploration of participants' answers, increasing the reliability of the findings.

what they understand, because these play an important role in their learning process; they may even prevent it, as recognized by Lucariello et al. (2014). So, research on conceptions of the students and their role in the learning process has become one of the main domains in education (Duit & Treagust, 2003; Vezzani et al., 2018). It is possible that previous conceptions cause alternative conceptions in the students while learning a new concept and those could be inconsistent with the new one. This justifies, in part, why the alternative conceptions are resistant to change and are kept even after careful instruction (Bostan, 2016; Chi et al., 2012; Denbel, 2014) and persist across ages and levels of education (Thijs & Berg, 1995).

Alternative conceptions pose significant challenge to both teaching and learning, as they can lead students to develop misleading perceptions of mathematical knowledge (Chhabra & Baveja, 2012). This occurs because students' learning is shaped by their prior beliefs and their conceptions about the nature of knowledge and learning itself (García-García & Dolores-Flores, 2021a; Greeno et al., 1996). Such embedded conceptions may not only diverge from scientifically accepted meanings but can also be in direct contradiction to them (Duit & Treagust, 2003). These conceptions tend to persist over time because they are often functional and operative in other domains or everyday contexts, which reinforces their viability from the learner's perspective (Fujii, 2014).

Moreover, alternative conceptions often encourage rote learning of facts and procedures, particularly in preparation for assessments. However, as pointed out by Duit and Treagust (2003), students frequently revert to their original conceptions in novel contexts. This highlights the importance of investigating these alternative conceptions, as understanding their nature is a critical step toward designing teaching strategies that foster conceptual understanding (Bezuidenhout, 2001; Bezuidenhout & Olivier, 2000; Chow, 2011; Denbel, 2014).

The study of alternative conceptions also allows the identification of patterns of mistakes and the analysis of their possible causes (An & Wu, 2012), as well as their relationships to newly introduced mathematical concepts (Fujii, 2014). In particular, Serhan (2015) suggests the importance of studying derivatives and integrals because the alternative conceptions associated with them could also become an impediment to the construction and understanding of future concepts (García-García & Dolores-Flores, 2021a). We add that it is important to study them because they appear when students try to make mathematical connections.

Although the concepts of derivative and integral are central to calculus–especially through the application of theorems and properties–, several studies have shown that students, both university level and preservice teachers or teachers in practice, experience substantial difficulties (Hayes, 2024; Lumbantoruan & Manalu, 2024; Mkhatshwa, 2024; Muñoz-Pinto et al., 2025; Santos et al., 2024). These difficulties are often rooted in students' inability to meaningfully connect symbolic, graphical, and numerical representations, as well as in limited understanding of the conceptual meanings associated with differentiation and integration (Galindo-Illanes & Breda, 2024; Pino-Fan et al., 2018; Rodríguez-Nieto et al., 2022, 2023, 2024). In addition, the scarcity of tasks that promote visual analysis, functional interpretation and real-world modeling contributes to superficial learning (Ledezma et al., 2024; Rodríguez-Nieto et al., 2024).

One particularly persistent difficulty involves the graphical interpretation of functions and their derivatives (García-García & Dolores-Flores, 2021a). For instance, Natsheh and Karsenty (2014) highlighted that some students fail to construct function graphs from the properties of their derivatives, due to a procedural and symbolic approach. Similarly, Fuentealba et al. (2018) and Ikram et al. (2020) also pointed out the difficulty of relating derivatives to characteristics such as monotony and curvature, as well as the poor ability to graph without algebraic expressions. Despite proposals such as that of Borji et al. (2024), focused on partial derivatives and visualization in several variables, the disconnect between theory and practice persists. Even the FTC, which establishes the relationship between differentiation and integration, is difficult to grasp because students still lack a solid foundation in functions, limits, continuity, and the power of graphical representations, underscoring the rapid urgency of improving pedagogical approaches to teaching these abstract concepts (Munyaruhengeri et al., 2024).

The literature reflects a long-standing interest in the study of students' alternative conceptions across a range of mathematical and scientific domains–from junior high school to university levels, and even among teachers–. These studies span subjects such as physics (Chhabra &

Baveja, 2012; Narjaikaewa, 2013; Mulhall & Gunstone, 2012), arithmetic (An & Wu, 2012; Kennedy, 2015), algebra (Chow, 2011; Lucariello et al., 2014), and calculus (Bezuidenhout, 2001; Bezuidenhout & Olivier, 2000; Denbel, 2014; Dolores, 2004; Dolores et al., 2019; Kaplan et al., 2015; Muzangwa & Chifamba, 2012; Özkan & Ünal, 2009; Serhan, 2015; Ubuz, 2007). While many calculus-related studies have focused on university-level students, some have addressed high school or pre-university learners, albeit to a lesser extent.

Focusing on the pre-university level is essential, since this is often where students begin to form deeply rooted ideas about calculus that may conflict with formal mathematics later on. These early alternative conceptions can hinder students' ability to grasp more advanced mathematical concepts once they transition to higher education. Therefore, the research question guiding this study is: *What alternative conceptions emerge among a group of pre-university students when making mathematical connections while solving derivative and integral tasks?*

FRAMEWORK

A common goal of school curricula in many countries is the ability to make mathematical connections with real life, other mathematical domains or scientific disciplines in mathematics class (Evitts, 2004; Özgen, 2013). This is because, among other reasons, it encourages a view of mathematics as an integrated field (Evitts, 2004; Jaijan & Loipha, 2012; Mwakapenda, 2008) and aims to improve the mathematical understanding of the students (Eli et al., 2011; García-García, 2024; Mhlolo, 2012). Therefore, understanding and mathematical connections are essential in mathematics class. However, they are preceded by the students' beliefs and conceptions (García-García & Dolores-Flores, 2018, 2021a) as they promote or interfere with the construction of mathematical connections and the achievement of mathematical understanding (**Figure 1**).

