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Analysis of Prospective Classroom Teachers' Teaching of Mathematical Modeling and Problem Solving*

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This qualitative research is based on mathematical problem solving and modeling. In this study I analyzed 39 junior year prospective classroom teachers' experiences while they were enrolled in the "Teaching Experience" course and visiting public primary schools. More specifically, I examined teaching mathematical problem solving and modeling by content analysis of the qualitative data. The participants were given a six-hours training about problem solving and modeling prior to data collection. The findings were as follows: prospective teachers i) became more familiar with the nature of problem solving after the teaching practice; ii) they had difficulties in choosing problems and in teaching due to insufficient experience; iii) they had difficulties in finding non-routine problem situations; iv) they formed sympathy for mathematics teaching through problem solving approach by using modeling; v) the pupils at the host schools were willing to engage in such activities, and they used positive expressions about their learning experiences.

Keywords: Mathematics Teaching, Prospective Teacher, Problem Solving, Modeling

INTRODUCTION

When encountered with a problem, it is crucial that it should be understood to develop a solution process. An individual cannot come up with a solution or develop a strategy for a problem that s/he cannot understand. The main objective of problem solving training is to develop problem solving skill (Altun, 2010). Modeling has an important place in developing a problem solving technique. Modeling process starts with a real world problem. While modeling a real world problem, one should act between reality and mathematics. A model for a problem should be developed by simplifying, constructing and idealizing. Expressing the model within the mathematics world forms the mathematical model. Modeling problems have a real, reliable and complex structure (Lesh and Doer, 2003).Models are conceptual systems that explain and define mathematical concepts, tools, relations, actions, forms and settings all of which contribute to problem solving cases. Models with a practical structure are processes that enable achieving clearly defined aims. Mathematical modeling focuses on the structure of problem situation. Modeling development process involves digitizing, organizing, systematizing, sizing, coordinating (Lesh and Harel, 2003). Modeling is an ability to analyze and verify mathematical explanations. It means deciding and understanding within process in mathematical situations. Modeling is not only a skill or ability but also willingness to use this competence. Motivation is an indispensible part of modeling. Relevant research shows that information is not enough alone for modeling. The student should choose the information, follow the relevant process and possess competence to make sense of modeling process. Lesh and Kelly (2000) mention a structure that reflects the modeling cycle for these competences as follows:

a. *Building a model on a situation related to real world and ability to understand the problem involves simplifying the situation and hypothesizing for the problem, presenting the amount that affects the situation, defining and naming the key variables, configuring the relation between the variables,*

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State of the literature

- Modeling problems have a real, reliable, and complex structure.
- Mathematical modeling has such effects as reinforcing the dimensions of learning like motivation, conceptual learning, comprehension and retention, contributing to the development of various mathematical qualities and abilities, and making the abstract mathematics world more concrete and comprehendable.

Contribution of this paper to the literature

- Even though there are a number of factors important for effective mathematics education (e.g., students, learning environment, teaching materials, etc.) the factor of growing to be an effective and efficient teacher is crucial in that the teacher manages and organizes all of the processes for teaching and learning.
- This study helps us to understand how teaching mathematics prodders work more effectively while developing classroom school teachers improve their skills on problem solving and modelling which are fundamental purposes in mathematics education.

looking for information and distinguishing between related and unrelated information.

b. *Skill to form a mathematical model from actual model involve expressing the related qualities and their characteristics mathematically, simplifying the related qualities to minimize numbers and complexity, choosing the right mathematical display and using graphs.*
c. *Skill to solve mathematical problems in a mathematical model requires using discovery strategies like dividing the problem in a problem, forming similar problems or similar relations, renewing the problem, analyzing the problem in different forms, using mathematical knowledge to solve the problem.*
d. *Skill to interpret the mathematical solutions in real situations requires interpreting the result, generalizing the solution developed for specific situations, analyzing the solution using appropriate mathematical language.*
e. *Skill to verify the solution requires detailed analysis, returning to the model process or to the related part of the model if not sure of the solution.*

For a successful modeling process, the three crucial points Lesh and Doerr (2003) emphasize can be summarized as follows: 1) Understanding, simplifying and configuring the problem and developing a mathematical model by using explanations, diagrams, formula, tables and doing analysis on them: This process involves defining variables and their relation within the problem. 2) Deciding about the variable relation, hypothesizing, organizing thinking, analyzing the information in the text in details, using a strategy and preparing the model in details are very important. 3) Interpreting the solution, building a system to meet certain requirements, analyzing the system, defining the solution, putting forth suggestions, verifying, approving and reflecting the solution, developing and applying different models. This process involves generalizing and evaluating the solution from different perspectives. Analyzing the modeling process closely, its structure gives us opinion about the modeling cycle developed by the relations between real world and mathematics worlds. This cycle starts when we encounter real world situations and continues with us moving within the mathematics world freely by explaining and simplifying these situations.

