MODESTUM
EURASIA Journal of Mathematics, Science and Technology Education, 2024, 20(3), em2414
ISSN:1305-8223 (online)
OPEN ACCESS
Research Paper
https://doi.org/10.29333/ejmste/14321

Challenges in geometric modelling–A comparison of students' mathematization with real objects, photos, and 3D models

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Received 12 October 2023 ▪ Accepted 07 February 2024

Abstract
Mathematical modelling aims at contributing to the involvement of reality in mathematics education. As an example, geometric modelling can be implemented by the use of real objects in modelling tasks. Still, (geometric) modelling tasks can be a challenge for students, especially in the transfer from reality to mathematics, which is referred to as mathematization. Since the representation of a real object in tasks might differ, the question arises, which challenges can be observed when working in different task settings. In a study with 19 secondary school students, the task settings (1) outdoors at the real object, (2) indoors with photos of the real object, and (3) indoors with a 3D model of the real object are compared. Based on video recordings, differences concerning the students' challenges are examined. The results highlight challenges in estimating and measuring when working at the real object, scale and perspective when working with photos and the transfer between representation and object when working with 3D models.

Keywords: geometric modelling, mathematizing, reality, tasks, contexts

INTRODUCTION

Mathematical modelling aims at integrating real-life situations and objects into mathematics classes. Doerr et al. (2017) describe the relevance of mathematical models in different disciplines as one of the reasons why modelling competencies nowadays are part of mathematics education curricula on an international level (see also Schukajlow et al., 2015). In addition, modelling in mathematics education can support students in understanding the relevance and application of mathematics in everyday life (cf. Blum & Leiss, 2007) and in the scope of possible future professional practices (cf. Hernandez-Martinez & Vos, 2018). By looking at modelling tasks from different mathematics disciplines, several potentials for the development and training of mathematical thinking and acting can be recognized, too: Tasks require, for instance, discrete mathematics (e.g., Greefrath et al., 2022), algebraic structures (e.g., Ramírez-Montes et al., 2021) or geometric skills (e.g., Zapata-Grajales et al., 2018). The latter, in particular, will be addressed in this article.

Reality provides numerous examples in which geometry can be discovered in the scope of mathematical modelling, such as buildings, which can be described by using plane and shape geometry. Hereby, approximation is required since most real objects are not perfect mathematical solids. The way in which real objects can be included in modelling tasks differs, e.g., by using different representations or the object itself (cf. Buchholtz, 2021; Jablonski, 2023). This distinction in representation is referred to as a *task setting* in the following.

Despite the high relevance of modelling, research results show that students encounter problems with open character and reference to reality of modelling tasks (cf. Blum, 2015). Even though, the number of identified challenges and obstacles is high, less is known about the role of particular challenges in relation to the task setting. This article aims at a comparison of

A preliminary analysis of data concerning mathematical modelling was published in journal "Educational Studies in Mathematics" in March 2023. This paper extends this focus by providing a detailed analysis of encountered challenges. The theoretical consideration as well as data analysis from this different contextual point of view is being reported for first time here.

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Contribution to the literature

- This article focuses on different modelling settings involving real objects and introduces different representations in the context of geometry.
- The article extends the comparison of different modelling settings to the perspective of challenges that students encounter when solving tasks in different settings.
- The results show differences in the observed challenges and thus implications for research and practice in the context of challenges in geometric modelling tasks can be drawn.

observed challenges during mathematical modelling processes in the following three settings

(1) outdoors at the real object,

(2) indoors by means of photographs, or

(3) indoors by means of a 3D model.

Based on a preliminary analysis and observation (Jablonski, 2023), it is hypothesized that students encounter different challenges when working on modelling tasks in these three settings. Hereby, a special focus will be placed on challenges during mathematization processes.

