

## Classification of open mathematical problems and their role in academic achievement and motivation of students

Guliyash B. Niyazova<sup>1</sup> , Vyacheslav V. Utemov<sup>2\*</sup> , Tatyana N. Savina<sup>3</sup> ,  
Lyudmila Zh. Karavanova<sup>4</sup> , Inessa S. Karnaukh<sup>5</sup> , Valeria L. Zakharova<sup>6</sup> , Elvira G. Galimova<sup>7</sup> 

<sup>1</sup> L.N. Gumilyov Eurasian National University, Nur-Sultan, KAZAKHSTAN

<sup>2</sup> Vyatka State University, Kirov, RUSSIA

<sup>3</sup> Ogarev Mordovia State University, Saransk, RUSSIA

<sup>4</sup> Peoples' Friendship University of Russia (RUDN University), Moscow, RUSSIA

<sup>5</sup> Plekhanov Russian University of Economics, Moscow, RUSSIA

<sup>6</sup> I.M. Sechenov First Moscow State Medical University, Moscow, RUSSIA

<sup>7</sup> Kazan (Volga region) Federal University, Kazan, RUSSIA

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### Abstract

Problems of developing subject education and science have become more acute during different social changes. The most problematic is the teaching of exact disciplines, including mathematical disciplines (mathematics, algebra, geometry), which form knowledge for working in high-tech industries. It is worth noting that the methodology of teaching mathematics has accumulated a large unsystematized portfolio of new techniques and methods. Thus, open problems aimed at developing students' creativity are used more actively in the lessons. Tasks that require students to use non-standard approaches to the solution affect academic success. But a teacher of mathematics needs to have a classification of problems in the educational process to choose tasks of an open type correctly. In this regard, the article deals with substantiating the classification of open problems for teaching mathematical disciplines and developing methodological guidelines for lessons that help increase learning motivation and academic achievement of students. The leading research methods are the following: observation of teachers' methodological work, talks with teachers, analysis of methodological developments, educational documentation and teachers' surveys. Statistical processing of the results using Wilcoxon W test. In 2020-2021 the authors carried out an experiment with 358 students from 16 classes. Based on its results, the authors of the article have identified a classification of open problems for teaching mathematical disciplines; developed and implemented methodological guidelines for lessons of mathematics that contribute to developing students' creativity in the academic progress. The authors evaluated the effectiveness of using the selected classification of open problems for teaching mathematical disciplines. They concluded about the increase of group motivation among students and academic performance in mathematical disciplines. The practical use of the classification of open problems allows the teacher to differentiate educational problems in mathematics, taking into account the relationship between their logical structure and the level of ability achieved by the student to solve problems in mathematics. Methodological guidelines developed by the authors can be used to help increase the learning motivation and academic achievement of students while preparing for mathematics lessons.

**Keywords:** methods of teaching mathematics, tasks in mathematics, creative tasks, levels of complexity of tasks

### INTRODUCTION

The mental and personal characteristics of today's generation of children significantly affect approaches used by teachers. The methodology of teaching school

subjects should correspond to the characteristics of developing a modern child. Today it is practically impossible to accompany child's effective cognitive activity not paying attention to the development of his creative abilities. Child's formed creativity is seen when

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 [guliyash\\_5@mail.ru](mailto:guliyash_5@mail.ru)  [vv\\_utemov@vyatsu.ru](mailto:vv_utemov@vyatsu.ru) (\*Correspondence)  [savinatn@yandex.ru](mailto:savinatn@yandex.ru)  [Karavanova\\_L54@mail.ru](mailto:Karavanova_L54@mail.ru)  
 [karnaukh.is@rea.ru](mailto:karnaukh.is@rea.ru)  [zkvaleria@mail.ru](mailto:zkvaleria@mail.ru)  [elyagalimowa@yandex.ru](mailto:elyagalimowa@yandex.ru)

### **Contribution to the literature**

- The authors of the research characterize tasks often encountered in the teaching of mathematics (creative tasks) used for educational purposes: heuristic tasks, practical or applied (practice-oriented) tasks, task situations and divergent tasks. These types of tasks make it possible to construct a task that causes students' cognitive involvement and preserve its didactic value - an open mathematical task.
- The authors formulate a classification of three types of mathematical problems of an open type. To determine the levels of complexity, they use an approach based on the types of logical operations in scientific creativity. The practical application of the proposed types allows the teacher to differentiate educational tasks in mathematics, taking into account the ratio of their complexity and the level of ability achieved by the student to solve problems in mathematics.
- Methodological recommendations for mathematics lessons developed by the authors are unique in their contribution to existing research in terms of illustrating the relationship between methods for developing students' creativity and teaching methods. Recommendations cover a set of mathematical problems of an open type with the allocation of their logical structure used to develop students' creativity in a mathematics lesson.

