

Assessment of the knowledge mobilized by prospective primary teachers in the design of geometrical tasks

Juan Pablo Vargas Herrera^{1*} , María Belén Giacomone² , Yuly Vanegas³ 

¹ Facultad de Ingeniería y Negocios, Universidad de las Américas, Santiago de Chile, CHILE

² Department of Economics, Science, and Law, university of San Marino, SAN MARINO

³ Departament d'Educació Lingüística, Científica i Matemàtica, Facultat d'Educació, Universitat de Barcelona, Barcelona, SPAIN

Received 05 June 2025 ▪ Accepted 10 October 2025

Abstract

Task design is a key component of professional development in teacher education, particularly during initial training, where essential knowledge for teaching is consolidated. This article aims to assess the professional knowledge mobilized by prospective primary school teachers when designing geometry school tasks. To this end, 250 prospective teachers were asked to design school tasks for teaching geometry in primary education based on real-life images. Their proposals were analyzed using a qualitative methodology, drawing on tools from the onto-semiotic approach and the didactic-mathematical knowledge and competences model. This model supported the categorization of the results through the facets of didactic suitability. The findings show that participants frequently mobilized knowledge related to the use of real-life representations, manipulatives, and students' interests, enabling them to create contextualized tasks and mathematical connections. However, significant gaps were observed in their consideration of assessment strategies, formulation of learning objectives, and instructional time planning. The study underscores the formative potential of task design in initial teacher training and the importance of fostering opportunities to develop integrated and interdisciplinary professional knowledge in prospective teachers.

Keywords: professional knowledge, geometry, school task, prospective teacher

INTRODUCTION

During the last decades, studies on the knowledge of prospective mathematics teachers have become relevant in the field of mathematics education (Alfaro-Carvajal & Fonseca-Castro, 2024; Chapman, 2015; Depaepe et al., 2020; Llinares et al., 2018). It is well established that this knowledge plays a crucial role in the teaching and learning process, as the mathematical knowledge students develop is largely influenced by the training and experience of their teachers (Castro et al., 2021; Fennema & Franke, 1992). Considering the complexity of the teacher education process, research in didactic of mathematics has focused on how to prepare future mathematics teachers, approaching this task from diverse conceptual frameworks and mathematical objects. Research has explored teacher education in calculus (Amaya et al., 2016; López-Martín et al., 2022),

statistics and probability (Ruz et al., 2020), geometry (Browning et al., 2014; Godino et al., 2018), mathematical processes (Alsina & Coronata, 2014; Oliveira et al., 2021), task design (Daher et al., 2022; de Sousa et al., 2020), mathematical modelling (Kara & Corum, 2023; Leikin et al., 2025), among others.

Despite this growth, most studies focus on secondary education, leaving primary and early childhood education underexplored (Charalambous & Pitta-Pantazi, 2016). In response, task design has emerged as a key activity in initial teacher training, recognized for its potential to improve teacher preparation (Blum & Niss, 1991; Ferrando et al., 2025; Stacey & Turner, 2015; Sullivan et al., 2015). Task design not only facilitates the discovery of connections between the real world and mathematics (Felton, 2010) but also facilitates the transfer of mathematical ideas and knowledge from one to another context (Salgado, 2017).

Contribution to the literature

- This study provides empirical evidence on the professional knowledge mobilized by prospective primary school teachers when designing school geometry activities. There are several investigations that analyze the professional knowledge of teachers at other stages, but not prospective primary school teachers. It introduces a methodological approach that combines a professional task based on real-life images with the analytical tools of the onto-semiotic approach (OSA) and the didactic-mathematical knowledge and competences (DMKC) model and contributes to the refinement of this model by illustrating how its dimensions-epistemic, cognitive, instructional, affective, and evaluative-emerge in the task design processes of future teachers.
- It highlights common strengths and gaps in preservice teachers' professional knowledge, especially regarding the use of manipulatives, student interests, and contextualization versus their limited attention to assessment strategies, learning goals, and time management.
- It underscores the formative potential of task design as a diagnostic and developmental tool in initial teacher education and suggests directions for integrating interdisciplinary knowledge in preservice teacher training programs.

There is consensus on the importance of teachers' competence not only in solving mathematical tasks, but also in selecting, modifying and formulating them for educational purposes (Mallart et al., 2018). It is known that adequate mathematics teaching depends on teachers possessing knowledge of both the content and the learning process, which will allow them to select appropriate tasks for their correct use in the classroom. By getting prospective teachers to relate their mathematical knowledge to their own experiences, better conditions can be created for their students (Lucey & Maxwell, 2016).

The literature on task design in mathematics teacher education emphasizes strengthening future teachers' knowledge to enable the creation of effective and engaging tasks (Watson & Ohtani, 2015). In this sense, several studies describe the link between teachers' level of knowledge and the quality of tasks produced. For example, Singer and Voica (2013) found that many tasks proposed by practicing teachers were incorrect or uninteresting (e.g., the activity "calculate $1.5 + 0.6$ "), highlighting the difficulty of constructing suitable tasks without specific knowledge.

According to Chapman (2012), prospective primary school teachers often struggle to develop engaging and useful mathematical tasks due to limited experience in creating problems and restricted mathematical knowledge. Given this scenario, it is crucial to investigate the didactic-mathematical knowledge mobilized by prospective primary school teachers when designing geometry tasks during their initial training. As Watson and Mason (2006) suggest, designing mathematical tasks engages teachers in mathematical exploration, reflection on teaching, and refinement of their own formative discourse.

Unlike previous studies that have focused on characterizing the general difficulties of prospective teachers in the creation of tasks (such as Chapman, 2012) or on analyzing didactic-mathematical knowledge in

specific training contexts (such as Vargas et al., 2024), the present study focuses on identifying the knowledge that is mobilized during the process of designing geometric school tasks in the context of initial training. As Henríquez and Verdugo (2023) point out, this is an aspect that needs to be studied in greater detail to better understand how these competencies are configured in professional practice.

In summary, task design is a professional action that should be explored and discussed in teacher education. From a cognitive perspective, the detail and content of tasks significantly impact learning; from a cultural perspective, they shape students' experience with mathematics and their understanding of mathematical activity; and from a practical perspective, they constitute the backbone of classroom work (Watson & Ohtani, 2015). In this sense, this article aims to address the following research question: What knowledge does a group of prospective primary education teachers mobilize when designing school tasks for the teaching of geometry?

For this purpose, the theoretical-methodological framework adopted is the DMKC model, proposed by the OSA (Godino, 2024), which allows for an in-depth analysis of the didactic-mathematical knowledge that emerges during the task design process. Through this approach, we seek to provide empirical evidence on how this knowledge is articulated in initial training, thus contributing to the understanding of an aspect still scarcely addressed in the specialized literature.

THEORETICAL FRAMEWORK

This section is structured into two parts. First, some advances on task design are presented, including classifications and results of previous research that serve as analytical references for this study. Second, aspects of DMKC model by the OSA are described.

Aspects of Task Design

Learning to design school tasks that promote the development of geometric thinking and that are in line with current curricular orientations is one of the great challenges of geometric education. Mallart et al. (2018) recognize the selection, creation, and adaptation of mathematical tasks as a teacher training opportunity. Guberman and Leikin (2013) argue that proposing a series of mathematical tasks to prospective teachers—covering mathematical concepts, properties, and procedures—helps them develop skills and knowledge tailored to the content.

However, despite the consensus on the importance of task design and selection in mathematics teacher education, it is also known that both in-service and prospective teachers have difficulties in differentiating routine from non-routine tasks and in choosing appropriate formats that enable the development of skills in their students (Galant, 2013). Studies such as that conducted by Knott et al. (2013) analyze how to help teachers use textbooks in the design of tasks that involve students in processes of justification and generalization.