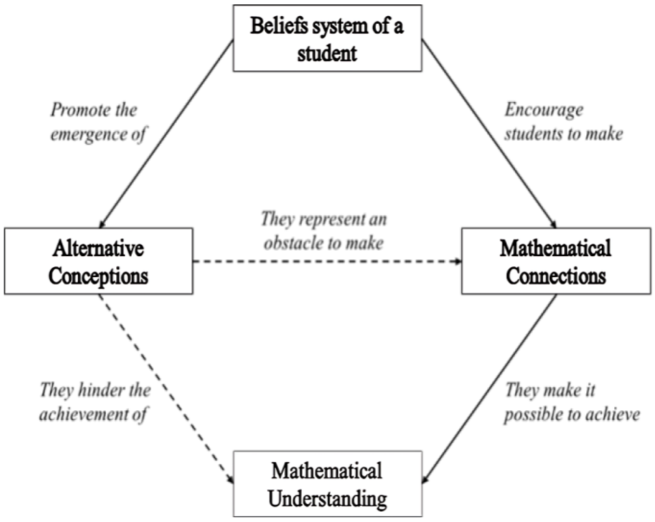

Figure 1. Relationship between beliefs system, mathematical connections and mathematical understanding (García, 2018)

In this research, mathematical connections play a fundamental role and are assumed from the position of García-García and Dolores-Flores (2018, 2021a), García-García (2024), Rodríguez-Nieto et al. (2022), Rodríguez-Nieto (2025) and are classified as: instruction-oriented, modeling, different representations, procedural, part-whole, implication, reversibility, feature, meaning, inter-conceptual, extra-conceptual, metaphorical, mnemonic-based metaphorical, and idealizing. These connections do not emerge in the order proposed above but depend on how the subject activates and models their mathematical practice and also the type of problems they are solving.

According to Kastberg (2002), a conception involves the communication of feelings and ideas about a concept and impacts the way students use this concept. In a similar way, Thijs and Berg (1995) defined conception as a personal idea about the meaning of a concept. This interpretation has some idiosyncratic characteristics even if the individual is scientific.

On the other hand, Amirali (2010) assumed that conceptions are cognitive and affective beliefs, conscious or not, personal meanings, mental images and preferences constructed from experiences inside and outside school. In this sense, we agree with Confrey (1990) when he said that students arrive on formal instruction with previous conceptions and beliefs as a result of their relationship with their context. These conceptions are critical for further learning because they interact with the knowledge that students find in class (Chow, 2011).

In this way, students use their beliefs and previous conceptions to try to give sense to a mathematical concept when it is introduced. These attempts conform to a collection of beliefs (Katsberg, 2002) that may be adequate to the previous system or constitute an impediment to assimilate the new concept. In the latter case, the students keep their previous system of beliefs, at least for a while. To the contrary, if the new concept reaches a higher status than the previous system then a conceptual exchange may take place (Treagust & Duit, 2009). In this case, the system of beliefs connected with the mathematical concept is reinforced and improved in a significant way allowing the possibility to make mathematical connections and, consequently, a mathematical understanding (**Figure 1**). However, this does not mean that the replaced conception is forgotten, it could be totally or partially restored at a future date (Treagust & Duit, 2009).

For Katsberg (2002), a student understands a mathematical concept when, based on the analysis of evidence, the system of beliefs attributed to the student is consistent with the culturally accepted beliefs about that concept, namely, with the meanings and knowledge accepted or negotiated by the mathematical community. However, the inconsistencies may be caused by the presence of alternative conceptions.

Konicek-Moran and Keely (2015) defined alternative conceptions as mental models conceived by an individual to try to explain the natural phenomena. In this way, these conceptions are the basis to build new and more complete conceptions even if they are apparently wrong or naïve. In general, the authors in mathematics education agree that alternative conceptions refer to a conception or idea that contradicts or is inconsistent with some aspects of the concept as per the negotiated or accepted scientific constructs (Chhabra & Baveja, 2012; Confrey, 1990; Narjaikaewa, 2013; Thijs & Berg, 1995) or differ from the knowledge proposed to be acquired (Mevarech & Kramarsky, 1997).

In the specific mathematical domain, Fujii (2014) considers alternative conceptions manifest when the students' conceptions are in conflict with the accepted meanings in mathematics. Therefore, their understanding is frequently limited by the incompatibility of the mathematical notions to be acquired and the knowledge of the students (Tirosh & Tsamir, 2004).

In this study, alternative conceptions are understood as students' conceptions that are inconsistent with what is accepted as true and has been socially constructed and negotiated within the mathematical community. Such conceptions may hinder the ability to make mathematical connections when solving tasks, thereby impeding the development of an adequate mathematical understanding. A student exhibits an alternative conception when their conception is mathematically incorrect or only partially accurate. In this context, the knowledge the student relies on is insufficient to correctly solve the given task.

From practical terms, alternative conceptions are inferred through students' observable actions and the verbal or written justifications they provide during task resolution. These may include, for instance, the misapplication of a familiar procedure, informal reasoning that conflicts with formal definitions, or the generation of explanations grounded in personal experience rather than instruction. These conceptions are not merely mistakes; rather, they represent coherent, albeit non-normative, cognitive structures that are often resistant to change. Analyzing these productions involves not only examining what the student concludes, but also how and why they arrive at such conclusions. This perspective assumes that students' conceptions is shaped by implicit conceptual frameworks that may coexist or compete with formal mathematical knowledge.

In this sense, a distinction is made between occasional mistakes and alternative conceptions. Mistakes typically refer to procedural or sporadic misunderstandings that do not necessarily reflect an underlying model. In contrast, alternative conceptions are more persistent, structured, and internally consistent beliefs that deviate from accepted meanings and interfere with conceptual learning. Furthermore, these conceptions are closely tied to students' beliefs and thoughts. Beliefs–understood as convictions about the nature of mathematics, learning, and oneself as a learner–influence how students interpret problems, justify their procedures, and construct meaning. Therefore, identifying alternative conceptions requires attention not only to students' answers but also to the implicit and explicit beliefs that guide their reasoning. Beliefs, thoughts, and conceptions are interdependent elements that collectively shape the emergence and persistence of alternative conceptions.

