Application of Graph Theory in an Intelligent Tutoring System for Solving Mathematical Word Problems

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This study is aimed to construct a model to transform word "motion problems" in to an algorithmic form in order to be processed by an intelligent tutoring system (ITS). First; categorizing the characteristics of motion problems, second; suggesting a model for the categories were carried out. In order to solve all categories of the problems, graph theory including backward and forward chaining techniques of artificial intelligence were utilized. The study outlines the adoption of graph theory in to the motion problems and put forth some evidence that the model solves almost all of the motion problems. In conclusion, the recommended model can be suggested to be used in educational software in the problem solving context.

Keywords: intelligent tutoring systems, graph theory, mathematics education, motion problems, problem solving

INTRODUCTION

The basic aim of mathematics education was described as "to bring mathematical knowledge and skills that are required by daily life to the individual, to teach students problem solving and to bring them a way of thinking that handles incidents including a problem-solving approach". For this reason, problem-solving skills take an important place among mathematical skills (Baykul, 2004; De Corte, 2004). Therefore, this issue was carried to the center of the mathematics curriculum at multiple levels from primary school. Indeed, Nation Council of Teachers of Mathematics (NCTM) standards also indicate that problem-solving skills have higher priority in teaching mathematics. Therefore, there is a consensus among mathematics educators on enhancing the problem solving skills of students being the primary objective of education (Cai, 2003; Karataş & Güven, 2004). In the mental process of the problem-solving process; understanding the information presented, constructing relationships between the information, creative and reflective thinking, analysis and the use of synthesizing skills are the main sub-processes (Soylu & Soylu, 2006).

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For the solution process of problems; Polya (1957) recommended a framework that contains the stages of understanding the problem, selecting a strategy for the solution, the application of the strategy and the evaluation of the solution. Within this framework in many studies students were indicated to encounter a number of difficulties in understanding the concepts that problems include and the relationships between the concepts while they are solving problems (Chiu & Klassen, 2008). According to Polya, one of the most important causes of these difficulties is that; students do not perceive problem solving as a hierarchic process. This idea was supported by some research studies. Verschaffel, De Corte, and Viersraete (1999) determined students' difficulties with the problem-solving process while they were solving word problems. They stated that; traditional arithmetic word problems cause students to solve word problems carelessly, superficially and by conducting routine processes while students identify the correct arithmetic operations that is needed to solve an extensive word problem.

In this sense, students often solve word problems using direct calculation. They do not think over the content of the problem sentence; because the problems can be solved without deliberating over the importance of problem content in most textbooks (Greer, 1997; Nancarrow, 2004). Thus, students do not often find a relationship between school mathematics and their daily lives. This is because either they have not been taught that there is a relationship, or because they have not been asked to solve problems which allow them to use their real-life knowledge while they make decisions about the solution. If students can be directed to understand the solutions of problems, and to accustom to use the problem solving steps, they can solve and conceptualize word problems more easily (De Corte, 2004; Nancarrow, 2004). In this sense Kilpatrick states that; "a student's success in problem solving depends on his development of the skills in problem-solving processes" (Kilpatrick, 1985).

Many efforts were provided for enhancing the problem solving (Cai, 2003; Chiu & Klassen, 2008; Kılıç & Samancı, 2005). In this sense motion problems were taken into consideration in problem solving domain of mathematics curriculums. Mathematics educators think that motion problems are valuable because of consisting different types of questions, different solution ways and supporting students' critical thinking with non-routine features. In addition, mathematics curriculums suggest that students can construct mathematical knowledge in accordance of constructivist approach. In this approach in some cases, using interactive educational software is offered for designing such constructivist learning environments. Therefore, the teaching motion problems in such a way may allow students to have experience in problem solving processes by educational software. In recent years the educational software tended to be designed by using ITS components. Of course, programmability and the usability of the software are also associated with the chosen topic. In this regard, motion problems within their nature serve students to deal with different kinds of problems with different characteristics. Some efforts were provided for analyzing the problems in order to carry them in executable form for software. These kinds of software are recommended to present causal feedback for students to understand and eliminate

State of the literature

- Problem solving is one of the difficult processes for mathematics instruction, because; students cannot perform hierarchically in the process.
- Graph theory is actively used in various areas biochemistry, engineering and computer sciences; that give hints about its use in problem solving.
- Intelligent Tutoring Systems are one of the favorite tools appearing as software for using in mathematics problem solving applications.