Several authors have addressed the design of tasks in the framework of the training of prospective teachers. For instance, Santa et al. (2018) have evidenced the emergence of mathematical processes such as visualization, conjecture and demonstration when future teachers propose tasks. For their part, Vieira et al. (2013) indicate that processes such as argumentation and demonstration are developed through the proposal of research-type tasks by prospective teachers. Other authors, such as De Gamboa et al. (2015), have analyzed the knowledge that prospective teachers mobilize in the management of instruction, as well as the didactic-mathematical knowledge that is evident in the solutions they propose to their own tasks. Isik and Kar (2012) observed that although the tasks created by prospective teachers often remain routine, the design process itself can foster more creative and meaningful teaching proposals. A strategy for creating attractive tasks is presented in Cáceres et al. (2010) propose that, through the prospective teacher's reflection on their own task, they can achieve significant improvements in their work.

To determine whether a mathematical task is appropriate, Stein and Smith (1998) proposed considering factors such as students' level, age, and prior knowledge. Their framework on levels of cognitive demand (memorization, procedures without connections, procedures with connections, and doing mathematics) has been widely adopted. More recent studies confirm that tasks positioned at higher cognitive demand levels foster heuristic problem-solving skills and deeper mathematical thinking (Wakhata et al., 2023).

Building on this framework, other recent studies highlight that low cognitive demand tasks (such as memorization and procedures without connections)

mainly lead students to reproduce facts or rules without engaging with underlying concepts. In contrast, high cognitive demand tasks require non-algorithmic thinking, integration of diverse representations, and sustained intellectual effort. When accompanied by appropriate instructional support, such tasks not only promote mathematical processes but also strengthen conceptual understanding, highlighting how cognitive demand directly shapes both the quality of engagement and the learning opportunities available to students (Prediger et al., 2024).

The design of school geometry tasks is a common teaching practice and provides a valuable context for analyzing the knowledge of prospective primary teachers, as it reveals their beliefs and conceptions about mathematics. Incorporating task design into initial training, with a focus on geometric objects, not only exposes future teachers to authentic teaching situations but also makes visible their conceptual difficulties, training gaps, and competence levels. As highlighted in Vargas et al. (2023), these experiences reveal a significant gap between school knowledge and the professional knowledge required, underscoring the need to systematically integrate task design as a core training strategy in teacher education programs.

Aspects Related to the Onto-Semiotic Approach

One of the central research approaches in mathematics didactics is aimed at identifying and analyzing the DMKC that teachers should possess or develop. In this line, several authors have proposed theoretical models that categorize the knowledge necessary for effective mathematics teaching, as well as the professional competencies that should be promoted in teacher education (Neubrand, 2018; Petrou & Goulding, 2011; Rowland, 2013).

From the perspective of the OSA, the DMKC model is proposed as an organizing and energizing element of both the knowledge and the competencies of the mathematics teacher (see [Figure 1](#)).

The DMKC model recognizes three dimensions for organizing mathematics teacher's knowledge: mathematical dimension, didactic dimension and meta-didactic-mathematical dimension. The mathematical dimension describes two types of knowledge that a mathematics teacher should have (Pino-Fan & Godino, 2015). First, there is common content knowledge, referring to the knowledge that a teacher should have about a particular mathematical object (e.g., polygons and polyhedra). This knowledge will allow him to solve tasks or problems posed in the curriculum and in the textbooks of the level to which he works (e.g., the classification of polygons included in the materials of the primary education course). Additionally, this knowledge is shared between the teacher and the students.

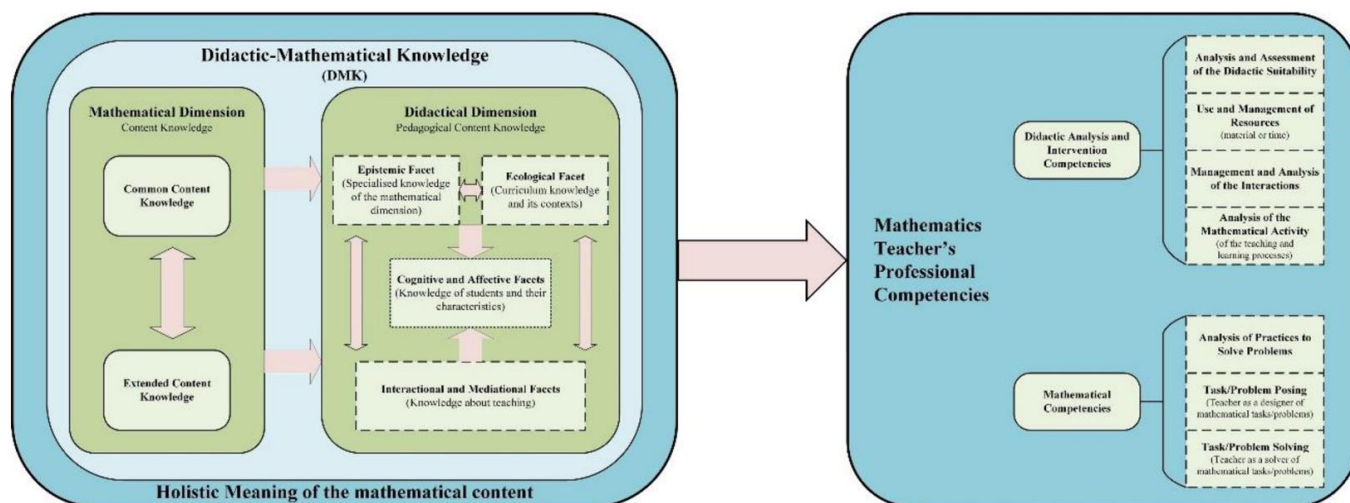


Figure 1. Model of DMKC of the mathematics teacher (Adapted from Pino-Fan et al., 2022, p. 7)

On the other hand, there is extended content knowledge. This type of knowledge refers to the teacher's understanding of a mathematical object, which, when taught at a specific moment (such as the recognition of plane figures and their characteristics in different positions in a second-grade geometry course), serves as a basis for mathematical content at higher educational levels (such as the classification of polygons in fourth grade). Expanded knowledge allows the teacher to establish learning trajectories, set challenging activities and tasks for students, link mathematical objects with other mathematical notions, and generate connections with elements both internal and external to mathematics.

In addition to the mathematical knowledge of the contents, the mathematics teacher must have a broad knowledge of the factors that can influence the teaching and learning processes of such contents. In this sense, the didactic dimension of the DMKC model describes six facets of this type of knowledge:

1. Epistemic facet: It refers to the specialized knowledge of mathematical dimensions. In the context of this study, it refers to the presence in the tasks proposed by prospective teachers of different representations, varied resolution strategies rich in mathematical processes, task proposals to argue and/or clarify and correct definitions and procedures.
2. Cognitive facet: It refers to the knowledge about the cognitive aspects of the students. In this study it will be understood as the statement of an evaluation system that considers the different levels of understanding that the proposed task may have, the inclusion of extension activities and the inclusion of previous knowledge in the task.
3. Interactional facet: It refers to the knowledge about the interactions that arise in the classroom, among the agents involved in the whole teaching-learning process. In this study we will analyze in

the proposed tasks, the declaration of group work, or the presence of moments of self-responsibility and the process of formalization of contents.

4. Mediating facet: It refers to the knowledge of resources and means that allow improving students' learning; it also includes the times designated for the teaching process. In the context of this study, we find the enunciation of the times required for each of the proposed activities and tasks, on the other hand, the inclusion of material resources.
5. Affective facet: It refers to knowledge concerning students' affective and attitudinal aspects. In this study, it includes the richness of the task in terms of motivational elements and student interest, the extent to which the task emphasizes the usefulness of geometry in everyday life, and the opportunities it provides for student participation in solving the proposed activities.
6. Ecological facet: It refers to the curricular, contextual, social, economic, etc. elements that influence the management of mathematics teaching and learning processes. Within the framework of this study, the achievement or not of intra- and extra-mathematical connections in the tasks designed by the future teachers is analyzed; the declaration of the level to which the proposed task is addressed and the real adequacy of the declared contents and objectives to the didactic proposal.

Finally, the meta-didactic-mathematical dimension of the DMKC model contemplates the teacher's knowledge that is necessary to systematize the reflection on his practice, which allows the teacher to be able to evaluate both the teaching and the learning implemented, to make judgments on these processes and to make improvement proposals for future implementations (Pino-Fan et al., 2022).