Explaining and simplifying can be said to concretize the solution and to enable one to see the problem structure concretely. Blum and Leiss (2005) mention about different levels for the solution of concrete problem situations we encounter which can be explained in traditional modeling cycle.

Level 0: Student cannot understand the problem situation and therefore cannot come up with a concrete solution.

Level 1: Student can only understand the real world situation but cannot express or relate it with mathematical ideas.

Level 2: Student can come up with a real model by configuring and simplifying the situation after researching the given real situation but cannot know how to express this model mathematically.

Level 3: Student can express not only the model but also the situation mathematically but cannot still work in mathematical world completely.

Level 4: Student can form problem situations from real ones, work with mathematical problems and achieve results.

Level 5: Student possesses experience during the mathematical modeling process and verifies the solution (Ludwig, Xu, 2008).

Henning and Keune (2005) analyze model development while solving a problem at three levels. The first is the ability to define and adopt the model process and the second is analyzing the problem and the abstract qualities of the problem, while the third is doing critical analysis on the model and expressing the criteria of model evaluation. The last level is the one that reveals the judgments on modeling and when modeling is best understood. According to Bonotto (2010), when children discover different strategies, they feel comfortable in reasoning processes. Moreover, mutual meanings can be achieved on solutions when the teacher discusses hypothesis and importance of alternative strategies with students. Positive results can be achieved on forming learning experience if tools are used in mathematical experiences, if students are given roles that enable students to do meaningful judgments in mathematical activities in and out of school and to live the discovery process, if appropriate learning environment is provided for students to share their knowledge and experience and to communicate, and if balance can be achieved in activities for problem presentation and solution.

Individuals that can solve problems efficiently follow thinking processes regularly and automatically. During individual or group work, it's better to enable students to ask themselves or to one another "*What are you doing? Why do you think like that?*" so that students can follow and control problem solving processes automatically (Van De Walle, Karp and Williams, 2010). When students are aware of their own or their peers' thinking processes or strategies, it will be easier for them to do analysis and model development while problem solving.

Mathematical modeling helps students to understand the real world best. It has such effects as reinforcing the dimensions of learning like motivation, conceptual learning, comprehension and retention, contributing to the development of various mathematical qualities and abilities, and concretizing the mathematics world (Blum and Ferri, 2009). Model development should have a certain level of continuity and every model should be analyzed in such a manner that will yield an answer to how it can simplify a situation better (Lesh, 2010). Efficiency of modeling problems and model development is concerned with the structure of group activities during which modeling are done. When modeling problems are designed for small group activities, it will ease the process if group roles are given during the problem solving process. Some roles that require group merging facilitate and boost the group work process. Group works with components like planning, observing, configuring andusing modeling problems that give students opportunities might contribute to the existing experience of the individual in mathematical thinking (English, 2006).

Efficient teaching of mathematics during first grade is crucial in formation of mathematical thinking. Teacher proficiency and teacher training are to be regarded important in achieving high quality for teaching mathematics. This research whas conducted to reveal the experiences and problems of prospective teachers during their training and application solution and modeling of mathematical problems in teaching mathematics.

METHODOLOGY

This is a qualitative study. The data were gathered through interviews. The participants were choosen through two criteria: ease of access and willingness to participate in the study voluntarily.

Participants

Thirty-nine prospective classroom teachers (10 male and 29 female) particpated in this study. At the time of the study the participants were enrolled in the "Teaching Experience" course and all of them were junior year students from the classroom teaching program of a state university in Turkey. In the Teaching Experience course prospective teachers visit schools and have a first hand experience within the classrooms under the teacher's supervision and guidence. The main objective in this course is to observe and try to identify and understand the main issues within the classroom and the school as a future teacher. The participants were given a six-hours training on problem solving, generating routine and nonroutine problem situations and modeling. Afterwards, they were asked to teach problem solving by modeling at their practicum schools where they taught as part of their "Teaching Experience" course.

Data Analysis

During the analysis process, an expert worked with the researcher. The data were analyzed using content analysis. Content analysis is based on processing the information carried by a message and the first step at this point is descriptive operation. Here, subjective and rough descriptions are to be sorted by revealing the characteristics of the message components objectively (Bilgin, 2006).The main objective in content analysis is to reach the concepts and relations that can explain the data. For this purpose, the data should be conceptualized, the concepts should be organized and the themes explaining the data should be put forward (Yıldırım and Şimşek, 2000). Various stages should be taken into account during content analysis. These stages are forming natural units in the meaning, doing arrangements in these units through classifications and categories, configuring the text to define the content and interpreting the data (Cohen, Manion and Morrison, 2007). During the research process, at the end of problem solving process based on modeling, the prospective teachers were asked to make evaluations about the lesson. The interview records for these evaluations were listed and then the data were analyzed by the expert and the researcher. The expressions directly showing the the participants' views about the modeling process were sorted out and listed in a table. The aim at this point was to organize the data to set the experiences of prospective teachers during problem solving and modeling process in detail. The table included the name of each prospective teacher, his/her

Table 1. The class levels, mathematical problem subjects and types chosen by prospective teachers as the basis of their problem situations

School classroom grade visited	Number of prospective teachers visiting	Number and Subject of Problems	Number and Types of Problems
1	4	4 measuring	13 non-routine problems
2	23	29 the four operations	
3	9	2 geometry	26 routine problems
4	3	4 fractions	

problem text, the problem subject, class level and the evaluation interview data. Afterwards, the table was developed into two separate tables. Table 1 above shows the class level, problem subject and problem type, while Table 2 shows the categories arising from the participants' evaluations and sample expressions for each category. However, the participant names were later replaced by numbers which were also given at the end of the sample sentences for evaluations in the "results" section below.