overcoming various problems and can be characterized by: the optimality of the proposed ideas, the effectiveness of reasoning, the originality of the answer and the degree of developing the solution. Taking into account that school age is a sensitive period for developing student's creativity, we underline the importance of each school subject in the process of forming creative skills.

Mathematical disciplines (mathematics, algebra, geometry) play a crucial role in the world's scientific and technological progress (Maass et al., 2019). High-quality mathematical education forms knowledge for working in high-tech industries and, as a result, it is important for the economic success of any state. Therefore, teaching methods used at school should increase the motivation for studying mathematical disciplines and emphasize the importance of mathematical education.

Considering the means of developing students' mathematical literacy, it is evident that a task is the main didactic tool (Hwang & Ham, 2021). The task used in the educational process requires the student to master cognitive actions and discover a solution method. In the future, the open solution method can be used to solve a similar range of applied problems. An important component for students' holistic formation is creative tasks. Creative tasks allow the student to generate independently new algorithms and sequences of actions. The result of this is the discovery of subjectively or objectively new knowledge, phenomena, objects, systems (Begishev et al., 2021). Tasks of a creative type used for educational purposes can be called tasks of an open type. These tasks have a vague condition. It is not clear enough how to act and what to use in solving, but the required result is clear. Such tasks involve a variety of solutions that are not straightforward. While solving them, you have to overcome the obstacles that arise. There are many options for solutions, but there is no concept of the right solution: the solution is either applicable to achieve the desired result, or not.

A mathematical problem of an open type should arouse students' interest, without losing the methodological value of the educational problem. Therefore, it is extremely important to determine educational creative tasks used in teaching mathematical disciplines and on the basis of the selected types, describe the classification of open mathematical problems with the allocation of their logical structure. It will allow the teacher of mathematics to carry out an effective selection of open tasks, taking into account the level student's cognitive development, his mental and personal characteristics. An important result of the correct design of open problems for a mathematics lesson is the development of the main characteristics of students' creativity. Therefore, the proposed approach to updating methodological techniques is the introduction into the pedagogical practice of the teacher's work to the classification of open problems for teaching mathematical disciplines.

### **LITERATURE REVIEW**

There is little research in the scientific-pedagogical, psychological and methodological literature on the issue of searching for the classification of educational creative tasks in mathematics as effective methodological methods for developing creativity and subject knowledge.

In the literature, we can single out research questions that characterize the significance of forming a wide range of student abilities in mathematics lessons. Wang and Feng, (2009), Gülmez-Dağ and Yildirim (2016) emphasize the necessity of mastering a wide range of methods of teaching mathematics by teachers. A math teacher can integrate thinking skill-building into every lesson, highlighting the importance of mastering math competence in students' lives (Putra et al., 2018). Tian (2021) concludes that with the gradual enrichment of the theory and practice of teaching, the proportion of network information teaching methods is increasing,

and traditional teaching methods are weakening. It is impossible to ignore students' initiative. Taking into account cognitive initiative leads to the development of collaboration and creativity (Liñán-García et al., 2021). The introduction of private information methods into the work of an algebra teacher allows increasing students' performance in the subject as a whole (Egara et al., 2022). At the same time, didactic tools and methodological techniques are more effective when they are used in a comprehensive way in mathematics lessons in a combination of methods that ensure the assimilation of subject knowledge and form student's thinking and metacognitive skills (Ozturk, 2021; Platonova et al., 2021; Yang et al., 2011). The combination of methods allows students to master and understand the content (Boaler et al., 2021; Dong et al., 2015). It also allows a productive and meaningful exchange between different sections of mathematical disciplines (Hawes et al., 2021). Husain et al. (2022) demonstrate an interesting research result. During the COVID-19 pandemic, creative learning methods during math classes improve students' understanding of mathematics at the high school level. Similar conclusions are confirmed when analyzing the experience of teaching mathematics comparing inclusive and specialized schools (Maurer et al., 2021). It is worth noting that these methods should be studied in higher education programs in the training of mathematics teachers (Lestari et al., 2018). Therefore, mathematics education creates favorable conditions for students to develop core competencies, thinking and creativity as a basis for academic success in general (Tong et al., 2021). According to Gilford (1967), creativity is an original kind of thinking - the so-called divergent ("going in different directions") thinking, which omits the variation of ways to solve the problem, leads to unexpected conclusions and results. Such thinking is opposed to convergent one.