THEORETICAL BACKGROUND: MATHEMATICAL MODELLING IN GEOMETRY

Geometry finds its origin in the environment: Deriving from the Greek, the word originally means "land survey" (Hwang et al., 2020). A lot has happened since then. Especially with the introduction of geometry into mathematics lessons, its purely practical application in the sense of measuring has receded partially into the background (cf. Jablonski & Ludwig, 2023). Nevertheless, a mathematical perception of the environment is reflected as a designated goal of geometry teaching alongside a scientific and problem-oriented focus. Real objects in particular play a role here:

> "One example is when learners are instructed about angle concepts during geometry class and apply new knowledge to the real world by solving real-life problems in surrounding contexts outside of school, for example, they measure the angles of objects they see on their way home from school (Crompton, 2015)" (Hwang et al., 2020, p. 1124).

Hereby, "the geometry ability is developed in the practice of our life and has significance in humans learning and daily life" (Zhao et al., 2018, p. 1). In the active engagement with reality, a strong potential for spatial ability is seen in the sense that real objects are converted into mental images and used to recognize shapes and sizes (Zhao et al., 2018).

In addition, geometry questions from reality show potential for mathematical modelling, too: As pointed out, reality provides several examples of geometric phenomena. However, it is rarely the *one* geometric object that describes its shape or symmetry perfectly. Approximating reality with geometric solids and shapes requires simplifications and structuring (*How can reality be simplified to narrow it down to a mathematical solid?*), as well as mathematization (*What data are needed, available and important for the usage of a mathematical model?*). All these activities can be found in the modelling cycle by Blum and Leiss (2007). Based on this cycle, Buchholtz (2021) provides an adapted cycle for outdoor modelling tasks at real objects (cf. **Figure 1**). An outdoor modelling task contains a mathematical question being linked to real objects. The task process happens directly on-site of the object and requires mathematical activities such as measuring (cf. Ludwig & Jesberg, 2015).

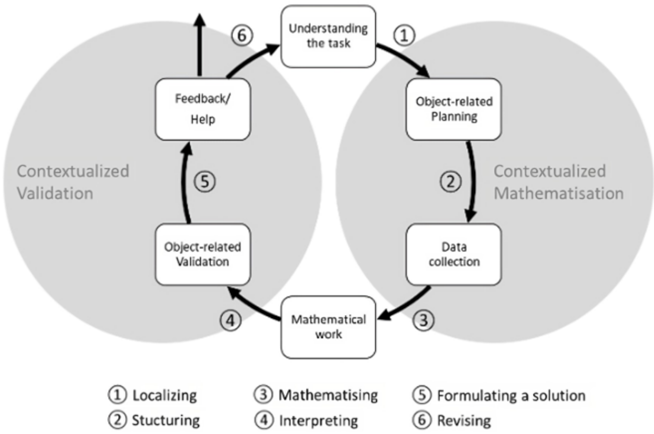

Figure 1. Adapted modelling cycle for tasks at real objects (Buchholtz, 2021, p. 145)

Buchholtz (2021) particularly puts emphasis on contextualized mathematization and contextualized validation. According to him, the steps of structuring and mathematizing on the one hand and the validation process on the other hand happen in the context of the real object. In contrast to the general cycle for mathematical modelling, the context of the object plays a major role and the data collection at the object is emphasized as a separate step. In the following, the focus will be laid on the stages being related to *contextualized mathematization* here.

Even though the adapted modelling cycle mainly focuses on outdoor modelling tasks, with the involvement of real objects, geometry modelling tasks related to real objects can be introduced in different ways. A commonly used example is the introduction of real objects through photos of the object brought to the classroom (e.g., Hartmann & Schukajlow, 2021). Hereby, one or more photos of the real object are provided, sometimes together with an object of reference since usually no data is given. In terms of object-related mathematization, students are requested to do estimations based on the photos and, in the case of an originally 3D object, make assumptions about perspective. Further attempts in geometry modelling are based on the development of 3D modelling and printing (e.g., Anđić et al., 2024), which enables a scaled 3D version of a real object (cf. Jablonski, 2023).