Contribution of this paper to the literature

- A usable categorization is developed for motion problems
- An programmable model is developed by applying graph theory for motion problems
- An artificial intelligence techniques and graphs are used together for educational purposes.

their mistakes. By this way students can have experience on the new kind of solutions using feedbacks in different stages of problem solving stages. This study recommends a model that transforms problems into a form that can be processed by ITS, with analyzing motion problems. A computer cannot think and make decisions like humans, especially issues like problem solving is not easy for software. Because the issue includes, analyzing the problems, working on problem sentences as natural language, finding different solution ways for the problems and organizing the computer recourses during these processes. For this purpose, considering the programmability of the problems, computer resources like memory, disc and data processing should be well organized. In this context, graph theory was used as a basic framework in this study. In the history of mathematics, Königsberg bridge problem which is solved Euler's solution in 1735 is considered to be the starting point of graph theory. Wate-Mizuno (2014) addressed that the first book about graph theory is published in 200 years later by D.König in 1936 which is named as Theorie der endlichen und unendlichen Graphen. Since that time, graph theory is actively used in various areas such as biochemistry (genomics), electrical and electronic engineering (communication network and coding theory), computer sciences (algorithm and computation) and operational research (scheduling).

A graph is a structure defined with the help of edges that expresses a set of points (nodes) and the relationships between these points G=(Y, E), and it can be expressed as a set of finite Y and E elements. Here, Y elements are referred to as vertices (vertex) or nodes. E elements are expressed as edges. Each edge in E adjoins two different nodes in Y. Nodes are shown by circles and edges are shown by lines (Dharwadker & Pirzada, 2011). A graph structure with its edges and vertices is shown on Figure 1.

This study was carried out by focusing on the solution of motion problems Y, E in graph structure and the relationships between these.

MATERIAL AND METHODS

The study comprises two stages about determining the characteristics of motion problems and suggesting the model that providing a solution in line with these characteristics.

The characteristics of the motion problems

In order to determine the characteristics of motion problems, the problems were compiled from the motion problems from 9th grade mathematics textbooks of Turkish Ministry of Education. Also questions about motion problems from the
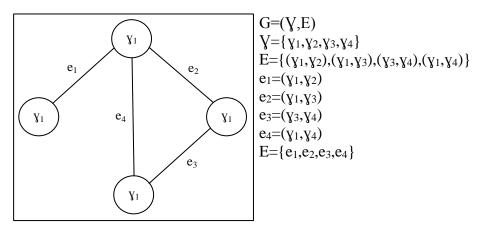

$G=(Y,E)$
$Y=\{Y_1,Y_2,Y_3,Y_4\}$
$E=\{(Y_1,Y_2),(Y_1,Y_3),(Y_3,Y_4),(Y_1,Y_4)\}$
$e_1=(Y_1,Y_2)$
$e_2=(Y_1,Y_3)$
$e_3=(Y_3,Y_4)$
$e_4=(Y_1,Y_4)$
$E=\{e_1,e_2,e_3,e_4\}$

Figure 1. An example graph with edges and vertices

Table 1. The variables used in the model

X	Path		
V	Velocity		
T	Time		
XT	The total path that the vehicles moved		
XG	The total of the path on which vehicles moved		
XK	The remaining path between the vehicles		
X1	The path which first vehicle moved	**X2**	The path which second vehicle moved
V1	Velocity of first vehicle	**V2**	Speed of second vehicle
T1	Time of the first vehicle moved	**T2**	Time of the second vehicle moved
X11	Path which first vehicle moved in first status	**X21**	Path which second vehicle moved in first status
V11	Velocity of the first vehicle in first status	**V21**	Velocity of the first vehicle in second status
T11	Time which first vehicle moved in first status	**T21**	Time which second vehicle moved in first status
X12	Path which first vehicle moved in second status	**X22**	Path which second vehicle moved in second status
V12	Velocity of the first vehicle in second status	**V22**	Velocity of the second vehicle in second status
T12	Time which first vehicle moved in second status	**T22**	Time which second vehicle moved in second status