Separately, it is worth noting the studies describing the use of STEM technologies in teaching mathematical disciplines. ElSayary (2021) points out that STEM technology has a positive impact on the attitudes, motivation and interests of students, which leads to the development of creative skills. Other studies have shown that implementing a STEM approach that includes both creative task-based learning is effective and able to improve student academic achievement in mathematics (Kong & Matore, 2022). Barana (2021) underlines the importance of modeling skills and creativity when solving mathematical problems. Basu and Nguyen (2021) note that realistic creative tasks and open questions in mathematics allows to form cross-subject connections. It also stimulates interest in studying the discipline (Duyen & Loc, 2022). Examples of stimulating interest make it possible to focus on memorizing mathematical algorithms and to form mathematical thinking (Mahmud et al., 2021). According to Guerra and Lim, (2017), formed mathematical

thinking allows using knowledge in mathematics in a real-life situation.

On the other hand, there are works indicating conditions for improving students' academic performance. Students with a large number of information resources, as a rule, show higher achievements in mathematics (Japelj Pavešić et al., 2022). The creative inclusion of messengers (Jere et al., 2019), comics (Rose et al., 2021), videos (Leikin & Kawass, 2005), interactive mathematical tools (Lukac & Sekerak, 2017) and other tools can improve student performance (Shaidullina et al., 2018; Shang et al., 2011). Another condition is the organization of collective interactions in solving problems. The results of the experiment show that joint educational work has a significant positive impact on the success of mastering the subject (Ali et al., 2021). When solving joint problems, strategies and approaches to decision-making are used, which is typical for creative activity (Budayasa & Juniati, 2019). These practices are more focused on students with fairly high achievements (Zuzovsky, 2013). Therefore, it becomes necessary to take into account the level of formation of cognitive skills when using creative tasks.

Kalimullin and Utemov (2017) describe open tasks as a means of developing the creativity of secondary school students. Academic success in mathematics also influences the attitude to solve problems in mathematics (Mohd & Tengku Mahmood, 2011). Utemov, and Masalimova (2017) describe a method for modeling creative mathematical problems, taking into account the levels of complexity of tasks in accordance with the system scale and the requirements for setting tasks at school.

Thus, we can note the scientific interest in considering an open mathematical problem that arouses students' interest, without losing the didactic value of the problem. However, the known research either describe the scattered results of using educational creative tasks, or the description is not limited to their use as methodological techniques in school mathematics lessons. Moreover, the classification of open problems for teaching mathematical disciplines can be used in teaching students in mathematics, taking into account the level of their ability to solve problems and features of cognitive development.

## MATERIALS AND METHODS

### Theoretical Basis of the Study

The theoretical basis of the research is theoretical approaches used in scientific creativity and their implementation in mathematics lessons as teaching methods. The task is a key element in the structure of learning activities. Its solution involves students' performance of certain educational actions and operations. In addition, the task requires the student to

carry out more or less detailed mental actions (productive or reproductive). Therefore, educational activity has a given structure, that is, it unfolds as a solution to its characteristic educational tasks (Kostyuk et al., 1973.). The purpose of this activity is the assimilation of theoretical knowledge. Using creative tasks during the lesson contributes to the change of the dominant from the reproductive scheme, which looks like this: ready-made knowledge → mastering knowledge → controlling the strength of knowledge assimilation - search cognitive activity: problem → multivariate search for a solution → choosing the optimal solution. Thus, the introduction of balanced creative tasks into the content of mathematical education, which ensures creative development and the assimilation of knowledge, is essential for the transition to a new scheme. In our research, we consider it appropriate to use mathematical problems of an open type for the development of creativity (Kalimullin & Utemov, 2017).

We focus on the following types of creative tasks: practice-oriented (applied, practical), divergent, task situations, heuristic tasks. Having characterized these types of problems, we single out the classification of open mathematical problems that arouse students' interest without losing the didactic value of the problem.