The following conclusion can be drawn: Geometry seems to be a relevant aspect for mathematical modelling in relation to reality and vice versa. On the one hand, the solution of modelling problems related to reality requires geometry knowledge. On the other hand, geometry can be discovered in reality and allows interesting questions in the scope of mathematical modelling. The relation between modelling and geometry seems to contribute to both the general aims of geometry teaching and learning and the enhancement of mathematical modelling. Therefore, the relevance to involve geometry modelling tasks in mathematics classes is evident.

Still despite this importance, the special character of reality-based tasks can be challenging for students. “Ignore the context, just extract all data from the text and calculate something according to a familiar schema” (Blum, 2015, p. 79): Potential difficulties students may encounter in solving modelling tasks can lead to such “fall back” strategies–also in the context of geometry. The number of difficulties is high since “every single arrow in the modelling cycle represents a demanding process […] The processes are demanding in different ways and depend on the specifics of the situation” (Jankvist & Niss, 2019). In the scope of contextualized mathematization, the choice and selection of relevant data seems to be an obstacle (cf. Schukajlow et al., 2023). In the state of the art, the focus is laid on findings concerning secondary school students’ challenges in mathematical modelling and in particular mathematization by reporting on the stages *object-related planning* and *data collection* (cf. Buchholtz, 2021). Further, it takes the role of different task settings involving real objects and their potential specifics against the background of geometric questions into consideration.

STATE OF THE ART: CHALLENGES IN MATHEMATIZING (REPRESENTATIONS OF) REAL OBJECTS

Schukajlow et al. (2023) describe modelling tasks with missing data as *modelling problems with an open initial state* and highlight the challenges to “structure, simplify, and idealize the given situation. As these activities occur at the beginning of the modelling process, it is crucial to overcome barriers in these activities in order to develop am meaningful solution.” (p. 419). This subdivision is based on a study with fifth graders from which the barriers *noticing missing information*, *gathering unknown data* and *making realistic assumptions* emerge (Krawitz et al., 2018).

In terms of the first two barriers, Stillman et al. (2013) report from an empirical study with secondary school students that a main obstacle for them is the management of an open task as well as gathering information and data. These tasks usually require the acquisition of missing information by different forms of data collection, e.g., measurements and estimations. In the context of measuring, Gurjanow and Ludwig (2020) analyze obstacles during secondary school students’ data collection processes by means of measurements. The authors identify the use of measuring tools to gather data and the conversion of units as learning barriers that occur when students take measurements at real objects. Estimating, in general, is described as a “unidimensional construct” with varying difficulty depending on the objects’ size and accessibility (i.e., touchable or not) (Hoth et al., 2022, p. 1861).

Buchholtz (2017) describes autonomous mathematization with a particular focus on real-world contexts as a challenge, especially in making assumptions. This observation is therefore linked to the third barrier identified by Krawitz et al. (2018). Making assumptions is taken into consideration as being “difficult for students and a potential reason why they fail in solving modelling problems” (Chang et al., 2019, p. 61). Chang et al. (2019) divide assumptions into *non-numerical* (based on extra-mathematical knowledge) and *numerical* (about missing quantities) assumptions, whereby the latter is hypothesized as more difficult since “estimation skills and strategies such as the reference point strategy” are needed (p. 61). A close connection to the previously description of data collection is seen here.

Even though there are numerous attempts to support and enrich modelling by digital technologies (e.g., Greefrath et al., 2018), a review study by Cevikbas et al. (2023) shows that, depending on how the tools are used, they “might obscure the meaning behind calculations and mathematicians in modelling approaches […] learners may focus solely on a certain approach to the modelling process and may not be aware of different ways to solve tasks.” (p. 12).

