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Building network of relationships between teachers' mathematical knowledge for teaching fractions and teaching practices

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Abstract

This study sought to explore a network of relationships between pre-service teachers' (PSTs') mathematical knowledge for teaching fractions (MKTF) and their teaching practices. It was based on the mathematical knowledge for teaching framework and the mathematical task framework that identify teacher knowledge domains and teaching practices required for quality teaching of mathematics. Data was collected from 171 PSTs using two instruments: MKTF test and teaching practices test. Though the results showed significant correlations among several domains of the PSTs MKTF, the study identified knowledge of content of fractions and students as the knowledge domain that appeared to find its focus in the future teachers' minds that connect to all their MKTF domains. Furthermore, out of the five constructs on teaching practices, the study identified: using representations and responding to student's requests for help as practices with the pronounced influence on the teaching practices of future teachers as they correlated with majority of the constructs defining the PSTs teaching practices. The findings of this study have implications for the training of PSTs in Ghana as well as countries with similar contexts.

Keywords: mathematical knowledge for teaching, teaching practices, mathematical task framework, network of relationships

INTRODUCTION

Teacher knowledge is the focus of which the content of teacher education and the knowledge possessed by teachers can be examined (Neubrand, 2018). It is reasonable that the content of teacher education impact on future teachers' knowledge which intends, impacts their teaching practices and consequently on students learning and achievements in mathematics (Fung et al., 2017; Hill et al., 2005; Schmidt et al., 2011; Tchoshanov et al., 2017). The results of the first teacher education and development study in mathematics (TEDS-M) showed that teacher knowledge has an effect on students' achievements in mathematics. Moreover, the results of trends in international mathematics and science study (TIMSS) are better in countries where teachers' knowledge is high (Schmidt et al., 2011). Researchers have agreed that teachers need strong subject knowledge, and pedagogical knowledge to be able to teach effectively.

With the existence of several teacher knowledge frameworks that describe the knowledge domains required for effective teaching, it is not clear how the domains of teacher knowledge are interrelated (Baumert & Kunter, 2013; Hashweh, 2005). According to Fennema and Franke (1992), teacher knowledge domains actively influence each other. Ball et al. (2008) also noted that many of the mathematics teaching demands require knowledge of the intersection of the six knowledge domains of the mathematical knowledge for teaching (MKT). Hurrel (2013) have re-conceptualized the six MKT domains by Ball et al. (2008) by showing connections between these knowledge domains. In their network analysis, Kooponen et al. (2019) identified interconnections between MKT domains.

Research (Ball et al., 2008; Hoover et al., 2016) also acknowledge MKT as a construct in teaching, which has an immense contribution to the quality of mathematics instruction. These discussions suggest that along teachers' mathematical knowledge, teachers' teaching practices are key in order to produce lessons in which learners will be exposed to high quality tasks that help them to learn concepts and procedures in mathematics with understanding (cf. Addae & Agyei, 2018). This in

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Contribution to the literature

- The study reveals a large network of relationships between and among five teaching practice constructs and six domains of mathematical knowledge for teaching fractions (MKTF). It identifies the "knowledge of content of fractions and students" as the knowledge domain that played a significant role in relation to the totality of MKTF domains.
- Among the teaching practice constructs, the study identifies the practices of "using representations" and "responding to student's requests for help" as the practices that appeared to find their focus in the pre-service teachers' (PSTs') teaching practices.
- Finally, the study identifies "special content knowledge of fractions" and "knowledge of content of fractions and curriculum" as the domains of mathematical knowledge that find their focus in PSTs' knowledge domains as having relationships with all PSTs' teaching practices.

turn produces in learners' self-confidence to engage in challenging mathematical tasks that are provided in a rich mathematics curriculum (NCTM, 2000). Charalambous (2008) explored five teaching practices (selecting and using tasks, using representations; providing explanations, responding to students' requests for help, and analyzing student's work/contributions) that pre-service mathematics teachers require for quality teaching but did not show how these five teaching practices were interconnected. However, he identified the five teaching practices as composite practices since according to him, each practice involves other practices. For example, he describes explanations to involve the use of representations to model the task for easy understanding.

The implication, therefore, is that these teaching practices are somehow interrelated though Charalambous (2008) did not explore that in his study. This study, therefore, sought to explore the relationships if any, among the five teaching practice constructs and the six domains of mathematical knowledge for teaching mathematics. Furthermore, the relationship between MKT domains and the five teaching practice constructs will also be explored. Similar to Charalambous' (2008) study, the branch of mathematics the study focused on was fractions. The choice of fractions was based on the fact that students' knowledge of fractions is important in order to obtain success in algebra and beyond (Van de Walle, 2010). Again, within the context of this study, West Africa Examination Council (WAEC) chief examiner's reports have identified weaknesses in students' performance in fractions for several years (WAEC, 2019). It is therefore quite critical that teachers teach and present fractions as fascinating and relevant and commit to the task of helping students to understand the big concepts and ideas in fractions. The study, therefore, focused on fractions as the mathematical content knowledge in order to investigate it in more depth how teachers' knowledge of this content is connected to their decisions and actions concerning the five teaching practices.