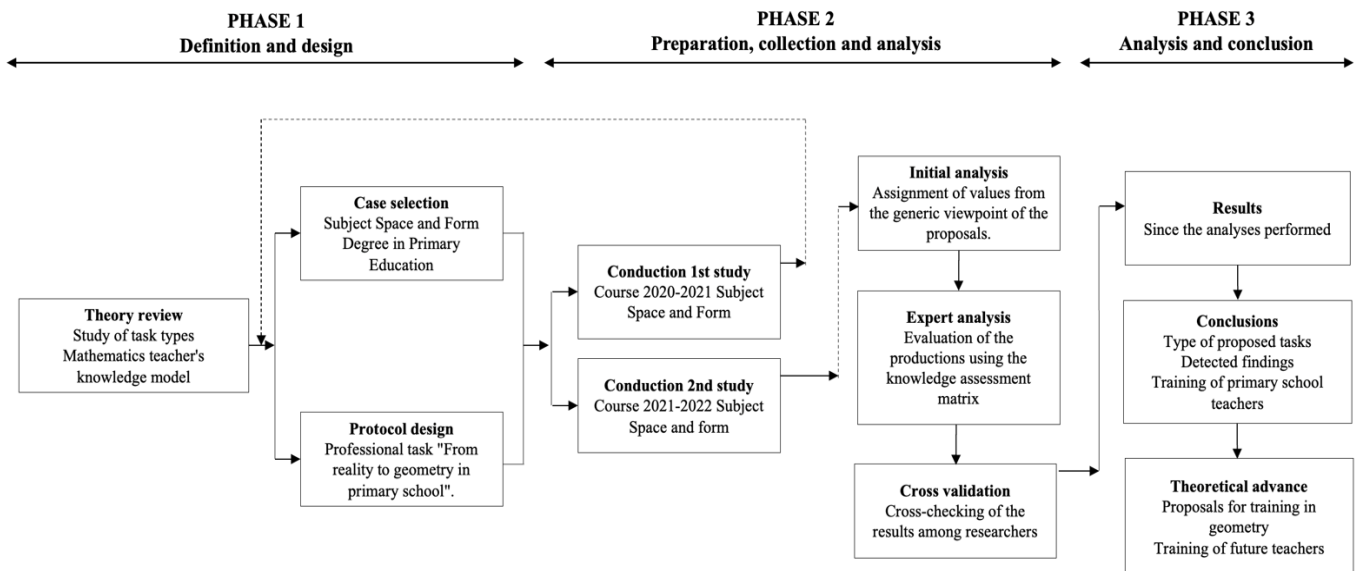


Figure 2. General structure of the research (Source: Authors' own elaboration)

The DMKC model has been used to study the didactic-mathematical knowledge and skills of teachers who teach mathematics. For example, Silva et al. (2024) analyzes the didactic-mathematical knowledge mobilized by prospective teachers when integrating educational robotics into primary school mathematics lessons, highlighting the potential of the DMKC model to characterize their teaching practices. Similarly, Llanos-Lagos et al. (2025) employ the model to examine the mathematical competencies of prospective teachers when teaching confidence intervals in a microteaching context, evidencing its usefulness for assessing both content knowledge and pedagogical decision-making. In the same way, Arana-Pedraza et al. (2020) analyze through the components and criteria of the model, the textbooks used in a linear algebra course for engineers. Finally, Vargas et al. (2023) use the model to determine the didactic-mathematical knowledge of future primary education teachers regarding the classification of polyhedra.

Considering the experiences that have been carried out from the definition and characterization of the DMKC model for the analysis and evaluation of the DMKC model of teachers, it is interesting to focus on the knowledge of future primary school teachers. The present study allows us to broaden the spectrum of contexts of implementation of this model and to approach a new subject (design of school tasks for the teaching of geometry) and a new population (prospective teachers of primary education) from this model of knowledge and competences.

METHOD

This study is framed as an instrumental case study (Yin, 2014) analyzing the design of geometry school tasks, carried out by a group of future teachers of primary education. The case corresponds to the practices

of two cohorts of students in the subject "space and shape", belonging to the same initial training program, distributed in the academic years 2020-2021 and 2021-2022.

The methodology followed three phases (see Figure 2). Phase 1 included a theoretical review structured on two axes: the literature on types of mathematical tasks and models of mathematics teacher knowledge and competencies. Subsequently, the groups of participants were selected and a professional task entitled "From reality to geometry in elementary school" was designed, which also functioned as the main data collection instrument.

Phase 2, corresponding to the preparation, implementation and initial analysis of the data, was based on the implementation of the professional task in two consecutive courses, maintaining homogeneous conditions of application to ensure the reliability of the results. The data collected were analyzed in three steps:

- (1) a first qualitative and quantitative assessment based on the professional experience of one of the researchers,
- (2) a systematic analysis using an evaluation matrix based on the DMKC model, and
- (3) a cross-validation process among researchers to ensure consistency of criteria.

Phase 3 consisted of the structuring of results, focused on identifying the types of knowledge mobilized by future teachers when designing geometrical school tasks. This phase enabled a detailed characterization of the knowledge evidenced and to discuss its emergence—or absence—in their productions, framed within the DMKC categories and linked to training proposals for strengthening didactic-mathematical knowledge in geometry.

CODING					MEDIATIONAL			ECOLOGICAL				
Educational level	Materials	It Propose an arithmetic procediment	It Propose an algebraic procediment	It Propose a geometric procediment	Time dedicated to the didactic propose	Material Resources	Total	Indicates level at wich directed	Evidences Intramathematics connection	Evidences Extramathematical connection	Adapted to the level to which it is directed	Pr co
Initial level	Wall Paints Cards	0	0	1	1	1	3	1	0	0	1	1
Not indicated	image Bags paper to draw	0	0	1	1	1	3	0	0	1	1	0

1	2	3	4	5	6	7	8	9	10
a b	a b c d e f	a b c d e	a b c d	a b c	a b c d e	a b c	a b c d e	a b c	a b c d e

Figure 3. Structure of the knowledge assessment matrix following the DMKC model (Source: Authors' own elaboration)

Instructional Design

For data collection, a professional task called “From reality to geometry in elementary school” was designed. Its aim was to elicit the didactic and mathematical knowledge mobilized by future teachers when designing school tasks for teaching geometry. To this end, participants were provided with a set of images that simulate real-world contexts and asked to develop didactic proposals with geometric meaning.

The professional task was structured in three parts. In the first, participants were presented with images of real objects and phenomena (e.g., protozoa, earth movements in space, etc.) and asked to design a geometry teaching task based on them. In the second, they had to consult the official primary curriculum of Catalonia to establish explicit links between the curricular contents of the *space and shape* axis and their designed tasks. In the third part—considered the focus of this study, as it provides the most relevant information—, each group selected one initial image and elaborated a complete classroom activity focused on a specific geometric concept and tailored to a chosen grade level.

Context and Participants

The research was carried out with a total of 250 second-year students enrolled in a primary education teacher training program at a Spanish university. The professional task was implemented in the course “space and shape”, the only subject in the curriculum specifically devoted to geometry and its didactics. Therefore, the previous knowledge of the participants comes mainly from their elementary and middle school education.

The data analyzed corresponds to the written productions generated within the framework of the

implementation of the professional task. A total of 57 protocols were collected through the Moodle platform. The implementation was carried out in online mode for both cohorts through synchronous sessions with the teacher (one of the authors of this work). A first preliminary submission was made (without feedback) and a few days later the final submission, after which the analysis of the assignment was made as a whole, as well as the feedback and correction of all proposals.

Data Analysis

The data analysis was carried out in three successive phases. First, the 57 proposals were examined by means of an exploratory analysis of their constituent elements. This made it possible to classify them as tasks of high or low quality, assigning each of them a score between 1 (lowest quality) and 5 (highest). This rating was assigned by one of the authors of this article, based on his experience in the design of professional tasks and considering that the proposal submitted by each group was an activity of the “space and form” course. For this rating, criteria such as level of innovation, context adequacy, internal coherence of the task, presence of introduction and problematization, as well as clarity in the indications for its solution were considered.