Entrance Exam to Higher Education to present and motion problems from different auxiliary sources were chosen. The characteristics were determined by searching and classifying of 476 motion problem questions. Before the final form of the classification was given, the above-stated resources were reviewed by five field experts.

In the process of classification of motion problems, problems were seen to diversify according to the peculiarities of the number of movements (single vehicle, two vehicles, etc.), the direction of movement (the same direction, the opposite direction) and the time of motion (at the same time, different time, and so on). The basic element in these parameters was determined as the number of vehicles. In fact, the number of vehicles has a direct influence on the path of the solution to the problem and different status may arise according to whether the number of vehicles is one or two. The variables used for different status related to the vehicle numbers are shown in Table 1.

Motion problems with single vehicle

Two different statuses can be encountered in motion problems with a single vehicle.

Single Status: Status of a vehicle taking a particular path at a particular time.

Example 1: How many kilometers does a vehicle travel in 5 hours at 60 km per hour?

Solution 1: It can be found by following the path A1→D1→M1.

The problem can be solved by being modeled as $X = V*T$ in M1 step.

Two Statuses: Repetition of the situation of a vehicle to take a particular path at a particular time.

Example 2: A vehicle takes 5 hours to return using the same path that it had taken 3 hours to drive when traveling at 50 km per hour. So, at what velocity was it traveling when returned on the same path?

Solution 2: It can be found by following the path A2→D2→M2.

The problem can be solved by being modeled as $XF = X1 - X2 = (V1*T1) - (V2*T2)$ in M2 step.

Motion problems with two vehicles

There are at least two statuses because of the number of movements in the solution of motion problems with two vehicles. These two situations turn into 8 models by ramifying with the parameters of the direction, the time and the starting point. However, with the help of the analysis of these models, two models were basically seen to emerge. In these models, the parameter of direction determines the model that will be chosen and the time parameter determines the rule base of the model that will be used in the solution of the problem. The starting point parameter allows determining whether the intermediate variable X, which indicates the distance between the vehicles in the questions regarding the vehicles catching each other, will take place in the model. Motion problems were investigated on the perspective of these parameters, a scheme occurs as outlined in Figure 2.

Thus, considering the above-stated parameters, the path to be followed for a sample question with two vehicles is as follows:

Example 3: The distance between cities A and B is 40 km. A pedestrian traveling at a velocity of 5 km per hour departs from city A towards city B and at the same time a cyclist traveling at a velocity of 15 km per hour departs from city B towards city A. After how many kilometers does the pedestrian meet the cyclist?

The solution can be found by following the path A2→D3→Y2→Z3→B6→M8

The problem can be solved by being modeled as XT= (V1*T1) + (V2*T2) in M8 step.

If motion problems and parameters were represented in different ways, the functions that were used to resolve motion problems are basically structured in three ways. These functions can be expressed as;

1. **X**=V*T for the structures with single vehicle and single situation, which is the fundamental solution to motion problems,
2. XF= (V1*T1) − (V2*T2) for the structures with two vehicles and the same directions, and with single vehicle and two situations,
3. XT= (V1*T1) + (V2*T2) for the questions with two vehicles and opposite directions.