Despite a focus on general challenges in mathematizing, first research attempts focusing on different settings of mathematical modelling are evident, as well. Especially the role of photos is examined in these works (Hartmann & Schukajlow, 2021) with a focus on potential barriers in terms of interpreting perspective (Schukajlow, 2013). In the context of the study further described in this article, activities in the modelling steps *simplifying and structuring* as well as *mathematizing* have already been examined (cf. Jablonski, 2023). The comparison of students working at the real object, with photos or a 3D model shows that the real object setting is dominated by taking measurements and discussions on how to measure available data (cf. Buchholtz, 2021). When working with photos, students place more emphasis on estimating and justifying perspective assumptions. For the work with a 3D model, students' work is focused on assumptions about scaling.

From these findings, it can be hypothesized that different task settings place different demands on students in terms of object-related planning and data collection, e.g., measuring, estimating, and perspective-taking. Thus, it is likely that students encounter different challenges in the task settings. Building on this hypothesis, the aim of this research is to investigate the challenges that students encounter during contextualized mathematization in different task settings and enrich previous research findings through this perspective. To do so, the following research question is formulated:

Which challenges in contextualized mathematization can be observed when students work on modelling tasks in the task settings real object, photos and 3D model?

METHODOLOGY

To answer the research question, a study with 19 students in grade 6-grade 8 in 2022 was conducted. At the time of the survey, the students attended an enrichment program to promote mathematical giftedness. This sample was used because it could be assumed that the students would engage with new task settings and solve them with the necessary willingness to exert effort so that a full comparison of each setting would be possible. The students were divided into six groups.

For the study, mathematical modelling was linked to questions on geometric objects from the daily environment. For each of these objects the following three representations as tasks were generated: photographs of the objects, 3D models of the objects brought to the classroom, and real objects in the daily environment. For the photo setting, a 1.75 m tall person was placed next to/in front of the objects. For the 3D model setting, the objects were scanned and scaled so that a LEGO figure besides the object would represent a 1.75 m tall person. The models were created by 3D print.

Each group worked on three tasks in the three chosen modelling settings. In the *Body of Knowledge* task, students were asked to determine the height of the sculpture, representing a sitting person, if it would stand up (see part a in **Figure 2**). In the *Stone* task (see part b in **Figure 2**), the students should determine the *Stone*'s volume. The *Rotazione* sculpture task (see part c in **Figure 2**) was to determine its surface. Three different task settings were defined for each of the objects:

Figure 2. (a) Task: *Body of Knowledge*–Setting: Outdoors at the real object; (b) Task: *Stone*–Setting: Indoors with photos; & (c) Task: *Rotazione*– Setting: Indoors with 3D model (Photos taken by the author)

Table 1. Group arrangement according to LSD

Groups	*Body of Knowledge*	*Stone*	*Rotazione*
A & D	Photo	3D model	Real object
C & F	Real object	Photo	3D model
B & E	3D model	Real object	Photo

1. *Outdoors at the real object:* The students solved the task outside directly at the real object. They had a folding ruler with them (see part a in **Figure 2**),
2. *Indoors with photos:* The students solved the task using a series of photos of the real object with a person as a possible reference. In addition to the photos, students had a ruler for measuring sizes (see part b in **Figure 2**).
3. *Indoors with 3D model:* To solve the task, the students were given a 3D representation of the real object, which had previously been printed to scale. A LEGO figure and a ruler were provided as a reference size and for measurement (see part c in **Figure 2**).

To ensure that each group worked on each object and in each setting exactly once, the objects and settings were arranged systematically in Latin Square Design (LSD) (Field, 2016; see **Table 1**).

LSD involves to nine different pairs of setting and object. With six groups solving three tasks each, each pair of setting and object was experienced by exactly two groups. This leads to a total of 18 task solution processes with six solution processes per task and six solution processes per setting each. While solving the tasks, the student groups were filmed. The video interactions during all 18 solution processes are the basis for an analysis concerning the following variables for both the solution process and product:

1. *Modelling step:* The processes were coded deductively according to the stage definitions of Buchholtz (2021): Contextualized mathematization, mathematical work and contextualized validation. The scenes categorized in the first stage were taken into consideration for the following step of data analysis.