METHODOLOGY

This is qualitative research where task-based interviews were used to collect data. According to Goldin (2000), this method involves "a subject (the problem solver) and an interviewer (the clinician), interacting in relation to one or more tasks (questions, problems, or activities) introduced to the subject by the clinician in a pre-planned way" (p. 519). Task-based interviews provide opportunities to develop the understanding of the conceptual knowledge of the students instead of simply evaluating it. According to Assad (2015), the protocol of the interview can be semi-structured which allows the interviewer to judge the appropriate response to the student's mathematical reasoning. Task-based interviews allow observation, record and interpretation of complex behavior and patterns of behavior, including the words used by the subjects (Goldin, 2000).

Design of Task-Based Interviews

We used a semi-structured questionnaire that included derivative and integral tasks in three different representations: algebraic, graphical and application problems. These tasks were taken from García-García and Dolores-Flores (2018, 2021a, 2021b) because they allowed for the exploration of mathematical connections when students' responses were mathematically consistent. However, they also enabled the identification of alternative conceptions when students' responses reflected mathematically incorrect or partially correct knowledge, resulting in an inability to solve the tasks consistently. These tasks were provided to the students on printed sheets.

For the algebraic representations, we proposed nine tasks. These included tasks involving the derivative at a point, the definite integral and, the application of the FTC through tasks requiring the derivative of the integral of a polynomial function and vice versa. In addition, we asked guiding questions such as: what is a derivative? What is an integral? How did you get your result? What do your results mean? Can you get the same result using another procedure?

We designed two tasks for the graphical representation. In the first, students were given a graph of a polynomial function $f(x)$ and asked to sketch the graph of its derivative $f'(x)$. In the second, they were provided the graph of $f'(x)$ and asked to students to sketch the original function $f(x)$.

On the other hand, four application problems were included. In the biological context, students were presented with a function modelling the total population of an animal species and were asked to describe mathematically the growth rate at both a specific instant and over a general time interval and vice versa. In the physical context, students worked with two graphs: one representing the position of an object over time, and the other representing the velocity of the object at a specific moment. In the first case, students were required to derive the velocity function at both an instant and over time; in the second, they were asked to reconstruct the position function considering a given initial value.

Throughout all tasks, the interview questions were formulated to encourage students to articulate their reasoning. This made it possible to detect indicators of alternative conceptions, such as incorrect interpretations of graphical features, misapplication of formulas, or conceptual confusion between accumulation and rate. These indicators were essential for identifying the nature of students' conceptions beyond their final answers.

Participants

This research was conducted at a pre-university school in the state of Guerrero, Mexico. The participants were selected based on the following criteria:

(1) they were enrolled at the pre-university level,

(2) they had successfully completed differential and integral calculus in the semester prior to data collection, and

(3) they voluntarily agreed to participate in the study.

Based on these criteria, a total of 25 students between 17 and 18 years of age participated. For the purposes of this study, we will refer to them as *S1*, *S2*, *S3*, …, *S25*.

Data Analysis

While a student solved each task, the interviewer asked auxiliary questions to identify the alternative conceptions of the students when they tried to solve them. In this study, describing an alternative conception involved analyzing the student's response, actions, or written work to determine the mental model or prior belief guiding their conception. For example, a student might apply a known procedure in an incorrect context, misinterpret mathematical symbols or graphs, or offer informal justifications that rely on personal experience rather than formal instruction. These behaviors were used as evidence to describe the alternative conception.

The activity was video, and audio taped for its analysis. All interviews were transcribed in their totality to analyze the narratives of the students together with their written production that students made during the interview. The first author of this paper and a PhD student with previous experience as an interviewer, and with complete knowledge of the goal of our research, conducted the interviews during four working days. The average length of each interview was 80 minutes.

Thematic analysis (Braun & Clarke, 2006, 2012) was used as a method to analyze the data. The main goal of this method is to identify patterns of meanings (themes) through a set of data obtained from the answers to the research questions. Among the advantages of thematic analysis, we draw attention to the possibility to use it with a wide range of frameworks and different research questions. It may also be used to analyze different types of data, that is, it allows working with a great deal of data or with little information; finally, it can be used to produce data-driven analysis or theory-driven analysis. This method was used in this research following the next six phases (Braun & Clarke, 2006):

Phase 1. *Getting familiar with the data.* A general reading of all the narratives of the students was made several times. During this process, some initial observations were made. This was important to get ideas for possible initial codes to infer the alternative conceptions of the students.

Phase 2. *Generating the initial codes.* We established initial codes for a first classification based on the reading of the narratives. We look for phrases or statements where the relationships established by the students between mathematical ideas, concepts, procedures, representations, theorems or meanings were inconsistent from the mathematics' point of view, that is to say, where alternative conceptions appear. For example, from the following excerpt from the interview with S6 the code was formed "an expression of the form $f\ (x)$, by itself means function".

> Interviewer: Here you can see an expression (the expression $f(x) = 3x^2$ is shown). What does the expression represent to you and what elements constitute it?
>
> S6: The up function, the equal sign that tells me it is an equality, a high unknown value with exponent 2.
>
> Interviewer: How do you know that is a function?
>
> S6: Because it has the f and in parentheses it has the x.