From the preliminary analysis it was possible to elaborate an evaluation matrix that later allowed systematizing the analysis from the DMKC model, aligned with the didactic suitability facets of the OSA.

Figure 3 presents, at the top, a fragment of the original matrix used to code the analyzed didactic proposals. This matrix organizes the information in different dimensions; however, due to space restrictions in this document, a more compact and summarized format is included in the lower part of **Figure 3**.

In this abbreviated version, the same information is represented by a numerical and alphabetical coding system: each column numbered from 1 to 10 groups different indicators, which are represented by letters (a, b, c, etc.). These letters correspond directly to the specific elements that appear in the original matrix, allowing a clear relationship to be maintained between the two representations and facilitating their interpretation in the subsequent analysis.

In the first column of the evaluation matrix, general elements of the proposal were coded: task, objective, concepts involved, educational level, materials used, and types of procedures (arithmetic, algebraic, geometric). The remaining columns contain indicators associated with the facets of the model, as shown below:

- 2a: Time dedicated to the proposal; 2b: Material resources.
- 3a: Indicates target level; 3b: Evidence of intra-mathematical connection; 3c: Evidence of extra-mathematical connection; 3d: Adapted to the target level; 3e: Proposes contents; 3f: Proposes didactic objectives.
- 4a: Statement with context; 4b: Presents pre-situational theory; 4c: Variety of tasks; 4d: Problematization; 4e: Others.
- 5a: Natural register; 5b: Arithmetic register; 5c: Associated image; 5d: Graphical representation
- 6a: Labels are used and explained; 6b: Correct algebraic symbols; 6c: No possible conflicts of interpretation.
- 7a: Statement is clear; 7b: Units of measurement are indicated; 7c: No errors; 7d: No disconnect between image and mathematical discourse; 7e: Fundamental definitions, properties, and procedures for the level.
- 8a: Mathematical objects are related to each other; 8b: Articulation between different meanings; 8c: Articulates different systems of representation.
- 9a: Group work; 9b: Autonomy; 9c: Discussion-formalization; 9d: Proposes an evaluation model; 9e: Evaluation criteria address sufficient and necessary elements.
- 10a: It is an interesting task; 10b: It allows to value the usefulness of mathematics in everyday life; 10c: The design contemplates student participation.

The second phase of the analysis consisted of individual assessments by each of the investigators to evaluate the knowledge evidenced in each of the written protocols. The investigators assigned a score of 1 if evidence of each element was found, or a score of 0 otherwise. The total possible score per protocol was 36 points. Subsequently, a cross-validation process was carried out among the investigators. When comparing

the scores obtained, discrepancies were discussed, and the categories of analysis were adjusted to achieve the highest possible level of agreement.

The last phase of this research consisted of the systematization and discussion of the results, in which the information was systematized from the scores obtained, considering both the type of tasks proposed and the didactic-mathematical knowledge evidenced. This analysis was articulated with theoretical discussions on the DMKC model, teacher training in geometry, and the challenges in primary education. Based on this triangulation, training proposals were developed to strengthen the teaching of geometry in initial training.

RESULTS

The results are according to the three dimensions of the DMKC model: mathematical dimension, didactic, and didactic-mathematical dimension (Pino-Fan & Godino, 2015).

Mathematical Dimension

Most tasks proposed by the prospective teachers (82.5%) were of low cognitive demand, that is, following Stein and Smith (1998), tasks based on routine procedures or with few geometric connections. These types of tasks also evidence common knowledge of the primary school teacher and, although they have a geometric focus, they are mainly composed of arithmetic-type activities such as basic operations between randomly selected numbers. They are mostly based on the calculation of areas, perimeters and volumes of geometric figures with randomly assigned values, but without a real connection (see proposal group 8-year 2020).

“It would be aimed at students in the initial cycle and would be based on image E (image of a choreography made with parts of the human body), as specified in the image would make them perform in groups of three or four people a choreography to represent geometric figures, such as triangles, squares, etc. What we intend is that they would see the different geometric figures and would be able to elaborate them with physical materials, which in this case would be the arms. Also, when they had invented four or five figures each time, I would have them calculate the perimeter of the geometric figure they had elaborated by measuring with a ruler the length of the arms and, if possible, the area of those different figures” (group 8-year 2020).

The common content knowledge observed in these proposals refers to the knowledge about geometric figures in prototypical position and their characteristics such as area and perimeter, as well as the identification

of geometric figures in known structures. However, no evidence of extended knowledge was detected. In this case, we expected prospective teachers to recognize the limitations of the human body in constructing geometric figures, for example, the impossibility of forming a regular polygon due to the non-congruence of body parts.

Another common type of knowledge is evident in the conversion of units, the equivalence between measurements and the basic transformations between measurement systems; this is based on the direct valuation of a given formula or, basically, the direct calculation of some algebraic expression knowing all its components.

“Calculate the area of the bacterium in the image remembering how to calculate the area of this type of geometric figures (ellipse) $A = \pi ab$ being a and b the semi-major and minor axes of the ellipse. The major semi-axis measures 3 cm and the minor semi-axis measures 2” (group 9-year 2021).

In this case, future teachers associated the figure of the bacterium with a more advanced geometric structure, such as an ellipse demonstrating extended knowledge. However, the inadequate use of this knowledge within the context for which the task is being designed, as well as the calculation proposal to be performed, maintain its proposal as a task of low cognitive demand focused particularly on common knowledge (calculation of the area of a geometric figure from a given formula).

The remaining 17.5% of proposals were classified as tasks of high cognitive demand at an initial level (procedures with connection). These required complex, non-algorithmic thinking and consistently demonstrated extended knowledge. They often involved unusual geometric figures, such as circular arcs, truncated 3D figures like pyramidal sectors, or atypical configurations of 2D and 3D figures.

An example of this type of task is found in group 11's 2021 proposal, where they used the image of Arawak backpacks.

“These cloth bags have geometric designs. However, there are two backpacks that do not have any geometric shapes. can you identify them? Using construction paper try to recreate them. is it possible to cover the entire backpack with any geometric figure? Justify” (group 11-year 2021).

The previous task is not based on an algorithmic procedure; on the contrary, it is a task based on experimentation and work with manipulative material. By giving a cardboard to the students and proposing to cover it with geometric figures, the aim is to confront the person who performs the action with a geometric

invariant (covering the plane), which, used correctly, would allow the students to develop the idea of tessellation, coverings and groups of figures that effectively cover the plane. The question that group 11 (year 2021) proposes about whether it is possible to cover the whole backpack with any geometric figure is an open question that gives infinite possibilities to the students and that evidences an extended knowledge about the configurations of geometric figures in 2D and their relationship with the covering of the plane.

Didactic and Meta-Didactic-Mathematical Dimension

To organize the results found in this dimension, examples of task proposals classified in each of the knowledge facets of the DMKC model will be presented.

Mediational facet

The use and description of time in the prospective teachers' proposals reflects their knowledge of the mediational facet, but also reveals their limited experience, as the time allocated for teaching some geometric elements often seems insufficient. For example, group 2 (year 2021) assigned 15 minutes to identify geometric figures in the Arawak backpacks and another 15 minutes to construct a geometric pattern, even though cognitively demanding tasks such as pattern creation would typically require more time than simpler visualization tasks. Almost all groups (94.7%) incorporated didactic materials. While some materials were unconventional—such as trees to represent cylindrical structures (group 1, year 2020) or the classroom itself to study volumes (group 3, year 2021) – they consistently show an effort to connect geometric concepts with tangible elements. This aligns with the literature reviewed, which highlights the use of tangible materials at early ages as a significant contribution to the development of geometric thinking.

Ecological facet

The mostly of the groups (86%) indicate the level at which the activity is aimed; however, there is a percentage of them (15%) that propose an activity that is not adapted to the declared level, generally activities that are too simple for a group with more advanced capabilities (see group 11-year 2020).