Considering the solution categories of motion problems, it is possible to divide motion problems into sub-problems categories using the parameters in each category. The sample problems for each category were outline in Table 2. The
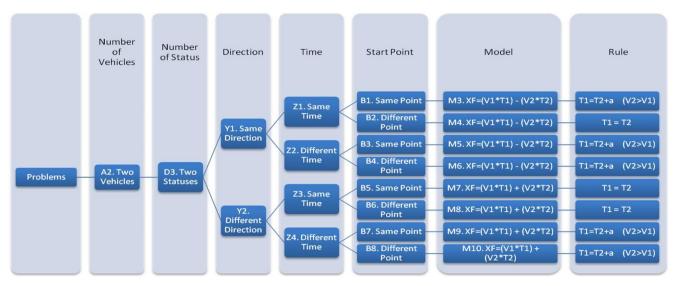

Figure 2. Model for motion problems with two vehicles

solution model was developed according to the variables on the problems such as number of vehicles, number of status, the directions of the vehicles, time of the motion and the start point of the vehicles. The three main categories of the problems and the examples for the categories were shown on Table 2.

After identifying the characteristics of motion problems, a common solution model was provided for each problem category. A graph structure expressing the

Table 2. Types of motion problems and applied models for solutions

Type	Analyses of the number of questions	Method of Solution	Model	Number of Vehicles	Number of Status	Direction	Time/ Start Point	Example
1. Type	86	**X=V*T**	A1→D1→M1	1	1	-	-	-	How many kilometers will a vehicle travel in 5 hours at 60 km per hour?
2. Type	180	**XT= (V1*T1) + (V2*T2)**	A2→D3→Y2→Z3→B5 →M7	2	2	D	S	S	Two cyclists whose velocities are 8 km/h different left from the same place at the same time and traveled in opposite directions. After an hour from the arrival, the distance between them is km. What is the velocity of slower cyclist?
			A2→D3→Y2→Z3→B6 →M8	2		D	S	D	The distance between cities A and B is 40 km. A pedestrian traveling at 5 km/h, left from city A and a cyclist traveling at 15 km/h left from city B at the same time and they were traveling towards each other. After how many kilometers will the pedestrian meet the cyclist?
			A2→D3→Y2→Z4→B7 →M9	2		D	D	S	A vehicle left city A traveling at 60 km/h. After two hours, if another vehicle leaves city A in the opposite direction at 70 km/h, what is the distance between them after 5 hours?
			A2→D3→Y2→Z4→B8 →M10	2		D	D	D	The distance between A and B cities is 600 km. A vehicle left city A traveling at 50 km/h. Another vehicle left city B traveling at 75 km/h. They were traveling towards each other. At what time will they pass each other?
3. Type	210	**XF= (V1*T1) - (V2*T2)**	A2→D3→Y1→Z1→B1 →M3	2	2	S	S	S	Two cars traveled from city A to city B. They left at the same time. The first car was traveling at a speed of 30 km/h and the second car was traveling at a speed of 40 km/h. If the second car arrives at city B two hours early, what is the distance between the two cities?
			A2→D3→Y1→Z1→B2 →M4	2		S	S	D	The distance between cities A and B is 120 km. Two vehicles left cities A and B at the same time towards each other traveling at speeds of 85/h and 70 km/h respectively. How long will it take the second vehicle to catch the first vehicle?
			A2→D3→Y1→Z2→B3 →M5	2		S	D	S	A vehicle left city A at a speed of 50 km/h. If another vehicle left city A at a speed of 80 km/h in the same direction after 3 hours, how long will it take the second vehicle to catch the first vehicle?
			A2→D3→Y1→Z2→B4 →M6	2		S	D	D	The distance between cities A and B is 50 km. A vehicle left city A at a speed of 70 km/h. If another vehicle left city B 3 hours later traveling in the same direction at a speed of 90 km/h traveling, how long will it take the second vehicle to catch the first vehicle?
			A1→D2→M2	1		-	-	-	A vehicle took 5 hours to return using the same path that it had traveled on for 3 hours at 50 km/h. How fast did the vehicle travel on the return journey using the same path?

D: Different; S: Same

steps and parameters for the solution as a network was used in the model.