This classification allows us to demonstrate the construction of tasks for each student, taking into account the level of ability to solve problems and the characteristics of cognitive development.

## **Research Methods**

To substantiate the classification of open problems for teaching mathematical disciplines, we used the following methods: observation of teachers' methodological work in mathematics lessons, talks with teachers, analysis of methodological developments, educational documentation and teachers' surveys. The measurements have not normal distribution. So, in order to compare pre-post of teachers' survey, Wilcoxon rank test is applied.

We monitored teachers' methodological work in order to analyze tasks that require students to use non-standard approaches. As the observed parameters, we took the issues of using creative tasks and their design for organizing learning: creative tasks, approaches to the design of creative tasks, taking into account students' cognitive characteristics.

To identify the difficulties of developing creative tasks, we talked to mathematics teachers. Content analysis of these conversations revealed the problem of low correlation between the use of creative tasks and students' academic performance and learning motivation.

To generalize and classify the types of mathematical non-standard tasks, we analyzed the methodological

developments of teachers (lesson notes, systems of tasks and exercises).

To identify the effectiveness of the developed classification of open mathematical problems, we conducted a survey of teachers supervising the experimental groups on the diagnosis of ILadanov's (2004) group motivation. The survey was offered in a blank form before and after the experiment.

To analyze the academic performance in mathematics in the experimental classes, we carried out an analysis of educational documentation (class journals) with the identification of averages in mathematics for the academic quarter preceding the introduction of the task classification and following the results of the academic quarter when the mathematics teacher applied the proposed classification.

The Jamovi (2.3.12) is used in order to perform all statistical process (The jamovi project, 2022). Before inferential statistics, we check that the measurements have normal distribution. According to Shapiro-Wilk test results,  $p$  is smaller than 0.05. So, all items in the questionnaire and the academic performance grade have not a normal distribution. To perform inferential statistics, the Wilcoxon  $W$  test is applied.

## **Experimental Research Base**

Approbation, generalization and implementation of the research results were carried out in two schools in the European part of Russia - innovative sites supervised by Vyatka State University, where we carried out an experimental study in order to evaluate the effectiveness of using the classification of open problems for teaching mathematical disciplines (2020–2021):

- by conducting experimental teaching of mathematics in grades 7–10 (358 students from 16 classes took part who studied mathematics using a selected classification: guidelines for conducting mathematics lessons that contribute to the development of students' motivation.
- by discussing the classification at didactic seminars with mathematics teachers as part of summing up the annual results of the work of innovation platforms (50 participants).

## **Research Stages**

The research had four stages.

The first stage was aimed at identifying the state of the problem not only in theory, but also in the practice of teaching mathematics to students in a general school. We studied and analyzed the psychological, pedagogical and methodological literature on the research problem (both in mathematics and in related school disciplines); carried out observation, talked and analyzed the work experience of mathematics teachers, methodologists in

order to study issues related to the use of creative mathematical problems in the process of teaching.

The second stage was devoted to the development of leading approaches to determining the types of educational mathematical problems for the development of creativity, implementing the differentiation of educational tasks. We offered the generalized results for discussion in the course of reports at conferences and seminars on the problems of methods of teaching mathematics at various levels.

Simultaneously with the second stage, we implemented the third stage. It was the experiment itself. 358 students from 16 classes took part in it under the guidance of mathematics teachers from two schools in the European part of Russia - innovation sites supervised by Vyatka State University. Teachers designed mathematical problems of an open type for lessons based on the selected classification. After that, a survey was conducted, and the results were analyzed.

At the fourth stage, we generalized the experience and evaluated the effectiveness of using the selected classification in teaching mathematics. We described types of mathematical open problems with the allocation of their logical structure used to develop students' creativity. They were described as methodological techniques for use by mathematics teachers for their phased introduction into the pedagogical practice of teaching school mathematics.

## RESULTS

As a result of the generalization of creative learning tasks used by mathematics teachers, talks to these teachers, and content analysis of their methodological developments, we have identified the following creative tasks used for educational purposes: practical or applied (practice-oriented), divergent, task situations and heuristic tasks. These types of tasks are presented in more detail.