“We would select image E (image of figures formed with the human body). In this case, I would ask the students to get into groups of 3 and for 5-10 minutes represent and write on a sheet of paper the different possibilities of geometric figures that they could represent. After this time, I would ask each group to represent one and show them the different elements that characterize the figure. For example, in the case of the triangle, I would tell them and point out the three sides, the

three vertices, etc. This activity would be aimed at upper cycle and was intended to introduce them to some of the different existing geometric figures, as well as the elements that compose them" (group 11-year 2020).

This activity exemplifies a proposal that does not match the intended level, as identifying geometric figures and their basic characteristics—such as the number of sides and vertices—corresponds to early primary school geometry. Since the task was aimed at the upper cycle, it would be too easy for these students.

On the other hand, most of the groups (73.7%) state the contents they wish to address, but only 24.6% of the groups include the didactic objectives proposed with the activity (an example was group 11-year 2020); in this sense, we consider that the non-declaration of these objectives is evidence of the lack of knowledge around the ecological facet since geometric elements are included in the task, but it constitutes a simple assignment of such object, without taking into account the characteristics and possible potentialities for the development of knowledge.

An expected result is the high percentage (70.2%) of proposals that present some extra-mathematical connection (relations between a mathematical idea and an object outside mathematics). This is explained by the fact that all the images provided to the future teachers represented real life situations and in known contexts. Only 12.3% of the groups propose some type of intra-mathematical connection (relationship between two or more mathematicians).

The groups that refer to extra-mathematical connections propose relationships between concepts such as area or perimeter and everyday elements such as feet or arms. While the groups that propose some type of intra-mathematical connection, establish different representations between mathematical objects, like for example group 7 (year 2020) who propose to convert the measurement of the COVID-19 virus from nanometers to micrometers for its dissemination in another country. The use of units of measurement and their transformation is therefore identified as an intra-mathematical connection.

Although in this research it is evident that most of the future teachers proposed some type of extra-mathematical connection, this is not what is usually obtained (see detail in Vargas et al., 2024). In this sense, it is important to highlight that the professional task assigned to the future teachers depended on a series of images and videos extracted from real contexts, thus inducing this type of connection. The quality of the connections, as well as their depth, were clearly determined by each group's understanding of the extra-mathematical element with which they worked.

Epistemic facet

Only 38.6% of the proposed tasks include adequate additional contexts, which support the designed task. In general, most of the groups assign random values to the images given to them and propose elements that accompany the statement but do not provide relevant information. In several proposals it was observed that, although there is a broad and well-structured combination of contextualization, exercises and applications -recognized as indicators of the situation-problem component of epistemic competence-, they also had limited knowledge to devise meaningful contexts or tasks that went beyond the following of a preestablished algorithm.

On the other hand, in terms of registers and representations, most of the proposed activities are in natural language (98%). The remaining percentage also includes some element related to numerical or algebraic language, such as group 9 (year 2021) with its proposal for the area of a bacterium using the formula for the area of an ellipse. Additionally, 19.3% of the groups proposed an activity with an associated image, by way of representation; however, the images are of simple reference since most of them do not present a connection between the image and the mathematical object being addressed.

Within the epistemic facet, knowledge related to language was expected, including the use of various modes of mathematical expression—verbal, graphic, symbolic—and processes such as translating between registers and using language accessible to learners. However, only 1.8% of the groups used and explained any labels within their activity design, indicating that participants' mathematical language remained elementary. None of the groups incorporated algebraic symbols beyond expressions intended for direct numerical calculation.

A significant proportion of the groups (49.1%) proposed activities in which potential conflicts of interpretation could arise, mainly due to the context in which they pose the geometric school task.

"To start the activity, the whole class will brainstorm ideas about the zodiac signs, to see what the students know about this topic. After exposing all the ideas, we will explain what the horoscope is and what dates are included in each of the figures, and then groups will be formed according to their zodiac sign. Once the groups are divided, a question will be asked: with what geometric figure would you identify the constellation that forms your zodiac sign? Then, with material provided by the school (plasticine and straws), they will build a new geometric figure from different geometric shapes,

symbolizing a new horoscope" (group 15-year 2020).

It is considered that group 15 (year 2020), incurs in a conflict of interpretation given the use of astrology as an element for geometric analysis. The intention of the use of the image of the constellations was the analysis of these from astronomy and the phenomena associated with the universe; however, assigning them to other extra-mathematical elements related to cultural beliefs could potentially create conflicts in the objective knowledge, as well as absence of connections between these ideas.

Norms are also part of epistemic suitability, requiring definitions and procedures to be clear, correct, and appropriate for the educational level of the activity. In this regard, it is possible to indicate that all the proposals presented by the prospective teachers contain well-formulated statements. In those proposals that include some type of measurement, the corresponding ones are always expressed; however, the assignment of units and dimensions is sometimes done in a random way and without a real consideration of reality. For example, group 9 (year 2020) proposed activities in which the distance between constellations was 3 cm, while group 4 (year 2021) assigned a circumference length of 100 m to sequoias.

However, 12.3% of the proposals contained errors. For instance, group 6 (year 2021) asked children to measure a tree using wooden sticks but mistakenly referred to the diameter when describing the tree's circumference.

"... If the measurement is correct and the sticks manage to go around the tree, the children will hold hands and hug the tree together with their hands intertwined. In this way, they will understand that the diameter of the tree is still a straight line that forms a round shape" (group 6-year 2021).

Finally, the epistemic facet includes a component related to the relationships established between knowledge. In this case, only 36.8% of the proposals present activities in which geometric objects are correctly related; however, when they include more than one geometric element, they do it separately, such as the study of angles and the classification of triangles or the identification of geometric figures and the relationship of some of them with the plane covering. Only 5.5% of the proposals articulate some meanings, such as group 14 (year 2021) who, through the study of Arawak backpacks and the detection of patterns in their manufacture, propose to identify regularities and symmetries in the classroom or the environment surrounding the students, thus relating symmetry and tessellation, in addition to the different representations that geometric figures can have.

Cognitive facet

The lack of extended knowledge, as well as the failure to establish relationships between different geometric knowledge justifies that only 17.6% of the total number of proposals included some type of previous theory for the development of the task. An example of a proposal that evidenced this type of knowledge can be seen in group 10 (year 2020), where an activity is proposed that begins with a previous training, and then introduces a variety of tasks. Through these tasks, they problematize the concept of angle classification by exploring constellations.

"First, the teacher will give an introductory explanation of what constellations are and then present the tool to be used for the activity: a protractor. What is it? Why is it used? Afterwards, each child will have to choose a constellation using IT, search for different constellations on the Internet and choose one of them. Then, the child will have to represent the chosen constellation on a piece of paper and, with the help of the protractor, measure the angles between the different points that form the constellation. Next, he will have to classify the angles as complete, null, right, acute, obtuse or flat. After performing this procedure, you will have to find out if it is present in your constellation" (group 8-year 2021).

In this proposal, it is possible to identify how the initial explanation on the use of the protractor and elements related to constellations allow the development of a task of angle measurement, another of angle representation and finally another of angle classification in relation to constellations. The fact that the task also requires the identification and location of what is constructed in some constellation (an activity that will require discussion about the existence or not of angles in that position in the universe), is considered a problematization of knowledge.

Finally, the cognitive facet includes the component of an evaluation system; in this case, only one group (1.8%) proposed evaluation criteria that address sufficient and necessary elements to determine whether or not the proposed objectives have been achieved (this case can be seen in detail in the proposal of group 6, year 2021).

"After the session, a rubric will be elaborated to evaluate the task performed by the whole group, specifying the aspects and points to be considered when assessing the work done during the 15 minutes, as well as the group presentations and creations, since we consider that the process is more important than the result. Finally, to conclude and complete the above evaluation table (Table 1), the values that will qualify each task

Table 1. Evaluation table

Criteria	1	2	3	4	Observations
Students work constantly and with a good organization.					
The students have an active participation, showing motivation and enthusiasm.					
The students share the responsibility in the task development.					
They listen and respect the comments, urgencies and opinions from the other team members.					
They are capable to make treatments to improve the results.					
They respect each other's and help to other to improve the work environment.					
They have roles definite, and it is development in an effective way.					

will go from 1 to 4 and will have the following significance: 1: Didn't work out well at all, 2: Didn't work out too well, 3: Worked out well, 4: Worked out very well" (group 6-year 2021).