Generating the model for the solution of problems

By examining the graphs derived from the literature, the aim of this section is to express a common drawing type which can provide solutions to different categories of motion problems. The solutions include parameters such as the number of vehicles, the number of statuses, the direction, time and the starting point of movement and various combinations of these parameters. Following basic principles for the graph structure was provided. These features were identified at the beginning of the study in order to provide a standard for graphs:

- The direction of the movement must be specified while moving on the graph.
- The direction of the vehicles (the same direction, the opposite direction, circular) in the problem can be specified on the edges. For example, if it is the same direction, (+) or the opposite direction (-) can be put on the connection edge.
- If a transaction is needed with a constant value on one of the variable nodes (X, V, T), a node to show it must be generated.
- There may be sub-questions in every node even if the basic nodes are X, V, T. For example, possible sub-questions for X (showing the path) nodes may be in the forms of the average path, the path between vehicles or the total path that vehicles traveled.
- The slots (frame nodes) including sub-questions, information about nodes and transactions must be created. These slots need to be created separately for each variable (X, V, T) or even for each node, if it is needed.

The development stage of the model is outlined below:

Preliminary model

The first graph models which were created according to the characteristics of motion problems and the basic features that are based on X=V*T.

In the first category; the traveled path was expressed as routing nodes which indicate the parameters of the velocity and time, with arrow directions indicating the transaction node. The operator in the transaction node enables us to obtain the result. In the second category; to find the value of the total traveled path, processing (stated in the transaction node) firstly the values of the velocity and the time for the first vehicle by arrow directions, the value of the first vehicle traveled is found. Later, the same transaction is applied for the second vehicle. Finally, the result is found by processing the values the vehicles traveled in the transaction that is stated by the arrow directions in the transaction node. Both Type2 and Type3 graph structures are shown in Figure 3.
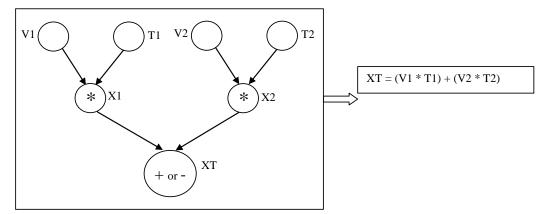

Figure 3. Graph structure for Type 2-3

In the Type 3 questions, to find the difference between the values of the path that the vehicles traveled, processing firstly by arrow directions the values of the velocity and time into the transaction that is stated in the transaction node for the first vehicle, the value of the first vehicle traveled is found. Then, the same transaction is processed for the second vehicle. Finally, the result is found by processing the values of vehicles traveled in the transaction that is stated by arrow directions in the transaction node.

Although the graph model that was created to express a certain number of related problems was successful, there were also some limitations as follows: Being unable to make exactly a common graph figure even for the same categories of problems, the need for a separate model creation for each categories of problems and that the way of reading the graph could not fully reflect the questions (having difficulty in the redaction of the model), lack of flexibility of use, lack of adaption for all categories of problems and having limitations for possible add-ons in different types of questions. Therefore, an improved structure to provide a standard for the graph drawings in one model was needed.

Enhanced model

To eliminate the limitations in the preliminary model, a common model was developed with a structure that would be able to use Forward Chaining and Backward Chaining, both of which have an important place in the application of artificial intelligence. While forward chaining is the application of an inductive approach towards conclusions from the data, backward chaining is the application of a deductive approach directly for the conclusion. After the experiences in using the preliminary model, a three-node graph which includes almost all types of motion problems and their combined adaptations was developed in the enhanced model. The basic graph structure of Enhanced model was illustrated in the circles on Figure 4. Forward chaining and backward chaining graph models for the solution of an Example 4 was shown on Figure 4.
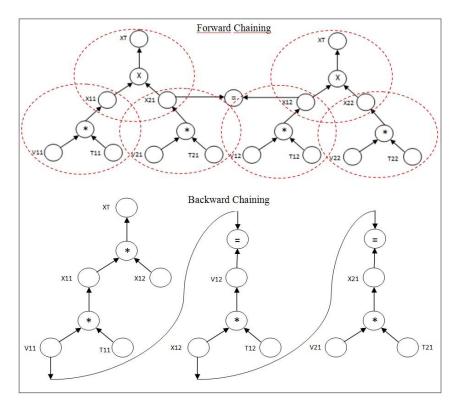

Figure 4. Graph model for problem solving with forward and backward chaining

Example 4: Two vehicles, whose velocity difference is 30 km/h, leave from points A and B traveling towards each other. After their departure, they cross at point C after 6 hours. The faster vehicle arrives at point B 4 hours after this encounter. What is the distance between A and B?