Phase 3. *Looking for themes.* We created, assigned and modified the codes to understand their relationships and

Table 1. Identified alternative conceptions associated with the derivative and integral tasks

Alternative conceptions	N
The integral of the derivative of a polynomial function is obtained by finding the derivative and integral separately.	20
The instantaneous velocity of an object is calculated using the formula $v = d/t$.	14
The value of $f'(a)$ is interpreted only as the value of y when x equals a.	10
In the expression $f(x) = 3x^2$, $f(x)$ is considered simply a function by itself.	10
A dx must be added after completing a derivative to represent its derivative nature.	5
It is interpreted that $f'(a)$ corresponds to a point on the tangent line of the curve.	3
The meaning of $p'(a) = k$ is that it takes k years for the population to grow.	2
The meaning of $p'(a) = k$ is that k is the speed of population growth per year.	2
The graphical interpretation of the derivative is seen as the tangent line that touches the curve at a maximum point.	1
Total	67

Note. N: Frequency

establish a family of codes (potential themes). Once the initial codes were established, we contrasted the excerpts associated with each of them to look for themes between the codes. This allowed us to cluster the patterns of responses of the students associated with specific alternative conceptions in themes. For example, from the answers of other students, the code described above was established as the next theme "in the expression $f(x) = 3x^2$, $f(x)$ is considered simply a function by itself".

Phase 4. *Reviewing the themes.* The themes were discussed with professionals in mathematics education with research experience from phase 1. The title or descriptions of some themes were modified. We also establish clusters of initial themes and eliminate those that do not have enough evidence to support the ideas of the students and we generate new themes when it is necessary.

Phase 5. *Defining and naming themes.* The themes that encompassed the main ideas of the students and that answer the research questions were defined. Likewise, the description of each alternative conception was made, associating to each of them representative extracts of the collected data set.

Phase 6. *Writing the report.* Finally, we will write the final report of the study.

To further support the identification of alternative conceptions, the framework of mathematical connections proposed by García-García and Dolores-Flores (2018, 2021a), García-García (2024), Rodríguez-Nieto et al. (2022), Rodríguez-Nieto (2025) was used as an analytical lens. Once themes related to alternative conceptions were established through thematic analysis, we examined whether and how these conceptions affected the students' ability to activate specific types of mathematical connections. This classification provided a structured basis to interpret not only what alternative conceptions emerged, but also how these conceptions interfered with or prevented mathematical connections.

RESULTS

The nine alternative conceptions reported below emerged from a systematic thematic analysis proposed by Braun and Clarke (2006). We coded segments of student discourse that reflected inconsistencies with accepted mathematical meanings, grouped them into categories, and defined them as themes based on recurring patterns across different representations and tasks. Each alternative concept described below corresponds to one of these themes (**Table 1**).

The alternative conceptions identified (**Table 1**) are explained below:

1. *The integral of the derivative of a polynomial function is obtained by finding the derivative and the integral separately.*

This alternative conception appeared in 20 students (80%) when solving tasks like $\int \left[\frac{d}{dx}(3x^2)\right] dx$. Rather than interpreting this expression through the lens of FTC, students sequentially applied procedural rules based on the order of operations and grouping symbols learned in earlier mathematics courses (e.g., arithmetic and algebra). They performed these operations in the following order: they first solve the math in parenthesis (derivative), and finish with the math in brackets (integral).

This alternative conception illustrates a procedural overgeneralization, where students extend rules for simplifying algebraic expressions to contexts where conceptual understanding is required. The mental model guiding this reasoning treats integration and derivation as isolated, mechanical processes rather than as inverse operations–a hallmark of understanding the FTC–. Instead of recognizing the reversibility encoded in the theorem, students' default to syntactic processing of symbols, privileging order-of-operations schemas over conceptual structure. This is not simply a mistake in execution, but a manifestation of an underlying alternative conception: that mathematical operations must always follow syntactic rules, even when conceptual relationships suggest a different approach.

For instance, 20 students offered $\int \left[\frac{d}{dx}(3x^2)\right] dx = 3x^2 + C$ (**Figure 2**) as an answer (**Figure 2**). This indicates that they do not use the FTC when polynomial functions satisfy the properties for their use. Instead, students perform the indicated operations considering the grouping symbols instead. In this particular case, the

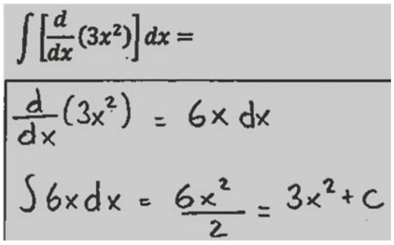

Figure 2. Answer of S10 to the integral of the derivative of a polynomial function (Source: Authors' own elaboration)

function $f(x)$ is continuous on the interval $[a, b]$, therefore, the FTC states that $\int \left[\frac{d}{dx}(f(x))\right] dx = f(x)$. In other words, the students could have noticed the reversibility of the derivative and the integral if they understood the FTC and, consequently, the result of the integral of the derivative would be the original function.

On the other hand, this alternative conception also means that students could consider this inequality $\frac{d}{dx}[\int f(x)dx] \neq \int \left[\frac{d}{dx} f(x)\right] dx$ as true when $f(x)$ is a polynomial function; the difference comes from the presence (or not) of the constant of integration C in their results (see excerpt of interview of S11).

> Interviewer: Do you consider that solving this operation (points the finger at $\int \left[\frac{d}{dx}(3x^2)\right] dx$) is the same or different than solving the previous one (points the finger at $\frac{d}{dx}[\int (3x^2)\, dx]$)?
>
> S11: It is different. There are integrals and derivatives in both of them, but in the first one you have to do one operation first and then the other one. In the second one, deriving results in an exact value, *but you get a constant when you integrate*, this means that the value may be inaccurate, and you have to add it.

2. *The instantaneous velocity of an object is calculated using the formula $v = d/t$.*

This alternative conception illustrates a misapplication of a correct formula in an inappropriate context, revealing a deep-seated misunderstanding of the concept of instantaneous rate of change. While the formula $v = d/t$ is accurate for calculating average velocity over an interval, it is not applicable to determining velocity at a specific instant, which requires an understanding of limits. Students' reliance on this formula reflects a conceptual conflation between average and instantaneous velocity–an ambiguity often introduced during early science instruction without sufficient emphasis on their distinction–. Consequently, students tend to invoke the familiar expression $v = d/t$, even when it is conceptually mismatched to the nature of the task.