Interactional facet

This facet refers to the modes of interaction in the teaching and learning of mathematics, which helps identify and resolve conflicts of meaning while promoting autonomy and communicative competence. In this study, 77.2% of the proposals included group work; 54.4% incorporated opportunities for individual reflection or autonomous work; and 35.1% provided moments for discussion or formalization, often through brainstorming or student-to-teacher conclusions. Notably, interactions were mostly student-centered, with one member often guiding the reasoning to reach a group consensus.

Affective aspect

Finally, regarding the knowledge on affective appropriateness, it is possible to observe that 43.9% of the proposed activities allow valuing the usefulness of mathematics in real life. This tendency is often aligned with tasks that precisely state their objectives and build a teaching and learning process of geometry through questioning, the use of tangible materials and a clear and detailed evaluation process.

In these tasks, it is evident how, through questions, the future teachers guide the construction and emergence of a geometric object. It is striking that in most of the proposals (59.6%) a unidirectional activity is developed in which the teacher is the one who leads and develops the task from beginning to end, postponing the role of students only to the execution of activities and their reflection on what is understood.

An interesting element (observable in group 6, year 2021 and in others) is the inclusion of evaluation elements and criteria that are not purely geometric, but that address affective and interactional elements such as vocabulary, teamwork, values education, communication and justification, and clear presentation of results. For the research team, this type of elements from different facets of knowledge, shows that the group of future teachers has knowledge and pays attention to other formative aspects such as education in values,

communication among peers and mathematical processes in general (even if at a low level).

DISCUSSION AND CONCLUSION

The results described above have made it possible to identify the knowledge mobilized by a group of prospective teachers when facing the challenge of designing school tasks for the teaching of geometry in primary education. The presence of different types of knowledge (common, extended and didactic-mathematical) according to the typology proposed by the OSA, distributed along the different facets that configure it: epistemic, cognitive, interactional, ecological, mediational and affective has been evidenced. In this sense, we confirm what has been stated in recent international studies on how the DMKC model can be used to characterize the skills mobilized by future teachers in different contexts, from educational robotics (Silva et al., 2024) to the teaching of statistical concepts such as confidence intervals (Llanos-Lagos et al., 2025) or, in our case, in the design of tasks for teaching geometry in primary education.

We agree with Sullivan et al. (2010) that involving future teachers in the design of school tasks is crucial, as such tasks largely determine learning opportunities for their students. This highlights the need to address skills that are currently lacking, which is a key responsibility of teacher training programs. Planning, designing, and selecting appropriate tasks is a central professional practice and should be integrated into all curricula. These conclusions align with recent research on cognitive demand, which shows that task design shapes students' opportunities for high-level engagement (Prediger et al., 2024; Wakhata et al., 2023), reinforcing the importance of providing prospective teachers with opportunities to develop their ability to select and adapt demanding geometry tasks.

The results found reinforce Braga and Belver (2016), note that designing and adapting tasks can be challenging for teachers, highlighting the need for training programs to develop the necessary knowledge and skills. In the tasks proposed by the groups of teachers across the two academic courses in this study, creating suitable geometry tasks proved complex, mobilizing knowledge across all facets of the DMKC. It required them to investigate, propose diverse solution

strategies, and, in the researchers' view, exercise creativity and innovation in tasks initially perceived as basic. This aligns with international research that shows how prospective teachers' mathematical and didactic-mathematical knowledge is activated and developed when they are asked to engage in authentic practices of task design (Ferrando et al., 2025; Leikin et al., 2025).

Asking future primary school teachers to design geometric tasks revealed the absence of knowledge and skills not only in the mathematical field, but also in other disciplines. This reinforces the need to incorporate in the formative processes an interdisciplinary perspective that allows us to establish links between diverse theories and sources of knowledge. In this sense, we agree with Shower (2017), who proposes training teachers in both disciplinary and pedagogical knowledge and incorporating models of reflection that integrate research results from different scientific fields in initial teacher training. The integration of theoretical models such as OSA and DMKC with recent international research provides a robust basis to rethink teacher training in geometry, positioning task design as a privileged site for developing professional knowledge and competencies.

Now, a difficulty detected when proposing geometric tasks in non-mathematical contexts is that of constructing interdisciplinary type tasks. Rebello et al. (2017) states that for there to be emergence of knowledge in an interdisciplinary type of task, it is necessary that the student has a robust scheme in the initial context; in addition, he/she must know how to apply these mathematical concepts in tasks of the other sciences. We agree that the groups that were able to propose more suitable tasks through the use of an additional context or discipline were those whose participants had previous knowledge in that discipline; thus, for example, the groups that knew about knitting backpacks or drawing and painting achieved more suitable tasks by including elements of knitting and/or painting for the development of the idea of tessellation in geometry.

Normally, the inclusion of additional contexts when proposing geometry tasks is done in a forced way, losing richness when using both geometric elements and the disciplinary elements of the context; this alerts us to the need for the primary school teacher to know, in addition to mathematical objects, other disciplines that allow him/her to enrich the teaching process.

The findings of this study point to clear implications for curriculum design in initial teacher education. The difficulties observed in prospective teachers when proposing appropriate geometry tasks reveal the need to explicitly incorporate systematic spaces for task design within training programs. These activities should not only focus on the reproduction of content but also on fostering connections between mathematical ideas, real-life contexts, and interdisciplinary perspectives.

Integrating frameworks such as the OSA and the DMKC model into curricula can support prospective teachers in developing deeper didactic-mathematical knowledge, while simultaneously promoting creativity, critical reflection, and the ability to adapt tasks to diverse classroom realities.

Recommendations and Limitations

We recommend that initial teacher training programs systematically provide opportunities for prospective teachers to design, adapt, and reflect on school tasks in authentic contexts, as this mobilizes diverse facets of didactic-mathematical knowledge. Explicitly integrating interdisciplinary contexts and theoretical frameworks such as the OSA and the DMKC model can further enrich teacher preparation, fostering creativity, innovation, and professional reflection. Future research could examine prospective teachers' task design in other mathematical domains (e.g., statistics, algebra) or investigate the impact of training interventions targeting specific knowledge facets.

However, certain limitations must be acknowledged. Data were drawn from a single teacher education program in Spain, limiting generalization to other institutional or cultural contexts. The analysis relied solely on written productions, without complementary data such as classroom enactments or interviews, which could provide deeper insight into the knowledge mobilized. Additionally, although cross-validation among researchers enhanced reliability, qualitative coding inherently involves some subjectivity. Future studies could implement similar professional tasks in different settings, triangulate data sources, include multiple programs, and enable international comparisons to strengthen the evidence base.

Author contributions: JPVH: conceptualization, data curation, formal analysis, writing-original draft, and writing-review & editing; MBG: conceptualization, methodology, and writing-review & editing; & YV: data curation, formal analysis, methodology, and writing-review & editing. All authors agreed with the results and conclusions.

Funding: No funding source is reported for this study.

Ethical statement: The authors stated that the participants and the university were informed that this study was conducted exclusively for educational and research purposes, with no economic or political interests involved. Participation was voluntary. Furthermore, one of the researchers is a full-time faculty member at the institution, with formal authorization to carry out research involving the student population, as described in the section on participants and context, with the aim of enhancing students' understanding of mathematical concepts.

AI statement: The authors stated that no artificial intelligence tools were used in the preparation or development of this manuscript

Declaration of interest: No conflict of interest is declared by the authors.

Data sharing statement: Data supporting the findings and conclusions are available upon request from the corresponding author.