TOWARDS A CONCEPTUAL FRAMEWORK FOR THE STUDY

Mathematical knowledge for teaching is a practice-based theory of mathematics teacher knowledge (Ball et al., 2008; Hill et al., 2008a, 2008b). Ball et al. (2008) have conceptualized MKT to include six knowledge domains: *common content knowledge (CCK), specialized content knowledge (SCK), horizon content knowledge (HCK), knowledge of content and students (KCS), knowledge of content and teaching (KCT) and knowledge of content and curriculum (KCC)*. Although MKT is quite a popular framework for describing teachers' knowledge, several questions still require further attention:

(a) What kind of teaching tasks require which domain?

(b) What is the relationship between MKT domains?

(c) What are the exact definitions of MKT domains? (Ball et al., 2008; Markworth et al., 2009).

Mathematical task framework (MTF) also identifies instructional tasks through three phases of the instructional process of planning, presenting, and enactment of tasks (Stein & Smith, 1998; Stein et al., 2000). Stein and Smith (1998) noted that students are involved in two types of thinking based on whether the students are made to memorize procedures in a routine manner (instrumental thinking), or students are made to think conceptually and make connections in a given task (relational thinking). This means that what pupils learn is shaped by how tasks are chosen and enacted during instruction. Charalambous (2008), drew on the MTF and identified some teaching practices under the three phases that instructional tasks pass through. These teaching practices as were applied in the study were: *selecting and using tasks; using representations; providing explanations; responding to students' direct or indirect requests for help; and analyzing students' work and contributions*, which are considered to enhance quality mathematics teaching. During the planning phase, teachers are supposed to perform certain practices (e.g., selection of instructional tasks, modifying/adapting instructional tasks, sequencing instructional tasks,

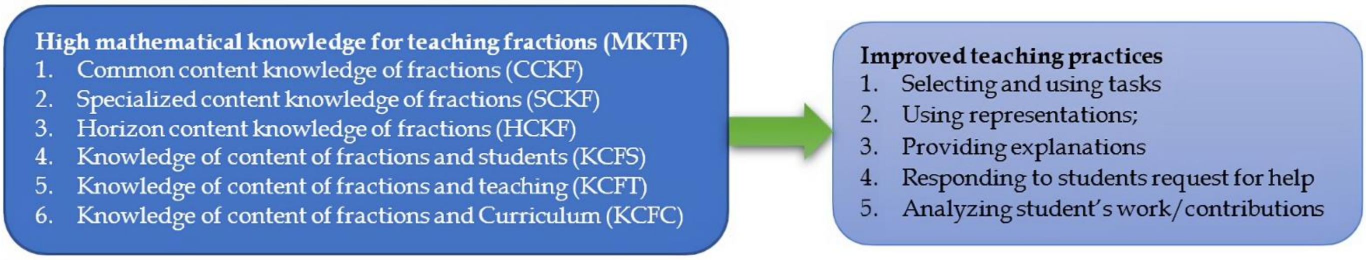

Figure 1. Conceptual framework of the study (Source: Authors' own elaboration)

anticipating students' errors or difficulties) and design a lesson plan.

At the presentation phase, teachers are required to use appropriate teaching practices (e.g., presenting definitions, giving explanations, providing examples and counter-examples, using analogies, using representations and manipulative, establishing connections among different ideas and representations, and simplifying tasks to support students' success) in order to present the content or tasks to their students. Moreover, at the enactment phase, teachers collaborate with pupils on assigned activities or tasks making use of certain teaching practices (e.g., responding to requests for help, following and analyzing students thinking, identifying student errors, understanding students' alternative approaches, asking probing questions, orchestrating the sharing of multiple ideas/solutions) (Fumador & Agyei, 2018). Charalambous (2008) noted that though these three phases are discussed and presented separately, there are no clear boundaries between them. Putting the two theories of MKT and MTF together, the study adapted a conceptual framework as shown in **Figure 1**.

From **Figure 1**, the study hypothesizes that there are interconnections among and between mathematical knowledge for teaching fractions (MKTF) domains and teaching practices constructs and that there are positive relationships between domains of MKTF; constructs on teaching practices; or MKTF domains and teaching practices constructs.

In this study the MKTF domains are explained, as follows:

- *Common content knowledge of fractions (CCKF)* is the competent knowledge of fractions, which includes knowledge of concepts, terms, definitions, rules and symbols used in fractions and common to all workers who use fractions.
- *Specialized content knowledge of fractions (SCKF)* is the knowledge about multiple solution strategies, making generalizations, determining why an algorithm works, or makes sense, explaining concepts by using suitable examples and representations to visualize fractions, making connections between different representations, and identifying practical definitions.
- *Horizon content knowledge of fractions (HCKF)* in this study refers to knowledge and understanding of how topics within the mathematics curriculum are related so they can make connections to topics when teaching fractions.
- *Knowledge of content of fractions and students (KCFS)* refers to knowledge of how students learn fractions including: knowledge of common errors, misconceptions and difficulties of students in learning fractions.
- *Knowledge of content of fractions and teaching (KCFT)* as used in the present study refers to knowledge of content of fractions and different teaching strategies needed to effectively teach fractions.
- *Knowledge of content of fractions and curriculum (KCFC)* is the knowledge about the contents and topics and the organization of these contents and topics that are needed for teaching fractions at the basic level.