RESULT

This section involves the data analysis of the interview with prospective teachers and their application samples in problem solving and modeling.

The problem solving application was carried out at practice classes in the context of "Teaching Experience" course and the majority conducted their practice at third grade classes, which allowed coherence among the research data because the problem situations were at similar levels. The prospective teachers were set free in deciding the problem subject and generally chose their problems from four operations problems. It can be thought that they preferred such a subject due to ease of access and prevalence of the subject. The prospective teachers were also set free in deciding routine and nonroutine problem situation. Thirty-three of them preferred nonroutine problem situations. It is remarkable that one third of the prospective teachers went for nonroutine problem situations in their first teaching of problem solving and modeling.

The analysis of the interview data has yielded eleven categories which show that the participants developed positive attitudes towards problem solving and modeling (Table 2). The categories show that modeling enriched the problem solving process; the prospective teachers had precious experiences from this process; they saw their shortcomings and stated that there were some points that they had difficulties with. Explaining, concretizing and simplifying are important components of modeling. As understood from the expression "When I first gave them the problem, they couldn't understand it clearly and couldn't solve it because they couldn't visualize the problem in their minds, but thanks to modeling, more students understood the problem and solve it" (26), the mathematical language a teacher use in solving concrete problem situations should involve these components or should let the students recognize these components. It is thought that prospective teachers' knowledge and experiences were effective in enriching their experiences in problem solving through modeling. The fact that the prospective teachers realized that modeling process is a social process and peer learning is an important part of this process was effective for them to develop positive attitudes towards problem solving and modeling. It is also deduced from the interview data that such expressions of the prospective teachers as "I did, applied, drew, made them solve, made sure or had difficulty" might have arisen from the fact that they hadn't undergone sufficient practice and that they saw themselves in the center of teaching process.

The following excerpts vividly reflect the nature of problem solving and modeling processes the prospective teachers had been going through.

(5) *Arda, who lives in a village, has to come to school with his mother because there is no service bus. With his mother, he arrives at school in total three quarters of an hour. One day, his mother asked him whether he could come home with his friends because she wouldn't be able to get her. He answered "Of course mom. I am grown up. I can come even alone." On the way back home, he lingered with his friends and it took him longer to arrive. Because his coming back lasted for two half hours, how long did it take him to go to and come back from school?* (See Figure 1).

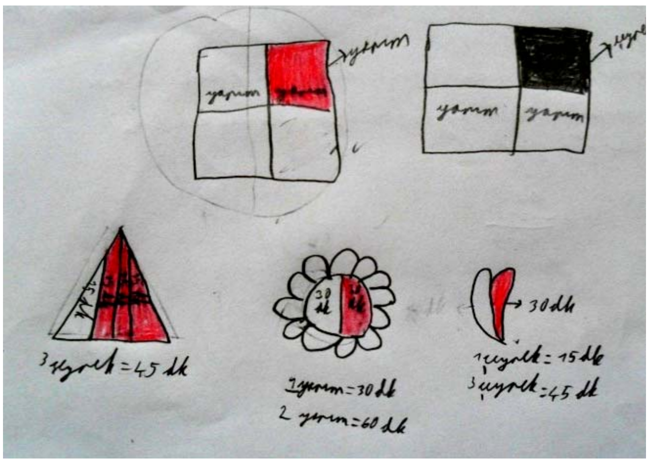

Figure 1. The problem generated by student #5

Table 2. The analysis table for the interview with the prospective teachers about their problem solving and modeling experiences