Table 2. Groups' results & duration needed sorted by object & setting (cf. Jablonski, 2023)

	Body of Knowledge	*Stone*	*Rotazione*
Real object	17.4 m/06:20 min	6.4 m³/11:20 min	29.8 m²/19:30 min
	15.4 m/19:30 min	6.2 m³/06:30 min	21.6 m²/07:00 min
Photos	11.5 m/25:40 min	2.6 m³/14:20 min	14.1 m²/14:30 min
	10.5 m/15:20 min	3.1 m³/14:30 min	11.0 m²/08:40 min
3D model	16.9 m /13:40 min	4.2 m³/24:00 min	21.6 m²/20:30 min
	17.0 m/13:10 min	5.3 m³/10:10 min	23.8 m³/28:40 min
Solution interval	[18-22 m]	[5-9 m³]	[17-26 m²]

Table 3. Challenges during contextualized mathematization (cf. Buchholtz, 2021) in settings real object (RO), photos (P), & 3D model (3D)

Challenge	Stage	Quantity		
		RO	P	3D
Problems in identifying important/unimportant data	*Object-related planning*	1	3	1
Problems in finding a unified strategy for structuring		3	0	2
Problems in recognizing perspective		0	4	0
Problems in understanding link between representation & object		0	0	4
Inaccurate estimations	*Data collection*	3	2	1
Inaccurate measurements/measurement errors		3	2	1
Errors in scale calculations		0	1	4
Problems in estimating/using reference person		0	2	2

2. *Challenges:* In addition, an inductive, qualitative content analysis according to Mayring (2000) was used to extract the challenges that occurred during the contextualized mathematization. Hereby, a challenge was defined by means of the following cases:
   a. *Internal challenge:* The group encounters a problem in their mathematizing for which they do not have a plan ready to be used (cf. problem-solving). It goes along with uncertainty, the explicit mentioning of the problem and/or the lack of agreement by the whole group concerning mathematization.
   b. *External challenge:* The group follows a mathematization strategy that is inadequate, unrealistic or leads to an incorrect result. In contrast to the first challenge-type, the students do not show any kind of awareness hereof.
3. *Time on task/mathematization:* Afterwards, the time on task for each group as well as the time needed for mathematization was taken into consideration.
4. *Solution quality:* Finally, the quality of the achieved result was analyzed. For this purpose, solution intervals for each task were created in advance, based on multiple solution processes by means of different models.

RESULTS

The presentation of the results starts by giving an overview of the achieved results and the groups' time on task in the respective setting and object (cf. **Table 2**; cf. Jablonski, 2023). On average, the groups need 13:40 minutes to solve a task at the real object, 14:30 minutes to solve a task with photos and 13:25 minutes to solve a task with a 3D model. Concerning the different task objects, the groups need about 14:30 minutes to solve the *Body of Knowledge* task, 12:50 minutes to solve to *Stone* task and 17 minutes to solve the *Rotazione* sculpture task. Of their total time on task, the groups spend about 64% (*real object*), 62% (*photos*), and 60% (*3D model*) in the contextualized mathematization.

The solution quality for the real object setting can be summarized, as follows: Three results (one for the *Rotazione* sculpture and two for the *Stone*) can be evaluated as correctly solved. The two results for the *Body of Knowledge* underestimate the result and one solution for the *Rotazione* sculpture exceeds the interval. For the photo setting, all achieved results underestimate the defined solution interval. In the work with the 3D model, students underestimated the height of the *Body of Knowledge* and the volume of the *Stone* in one case. All remaining results fit in the solution interval.

Table 3, in addition, gives an overview of the observed challenges. They derive from the coding process of those scenes being related to contextualized mathematization (cf. Buchholtz, 2021). They are distinguished according to the related stages *object-related planning* and *data collection*. The challenges are ordered in their quantity (number of groups that encountered the challenge in the particular setting).