The teaching practices constructs are also explained, as follows:

- *Selection and using instructional tasks* refer to the ability to choose, adapt and sequence instructional tasks that challenge the cognitive demands of students to learn and make connections for conceptual understanding.
- *Providing explanations* is a teacher's ability to deliver clear explanations that help students comprehend the mathematics being taught. This is whereby a teacher crafts and presents clear mathematics examples, counter-examples, and makes analogies that are understandable to students.
- *Using representations* is the ability to enhance student learning by working with and around representational modes.
- *Analyzing students' work and contributions* is the ability to evaluate students' explanations and decipher what they say determining the soundness of students' mathematical strategies and non-routine approaches to problem-solving, and determining what students know and their knowledge gaps based on their work and contributions or their errors.

- *Responding to student's requests for help* refers to a teacher's ability to respond and attend to students' requests either expressed directly or indirectly.

RESEARCH DESIGN AND QUESTIONS

The study employed a correlational research design to collect and analyze data obtained from the respondents. The study addressed three main research questions:

1. What relationship exists among the domains of pre-service teachers' (PSTs') mathematical knowledge for teaching?
2. What relationship exists among the five constructs of teaching practices reported by PSTs?
3. What is the relationship between PSTs' mathematical knowledge of teaching domains and their teaching practice constructs?

Respondents

The targeted population of the study comprised of pre-service mathematics teachers in the 46 public colleges of education in Ghana. The accessible population was made of pre-service mathematics teachers from five colleges of education that were conveniently selected. The five colleges were conveniently sampled in order to prevent the risks of travelling long distances in the midst of COVID-19 pandemic. The stratified random sampling procedure was used to select 171 out of 1,445 pre-service mathematics teachers from the five colleges of education to constitute the sample for the study.

Instruments

Two tests: Mathematical knowledge for teaching fractions test and teaching practices test were used to collect data from the 171 PSTs about their MKTF domains and their teaching practices constructs respectively. These instruments are discussed in the subsequent sections below.

Mathematical knowledge for teaching fractions test

The researcher adapted the online sample of the learning mathematics for teaching (LMT) test items by Hill et al. (2004) to examine PSTs' MKT in fractions. The online LMT sample test items contain 64 test items on number, algebra and operations. Upon analyses of this instrument, several of the items were found to be irrelevant to the current study. Thus, 11 items pertaining to fractions were selected, modified and used in the study. The online LMT test items contain items that could measure four knowledge domains of MKT: CCK, SCK, KCT, and KCS. Through a review of previous studies (Avcu, 2019; Ball et al., 2008; Cole, 2012; Sugilar, 2016; Shulman, 1986), which highlighted the concepts and skills that instructors must master in order to teach fractions properly, we were able to expand the LMT items by 33 to cover all the six knowledge domains of MKT (Ball et al., 2008) adequately.

The revised and adapted LMT test items known as mathematical knowledge for teaching fractions test comprised of closed-ended questions. PSTs' responses for each item on the MKTF test were scored dichotomously on a 2-point scale: 0 for a wrong response and 1 for a correct response. In all, 31 out of 44 items of the MKTF test were scored and grouped along the six MKTF domains: CCKF (eight items), SCKF (six items), HCKF (three items), KCFT (three items), KCFS (four items), and KCFC (seven items). The total score of each MKTF domain was standardized to the same scale maximum value of eight points for easy comparison. A score of four was considered as the average score point value. Getting a score of four and above was considered to be a high MKTF score while getting a score below four was considered as a low MKTF score. The Kuder-Richardson reliabilities of the MKTF domains ranged from 0.64 to 0.82 (*CCKF*, $\alpha = 0.72$; *SCKF*, $\alpha = 0.70$; *HCKF*, $\alpha = 0.82$; *KCFT*, $\alpha = 0.64$; *KCFS*, $\alpha = 0.73$; and *KCFC*, $\alpha = 0.79$), exceeding the acceptable threshold value of 0.60.

Teaching practices test

Teaching practices were measured to include how teachers demonstrate the three skills of noticing, evaluating and performing the five teaching practices of selecting and using tasks, using representations, providing explanations, responding to student's requests for help and analyzing student's work/contributions. Teaching practice test (Charalambous, 2008) was adapted for the study. Charalambous (2008) explored performance of 20 PSTs in five selected teaching practices through the use of an interview guide that consisted of 24 items. The adapted teaching practices test in this study, however, comprised of 27 test items, which respondents were asked to provide answers at some points concerning what they noticed, how they interpret and how they would have performed such practices. The test was accompanied by a lesson script that contained the five teaching practices as used by a teacher in a lesson on division of fractions. The PSTs were asked to read the lesson script and answer test questions on what teaching practices they notice, how they interpret or evaluate and how they will perform the teaching practices they observed.