Category	Statement
Having difficulty	*At first, I had difficulty in explaining the questions to the students while analyzing the questions with them. It is really hard to explaining according to their level, but really interesting ideas come out of the kids and even sometimes they lead us (33).* *Because the students thought that this application is an exam, we had some problems. They tried not to make a mistake while solving the problem. After modeling stage, I saw that some students erased their solution and wrote the correct one (38).* *I had some difficulties in answering some questions students asked during problem solving stage. I pulled myself together at once and gave the answers. I had some problems during modeling stage. I found and corrected my mistakes in drawing and writing (2).* *The question was solved through operations class-wide and the students had difficulties in modeling because they weren't accustomed to solving problems in this way (34).*
Analysis	*At the analysis part of solution, I divided the question into three stages. At the first stage, the stage of "I gave my brother four pencils", I brought four pencils and asked the student to draw a pencil on the board and to give them to his/her brother. At the second stage, the stage of "You have five pencils left", I asked the student to draw 5 pencils on the board and told him "You have this number left". At the last stage, the stage of "How many pencils did you have total", when I said "I brought nine pencils and gave four of them to my brother and these are left, so how many pencils are left?", they understood the question and when I got the answer "nine", I made him draw nine pencils on the board (3).* *Some students preferred solving the questions by diving it into pieces. The separated the data as what is given and what is wanted. They merged the data and applied the operation and got the solution. Some even concretized the data they had broken into pieces by drawing figures, making the problem more solvable (11).* *The question first seemed complex, but later when the problem was retold to the students by simplifying and analyzing, it became simpler and the majority understood the problem (19).*
Different modeling	*Later, I modeled the problem and solved it on the board. While modeling, I explained the solution step by step. After that, I asked everyone to turn their study paper back and to do it as what I had told on the board or in different models (4).* *In some problems, I noticed that the students also modeled the questions in various ways (23).*
Simplifying	*They formed a simpler table by decreasing the complexity using "some parts given and the rest wanted" method (6).* *Most of the class, especially the active students, said that the problem was solved more easily after modeling (8).* *The students understood the problem after modeling and they were no longer confused. Everybody was able to model what the problem required and the result (36).* *As the question was simplified during modeling stage and the students were encouraged to interpret, the question became easier for them (12).* *However, thanks to modeling we did later, the problem was grasped and answered by more students (26).* *They understood the questions better when we came to modeling stage and solved the problem faster (14).*
Explanation	*I asked some students "Why did you go this way? Why did you do like this?" and wanted them to explain. We solved it together and checked the ones done through modeling and crosschecked (29).*
Problem choosing	*I had planned to hand out the material to everyone and to do interpretation on the model together, but it didn't go that way. May be because the question was below their level, they explained and solved the question before I could analyze it step by step, interpret and simplify it (21).*
Peer learning	*During modeling process, the students tried to explain the understood parts to one another. They told me where their friends got stuck and wanted me to help them. They did group work by doing explanations. Sometimes they told their friends where they did mistakes (20).* *Some came to me and wanted to see what their friends had done, but I don't think that they (who do by looking at their friends) can't learn. Looking at one another enables them to learn better (22).*
Motivation	*Because even those uninterested in the lesson try to do something (37).* *The children were very keen to ask something, which helped me determine what they couldn't understand (17).*
Teacher modeling	*Later I asked how many more melons the grocer had and they answered seventeen. I drew seventeen melons on the board and they drew seventeen figures they chose on the papers in front of them. When I asked how many melons there were total, they said twenty-four. While I drew twenty-four melons on the board, they drew various figures on their notebooks. Here, for convenience to the operation in the next step of the question, I drew twelve melons on one line and the other twelve on the line below (16).* *At the end of the lesson, I went over the questions to make it better understood by drawing boxes with matches in and asking questions (25).*
Enjoyable	*By modeling the question, I saw them having fun while solving it. Every one of them was trying to come up with new ideas (23).* *The students said that it was the first time they had ever solved a problem and that they had a lot of fun (13).*
Making more concrete	*Some of them concretized the broken data by drawing pictures and turned them into a more solvable state. They drew houses, bakeries, groceries, and roads and located the data on figures (11).* *Modeling with drama was both fun and attractive. It is an effective method in providing permanent learning because they learn by doing and living it (35).* *They said that they solved fewer questions compared to the other lessons but understood much better (30).* *Having a cube model in their hands was another advantage for them (12).* *When I wanted them to solve it without giving any material, they couldn't understand how to do it; however, with the material in their hands, counting the matches one by one and putting them into boxes, they understood where the remaining came from (25).*

Analyzing the modeling done by Melike (a pupil), it is seen that prior to modeling, she made a mistake but when she did modeling, she used a model appropriate for the solution of the problem.

(7) *A dancer first goes 5 steps forward and then 2 steps back on a line. The dancer is now 18 steps away from the start point, so how many steps did s/he take total?* (See Figure 2)

(11) *Özkan's house is 150 m to his house. After he left school, he came home after he bought some bread from the bakery 100 m to school and so he walked total 400 m. how far is his house to the bakery?* (See Figure 3)

In the other problems, students first did calculation then used to model, but as seen in the sample, they first draw the figure appropriate for the solution and didn't prefer doing calculation first. The prospective teacher stated that the students had difficulty in understanding the problem without forming the model.

(12) *I want to decorate the corners of my cube moneybox with beads. I have got three dozens of beads and if I put three beads on each corner, how many beads will be left in my hand?* (See Figure 4)

The prospective teacher points out that they tried to reveal the variables of the problem with questions "How many beads are there in our hands? How many beads are we putting at every corner of the moneybox?"; real objects were used for the solution and some students showed their modeling with real objects even on paper.