### Applied or Practical Tasks

The main strategy of applied or practical problems in the teaching of mathematical disciplines is based on implementing the semantic relationship of the school discipline with the future professional activity. Such tasks are aimed at forming professional competencies, universal learning activities and cognitive activities in the course of mastering the subject, rather than developing the characteristics of the student's creativity. Therefore, we can call these tasks practice-oriented tasks. But they are not fully open mathematical problems.

**Example № 1.** The door is shaped like a semicircle lying on a rectangle. Describe the possible characteristics of the door so that, for a given perimeter, the doorway is as large as possible.

The proposed task is applied. It can be characterized as practice-oriented. To solve it, it is necessary to use analysis for an extremum using a derivative.

All the required data are given in the problem, the solution method is defined in the form of sequential operations based on the algorithm. Therefore, this task is not aimed specifically at developing creativity. It does not give the opportunity to manifest creativity in the mathematics lesson.

### Divergent Tasks

Divergent tasks have many options for correct answers and, as a result, multiple solutions. We have the prepositional type of problems when the problem has not yet been disclosed and the way to solve it is unknown. In classical mathematical teaching, divergent tasks are used rarely. At the same time, divergent tasks are very effective for the development of creativity, since the variety of answers and problem solutions creates optimally favorable conditions for the realization of the student's creative potential, allowing him to show fluency, flexibility, and originality in mathematics lessons.

**Example № 2.** On Friday evenings after school, Vasya can choose any sequence of things to do. He can start with a workout or go to the library. After that, he can do his homework or tidy up the house or read a textbook, or watch a film, or go for a run. How many ways of spending the evening does Vasya have?

To solve the problem, we introduce indicators: training - T; library - L; homework - H; cleaning - C; reading a book - R; watching a film - W; jogging - J. To structure possible sets of actions, you can use a hierarchy in the form of a decision tree. Note that in addition to linear enumerations in the "TLHC" or "RW" format, there may be options for combining actions in the "T+L" format, i.e. when two actions can be performed at the same time. Thus, we find a number of ways to spend the evening, different from those previously found. Then after building a decision tree, it is possible to highlight new ways of combining actions, which lead to new answers. In divergent tasks, we often have the specificity of the task conditions. There is no insufficiency or redundancy of data in the formulation of the task. Therefore, we can assume that we have partially open mathematical problems.

### Task Situation

A task situation is a set of objects of the subject being mastered and relationships between objects that stimulate the student to productive actions. Traditionally, when solving text problems, it is necessary to make a transition from the plot of the problem to an analytical (algebraic or geometric) generalization in the form of a model. Then it is necessary to choose a solution algorithm from the already known methods of working

with this type of model. In applied activity, there is an additional mental transition from the situation to the plot and only after that to a possible analytical model. This transition allows forming creative thinking directly in the classroom. Note that the transition from the task to the model is important for maintaining other didactic goals and allows to adhere to the traditional methodology of teaching problem-solving.

**Example № 3.** Surprisingly, Ivan Petrovich argued today that he could determine the width of the river from a photograph. Do you think Ivan Petrovich can win the argument? In which cases? Justify your conclusions.

Let's pay attention to the condition of the problem. It is necessary to select mathematical objects in it. Objects can be described as a system with many relationships between them. Therefore, we can talk about operating with subject-object relations.

### Heuristic Task

The educational heuristic task must necessarily meet the following criteria requirements: latency, uncertainty, accessibility and the presence of a semantic context. The degree of certainty of the educational heuristic task determines the situation of forming intellectual and creative activity: orientation, search, transformation and integration. The sequence of situations of intellectual and creative activity is determined by the logic of the process of its formation, which goes through a number of stages. Considering the level of the intellectual and creative activity of a particular student, the teacher can change the intellectual and creative potentials of the educational heuristic task. So, for example, a teacher can enter or delete any information, change the way the task is presented. In other words, a teacher can change the degree of certainty of the task in order to stimulate the development of students' intellectual and creative activity. It should be noted that the educational heuristic task does not contain the main requirement for open tasks - a hidden contradiction, which very often focuses tasks only on well-motivated students.

**Example № 4.** To write numbers, you can use the knots on the shoelaces, correlating the number of knots on the shoelace to the written number. For example, the ancient Incas in South America wrote down the numbers. To add two numbers, it is enough to tie additional knots on the first bunch of shoelaces, equal to the number of knots on the second bunch of shoelaces. To subtract means untie. What should you do if you need to multiply the number by 2 or 3 with the help of knots? Describe how the ancient Incas could act with such multiplication.