The test, known as the teaching practices test, consisted of closed-ended questions. PSTs' responses for each item of the teaching practices test were scored dichotomously on a 2-point scale of 0 and 1, where a zero (0) score point indicated a wrong response and a one (1) score point indicated a correct response. Twenty out of twenty-seven items of the teaching practices test were scored and grouped along the five constructs of teaching

Table 1. Descriptive statistics of the six MKTF domains (n=171)

MKT domain	Mean	Standard deviation
Horizon content knowledge of fractions (HCKF)	4.83	3.342
Knowledge of content of fractions and curriculum (KCFC)	4.54	2.350
Knowledge of content of fractions and teaching (KCFT)	3.58	2.020
Special content knowledge of fractions (SCKF)	3.36	2.109
Common content knowledge of fractions (CCKF)	3.22	2.381
Knowledge of content of fractions and students (KCFS)	2.91	2.051

Table 2. Descriptive statistics of the five teaching practices (n=171)

Teaching practices	Mean	Standard deviation
Tasks	2.17	1.371
Explanations	2.04	1.441
Analyzing	2.03	1.708
Representations	1.48	1.699
Requests	1.26	1.320

practices: selecting and using tasks (six items), using representations (four items), providing explanations (four items), responding to students' direct or indirect requests for help (three items), and analyzing students' work and contributions (three items). For easy comparison of the scores of teaching practices constructs, the total score of each construct was standardized to the same scale maximum value of six points. Getting a score of three was considered the average score point value. A high score in teaching practices was therefore interpreted to mean getting a score of three and above while a low score in teaching practices was interpreted to mean getting a score below three. The Kuder-Richardson reliabilities of three of the teaching practices constructs (providing explanations, $\alpha = 0.61$; analyzing student's work/contributions, $\alpha = 0.81$; and using representations, $\alpha = 0.68$) ranged from 0.61 to 0.81; exceeding the acceptable threshold value of 0.60, whiles the Kuder-Richardson reliabilities for the remaining two teaching practices constructs (selecting and using tasks, $\alpha = 0.54$; and responding to students requests for help, $\alpha = 0.51$), which did not meet the acceptable threshold of 0.60; where later accepted by the researchers as having moderate reliabilities based on Hinton et al. (2014), guide concerning appropriate cut-off points for reliability coefficients.

Data Analysis

This study utilized the positivist approach to analyze numerical data about MKTF and teaching practices from a sample of 171 PSTs. The researchers used both descriptive (mean and standard deviation) and inferential (correlation) statistical methodologies to analyze and explore the explanatory linkages between variables in the study with the goal of understanding the relationship between PSTs' MKTF and their teaching practices.

RESULTS

Before finding the correlations between the variables, we performed descriptive analyses to obtain the mean and standard deviation of MKTF domains and the teaching practices constructs. The results of the descriptive analyses are shown in **Table 1** and **Table 2**.

Table 1 shows the descriptive statistics of the six domains of MKTF for all the 171 PSTs.

The mean scores of MKT domains ranged from 0.291 to 4.83. The results show a spread in the scores of the various domains of MKTF, which ranged from 2.020 to 3.342. The mean scores (*HCKF*, M=4.83; *KCFC*, M=4.54) of the PSTs indicated that PSTs, on the average, performed high in these two MKTF domains compared to the average score point value of four. However, compared to the average score point value of four, the PSTs mean scores (*KCFT*, M=3.58; *SCKF*, M=3.36; *CCKF*, M=3.22; and *KCFS*, M=2.91) in these four domains were low. Moreover, PSTs obtained the least average score in KCFS.

Table 2 present the descriptive statistics of the five constructs of teaching practices for all the 171 PSTs.

From **Table 2**, PSTs obtained average scores in the five constructs of teaching practices ranging from 1.26 to 2.17. The PSTs scores showed a spread in the scores of the five constructs of teaching practices, which ranged from 1.320 to 1.708. The mean scores (tasks, M=2.17; explanations, M=2.04; analyzing, M=2.03; representations, M=1.48; and requests, M=1.26) of the PSTs indicated that PSTs, on the average, performed low in all the five teaching practices constructs compared to the average score point value of three. The least average score in teaching practices of the PSTs was scored in responding to student's requests for help.

Correlations Between Mathematical Knowledge for Teaching Fractions Domains

To explore the relationship between PSTs' mathematical knowledge for teaching domains, correlation analyses were performed. **Table 3** shows the results of these analyses.

From **Table 3**, the results have shown significant positive correlations between PSTs KCFC and four

Table 3. Correlations among PSTs MKTF domains (n=171)

		CCKF	SCKF	HCKF	KCFT	KCFS	KCFC
CCKF	Pearson correlation	1					
	Sig. (p-value)						
SCKF	Pearson correlation	.330**	1				
	Sig. (p-value)	.000					
HCKF	Pearson correlation	.116	.252**	1			
	Sig. (p-value)	.132	.001				
KCFT	Pearson correlation	-.069	.048	.249**	1		
	Sig. (p-value)	.372	.530	.001			
KCFS	Pearson correlation	.333**	.530**	.341**	-.151*	1	
	Sig. (p-value)	.000	.000	.000	.048		
KCFC	Pearson correlation	.248**	.446**	.471**	.088	.422**	1
	Sig. (p-value)	.001	.000	.000	.251	.000	

Note. **Correlation is significant at the 0.01 level (2-tailed) & *Correlation is significant at the 0.05 level (2-tailed)

Table 4. Correlations among PSTs teaching practices constructs (n=171)

		Tasks	Representations	Requests	Analyzing	Explanations
Tasks	Pearson correlation	1				
	Sig. (2-tailed)					
Representations	Pearson correlation	.487**	1			
	Sig. (2-tailed)	.000				
Requests	Pearson correlation	.239**	.346**	1		
	Sig. (2-tailed)	.002	.000			
Analyzing	Pearson correlation	-.013	.135	.322**	1	
	Sig. (2-tailed)	.862	.079	.000		
Explanations	Pearson correlation	.093	.245**	.017	.059	1
	Sig. (2-tailed)	.226	.001	.824	.446	

Note. **Correlation is significant at the 0.01 level (2-tailed)

domains of MKTF: CCKF (r=0.248, p=0.001); SCKF (r=0.446, p=0.000); HCKF (r=0.471, p=0.000); and KCFS (r=0.422, p=0.000). This means an increased knowledge in KCFC was positively related to an increase in knowledge of CCKF, SCKF, HCKF, and KCFC. However, the PSTs' KCFC did not show any significant correlation (r=0.088; p=0.251) with their KCFT.