(15*) Books will be collected from sister schools for the needy. One of the schools gave 126, while another gave 234 and another gave 180 books. How can we allot the total equally to 6 schools?* (See Figure 5)

The students needed concretizing despite big numbers anddid modeling by keeping tally. As seen in the samples, the students used the modeling to verify their calculations.

(16) *A grocer has 7 melons. Buying 17 more melons, s/he puts half of the melons into boxes in twos and the other half into boxes in threes. How many boxes does he use in total?* (See Figure 6)

The model shows that the student analyzed the data by using a table. The student made use of visual expressions instead of numbers while doing calculation and continued the solution process by putting the objects into the table.

(20) *A waterman sells 20 bottles of water in an hour. After selling water for 5 hours, 1/5 of the bottles he has sold are returned due to leaking. After that, he sells water for 3 more hours. How many bottles of water has he sold in total?* (See Figure 7)

In the modeling sample above, it is seen that s/he formed a model that reflects the order of his/her operation order and used the model for verification again, which might have arisen from their habit of doing the calculation first and not using modeling frequently in problem solving process.

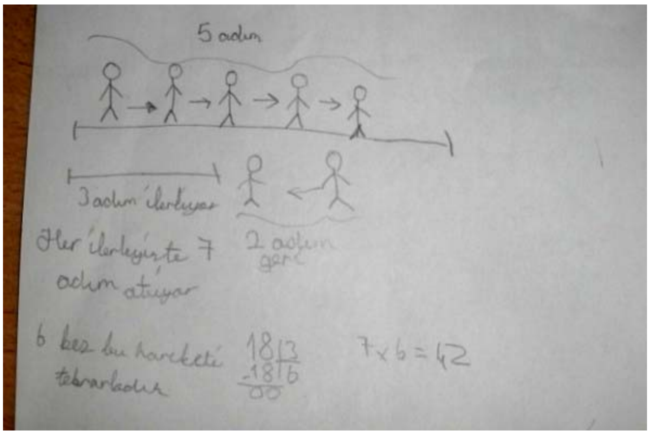

Figure 2. The problem generated by student #7

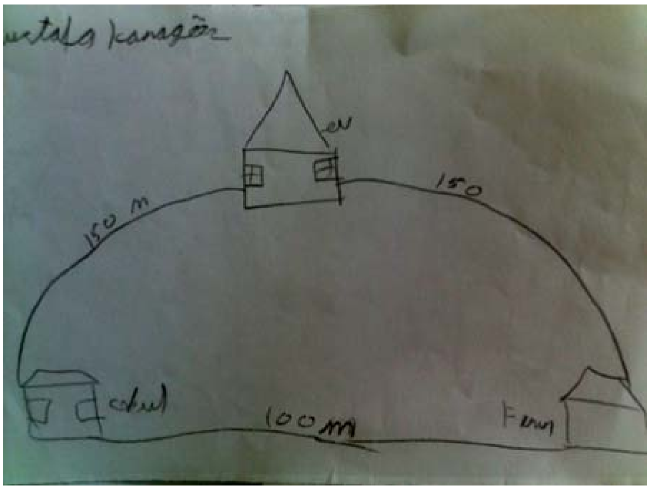

Figure 3. The problem generated by student #11

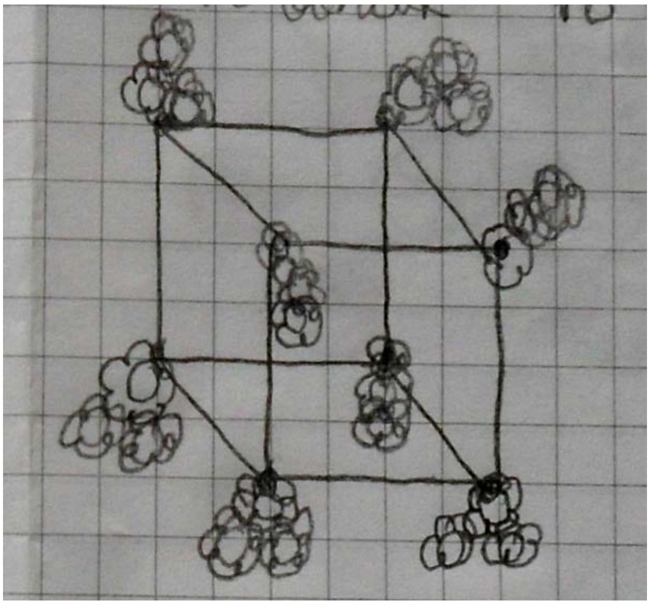

Figure 4. The problem generated by student #12

(26) *Emre, being bored, decides to play a game in which he is supposed to rescue a princess 10 steps away. Every step is 200 m.*

Emre will move on using a die. Throwing the die, if the smiling face comes, he will move on 1 step more than the number on the die, but if the crying face comes, he will move back as far as the number on the die. Upon the first throw 5 comes and 2 comes upon the second throw. How many meters are left for Emre to reach the princess at the final situation? (See Appendix 1)

The prospective teacher stated in his expression "I first tried to help the students understand the problem. To make a problem more concrete, I tried to simplify it by relating it to such games they play in real life as Ludo and data analysis tables" that's how he prepared the students for the solution with his/her applications during problem solving and that the students' solutions reflected the classroom activities. In this problem, it is remarkable that the students found the problem expression complex and they wanted extra information from the prospective teacher.