This alternative conception appeared in 14 students (56%) when they were asked to find the velocity of a

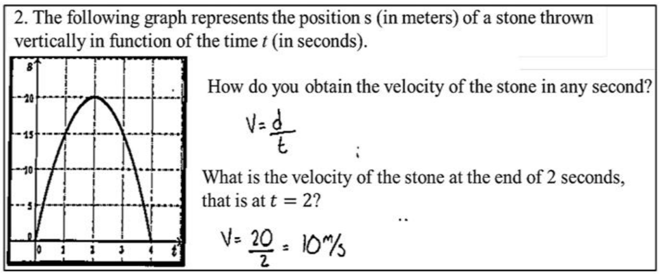

Figure 3. S10 using the formula $v = d/t$ to find the instantaneous velocity (Source: Authors' own elaboration)

stone in any instant x given the graph of the position or trajectory of the stone (in meters per second). First, they were asked for the algebraic representation to solve the velocity in any instant of time and then at the instant $t = 2$ seconds. The students obtained a velocity of $10\ m/s$ at $t = 2$ because they used the formula $v = d/t$ to find it (see **Figure 3**). This result is clearly inconsistent with the mathematical point of view because it is different from zero, which is the real velocity in the maximum height of the stone.

3. *The value of $f'(a)$ is interpreted only as the value of y when x equals a.*

This alternative conception reveals a semantic conflation between the function and its derivative, rooted in a superficial understanding of function evaluation. Students treat $f'(a)$ analogously to $f(a)$, implying that both expressions denote coordinate points on the graph of f. In this sense, ten students (40%) assumed that the only meaning of $f'(a)$ is a value for y when x takes the value of a and, consequently, it only represents the graph of a point.

This reflects a limited representational flexibility, where students struggle to distinguish between different mathematical objects–namely, the function and its derivative–and their associated graphical interpretations. As shown in the interview with S13, there is a belief that $f'(a)$ yields a point on the original graph, when it actually describes a property of the graph at that point–the slope of the tangent–.

> Interviewer: Could you find the derivative when $x = 1$ (points the derivative function $f'(x) = 6x$ obtained previously)?
>
> S13: (S13 does the corresponding operations)
>
> Interviewer: What does this mean?
>
> S13: The value of y is 6 when the value of x is 1; the coordinates are one comma six (the student writes (1, 6)).
>
> Interviewer: Could we say that it is a point of the graph?
>
> S13: It is going to be one of the points of the line.

This alternative conception may stem from instruction that emphasizes procedural skills (like evaluating functions) over conceptual understanding of rate of change, supporting a procedural conception of derivatives. On the other hand, the student's mental model is internally consistent–they correctly compute $f'(1) = 6$ and state it corresponds to the point $(1, 6)$–but this consistency is built upon a flawed conceptual base. This coherence within an incorrect framework is a hallmark of alternative conceptions.

4. *In the expression $f(x) = 3x^3$, $f(x)$ is considered simply a function by itself.*

Ten students manifested this alternative conception when they associated the letter "f" followed by the letter x in parenthesis to the concept of function without considering the complete elements of the expression. In this sense, students identify the presence of $f(x)$ as a sufficient indicator of *function*, without interpreting the meaning of the entire expression.

It is possible that the letter f taken from the word function generates this idea in the students; however, this alternative conception could prevent the understanding of the meaning of this concept. It is possible that this occurs because the students associate their knowledge of algebra to give meaning to the literals in the algebraic symbolism of some concepts, the concept of function in this specific case. For example, if we write y instead of $f(x)$ in the algebraic expression after the equal sign, S6 would interpret it as a second-degree equation (see excerpt of S6).

> Interviewer: You can see here the expression $f(x)$ (points out the expression $f(x) = 3x^2$). What does this expression represent to you and which elements compose it?
>
> S6: It is composed of the function, the equal sign that tells me this is equality and an unknown square variable.
>
> Interviewer: How do you know it is a function?
>
> S6: Considering the f followed by the x in parenthesis.
>
> Interviewer: Would it still be a function if we only write y equals three x square ($y = 3x^2$) instead of $f(x)$?
>
> S6: It would be a second-degree equation.

This excerpt of S6 shows that the student considers at least two different mathematical objects in the expression $f(x) = 3x^2$, the function represented by $f(x)$ placed before the equal sign and the quadratic equation placed after the equal sign. S10 recognized that is considered the expression $f(x)$ as a function only because it has the letter *f*. Such conception prioritizes surface features over underlying mathematical relationships and illustrates how students' interpretations are shaped by their exposure to notation rather than to meaning. This type of conception may hinder the ability to work flexibly with function representations, a critical skill in calculus.

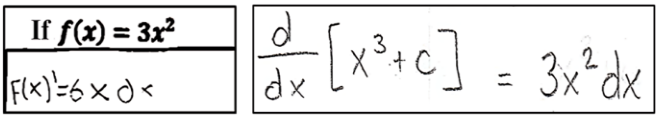

Figure 4. Students S3 and S10 adding dx to the result of the derivative (Source: Authors' own elaboration)

5. *A dx must be added after completing a derivative to represent its derivative nature.*

This alternative conception stems from a misinterpretation of Leibniz's notation, treating the differential dx as a quantity that must accompany any derivative result. Rather than understanding dx as part of the limit-based ratio in dy/dx, students treat it as a literal symbol that must appear in the final expression. Five students who manifested this alternative conception indicate that the differential of x should be added to the result of a derivative (**Figure 4**). In this sense, it manifests when dx is interpreted as the result of the derivative instead of finding the derivative with respect to the variable x, that is, they don't understand that dx represents an infinitesimal change.

This alternative conception could become an impediment in finding the integral of a function because the meaning of dx is important to find double or triple integrals or when the function has more than one variable. It could even cause difficulties to find partial derivatives of any order. On the other hand, as mathematics becomes more formalized, students lacking clarity in symbolic representation may develop rigid or incorrect rules to compensate, resulting in this kind of persistent alternative conception.