As the backward chaining technique follows the sub-nodes which are only necessary in order to find the result; starting from the desired final result node it scans the required nodes for final result. This kind of travelling among the nodes is quite fast and provides reaching the result directly. However, even if this situation provides solution quickly in designing software, it will lead to different possible solutions being neglected and even cause the system to enter into an infinite loop for the problems whose solutions cannot be found directly.

The forward chaining method tries to reach the final result by determining the blank sub-nodes and starting respectively from these blank nodes repeatedly. As a result, the system starts again for each blank sub-node, and provides the system to be operated more slowly while acting in software. However, this disadvantage actually enables the system to find all the more stable and different possible solutions. In the enhanced model, by conducting forward and backward chaining together, possible solution ways can be reached.

Final model

In the final model, variables were presented on different nodes according to their values and the problem was illustrated in graph components. Therefore, a table of the representations was prepared after the developing the basic graph structure. In accordance with the general graph features, the representations and meanings of the structure were shown in Table 3.

Graph structure is formed in two-vehicle cases in motion problems at the same time in opposite direction use of the basic graph structure was shown on Figure 5.

The proposed model was tested on various categories of motion problems. Some adjustments were needed to maximize the solution power of the model. However,
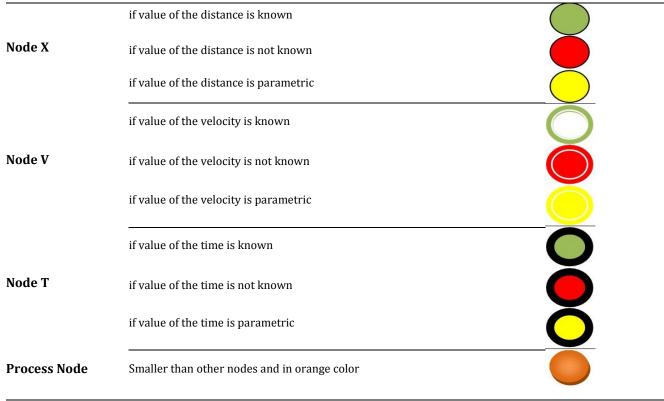
Table 3. The meanings of the nodes relative to the used graph

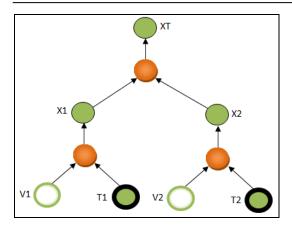|		
Node X	if value of the distance is known	
	if value of the distance is not known	
	if value of the distance is parametric	
Node V	if value of the velocity is known	
	if value of the velocity is not known	
	if value of the velocity is parametric	
Node T	if value of the time is known	
	if value of the time is not known	
	if value of the time is parametric	
Process Node	Smaller than other nodes and in orange color	

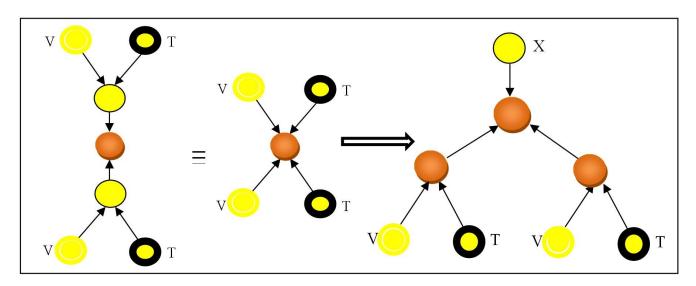**Figure 5.** Graph model for two vehicles in opposite direction at time same time

Figure 6. Graph model for two vehicles in opposite direction at time same time

these adjustments revised with the basic graph model, some other supplements were still needed. In particular, some supplements were made to the basic graph structure in parametric problems (the values of variables were not directly numerical but symbolized by letters) and where vehicles were evaluated relative to each other. The supplementary graph is needed to find the parameter in the parametric problems, which may be called as "equivalence graph" and after inserting the equivalence graph to the general graph, the resulting structure was shown in Figure 6.