(36) *Once upon a time, there were 21 raindrops. When they fell on the ground, 3 of them used to come together to wet a stone. Because there were 10 stones on the ground, how many of them wouldn't be wet?* (See Appendix 2)

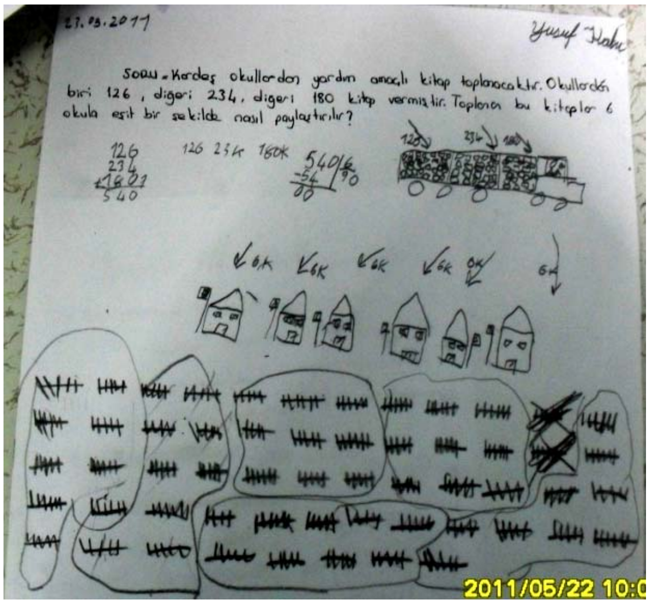

Figure 5. The problem generated by student #15

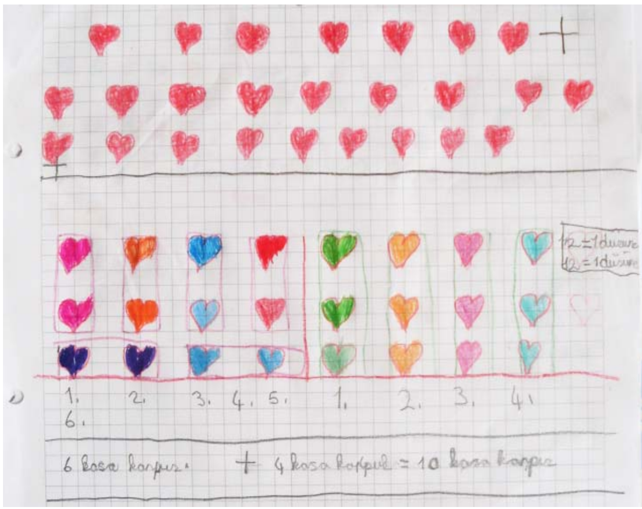

Figure 6. The problem generated by student #16

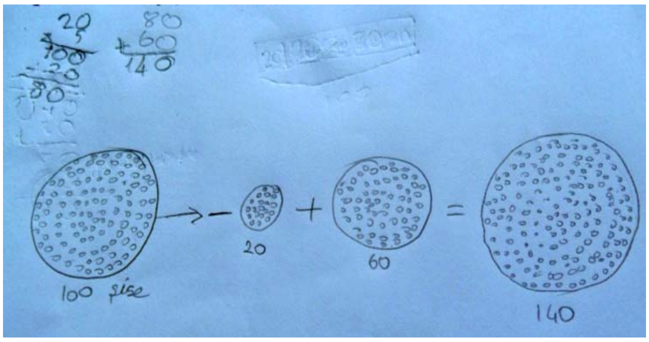

Figure 7. The problem generated by student #24

DISCUSSION AND CONCLUSION

During the data analysis, there are 18 categories determined as 'having difficulty, analysis, different modeling, simplifying, explanation, problem choosing, peer learning, motivation, teacher modeling, enjoyable, concretizing.'

During their training, the prospective teachers were trained in routine and nonroutine problem situations. They were set free in using whether routine or nonroutine problems in problem solving applications and twenty-six of them used routine while thirteen used nonroutine problem situations. They stated that they preferred using routine problem situations because nonroutine problem situations were hard to form and they are scarce in the market.'I used a routine problem because nonroutine problems are scarce in mathematics books (7).' However, 13 prospective teachers worked on nonroutine problem situations they formed themselves.