REFERENCES

- Alfaro-Carvajal, C. R., & Fonseca-Castro, J. (2024). Specialized knowledge of prospective mathematics teachers on the concept of mathematical proof. *Uniciencia*, 38(1). <https://doi.org/10.15359/ru.38-1.5>
- Alsina, A., & Coronata, C. (2014). Los procesos matemáticos en las prácticas docentes: Diseño, construcción y validación de un instrumento de evaluación [Mathematical processes in teaching practices: Design, construction and validation of an assessment instrument]. *Edma 0-6: Educación Matemática en la Infancia*, 3(2), 23-36. <https://doi.org/10.24197/edmain.2.2014.23-36>
- Amaya, T., Pino-Fan, L., & Medina, A. (2016). Evaluación del conocimiento de futuros profesores de matemáticas sobre las transformaciones de las representaciones de una función [Assessment of future mathematics teachers' knowledge of transformations of function representations]. *Educación Matemática*, 28(3), 111-144. <https://doi.org/10.24844/em2803.05>
- Arana-Pedraza, R. A., Ibarra, S., & Font, V. (2020). Conocimientos y competencias didáctico-matemáticas del profesor de matemáticas en ingeniería: Un primer acercamiento [Didactic-mathematical knowledge and skills of the engineering mathematics teacher: A first approach]. In Y. Morales-López, & A. Ruiz (Eds.), *Educación matemática en las Américas 2019* (pp. 928-935). Comité Interamericano de Educación Matemática.
- Blum, W., & Niss, M. (1991). Applied mathematical problem solving, modelling, applications, and links to other subjects-State, trends and issues in mathematics instruction. *Educational Studies in Mathematics*, 22, 37-68. <https://doi.org/10.1007/BF00302716>
- Braga, G., & Belver, J. L. (2016). El análisis de libros de texto: Una estrategia metodológica en la formación de los profesionales de la educación [Textbook analysis: A methodological strategy in the training of education professionals]. *Revista Complutense de Educación*, 27(1), 199-218. https://doi.org/10.5209/rev_RCED.2016.v27.n1.45688
- Browning, C., Edson, A., Kimani, P., & Aslan-Tutak, F. (2014). Mathematical content knowledge for teaching elementary mathematics: A focus on geometry and measurement. *The Mathematics Enthusiast*, 11(2), 333-384. <https://doi.org/10.54870/1551-3440.1306>
- Cáceres, M. J., Chamoso, J. M., & Azcárate, P. (2010). Analysis of the revisions that pre-service teachers of mathematics make of their own project included in their learning portfolio. *Teaching and Teacher Education*, 26, 1186-1195. <https://doi.org/10.1016/j.tate.2010.01.003>
- Castro, W. F., Durando, J., & Pino-Fan, L. R. (2021). Preservice teachers' argumentation and some relationships to didactic-mathematical knowledge features. *Eurasia Journal of Mathematics, Science and Technology Education*, 17(9), Article em2002. <https://doi.org/10.29333/ejmste/11139>
- Chapman, O. (2012). Prospective elementary school teachers' ways of making sense of mathematical problem posing. *PNA*, 6(4), 135-146. <https://doi.org/10.30827/pna.v6i4.6137>
- Chapman, O. (2015). Mathematics teachers' knowledge for teaching problem solving. *LUMAT: International Journal on Math, Science and Technology Education*, 3(1), 19-36. <https://doi.org/10.31129/lumat.v3i1.1049>
- Charalambous, C., & Pitta-Pantazi, D. (2016). Perspectives on priority mathematics education: Unpacking and understanding a complex relationship linking teacher knowledge, teaching, and learning. In L. English, & D. Kirshner (Eds.), *Handbook of international research in mathematics education* (pp. 19-59). Routledge.
- Daher, W., Baya'a, N., & Jaber, O. (2022) Understanding prospective teachers' task design considerations through the lens of the theory of didactical situations. *Mathematics*, 10, Article 417. <https://doi.org/10.3390/math10030417>
- de Gamboa, G., Badillo, E., & Ribeiro, M. (2015). El horizonte matemático en el conocimiento para la enseñanza del profesor: Geometría y medida en educación primaria [The mathematical horizon in knowledge for teacher teaching: Geometry and measurement in primary education]. *PNA*, 10(1), 1-24. <https://doi.org/10.30827/pna.v10i1.6093>
- de Sousa, J. R., Gusmão, T., Font, V., & Lando, J. (2020). Task (re)design to enhance the didactic-mathematical knowledge of teachers. *Acta Scientiae*, 22(4), 98-120. <https://doi.org/10.17648/acta.scientiae.5711>
- Depaepe, F., Verschaffel, L., & Star, J. R. (2020). Expertise in developing students' expertise in mathematics: Bridging teachers' professional knowledge and instructional quality. *ZDM Mathematics Education*, 52(2), 179-192. <https://doi.org/10.1007/s11858-020-01148-8>
- Felton, M. (2010). News & views: Is math politically neutral? *Teaching Children Mathematics*, 17(2), 60-63. <https://doi.org/10.5951/TCM.17.2.0060>
- Fennema, E., & Franke, M. L. (1992). Teacher knowledge and its impact. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 147-164). Macmillan.
- Ferrando, I., Barquero, B., & Segura, C. (2025). Does training matter? Effect of training strategies on how pre-service teachers pose and assess modelling

- problems. *ZDM Mathematics Education*, 57(2), 275-288. <https://doi.org/10.1007/s11858-025-01658-3>
- Galant, J. (2013). Selecting and sequencing mathematics tasks: Seeking mathematical knowledge for teaching. *Perspectives in Education*, 31(3), 34-48.
- Godino, J. D. (2024). *Enfoque ontosemiótico en educación matemática: Fundamentos, herramientas y aplicaciones* [Ontosemiotic approach in mathematics education: Foundations, tools and applications]. McGraw Hill-Aula Magna.
- Godino, J. D., Giacomone, B., Font, V., & Pino-Fan, L. (2018). Conocimientos profesionales en el diseño y gestión de una clase sobre semejanza de triángulos. Análisis con herramientas del modelo DMKC [Professional knowledge in the design and management of a lesson on triangle similarity. Analysis using DMKC model tools.]. *AIEM. Avances de Investigación en Educación Matemática*, 13, 63-83. <https://doi.org/10.35763/aiem.v0i13.224>
- Guberman, R., & Leikin, R. (2013). Interesting and difficult mathematical problems: Changing teachers' views by employing multiple-solution tasks. *Journal of Mathematics Teachers Education*, 16, 33-56. <https://doi.org/10.1007/s10857-012-9210-7>
- Henríquez, C. & Verdugo, P. (2023). Diseño de tareas en la formación inicial docente de matemáticas que involucran las representaciones de una función [Design of tasks in initial mathematics teacher training involving the representations of a function]. *Educación Matemática*, 35(3), 178-208. <https://doi.org/10.24844/EM3503.06>
- Isik, C., & Kar, T. (2012). The analysis of the problems posed by the pre-service teachers about equations. *Australian Journal of Teacher Education*, 37(9), 93-113. <https://doi.org/10.14221/ajte.2012v37n9.1>
- Kara, M., & Corum, K. (2023). Pre-service teachers as researchers: A mentorship model. *International Journal of Education in Mathematics, Science and Technology*, 11(1), 237-251. <https://doi.org/10.46328/ijemst.2351>
- Knott, L., Olson, J., Adams, A., & Ely, R. (2013). Task design: Supporting teachers to independently create rich tasks. In C. Margolinas (Ed.), *Task design in mathematics education: Proceedings of ICMI study 22* (pp. 599-608).
- Leikin, R., Boriskovsky, M., Ovodenko, R., & Miskin, M. (2025). Problem posing or mathematical modeling? The process of expert instructional design. *ZDM Mathematics Education*, 57(2), 333-350. <https://doi.org/10.1007/s11858-025-01668-1>
- Llanos-Lagos, E., Pino-Fan, L., Lugo-Armenta, J., Parra-Urrea, Y., Zapata, J., & Umanzor, R. (2025). Mathematical competencies of prospective teachers when teaching confidence intervals in a microteaching context. *Eurasia Journal of Mathematics, Science and Technology Education*, 21(9), Article em2701. <https://doi.org/10.29333/ejmste/16845>
- Llinares, S., Sánchez-Matamoros, G., & Aguilar, M. S. (2018). *El conocimiento didáctico-matemático del profesor: Avances y retos* [The teacher's didactic-mathematical knowledge: Advances and challenges]. Graó.
- López-Martín, M., Aguayo-Arriagada, C., & García López, M. (2022). Preservice elementary teachers' mathematical knowledge on fractions as operator in word problems. *Mathematics*, 10(3), Article 423. <https://doi.org/10.3390/math10030423>
- Lucey, T., & Maxwell, S. (2016). Teaching mathematical connections to financial literacy in grades K-8: Clarifying the issues. *Investigations in Mathematics Learning*, 3(3), 46-65. <https://doi.org/10.1080/24727466.2011.11790306>
- Mallart, A., Font, V., & Diez, J. (2018). Case study on mathematics pre-service teachers' difficulties in problem posing. *Eurasia Journal of Mathematics, Science and Technology Education*, 14(4), 1465-1481. <https://doi.org/10.29333/ejmste/83682>
- Neubrand, M. (2018). Conceptualizations of professional knowledge for teachers of mathematics. *ZDM Mathematics Education*, 50, 601-612. <https://doi.org/10.1007/s11858-017-0906-0>
- Oliveira, H., Polo-Blanco, I., & Henríquez, A. (2021). Exploring prospective elementary mathematics teachers' knowledge: A focus on functional thinking. *Journal on Mathematics Education*, 12(2), 257-278. <http://doi.org/10.22342/jme.12.2.13745.257-278>
- Petrou, M., & Goulding, M. (2011). Conceptualizing teachers' mathematical knowledge in teaching. In T. Rowland, & K. Ruthven (Eds.), *Mathematical knowledge in teaching* (pp. 9-25). Springer. https://doi.org/10.1007/978-90-481-9766-8_2
- Pino-Fan, L., & Godino, J. (2015). Perspectiva ampliada del conocimiento didáctico-matemático del profesor [Expanded perspective of the teacher's didactic-mathematical knowledge]. *Paradigma*, 36(1), 87-109.
- Pino-Fan, L., Castro, W., & Font, V. (2022) A macro tool to characterize and develop key competencies for the mathematics teacher's practice. *International Journal of Science and Mathematics Education*, 20, 1-26. <https://doi.org/10.1007/s10763-022-10301-6>
- Prediger, S., Erath, K., & Quabeck, K. (2024). Effects of interaction qualities beyond task quality: Disentangling instructional support and cognitive demands. *International Journal of Science and Mathematics Education*, 22, 885-909. <https://doi.org/10.1007/s10763-023-10389-4>
- Rebello, N. S., Cui, L., Bennett, A. G., Zollman, D. A., & Ozimek, D. J. (2017). Transfer of learning in problem solving in the context of mathematics and