The task does not imply a control solution. It is enough to pay attention to the specificity of the conditions of the problem, the absence of blurring. Heuristic tasks develop creative abilities of the individual. The outlook of the student is a base for it.

Therefore, it is rather problematic to build a system of teaching mathematics aimed at developing creativity.

### Classification of Open Problems for Teaching Mathematical Disciplines

Having considered the above types of educational creative tasks used in mathematics lessons, we single out the classification of open mathematical problems that arouse students' interest, without losing the didactic value of the task. The classification is based on the key property of mathematical and creative thinking - types of logical operations:

- Tasks for combining systems and their elements: selection of an element or several ones from the system; comparison of elements and systems; systematization of elements; changing elements; introduction of elements into the system; construction; design; classification of elements and systems; analysis of the structural and functional resources of the system.

**Example №5.** Grandfather Ivan has two gardens with an area of 3 acres. In every garden you can grow carrots and cabbage. Vegetable gardens can be divided in a free ratio between these vegetables. We know the productivity of carrots and cabbage in every garden. What other conditions of the problem need to be added in order to be able to calculate the greatest income of Ivan's grandfather?

- Tasks for determining cause-and-effect relationships: determining the causes; definition of consequences; proof; rebuttal; definition of regularity; definition of a new function; determination of links between elements of the system; forecasting.

**Example №6.** The problem of measuring the mass of many objects led to the emergence of various methods of analytical calculations of their mass without specific "weighing". We know that the mass of the Earth is  $5.9742 \times 10^{21}$  tons. Through the attraction formula, scientists calculated its weight. How could scientists know that the center is the liquid core?

- Tasks for performing research operations: formulation of problems; drawing up a research plan, a plan for solving the problem; planning and conducting observations, measurements and experiments; self-analysis and self-assessment of performance.

**Example №7.** Athanasius found 5 pieces of chain with 3 rings in each piece. He thought about the minimum number of rings that would need to be opened and re-forged in order to put the chain back together. 4 rings might be because one ring connects two sections to each other. But the correct answer is 3 rings: you can take the rings of one piece of the chain and use them to connect the remaining 4 pieces of the chain. Formulate

your problem, in the solution of which the reduction in the number of parts should be used by reducing one part.

Thus, the considered types of creative tasks for teaching mathematical disciplines and the selected classification, built on the principles of logical operations, can be used by mathematics teachers to design tasks for each student, taking into account their level of ability to solve problems and features of cognitive development. These are tasks that arouse the interest of the student, without losing the didactic value of the task.

### Analysis of Using the Classification of Open Mathematical Problems

To assess the effectiveness of using the selected classification of open problems for teaching mathematical disciplines in 2020–2021, we involved 358 students from 16 classes studying mathematics in grades 7–10 of two schools in the European part of Russia, innovative sites supervised by Vyatka State University in 2020–2021. Teachers were asked to apply the proposed methodological guidelines for conducting mathematics lessons that contribute to students' development of creativity and academic achievement during the academic quarter in the classroom.

For a qualitative analysis of the results of the study, we used the method of diagnosing group motivation by

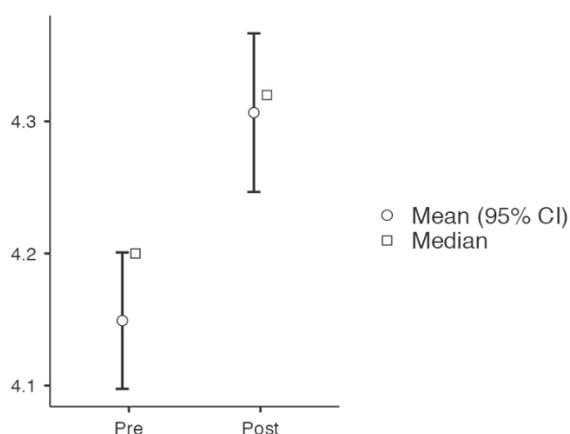
Ladanov (2004). Mathematics teachers were asked to assess the degree of formation of group motivation in their class before the start of the academic quarter preceding the introduction of the classification of tasks and following the results of the academic quarter. The technique is compiled according to the type of semantic differential. The questions include the teacher's assessment of 25 items that positively and negatively characterize group motivation on a rating scale in points from 1 to 7 (**Table 1**).