PSTs KCFS positively correlated with: CCKF (r=0.333, p=0.000); SCKF (r=0.530, p=0.000); HCKF (r=0.341, p=0.000); and KCFC (r=0.422, p=0.000) and negatively correlated with KCFT (r=-0.151, p=0.048). This means that an increased knowledge in KCFS of the PSTs was positively connected to an increased knowledge in their CCKF, SCKF, HCKF, and KCFC; and negatively connected to a decrease in their KCFT. The study has therefore shown that the PSTs' KCFS was connected to all the domains of MKTF.

The results of the study have further shown: PSTs' KCFT as having positive correlation (r=0.249, p=0.001) with their HCKF and having negative correlation (r=-0.151, p=0.048) with KCFS; PSTs' HCKF as having positive correlations with KCFC (r=0.471, p=0.000), KCFS (r=0.341, p=0.000), KCFT (r=0.249, p=0.001), and SCKF (r=0.252, p=0.001); PSTs' SCKF as having positive correlations with CCKF (r=0.330, p=0.000), HCKF (r=0.252, p=0.001), KCFS (r=0.530, p=0.000), and KCFC (r=0.446, p=0.000); and PSTs' CCKF as having positive correlations with SCKF (r=0.330, p=0.000), KCFS (r=0.333, p=0.000), and KCFC (r=0.248, p=0.001).

The study identified significant correlations between MKTF domains. This study has identified KCFS as the domain of MKTF that formed the center of the relationships between MKTF domains since it was the only domain that was linked to all the other domains. From the perspective of PSTs' MKTF domains, the knowledge about the content of fractions and students (KCFS) therefore appeared to find its focus in the future teachers' minds as the domain that correlate with all other MKTF domains. The result has shown PSTs' KCFT as the domain that least correlated with the domains of MKTF and negatively correlated with the PSTs KCFS domain.

Correlations Between Teaching Practices Constructs

Correlation analyses were further performed to explore the relationship between PST's teaching practices constructs. The results of these analyses are presented in **Table 4**.

From **Table 4**, the results: representations (r=0.487, p=0.000); and requests (r=0.239, p=0.002) showed significant positive correlations between the PSTs' practice of selecting and using tasks and their practices of using representations and responding to students' requests for help respectively. This means that a good

Table 5. Correlations between PSTs' mathematical knowledge for teaching fractions and their teaching practices

		Tasks	Representations	Explanations	Requests	Analyzing
CCKF	Pearson correlation	.207**	.350**	.254**	.144	.326**
	Sig. (p-value)	.007	.000	.001	.060	.000
SCKF	Pearson correlation	.187*	.417**	.169*	.309**	.327**
	Sig. (p-value)	.014	.000	.027	.000	.000
HCKF	Pearson correlation	.145	.294**	.255**	.258**	.299**
	Sig. (p-value)	.059	.000	.001	.001	.000
KCFT	Pearson correlation	-.053	.023	.225**	.098	.261**
	Sig. (p-value)	.494	.765	.003	.202	.001
KCFS	Pearson correlation	.382**	.499**	-.005	.456**	.272**
	Sig. (p-value)	.000	.000	.949	.000	.000
KCFC	Pearson correlation	.257**	.369**	.274**	.268**	.211**
	Sig. (p-value)	.001	.000	.000	.000	.006

performance in selecting and using tasks is positively connected to having a good performance in the practices of using representations and responding to student's requests for help. This is meaningful since a teacher needs to be able to select appropriate tasks first before using the right representations to present the tasks to students and also respond to students' challenges that they faced in learning to understand the tasks.

The correlation analysis also revealed a significant positive relationship between the PSTs' practice of using representations and their practices of selecting and using tasks (r=0.487, p=0.000); responding to students' requests for help (r=0.346, p=0.000); and providing explanations (r=0.245, p=0.000), respectively. This indicates that an improvement in PSTs' practice of using representations is connected to having an improvement in their practices of selecting and using tasks, responding to student's requests for help and providing explanations. This makes meaning as the practice of using representations is a composite practice, which also involves providing explanations, selection and using tasks and responding to student's requests for help.

Between the PSTs' teaching practices, the results again showed significant positive correlations between the PSTs' practice of responding to student's requests for help and their practices of selecting and using tasks (r=0.239, p=0.002); using representations (r=0.346, p=0.000); and analyzing students work and contributions (r=0.322, p=0.000). This is true since a teacher has to be able to respond to student's requests for help, which has a bearing on the teachers' selection and use of tasks, use of representations to be able to assist the students to overcome their challenges and also be able to analyze student's work and contributions in order to identify students' challenges and offer them the needed assistance.