The challenges being linked to *object-related planning* can be organized, as follows: Three of the six groups have problems in *identifying important and unimportant data and information* in the photo setting, whereas it only plays a minor role in the observation of the other settings. For example, one group wants to determine the

Table 4. Overview of Kendall's tau correlations

Variable		Challenges (total)	Challenges (data collection)	Challenges (planning)	Time on task	Solution quality
Challenges (total)	Kendall's tau	-				
	p-value	-				
Challenges (data collection)	Kendall's tau	0.754***	-			
	p-value	<.001	-			
Challenges (planning)	Kendall's tau	0.763***	0.348	-		
	p-value	<.001	0.103	-		
Time on task	Kendall's tau	0.546**	0.335	0.670***	-	
	p-value	0.003	0.080	<.001	-	
Solution quality	Kendall's tau	0.267	0.254	0.318	0.257	-
	p-value	0.162	0.199	0.109	0.152	-

Note. *p<.050; **p<.010; & ***p<.001

volume of the reference person given in the photo to determine the volume of the *Stone*. In addition, *problems in recognizing perspective* is a challenge only observed in the photo setting. During the work with a 3D model, four of the six groups have *problems in understanding the link between the representation and the real object*. One group attempts to stop the modelling task after calculating the size of the *Body of Knowledge*'s 3D model representation without seeing that it is only material for calculating the real size. *Problems in finding a unified strategy for structuring the process*, i.e., students being unable to agree on a common strategy and start multiple processes simultaneously, are observed in the real object and 3D model setting.

Analyzing the challenges being related to *data collection*, it can be observed that *inaccurate estimations* and *measurements* occur in all three settings with a focus on the real object setting. This is especially the case when estimating the height of the *Body of Knowledge* and the *Rotazione* sculpture, which are too high to be measured directly. *Errors in scale calculations* as well as *problems in estimating/using the reference person* are only relevant in the 3D model and photo setting, whereas the first is dominant in the 3D model and the latter is relevant for two groups in both settings. Problems in scale calculations can be focused on the handling of scale in terms of area and volume, e.g., one group estimates the scale factor to determine the surface of the *Rotazione* sculpture with the 3D model but multiplies the area of the object's representation only once by the scale factor.

In terms of an outlook concerning the identified challenges, possible relations between the challenges encountered and the time on task, respectively solution quality are identified. Therefore, the number of different problems in the groups (in total, in object-related planning and in data collection) is correlated with their time on task and the result quality. **Table 4** presents these results. Hereby, all 18 solution processes are equally taken into consideration without distinguishing different settings since the number of cases would be too small. Still, with the previously identified differences between the settings, possible relations will be taken into consideration in the discussion.

According to Gilpin (1993), values for Kendall's tau τ above 0.330 can be related to values for Pearsons's r exceeding 0.500, which marks a medium effect. With this interpretation, for the 18 task processes, a medium correlation is obtained between the total number of encountered challenges (respectively the number of encountered challenges in planning) and the time on task with Kendall's tau τ=0.546 (and τ=0.670). For the number of challenges encountered in the data collection and the time on task, Kendall's tau τ=0.335 can be reported, assuming that a bigger sample size might lead to significance, too. Concerning correlations with solution quality, no remarkable effects can be reported.

DISCUSSION & CONCLUSIONS

In order to answer the question of which challenges in contextualized mathematization can be observed when students work on modelling tasks in different settings, a study with 19 secondary school students was conducted. By means of 18 solution processes in six groups, the three different modelling settings outdoors at the real object, indoors with photos and indoors with 3D models were compared concerning the occurrence of challenges. The focus was laid on contextualized mathematization with its components *object-related planning* and *data collection* since a relation to the work with real objects (and their representations) was seen particularly here. Besides a focus on the encountered challenges, the time on task as well as the solution quality were taken into consideration, as well.