In the sample of group motivation items, the level of significance of the following 16 items has increased to 0.01 level and 3 items at 0.05 level. The greatest increase was in the item "The ability to show independence in solving tasks assigned by class members" ( $w=0$  and  $p<0.01$ ). Then second order is "Personal understanding of learning goals and their acceptance" ( $w=0$  and  $p<0.01$ ). The item "Positive attitude of students towards the teacher" ( $w=0$  and  $p<0.01$ ) is third order. There are similar increases in "High activity of members", "Trusting relationship between class members and the teacher", "Conditions for expressing the creative potential of students", "Good psychological climate in the class" and "Desire for self-realization in class members". Differences in "Recognition of the teacher's authority", "High level of control over the actions of each member of the class" and "Acceptance of moral standards of behavior within the class" are significant at

**Table 1.** Comparison of group motivation factors according to the assessments of mathematics teachers before and after the experiment using the statistical Student's t-test

Factors characterizing group motivation	Before	After	Wilcoxon W
1. High level of class cohesion	4.38 (1.09)	5.00 (0.97)	0.00**
2. High activity of members	3.69 (1.45)	4.81 (1.64)	0.00**
3. Normal interpersonal relationships in the class	4.50 (1.37)	5.25 (1.29)	4.5**
4. Absence of conflict relations in the class	4.00 (1.21)	4.38 (0.89)	11
5. High level of group compatibility	3.94 (1.24)	4.88 (1.63)	2.00**
6. Personal understanding of learning goals and their acceptance	4.13 (1.54)	5.44 (1.71)	0.00**
7. Recognition of the teacher's authority	4.19 (1.38)	4.50 (1.32)	0.00*
8. Respect for teacher competence	4.06 (1.34)	5.13 (1.59)	0.00**
9. Recognition of the teacher's leadership	4.31 (1.30)	5.38 (1.46)	0.00**
10. Trusting relationship between class members and the teacher	3.44 (1.37)	4.56 (1.71)	0.00**
11. Participation in collective decision-making by class members	3.88 (1.26)	4.94 (1.65)	0.00**
12. Conditions for expressing the creative potential of students	4.25 (1.24)	5.38 (1.50)	0.00**
13. Desire to accept the responsibility of class members for the work performed	4.06 (1.39)	4.75 (1.24)	0.00**
14. Good psychological climate in the class	3.69 (1.35)	4.81 (1.56)	0.00**
15. High level of control over the actions of each member of the class	3.81 (1.17)	4.44 (1.09)	4.5*
16. Presence of an active life position within the class	3.69 (1.35)	4.81 (1.42)	0.00**
17. Desire for self-realization in class members	4.13 (1.15)	5.25 (1.65)	0.00**
18. High degree of coordination of actions among class members	3.19 (1.17)	4.00 (1.71)	0.00**
19. Formation of common group values	4.31 (1.30)	4.50 (1.16)	8
20. No stress in the class	3.94 (1.12)	4.81 (1.33)	0.00**
21. Desire to work in the class	3.44 (1.21)	4.50 (1.37)	3**
22. Positive attitude of the teacher towards students	3.75 (1.39)	4.75 (1.69)	3**
23. Positive attitude of students towards the teacher	3.75 (1.39)	5.00 (1.93)	0.00*
24. Acceptance of moral standards of behavior within the class	4.06 (1.39)	4.75 (1.00)	12*
25. The ability to show independence in solving tasks assigned by class members	3.94 (1.29)	5.31 (1.40)	0.00**

\* - significant differences at  $p \leq 0.05$   
 \*\* - significant differences at  $p \leq 0.01$



**Figure 1.** Box-plot for pre-post of students' academic performance

0.05 level. In some items, differences pre- and post-measurements are not statistically significant. These are "Absence of conflict relations in the class" and "Formation of common group values". Regarding the classroom setting as a whole, it can be stated that the students' motivation grows, and the mood becomes more pleasant.

For the analysis of academic performance in mathematics, we selected in the experimental classes in which teachers worked average marks in mathematics for each student in the academic quarter prior to the introduction of the classification of tasks and average marks based on the results of the academic quarter. When the box plot (Figure 1) is examined, the post performances of the students differ according to their first performances. The results of the analysis of the average grades of 358 students showed an increase in the average grade from 4.15 to 4.31 on a five-point scale (by an average of 3.6%). Testing for the statistical significance of the change using Wilcoxon W confirmed the increase in mean score ( $W=9226$  at  $p \leq 0.01$ ).