Additionally, the results (r=0.322; p=0.000) revealed a significant positive correlation between PSTs' practice of analyzing student's work and contributions and their practice of responding to student's requests for help. This means that a teacher should have the ability to analyze student's work and contribution, which is connected to the ability to respond to student's requests for help. This is meaningful since a teacher need to analyze first student's work and contributions and identify challenges faced by these students in order to assist them to overcome their challenges. Moreover, between the PSTs' teaching practices, the correlation (r=0.245; p=0.001) showed a significant positive relationship between the PSTs' practice of providing explanations and their practice of using representations. This makes sense since providing explanation is a composite practice, which also involves using representations to help students understand mathematical tasks.

The study has found significant positive correlations existing between PSTs' teaching practices constructs. The study also revealed PSTs' practice of analyzing student's work and contributions and providing explanations as the practices that least correlated with the PSTs' teaching practices constructs. Moreover, from this study and the perspective of PSTs' teaching practices, the practices of using representations and responding to students' requests for help appeared to have a great influence on the teaching practices of PSTs since they correlated with majority of the PSTs' teaching practices.

Correlations Between Mathematical Knowledge for Teaching Fractions Domains and Teaching Practices Constructs

Further correlation analyses were performed in order to explore the relationship between PSTs' mathematical knowledge for teaching domains and teaching practice constructs. The results of these analyses are shown in **Table 5**. From **Table 5**, the results showed significant positive correlations between the PSTs' CCKF domain of MKTF and their teaching practices of: selecting and using tasks (r=0.207, p=0.007); using representations (r=0.350, p=0.000); providing explanations (r=0.254, p=0.001); and analyzing students' work and contributions (r=0.326, p=0.000) respectively. This suggests that having an improved CCKF is connected to an improvement in PSTs' selection and using tasks, using appropriate representations, providing better explanations, and analyzing students' work and contributions.

The correlation analyses revealed a significant positive relationship between PSTs' SCKF domain of MKTF and their teaching practices: selecting and using tasks (r=0.187, p=0.014); using representations (r=0.417, p=0.000); providing explanations (r=0.169, p=0.027); responding to students' direct or indirect requests for help (r=0.309, p=0.000); and analyzing students' work and contributions (r=0.327, p=0.000). This means that having an improved SCKF is connected to having an improved performance in all the five teaching practice constructs.

The results also showed significant positive correlations between the PSTs' HCKF domain of MKTF and four of the constructs of teaching practices: using representations (r=0.294, p=0.000); providing explanations (r=0.255, p=0.001); responding to students' direct or indirect requests for help (r=0.258, p=0.001); and analyzing students' work and contributions (r=0.299, p=0.000), respectively. This appears to suggest that a high HCKF is needed to be able to perform the teaching practices of using representations; providing explanations; responding to students' direct or indirect requests for help; and analyzing students' works and contributions.

With respect to PSTs KCFT domain of MKTF and the teaching practices constructs, the results revealed significantly low positive correlations between the PSTs' KCFT and their teaching practices of providing explanations (r=0.225, p=0.003); and analyzing students' work and contributions (r=0.261, p=0.001) respectively. This suggests that PSTs improved knowledge in KCFT was connected to the effective performance of their teaching practices of providing explanations; and analyzing students' work and contributions.

The correlation analyses further showed a significant positive relationship between PSTs' KCFS domain of MKTF, and four teaching practices constructs: selecting and using tasks (r=0.382, p=0.000); using representations (r=0.499, p=0.000); responding to students' direct or indirect requests for help (r=0.456, p=0.000); and analyzing students' works and contributions (r=0.272, p=0.000), respectively. This means that having an increased knowledge in KCFS was linked to successfully performing the teaching practices of selecting and using tasks; using representations; responding to students' direct or indirect requests for help; and analyzing students' work and contributions.

With regard to the PSTs KCFC domain of MKTF and the teaching practices constructs, the results revealed significant positive correlations between PSTs' KCFC and the five constructs of the PSTs teaching practices: selecting and using tasks (r=0.257, p=0.001); using representations (r=0.369, p=0.000); providing explanations (r=0.274, p=0.000); responding to students' direct or indirect requests for help (r=0.268, p=0.000); and analyzing students' works and contributions (r=0.211, p=0.006), respectively. This indicates that a high knowledge KCFC was connected to a PST's ability to effectively perform all the five teaching practices constructs.

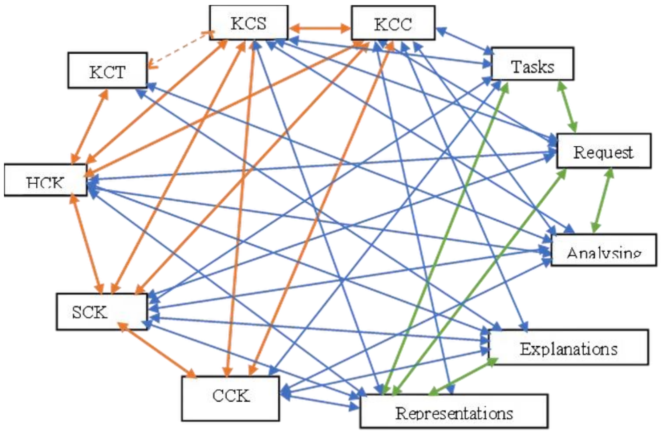

Figure 2. Network of relationships for conceptualization of PSTs MKT & teaching practices (Source: Authors' own elaboration)

From this study, we have identified that there are interconnected relationships among MKTF domains and teaching practices constructs. Again, SCKF and KCFC domains of MKTF find their focus in PSTs' knowledge domains since they correlated with all PSTs teaching practices constructs. From this perspective, it appears SCKF and KCFC knowledge domains are greatly required to perform teaching practices that will bring about mathematics teaching quality.