While presenting mathematical concepts, teachers transfer their own understanding and teaching language to their students; therefore, mathematical communication and mathematical language bear grave importance in learning mathematics (Barmby, Bilsborough, Harries, Higgins, 2009). The expressions of prospective teachers reveal that they had difficulty in the problem solving process, in some cases the students couldn't understand what the prospective teachers were trying to explain and the problem situations weren't appropriate for the class level. Olkun, Şahin, Akkurt, Dikkartin and Gülbağcı (2009) state that in order for a teacher to estimate the difficulty level of a problem, s/he should do some practice and enable her/his students to do generalizations through appropriately structured activities. It is crucial that the right mathematical language be used in learning mathematics and that prospective teachers spend more time and do more practice to possess the right and sufficient amount of experience.

The prospective teachers stated that modeling during problem solving contributed to peer learning and made the students more motivated and enthusiatic during the problem solving process. They also added that the modeling process made the problem solving process more concrete and enjoyable.Reys, Lindquist, Lambdin and Smith (2009) emphasize that in oder to organize mathematical ideas, language is an important tool and that speaking, writing, doing explanations, joining discussions about mathematical ideas contribute to students' experience. They also add that helping students communicate by using tables, graphs, estimation, drawings and symbols fascilitates and improve their learning. During modeling process, students frequently speak loudly, share and present their solutions to the class.Modeling is such a process that accelerates the communication process of students, increases their attention to focusing on the way to the result and enables focusing. Discussions in mathematics classes are necessary to achieve individual understanding and to share the solutions and interpretations (NCTM, 2000).

In their expressions, it is remarkable that the prospective teachers considered their own solutions and drawings and schemes for those solutions when modeling was mentioned. I tried to simplify the modeling and analyze it for them to do the modeling successfully. I explained the problem step by step by making sense of the variables in the question. I rearranged the problem by determining what was given and what was wanted in the problem. I drew an explanatory table on the board (20). The prospective teacher, seeing him/herself in the center of the course in which, in fact, student activities and discovery processes should have been the case, used the words "I analyzed, explained, rearranged" and revealed how they reflected their own analysis process to the classroom environment about how they perceived modeling. Also, the fact that some students formed a model other than their teacher used and therefore approached to the question with a different perspective amazed the prospective teachers. Different mathematical presentations lead to different perspectives. Two different types can be mentioned in mathematics presentation: the first being using graphs, diagrams and tables while the second beingthe teacher depended mainly on words and explanations. Mathematical concepts can be given via both presentations, but the latter reflects one's own experience and mathematical concepts in store; however, relying mainly on presentations supported by visual aids is crucial in mathematics teaching (Barmby, Bilsborough, Harries and Higgins, 2009). During mathematics teaching process, meanings of the words, explanations and symbols used by the teacher should be talked about and discussed in the classroom (Dickson, Brown and Gibson, 1984).

The prospective teachers stated that some of their students live the experience of solving problems by modeling for the first time. The students stated that in their previous mathematics lessons, they had solved more problems and in that lesson they solved fewer problems but using tools and modeling they understood the solution process better. Van De Walle, Karp and Williams (2010) state that at this point, students will develop their own learning strategies and solution processes if their teachers choose high quality problems and provide them with rich learning environment.

It is crucial to provide such environment to students that will encourage them to solve problems. Students should be feeling themselves free and comfortable in revealing all their ideas. They should be able to take risks, try new strategies and do different explanations. Even if their explanations aren't correct, they should be discussed in the classroom, reexplained and used as a learning tool. In this way, they will be able to evaluate their steps and results (Sherman, Richardson, Yard, 2005). In this research, the prospective teachers, as frequently seen in their expressions like "After we started modeling, the reactions of the students amazed me and made me happy. As we continued, they said among themselves "aa yes, right, how did I make this mistake...", realised that, as an important component of modeling, explanation helped the students make better sense of what was given and wanted and achieve correlational understanding during problem solving.

Analyzing the modeling samples of the prospective teachers in practice classes, it is seen that the students used various figures, schemes and even real objects. Whether routine or nonroutine problem situations, all the students' modelings were analyzed and it was found that they were appropriate for the solution process. As can be understood from the expression "Prior to modeling, the question wasn't been able to be answered and the classroom started get disturbed. It was clear that it was time for modeling. The students got their modeling clay and mushrooms. After reading the question again and doing the necessary analysis, we did the modeling of sorting the necessary out of unnecessary and subtraction-multiplication-addition operations with mushrooms and spots (28)", the prospective teachers stated that most of the students didn't have any difficulties in the modeling process andsolved the problems enthusiatically. This experience in mathematics teaching practices is crucial in that they saw the relation between theory and practice and they gained the proficiency for future encounters with similar situations.

With their expressions "The problem was simplified in modeling stage during which the students started to understand better. They drew models for half and quarter given in the question. After locating the data on the models, they went for the solution easily (5). But they had some difficulties in division operation so we did the division by modeling together (15).” the prospective teachers revealed their students experienced an easier and more relaxed solution process when they analyzed and made sense of the variables of the problems. Modeling provides a frame for the solution to understand and interprete what has been said and the activities during the problem solving process. Rather than being busy with a general strategy during problem solving, modeling enables them to focus on the problem (Carpenter, Fennema, Franke, Levi, Empson, 1999). Simplifying and interpretation in modeling are closely related with the analysis of variables of the problem. Baykul (2009) states that not only information in a problem about what is given and what is wanted and the concepts about these two but also exercises that determine the relation between the known and the unknown and enable theis relation to be written play an important role in a succesful problem solving process.