6. *It is interpreted that $f'(a)$ corresponds to a point on the tangent line of the curve.*

This alternative conception is closely related to conception number 3, but it introduces a new interpretive layer: identifying $f'(a)$ with the point of tangency itself rather than with a numerical rate (slope). This conception appears to stem from an intuitive association between the geometric image of a tangent and the derivative, without internalizing that $f'(a)$ quantifies the slope at that point, not its location.

For instance, even S11 recognize the relationship between $f'(a)$ and the tangent line, this conception is inconsistent with its meaning as the slope of the tangent line of $f(x)$ in $x = a$ because his idea is associated with the point of tangency (see excerpt of S11).

> Interviewer: Could you find the derivative of $f(x) = 3x^2$ in $x = 1$?

S11: Yes (S11 does the operations). It is only six.

Interviewer: What does this mean?

S11: […] It would be the point of intersection or the point where the tangent line touches the curve.

This alternative concept may emerge from the fact that students assume that the tangent line of a curve is made of points, so $f'(a)$ could mean the point where the tangent line and the curve meet, the point of tangency. On the other hand, this alternative conception may not only prevent the understanding of the real meaning of $f'(a)$, but also the meaning of $f''(a)$ and their associated conceptions.

This alternative conception signals a transitional understanding: students recognize a connection between derivatives and tangents but cannot yet separate geometric intuition from analytical meaning.

7. *The meaning of $p'(a) = k$ is that it takes k years for the population to grow.*

This alternative conception illustrates a misattribution of units and context, where students focus on the input variable (time in years) and project it onto the output without considering the functional relationship. So, this alternative conception emerges from solving problems involving biological concepts. The students were given the function $p(t) = 2t^3 - t^2 + 100$ that models the total population of some animal species after t years. They were asked to find the speed of growth in t=2, that is, to find $p'(2)$. The students explained that the meaning of $p'(2) = 20$ is that it takes 20 years for the population of animals to grow (see excerpt of S20), ignoring that $p'(2)$ represents the rate of change of the population at time 2 years.

The students focused on the time *t* and associated the result with this data without considering that they were asked to find the speed of growth in this specific instant. They seem to reason that the result of substituting the value of 2 must be expressed in years because the meaning of *t* is time, and they were told the time was measured in years.

Interviewer: What does this result mean to you (points the result of the derivative at $t = 2$, that is, $p'(2) = 20$)?

S20: That it takes 20 years.

Interviewer: Could you explain to me what you are saying in more detail?

S20: Because t is time and it is measured in years; you substitute two years, square two years and get four. Then four times six, 24 years, minus two times two or four years. The result is 20 years.

Interviewer: What is the meaning of this result in terms of the phenomena?

S20: That the animal species grows in 20 years.

S20's interpretation–it takes 20 years–demonstrates a linearization of time as the dominant feature, ignoring that $p'(2)$ refers to the rate of change at a specific time, not a duration. This conception is symptomatic of a pre-functional view of change, where quantities are interpreted in static, contextualized terms rather than as variable-dependent quantities. It also illustrates the influence of everyday language in shaping mathematical meaning, as expressions like *it takes time to grow* are internalized and projected onto symbolic results.

This alternative conception expressed by S20 could trigger mistaken *explanations* of future results of application problems. For example, it can be observed in his written production that he first derives the function $p(t)$ and then finds $p'(2)$ correctly, but he does not associate the accurate meaning with this result and an alternative conception appears. This kind of over-contextualization can lead students to systematically misinterpret derivatives in applied settings, making this a robust and coherent alternative conception.

8. *The meaning of $p'(a) = k$ is that k is the speed of the population growth per year.*

This conception, though closer to the normative meaning, still misrepresents the derivative as a long-term constant rate rather than an instantaneous one. Students interpret the rate of change as an average across future intervals, leading to extrapolations like 20, 40, 60, etc.

This alternative conception appears when the students solved the problem described previously. However, in contrast with the previous one, two of the students who manifested this concept believe that the meaning of the result $p'(2) = 20$ is that the population of animals grows at a velocity of 20 animals per year (see excerpt of S4). This result, as the previous one, is not only closely linked to the variable *t* as a time, but also to their conception of average velocity. The response of the student implies that there will be 20 animals after one year, 40 after two years, 60 after three years, and so on.

This alternative conception, as a result of the meaning assigned to the average velocity, limits their understanding of instantaneous velocity in the phenomena described previously. Such conception reflects the transfer of uniform motion models from physics to biological growth, disregarding the possibility of nonlinear or varying rates. This is consistent with the tendency to interpret derivatives through arithmetic sequences or linear growth models, which are more intuitive and familiar to students.

Interviewer: Can you solve the following question (indicates the question where S4 was asked to evaluate the function $p'(t)$ at $t = 2$)?

S4: Yes (he reads the question and substitutes the value of $t = 2$ in the derivative found previously). It is 20.

Interviewer: What is the meaning of 20 in the problem?

S4: The population's speed of growth will be 20 per year.

In contrast to alternative conception number 7, this one illustrates a misapplication of the concept of average rate of change, rather than an issue with units. However, both conceptions highlight students' struggles to internalize the idea of instantaneous change as a local property of a function.

9. *The graphical interpretation of the derivative is seen as the tangent line that touches the curve at a maximum point.*

This alternative conception likely arises from an overemphasis on the visual connection between tangents and extrema in instructional examples. Students may associate maximum points with important locations, assuming the derivative is somehow represented by a tangent at that specific point.

For instance, the first thing S12 does to draw the graph of the derivative given the graph of a polynomial function $f(x)$ is to sketch a straight line that touches $f(x)$ at the maximum point of this function (**Figure 5**). S12 originally assumed that this line represents the derivative function but rectifies after the auxiliary questions of the interviewer and sketches an acceptable graph of $f'(x)$.