Network of Relationships

The domains of MKTF of PSTs' and their mathematics teaching practices were joined into a single large network. The network show six different domains of MKTF and five different constructs of teaching practices. **Figure 2** represents how the six domains of MKTF, and the five constructs of teaching practices are interconnected and form a large network. In **Figure 2**, there are arrows used to connect variables that show significant correlations. The arrows are organized into three colors and the colors show whether connections were between MKTF domains (red); teaching practices constructs (green); or MKTF domains and teaching practices constructs (blue). A continuous arrow shows a positive correlation, and a dotted arrow shows a negative correlation. The network analysis shows significant relationships between MKTF domains, between teaching practice constructs and between MKTF domains and teaching practices, which could be used for future conceptualization of teacher's MKT and teaching practices.

With respect to PSTs MKTF domains, the network analysis showed that PST's CCKF positively related to three domains of MKTF (SCKF, KCFS, and KCFC) and four teaching practices constructs (representations, explanations, analyzing and tasks). The analysis also showed that the PST's SCKF domain had significant positive relationships with four MKTF domains (CCKF, HCKF, KCFS, and KCFC) and all the five constructs of teaching practices (representations, explanations, analyzing, requests and tasks). The results further revealed that PST's HCKF positively related with four domains of MKTF (SCKF, KCFT, KCFS, and KCFC) and four constructs of teaching practices (representations, explanations, analyzing, and requests). Additionally, the analysis showed PST KCFT as having significant positive relationships with one MKTF domain (HCKF) and two teaching practice constructs (explanations and analyzing). PST's KCFT also showed significant negative relationship with their KCFS domain of MKTF. Furthermore, the results showed that PSTs KCFS positively related with four MKTF domains (CCKF, SCKF, HCKF, and KCFC) and four teaching practices constructs (representations, analyzing, requests and tasks). PST's KCFS again showed a negative relationship with their KCFT domain of MKTF. Moreover, the network analysis showed PSTs KCFC as having significant relationships with four domains of MKTF (CCKF, SCKF, HCKF, and KCFS) and all the five constructs of teaching practices (representations, explanations, analyzing, requests and tasks). Concerning the MKTF domains, the results, therefore, showed PST's KCFT domain of MKTF as having the weakest link in the network of relationships whilst PST's SCKF, KCFS, and KCFC were the strongest links in the network analysis. This is true since knowledge of content, students and curriculum are needed first by teachers before they can understand the knowledge of effective teaching.

Regarding PSTs teaching practices, the network analysis showed that PST's practice of selecting and using tasks was positively related to two constructs of teaching practices (representations and requests) and four domains of MKTF (CCKF, SCKF, KCFS, and KCFC). The results also showed that PST's practice of responding to student's requests for help had significant positive relationships with three teaching practice constructs (tasks, representations, and analyzing) and four MKTF domains (SCKF, HCKF, KCFS, and KCFC). The results further showed that PST's practice of analyzing student's work and contributions have positive relationships with only one teaching practice domain (requests) and all the six domains of MKTF (CCKF, SCKF, HCKF, KCFT, KCFS, and KCFC). Additionally, the results of the network analysis revealed that PST's practice of providing explanations had significant positive relationships with one teaching practice construct (representations) and five MKTF domains (CCKF, SCKF, HCKF, KCFT, and KCFC). Moreover, the network analysis showed that PST's practice of using representations was positively related to three constructs of teaching practices (tasks, requests and explanations) and five domains of MKTF (CCKF, SCKF, HCKF, KCFS, and KCFC). Regarding the teaching practices constructs, the network analysis has shown PST's practice of using representations as having the strongest link in the network. This is true since a teacher's ability to select and use representations depends on his/her repertoire of knowledge. Moreover, a teacher uses representations in order to perform other teaching practices (e.g., providing explanations and responding to student's requests for help).

DISCUSSION

This study aimed at building a network of relationships between PSTs' MKTF and their teaching practices. The study utilized the mathematical knowledge for teaching framework and MTF to identify six teacher knowledge domains in fractions (CCKF, SCKF, HCKF, KCFT, KCFS, and KCFC) and five teaching practices (selecting and using tasks; using representations; providing explanations; responding to students' direct or indirect requests for help; and analyzing students' work and contributions*)* required for quality teaching of mathematics. More specifically, the focus of this study was to examine PSTs' mathematical knowledge required for teaching fractions, their teaching practices and their interconnections.

From the perspective of PSTs, the types of knowledge related to the six MKTF domains are interconnected, which does not support the present "egg model" structure of Ball et al. (2008) conceptualization that present the teacher knowledge domains without showing any relationships between them. The study, however, presents findings that are consistent with Kooponen et al. (2019) who have also identified relationships between MKT domains. This supports the views of Fennema and Franke (1992) that teacher knowledge domains are connected to each other. This study has demonstrated from correlation analysis that there are connections between the domains of teacher knowledge in fractions. From this study, we identified the domain of KCFS as being at the center of the relationships between MKTF domains since it was the only domain that was linked to all the other domains. This means that KCFS played a significant role in relation to the totality of the MKTF domains.