- physics. In D. Jonassen (Ed.), *Learning to solve complex scientific problems* (pp. 223-246). Routledge. <https://doi.org/10.4324/9781315091938-10>
- Rowland, T. (2013). The knowledge quartet: The genesis and application of a framework for analysing mathematics teaching and deepening teachers' mathematics knowledge. *Sisyphus-Journal of Education*, 1, 15-43. <http://doi.org/10.25749/sis.3705>
- Ruz, F., Molina-Portillo, E., & Contreras, J. M. (2020). Evaluación de conocimientos sobre el contenido de estadística descriptiva de futuros profesores de matemáticas [Assessment of knowledge of descriptive statistics content of future mathematics teachers]. *AIEM. Avances de Investigación en Educación Matemática*, (18), 55-71. <https://doi.org/10.35763/aiem.v0i18.268>
- Salgado, F. J. A. (2017). The role of context and context familiarity on mathematics problems. *RELIME-Revista Latinoamericana de Investigación en Matemática Educativa*, 20(3), 265-292. <https://doi.org/10.12802/relime.17.2031>
- Santa, Z., Jaramillo, C., & Gualdrón, É. (2018). Colectivo de profesores con doblado de papel en tareas de geometría escolar [Group of teachers using paper folding in geometry homework]. *Bolema: Boletim de Educação Matemática*, 32(62), 1092-1112. <https://doi.org/10.1590/1980-4415v32n62a17>
- Shawer, S. (2017). Teacher-driven curriculum development at the classroom level: Implications for curriculum, pedagogy and teacher training. *Teaching and Teacher Education*, 63, 296-313. <https://doi.org/10.1016/j.tate.2016.12.017>
- Silva, R., Costa, C., Freitas, Y., Martins, F., & Cebrián-de-la-Serna, M. (2024). Educational robotics and primary school mathematics teaching: An analysis of pre-service teachers didactic-mathematical knowledge. *Eurasia Journal of Mathematics, Science and Technology Education*, 20(10), Article em2515. <https://doi.org/10.29333/ejmste/15199>
- Singer, F.M., & Voica, C. (2013). A problem-solving conceptual framework and its implications in designing problem-posing tasks. *Educational Studies in Mathematics*, 83, 9-26. <https://doi.org/10.1007/s10649-012-9422-x>
- Stacey, K., & Turner, R. (2015). The evolution and key concepts of the PISA mathematics frameworks. In K. Stacey, & R. Turner (Eds.), *Assessing mathematical literacy* (pp. 5-33). Springer. https://doi.org/10.1007/978-3-319-10121-7_1
- Stein, M. K., & Smith, M. S. (1998). Mathematical tasks as a framework for reflection: From research to practice. *Mathematics Teaching in the Middle School*, 3(4), 268-275. <https://doi.org/10.5951/MTMS.3.4.0268>
- Sullivan, P., Clarke, D., Clarke, B., & O'Shea, H. (2010). Exploring the relationship between task, teacher actions, and student learning. *PNA*, 4(4), 133-142. <https://doi.org/10.30827/pna.v4i4.6163>
- Sullivan, P., Knott, L., & Yang, Y. (2015). The relationships between task design, anticipated pedagogies, and student learning. In A. Watson, & M. Ohtani (Eds.), *Task design in mathematics education. New ICMI study series* (pp. 83-114). Springer. https://doi.org/10.1007/978-3-319-09629-2_3
- Vargas, J., Vanegas, Y., & Giménez, J. (2023). Análisis de conocimientos didáctico-matemáticos sobre clasificación de poliedros con futuros maestros de educación primaria [Analysis of didactic-mathematical knowledge about the classification of polyhedra with future primary education teachers]. *Paradigma*, 44(4), 293-320. <https://doi.org/10.37618/PARADIGMA.1011-2251.2023.p293-320.id1404>
- Vargas, J., Vanegas, Y., & Giménez, J. (2024). Conexiones extra-matemáticas que establecen futuros maestros de Educación Primaria al diseñar tareas escolares geométricas [Extra-mathematical connections established by future primary school teachers when designing geometric school assignments]. *AIEM - Avances de investigación en educación matemática*, 25, 57-80. <https://doi.org/10.35763/aiem25.6441>
- Vieira, G., Monteiro, R. & Gómez, N. (2013). Simetria no ensino fundamental através da resolução de problemas: Possibilidades para um trabalho em sala de aula [Symmetry in elementary education through problem-solving: Possibilities for classroom work]. *Bolema: Boletim de Educação Matemática*, 27(46), 613-630. <https://doi.org/10.1590/S0103-636X2013000300018>
- Wakhata, R., Mutarutinya, V., & Balimuttajjo, S. (2023). Exploring the impact of Stein et al.'s levels of cognitive demand in supporting students' mathematics heuristic problem-solving abilities. *Frontiers in Education*, 8. <https://doi.org/10.3389/educ.2023.949988>
- Watson, A., & Mason, J. (2006). Seeing an exercise as a single mathematical object: Using variation to structure sense-making. *Mathematics Thinking and Learning*, 8(2), 91-111. https://doi.org/10.1207/s15327833mtl0802_1
- Watson, A., & Ohtani, M. (2015). Themes and issues in mathematics education concerning task design: Editorial introduction. In A. Watson, & M. Ohtani (Eds.), *Task design in mathematics education. New ICMI study series* (pp. 3-15). Springer. https://doi.org/10.1007/978-3-319-09629-2_1
- Yin, R. (2014). *Case study research: Design and methods*. SAGE.