In this structure, as a result of parametric values from below, for example when there is a need to find X1 and X2, the parametrical expressions V1*T1 and V2*T2 are directly equalized to XT, and X1 and X2 are subtracted. Due to the subtraction, we called this operation as "absorption rule". Process operators coming from below are connected to the transaction node going to XT by skipping X1 and X2. The absorption rule is used in the structure created by adding the equivalence graph to the general graph structure.

Though some requirements were provided by supplement graphs for different categories of problems, the fact that the basic graph was not changed. The experimental results on the final graph put forth some evidences about the stability of the model. In light of the examples given, the different operation steps and 476 different motion problems and with the experiments that have been carried out by

three different experts, it is concluded that the model provides successful and valid for the solutions for the different categories of motion problems. Two different examples are provided in example 5 and example 6 and depicted in Figure 7.

An example problem about using XT and XK were inserted to general graph and form the new structure was shown on Figure 8. It involves problems where the path between vehicles was needed accept the equivalence graph, with the opposite direction type motion.

> Example 5: The distance between A and B cities is 720 km. Two vehicles, with velocities of 70 km/h and 50 km/h, leave at the same time and in opposite directions from these cities. Find the distance between the vehicles after 5 hours?
>
> Example 6: Part of a 500 km route is asphalt road and part is less quality road. A vehicle completes the route in 8 hours at 40 km/h on the less quality road and at 70 km/h on the asphalt road. Find the length of the asphalt route?

The final graph model applied for Example 5 and Example 6 was shown on Figure 8.

The solution for the problem in Example 5 is XT= (V1*T1) + (V2*T2). By following the transactions of number 1➔2➔3➔4, firstly the path that the first vehicle traveled (X1) and then the path that the second vehicle traveled (X2) is

For two vehicles (Example 5)
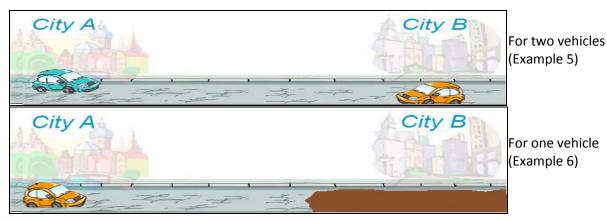
For one vehicle (Example 6)

Figure 7. Move in opposite directions at the same time and single vehicle movement
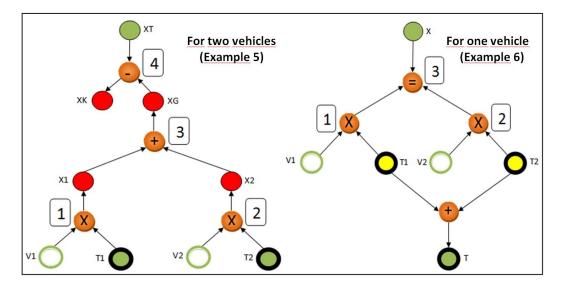

Figure 8. Graph model with forward chaining (Final model) of Example 5 and Example 6

found. After finding the sum of these, the total path that the vehicles traveled (XT) is found. It is subtracted from the total path and the path between the vehicles (XK) is found.

The solution for the problem in Example 6 is X= V*T is the solution to this problem. First of all, T1 and T2 values must be parametrically determined in the graph structure. By following the transactions of number 1➜2➜3, firstly the path that the first vehicle traveled (X1) and then the path that the second vehicle traveled (X2) is parametrically found. After equalizing these parametric values to each other, first the parameter variable (T value) and then the length of the asphalted part of the path (X2) is found.