Thus, it can be argued that using a dedicated classification of open problems for teaching mathematical disciplines contributes to an increase in group learning and academic performance.

## DISCUSSIONS

In the course of the research, we characterized educational creative tasks: practice-oriented (applied, practical), divergent, task situations, heuristic tasks. These types of problems on their basis allow constructing an open mathematical problem that arouses students' interest, without losing the didactic value of the problem. We made a classification of three types of mathematical problems of an open type. To determine the levels of complexity, we took an approach based on the types of logical operations used in scientific creativity. The practical application of the proposed types allows the teacher to differentiate educational tasks in mathematics, taking into account the ratio of

their complexity and the level of ability achieved by the student to solve problems in mathematics.

Experimental teaching of mathematics took place in grades 7-10 of a general school with the participation of 358 students from 16 classes studying mathematics using a selected classification. Here methodological instructions for conducting mathematics lessons were used.

We evaluated the effectiveness of using the selected classification of open problems for teaching mathematical disciplines. The results of the quantitative and qualitative analysis showed an increase in students' motivation and involvement in learning and academic performance. According to the findings of the research that was carried out by Wahyudi and Marsyidin (2019), the usage of open problems had a beneficial impact on the mathematical abilities of students. As a result of another study (Sarifuddin et al., 2021) using open mathematics problems, there was a positive improvement in the mathematics achievement of high school students.

This approach allows complementing the methodological techniques used in the lesson of mathematics. The technique consists in designing tasks for the student, taking into account his level of ability to solve problems and the features of cognitive development. It is one of the significant variables that contributes to the emotionally supportive environment of the classroom and the accomplishments of the students. Both the routines that instructors carry out in the classroom and the atmosphere that they cultivate have an effect on the students' attitudes of both the teachers and their fellow classmates (Przybylska, 2014).

According to Klein and Leikin's (2020) research, there are challenges that arise when teachers attempt to use open mathematics problems. Effective use of open math projects by math teachers was possible with the correct kind of direction, training, and support. The problem-solving abilities of future teachers-in-training were developed in active learning environments thanks to the utilization of open mathematics tasks (Litster et al., 2020).

Students' meta-cognition is positively impacted when open mathematics problems are used in class in an effective manner, which also has a favorable influence on the culture and atmosphere of the classroom (Hancock & Karakok, 2020). According to the findings of this investigation, there was a statistically significant rise in the component under "Good psychological climate in the class" In this regard, the findings of the study provide evidence of parallelism.

According to research carried out by McLure et al. (2022), both the emotional climate of the classroom and the interpersonal interactions that develop between the instructor and the students have a favorable impact on the students' attitudes about the subject matter being

taught. According to the teachers who participated in this research, "Normal interpersonal relationships in the class" improved as a direct result of the instructors' methods of instruction. It is comparable to the findings of the study that was discussed before.

Therefore, the discussed aspect is the choice of the volume of using educational open problems in mathematics lessons in comparison with traditional problems. On the one hand, tasks allow forming an interest in the study of mathematics and seeing their real significance in life, showing the student the applied application of mathematics. On the other hand, the limited time of the lesson does not allow deviating from the topic and its subject content, and hence the tasks for developing knowledge.

## CONCLUSION

Thus, a significant result of the study is an illustration of the relationship between the methods of developing students' motivation and methodological methods of teaching mathematical disciplines. A set of types of mathematical problems of an open type is distinguished with the allocation of their logical structure used to develop students' creativity in a mathematics lesson.

In general, we can identify three types of mathematical problems of an open type: problems for combining systems and their elements, problems for determining cause-and-effect relationships, problems for performing research operations. These types of mathematical problems allow the teacher to design problems, taking into account the level of ability achieved by the student to solve problems and his features of cognitive development.

On the basis of the selected classification of open mathematical problems, we developed methodological guidelines for conducting mathematics lessons that contribute to the development of students' creativity and academic performance. It should be noted that students also increase motivation and involvement in the study of the discipline, as well as academic performance. Besides, using the classification of open mathematical problems makes it possible to compensate for the existing trend towards a decrease in the level of development of students' creativity.

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