From the identification of the challenges in relation to the task setting, it can be seen that most are predominantly observed in one or two task settings. For the real object settings, the main focus can be seen in inaccurate estimations and measurements (cf. Gurjanow & Ludwig, 2020). In line with the findings of Buchholtz (2021), this setting poses special demands in terms of data collection. The other settings involve these challenges, too, but were observed in fewer cases. For the work with photos, the challenges can be seen in identifying important and unimportant data as well as recognizing perspective. Having a two-dimensional representation of a 3D object seems to be a challenge for students in terms of *object-related planning*. The results, especially in terms of the problems identifying the perspective are in line with the findings of Schukajlow (2013). Being relevant in both the photo and 3D model setting, problems in scale calculation and the use of the reference figure are observed. In addition, working with a 3D model, the students show problems in the understanding of the link between representation and the real object, which is not the case in the other settings. Even though a 3D representation is given here, the link between the scaled representation and reality places special demands. Potentially, the work with 3D models is less common to the students since photos and real objects are part of their everyday life. In contrast, students do not face problems in finding a unified strategy for structuring in this setting, which is encountered in the real object and 3D model setting, both potentially enabling more possibilities to solve the task. The observations can enrich the previous observations concerning the different settings in Jablonski (2023): Besides different emphases of the settings in terms of activities, different challenges related to these activities can (partly) be observed in the settings.

From the correlations, in addition, it can be seen that the time on task and the encountered challenges are related to each other with a medium effect using Kendall's tau τ. Still, the data do not allow for reporting any remarkable differences concerning the total number of challenges and the time on task between the settings. Moreover, no remarkable relation between the challenges linked to contextualized mathematization and the solution quality could be found. Potentially, challenges in other modelling steps might have a higher influence on the results' quality.

From the results, particular demands of the settings derive:

(1) measuring and estimating when working at the real object,

(2) perspective and scale when working with photos, and

(3) scale as well as the transfer of representation and reality when working with a 3D model.

Consequently, different kinds of preparations and hints can become relevant for the use of different settings in mathematics education. For the real object setting, it would be beneficial to strengthen measuring skills in advance (cf. Gurjanow & Ludwig, 2020), whereas the photos and 3D model setting could be supported through hints concerning perspective for the first and scale for the latter. To assist in the 3D model setting, it would make sense to let students experience the process of creating a 3D model in order to understand the link between the representation and the original object.

The limits of these results are the potential positive selection of mathematically gifted and interested students as well as the given tasks. For the first, it might be the case that the students had less difficulties because of their special relation to mathematics. Other students might have shown additional problems in solving the tasks. Still, it can be assumed that the identified challenges are also relevant for other students. As these results were still obtained with the same sample as in Jablonski (2023), they should only be interpreted as a supplement to the observational perspective and not as a validating study. Accordingly, confirmatory studies based on the data, which focus on larger samples, are still pending. Furthermore, the role of the identified challenges in modelling tasks from other topics, e.g., discrete mathematics and algebra, would be of high interest. Through the exclusive observation of geometry-specific tasks in this study, the kind of challenges, e.g., problems with measurements, perspective, and scaling, was influenced. Thus, future research could extend the findings in following the question of the results' transferability.

Funding: The study "Modelling, argumentation and problem-solving in the context of outdoor mathematics" is funded by Dr. Hans Messer Stiftung.

Ethical statement: The author stated that the highest principles of the Committee on Publication Ethics were followed when conducting the research and reporting the research results in the article. The author stated that the study did not require institutional ethics committee approval. The author further stated that consent was obtained from the participants before the research was conducted; and that the participants were informed about the research, anonymity and their rights about withdrawing from the study without adverse repercussions.

Declaration of interest: No conflict of interest is declared by the author.

Data sharing statement: Data supporting the findings and conclusions are available upon request from the author.

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