The sketch by S12 confirms that this is not a random error, but a consistent pattern of interpretation: students treat the derivative as a visual object that is tangentially connected to salient points on the original function, rather than as a new function derived from analyzing slope behavior across an interval.

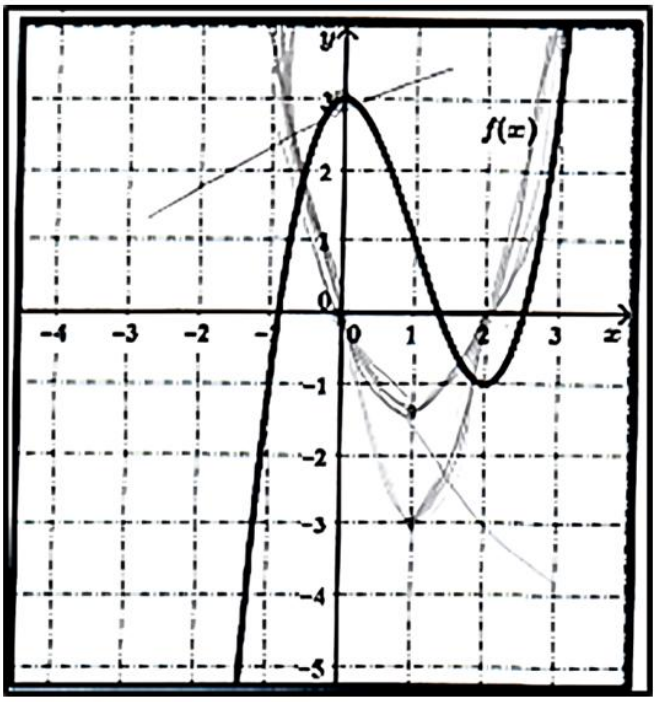

Figure 5. Sketch of $f'(x)$ made by S12 (Source: Authors' own elaboration)

DISCUSSION

The research question in this paper was: *What alternative conceptions emerge among a group of pre-university students when making mathematical connections while solving derivative and integral tasks?* We identified nine alternative conceptions (**Table 2**) through the thematic analysis; we also presented some possible causes (within mathematics education) for the alternative conceptions identified, as well as possible mathematical connections that could hinder them.

These results show that the students' mathematical understanding about derivatives and integrals is limited, as reported by Denbel (2014). Our results are similar to those of Kaplan et al. (2015) who also found that the students presented alternative conceptions associated with the instantaneous velocity in application problems in physics, that is, students find the instantaneous velocity using the formula $v = d/t$. On the other hand, the students that manifested the first alternative conception in **Table 2** used the formula $\frac{d}{dx}ax^n = anx^{n-1}$ to find the derivative; however, if they do not understand the meaning of this formula and the context where it can be used, then they could get results such as: the derivative of the function $y = x^x$ is xx^{x-1}, as reported by Muzangwa and Chifamba (2012). This leads the students to a misunderstanding of the FTC. Moreover, if the students do not understand the FTC, then they could even think that the meaning of $\int_{-1}^{1}\frac{1}{x}dx$ is the area under the curve $f(x) = \frac{1}{x}$ in the interval $-1 \leq x \leq 1$, as reported by Muzangwa and Chifamba (2012). This is not the real meaning of this integral because the function is not continuous at $x = 0$.

Instead, the idea surrounding the alternative conceptions "the value of f´(a) is interpreted only as the value of y when x equals a" and "It is interpreted that $f'(a)$ corresponds to a point on the tangent line of the curve" is the extrapolation of the notion of function as a corresponding rule to the meaning of derivative function at a point. This guides the students to think that the meaning of $f'(a)$ and $f(a)$ is the same. They are assuming that "a function is a corresponding rule such that to each value of x, there is assigned exactly one value of y". This is a correct idea, but with a limited understanding of the concept of function with domain in real numbers, as reported by Cuevas and Delgado (2016). The concept of function is a central topic in calculus, although it is difficult to understand for some students (Doorman et al., 2012; Elia et al., 2007; O'Shea et al., 2016). For example, the students usually have problems giving a correct definition of this concept and solving problems about functions involving conversions between different representations (Elia et al., 2007). In this sense, we can partially explain that the participants of this study state that "In the expression $f(x) = 3x^2$, $f(x)$ is considered simply a function by itself" turning out to be an alternative conception.

To avoid these obstacles and allow a flexible use of functions, Best and Bikner-Ahsbahs (2017) suggest constructing their meaning of function both as a rule of correspondence and covariation within and among all types of possible representations, and also between different functions. On the other hand, the alternative conceptions "the value of f´(a) is interpreted only as the value of y when x equals a" and "the instantaneous velocity of an object is calculated using the formula $v = d/t$ trigger other alternative conceptions while working with tasks involving graphics and application problems in biology". This partially explains the alternative conceptions "the meaning of $p'(a) = k$ is that it takes k years for the population to grow", "the meaning of $p'(a) = k$ is that k is the speed of population growth per year" and "the graphical interpretation of the derivative is seen as the tangent line that touches the curve at a maximum point" (**Table 2**) identified in this study.

Likewise, alternative conceptions identified in this study obstruct the students from making some mathematical connections[1] (**Table 2**), which are fundamental to achieve mathematical understanding.