In the evaluation interview with the prospective teachers for the application process, it is seen that the prospective teachers had certain problems during the application process; e.g. their counsellor teachers at practice schools didn’t provide them with adequate practice opportunity saying “I can’t catch up with my own lessons”, so they had to hurry up with their modeling and couldn’t get sufficient feedback about the application. Considering the age group for the mathematics teaching in terms of primary school teaching, rich learning and teaching experiences are required and should not be underestimated.

The modeling process cannot be successful without effective planning and effective communication between the participants. Encouraging the students to participate in modeling activities and enabling them to share their mathematical ideas within a group might be more effective than the lecture of the teacher during the problem solving process. Besides communication between the teacher and the student, communication between the researcher and the teacher has a role to strengthen the modeling process before and after modeling. Careful planning and regular interviews with the teachers will yield a successful modeling process (English, 2003). A research by Watters, English and Mahoney (2004) indicates that the students of the teachers who had done modeling show improvements in their social skills and inquisition skills and they also improved in interpreting tables and graphs.

Consequently, it is crucial to gain the accurate experience during mathematics teaching process for which prospective teachers should be sensitive during practice processes. In this research, it is seen that the prospective teachers could gain important experience in problem solving and using modeling in problem solving as a result of their practices. Their expressions reveal that the positive reactions they gained from their students encouraged them to use modeling.In this study, the prospective teachers found opportunity to practise what they had learned in theory and to evaluate themselves.In this context, it should be noted that their lecturers at university and administrators and counselor teachers at practice schools have great responsibility in preparing them successfully for real life experiences.In order to achieve proficiency in prospective teachers’ mathematics teaching, they should be given accurate feedbacks at every stage of their training processes so that their students can get positive gains in terms of mathematics.This research was carried out to reveal the prospective teachers’ experiences, problems and gains through training and practice processes in solving and modeling mathematics problems. Further researches can be conducted on the experiences of prospective teachers in the course of cooperation between practice and mathematics teaching lesson by giving them practice opportunities in different subject areas. By exposing prospective teachers to a longer observation process, their experiences, how they use them throughout their teaching career and the problems they encounter might be focused on. In the context of mathematics teaching lesson, prospective teachers can be sent for practice in the name of “Mathematics Teaching Practice” and the quality of their mathematics training can be analyzed.

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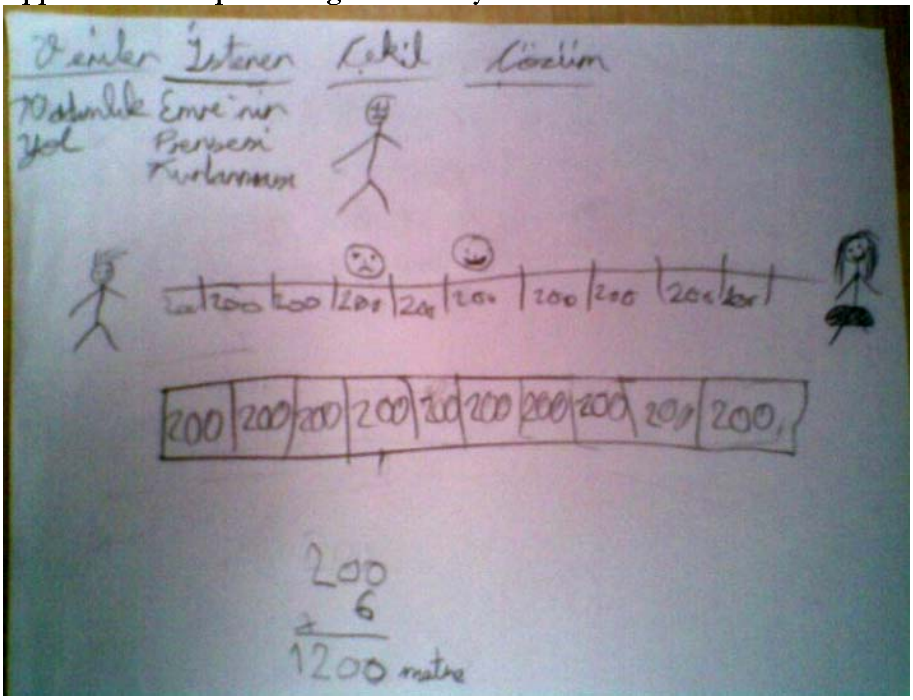
* **ERRATUM**

The initially published version of this paper erroneously contained the title that was given before revisions as follows: **Analysis of Primary School Student Teachers' Applications of Solving and Modeling in Mathematics Questions**

Appendix 1. The problem generated by student #26



Appendix 2. The problem generated by student #36