The result also demonstrates that the six MKTF domains exist in a hierarchical sequence in the minds of PSTs, and they revealed that KCFS was more like a background knowledge, which is needed in the development of other domains. This means that a teacher must develop first KCFS before developing the other domains. This study implied that in teaching the contents of mathematics it is important to teach first the errors and misconceptions that learners make, which will help them eliminate such misconceptions and errors in learning the content. In this study, knowledge of teaching fractions comes last in the hierarchy of Knowledge domains. This support O'Meara (2011) who noted that knowledge of teaching mathematics comes last, in the hierarchy of Knowledge domains that are needed in teaching mathematics. This means that teachers need the other knowledge domains before they can understand KCFT. Based on the finding that there are connections between the mathematical knowledge domains, the researchers argue that in teaching content and pedagogy, the PSTs should be made to explicitly acquire the six knowledge domains of MKT about a particular strand/topic before moving on to another strand/topic in mathematics. We support the views of researchers (O'Meara, 2011; Wu, 2005) about the scheduling of content knowledge and pedagogical knowledge in teaching mathematics to PSTs'. However, we propose that the scheduling of content knowledge and pedagogical knowledge should be done strand/topic by strand/topic to help prospective teachers to wholistically acquire mathematical knowledge for teaching domains about each topic/strand.

The study has found the PSTs' teaching practices as composite practices since the practice of one involves another, which was shown by the correlations existing between these teaching practices. This support Charalambous (2008) who identified the teaching practices involved in the instructional process as composite practices. From this study and the perspective of PSTs' teaching practices, the practices of using representations and responding to student's requests for help have been identified as the practices that appeared to find their focus in PSTs' teaching practices since they correlated with majority of the PSTs' teaching practices. This means that one cannot teach mathematics without using representations and also responding to student's requests for help. From this perspective, the teaching practices required to bring about mathematics teaching quality appeared to find its focus in two questions in PSTs' minds: "How can we select and use appropriate representations?" and "How can we appropriately respond to student's requests for help?" Despite, the relevance of PSTs' use of representations and responding to student's requests for help as having focus on the teaching practices of the PSTs, the results of the means (representations, M=1.48; requests, M=1.26) showed that PSTs scored low in using representations and responding to students' requests for help, indicating that they have low knowledge in these two constructs of teaching practices. It is therefore suggested that more effort should be put in place towards increasing PSTs' teaching practices of using representations and responding to student's requests for help.

The study identified significant correlations between PSTs' MKTF and their teaching practices. This is consistent with Charalambous (2008) study, which also identified significant relationships between PSTs' MKT and their teaching practices. This study support researchers (Ball et al., 2008; Hoover et al., 2016) who have acknowledged MKT as a construct in teaching, which has an immense contribution on the quality of mathematics instruction.

CONCLUSIONS, IMPLICATIONS, RESEARCH LIMITATIONS, AND FUTURE WORK

This study was not without limitations. Teaching practices in this study were explored using teaching approximations, thus identifying and interpreting teaching practices contained in lesson scripts. It would have been more appropriate to use observation data instead of using tests in which PSTs were made to read, identify and interpret the appropriateness of the teaching practices contained in lesson scripts. However, the use of a test enabled us to obtain data from PSTs about the same teaching practices for easy comparison that would have been difficult if observation data was used. Future research is therefore needed to use both observation data and lesson scripts with accompanying tests to explore the teaching practices of PSTs in order to compare whether PSTs' performance in the test is similar to their performance in the observation of teaching data. The study only explored MKT and teaching practices in fractions. It is not clear if the same results will be obtained using different topics in mathematics. This, to some extent limits the generalization of the results to other areas in mathematics. Future research is needed using other topics in mathematics to examine the relationship between teachers' MKT and their teaching practices.

Notwithstanding these limitations, the findings provide some insights into how PSTs in Ghana and countries with similar contexts could conceptualize MKT and teaching practices. The findings show that there are connections: between the domains of MKTF; the constructs for teaching practices; and between MKTF domains and the teaching practices constructs, which suggest that teacher knowledge is key to the enhancement of quality mathematics teaching, which consequently could lead to improved students' performance. The findings here confirm the hypothesis that an increase in MKTF corresponds to increased levels in teaching practices and that there are positive relationships between: domains of MKTF; constructs on teaching practices; or MKTF domains and teaching practices constructs. These findings suggest that teaching of content and pedagogy at the teacher training institutions should be done in the manner that PSTs would be made to explicitly acquire the six knowledge domains of MKT about a particular strand or topic before moving on to another in mathematics. The study also identified that SCKF and KCFC are the domains of MKTF that are greatly linked to PSTs' knowledge domains and the teachers' teaching practices, while using representations and responding to student's requests for help are key elements for effective teaching.

The implication is that in training PSTs, efforts should be made to develop their knowledge of content and students fully; this will influence the development of the special content knowledge and knowledge of content and curriculum, which consequently influence their teaching practices especially the central practices of responding to students' requests for help and using representations.

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