Table 2. Alternative conceptions associated with derivative and integral tasks, origin, and consequences

Origin	Alternative conception	Affects understanding of	Hinders mathematical connections
Hierarchy of multiple operations in arithmetic	The integral of the derivative of a polynomial function is obtained by finding the derivative and the integral separately.	FTC	Reversibility, implication (Eli et al., 2011; García-García & Dolores-Flores, 2021a, 2021b)
Formula to find the velocity in physics	The instantaneous velocity of an object is calculated using the formula $v = d/t$.	Average velocity and instantaneous velocity	Procedural, meaning, extra-conceptual (García-García & Dolores-Flores, 2021b; García-García, 2024)
Concept of function	The value of $f'(a)$ is interpreted only as the value of y when x equals a.	First and second order derivatives. Graphically $f(x) \neq f'(x)$. The meaning of the derivative at a point in application problems.	Meaning, different representations, procedural (Businskas, 2008; García-García & Dolores-Flores, 2018, 2021a)
The meaning of literals	In the expression $f(x) = 3x^2$, $f(x)$ is considered simply a function by itself.	The meaning of expressions using different variables	Meaning, feature (Eli et al., 2011; García-García & Dolores-Flores, 2021a, 2021b)
Concept of derivative using the notation $\frac{dy}{dx}$	A dx must be added after completing a derivative to represent its derivative nature.	The meaning of differential in partial derivatives or in double or triple integrals	Meaning, procedural (Businskas, 2008; García-García & Dolores-Flores, 2021a, 2021b)
Points that constitute a tangent line	It is interpreted that $f'(a)$ corresponds to a point on the tangent line of the curve.	The concept of first and second derivative in a point. Graphically $f(x) \neq f'(x)$. The meaning of the derivative at a point in application problems.	Meaning, different representations (Businskas, 2008; García-García & Dolores-Flores, 2018, 2021a).
Meaning of the literal	The meaning of $p'(a) = k$ is that it takes k years for the population to grow.	The concept of the derivative at a point in application problems	Meaning, extra-conceptual (García-García & Dolores-Flores, 2021b; García-García, 2024)
Concept of average velocity	The meaning of $p'(a) = k$ is that k is the speed of population growth per year.	The meaning of velocity at a specific point in time	Meaning, extra-conceptual (García-García & Dolores-Flores, 2021b; García-García, 2024)

[1] Are true relationship between two or more ideas, concepts, definitions, theorems, or meanings with each other (García-García & Dolores-Flores, 2018). Also, these relationships are useful when trying to improve mathematical understanding (Businskas, 2008). The usefulness of the relationships between mathematical concepts in achieving mathematical understanding is assessed by the expert (García-García & Dolores-Flores, 2021a).

Table 2 (Continued). Alternative conceptions associated with derivative and integral tasks, origin, and consequences

Origin	Alternative conception	Affects understanding of	Hinders mathematical connections
Concept of tangent line	The graphical interpretation of the derivative is seen as the tangent line that touches the curve at a maximum point.	The graphical meaning of the first and second derivative	Different representations, implication (Businskas, 2008; Eli et al., 2011; García-García & Dolores-Flores, 2021a)

This highlights the importance of these results and invites us to think about the role of teachers as a possible source of alternative conceptions because they can promote false ideas about previous concepts in students, for example, in algebra and arithmetic. Therefore, we agree with An and Wu (2012) who indicated that it is necessary to provide feedback to deal with these alternative conceptions in school situations. However, this would not be enough because some of these alternative conceptions have proven resistant to change. In this regard, Read (2004), Lucariello et al. (2014), Kennedy (2015), Bostan (2016), and Dolores et al. (2007) suggest the necessity to use methods that promote the conceptual change to replace these ideas in the students. Pajares (1992) considers the promotion of change of beliefs should be done when the existing ones are unsatisfactory.

With regard to this conceptual change, Kennedy (2015) highlights the importance to make sure that the student is really changing his thoughts and not only giving the "right" answer because this could promote the memorization with the only purpose to pass a test, but keeping the previous conceptions for other contexts, as recognized by Libarkin (2001). This means that promoting conceptual change is not an easy task (Lucariello et al., 2014). Mulhal and Gunstone (2012) suggest the introduction of processes of problem-solving in the classroom soon after the development of previous concepts in the students.

CONCLUSION

The alternative conceptions identified in the group of 25 pre-university students indicate that they could prevent the understanding of some more advanced concepts of calculus, such as partial derivatives, successive derivatives, graph of the first and second derivative, double and triple integrals, and hinder the understanding of the derivative and integral in application problems in physics, biology or other disciplines; specifically, the meaning of position and velocity in physics and the total population and speed of growth in biology. As a consequence, the students cannot make mathematical connections when they solve tasks that involve those mathematical and extra-mathematical concepts.

The fact that most of the 25 participants perform well academically explains why there are only 67 identified alternative conceptions. In contrast, many of the participants demonstrated mathematical connections similar to those reported by García-García and Dolores-Flores (2018, 2021a, 2021b), which are not presented here due to space limitations. We believe this result suggests that students with lower academic performance may exhibit a greater number of alternative conceptions. Thus, further investigations could focus on the identification of these alternative conceptions related to the FTC in pre-university and university students with low academic performance both in calculus and in other mathematical domains.

The findings of this research provide mathematics educators with concrete examples of how students may misconceive core ideas in calculus, allowing teachers to anticipate such conceptions in the classroom. By being aware of these alternative conceptions, educators can adapt their instruction to explicitly address and confront students' misunderstandings through targeted explanations, visual representations, or problem variations that emphasize conceptual relationships. In particular, teachers may use these conceptions as diagnostic tools to guide formative assessment and to design learning experiences that foster deeper mathematical connections, both within and beyond calculus.

One limitation of this study lies in the participant population, as a larger sample could help identify additional alternative conceptions that have not yet been reported in the literature. Therefore, future research should not only aim to identify alternative conceptions across a broader range of participants (including students at different educational levels, pre-service teachers, and in-service teachers) and for various mathematical concepts but also focus on the design of instructional approaches that support the transition toward the use of mathematical connections, thereby enhancing mathematical understanding.

Author contributions: JG-G: designed the research, collected and analyzed the data, and wrote the first draft of the manuscript; **CAR-N:** improved the literature review, analyzed the data, and revised the manuscript overall; **GS-B:** analyzed the data, improved the writing of the paper, and translated it into English; & **AZ-B:** analyzed the data and revised the final draft for improvement. All authors agreed with the results and conclusions.

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