DISCUSSION AND CONCLUSION

In this study, the suggested model can be thought as a new application area of graph theory. In this sense; graph theory applications in mathematics, science and other fields of technology are becoming increasingly important. It is used to solve many complex problems in biochemistry (Butenko & Wilhelm, 2006; Bonnici et al., 2013; Lancia et al., 2001), computer science (Shirinivas, Vetrivel, & Elango, 2010), GSM technology (Pirzada & Dharwadker, 2007), medical education (Giani et al., 2007), mining (Ye et al., 2012), architectural design (Poyias & Tuosto, 2015; Roth & Hashimshony, 1988; Zboinska, 2015), geologic models (Hirsch & Schuette, 1999; Phillips, Schwanghart, & Heckmann, 2015), urban planning (Almeida, Morley, & Dowman, 2007; Foltête, Girardet, & Clauzel, 2014), banking (Lin, Tzeng, & Chin, 2011) and simulation of electrical systems (Šarga et al., 2012; Benchouia et al., 2014). In sum, it is seen that graph theory is being used for the solution of many complex and extensive problems in daily life. However, a limited number of studies based on graph theory can be seen in education. In this context; graphs have been used for modeling students' using on the web for educational purposes (Gwiszdka & Spence, 2007; Juvina & Herder, 2005; Juvina & van Oostendorp, 2006; Somyürek, Güyer, & Atasoy, 2008). In such a research study, Sümersan Seyhanlı (2007) has focused on the use of the graph theory by the teacher in teaching probability subject. In this study, the teacher has benefited from the visual representation of probability problems (representation of all the possibilities in the tree structure) in order that students could better understand the problems. In this way, students have been able to find easily the solution identifying the known and the unknown data in the problems. Students who worked the subject of probability with the graph theory have outperformed than those who processed traditionally.

This study suggested a model for solution of motion problems with ITS. Two important conclusions can be asserted for this model. First; this model can be applied on almost all categories of motion problems and second; this model can facilitate the programming of motion problems in an ITS. By providing an ITS, the suggested model can detect the difficulties in the process of solving motion problems can be detected in the steps. By detailed analysis the solutions for motion problems can be based on suggested model based on graph theory. Each variable in motion problems was considered as a node evaluating the graphs as basic structures defined with the help of edges. The model expresses a set of points (nodes) and the relationship between these points (Nabiyev, 2012) producing solutions on the basis of relationships between these nodes.

In this study, artificial intelligence techniques and the graph theory together have been used to solve the problem beyond the understanding of the problem. Unknown nodes have been provided to be found by the help of known nodes (variables in the problems) in the process of the solution of the problem in generated graph structures. All possible solution ways of motion problems can be modeled with the graph structures. Therefore, the model can support students to achieve easily

complex cases in the process of problem solving. In this process, the students can be directed towards their solutions by determining their mistakes by the software.

An important contribution of this study is that; it suggests a structure where word problems are to be modeled in software. The graph structure which has been developed for the solution of motion problems in the scope of the study can produce solutions for problems including variables of the number of vehicles, the number of situations, the direction, the time and the starting point of moving, and several other variations of these variables. However, there is another important question that needs to be asked in this process. It is the question "What is the programmability of the graph structure?" This question gives programmers an idea related to the productivity of the graph structure. In this sense, programmers usually use artificial intelligence techniques developed for intelligent systems. The optimal algorithm for the solution should be preferred during programming. This is because it is one of the most fundamental realities of the software world that a problem can be solved through various algorithms and also similar problems can be solved by different algorithms. The important point here in this regard is that; a prospective program produces the most productive result using the least of the computer's resources. This can be shortly summarized as the minimum level of memory, using low disc space and producing solution at maximum speed. The provided graph model may be shown as one of the optimal solution methods that will be chosen for the software about the solution of motion problems. Furthermore, being found as the most suitable solution for the question with forward and backward chaining algorithms in the graph structure indicates the programmability of this model.

For future work; an ITS on the basis of suggested model will be constructed for students' use in problem solving context. Therefore, ITS in the future will contribute to eliminate the difficulties experienced by students in problem solving process. Thus, this study can support on the efforts about the aim of "developing problem-solving skills", which is one of the general aims of teaching mathematics.

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