Eurasia Journal of Mathematics, Science & Technology Education, 2014, **10**(4), 313-327

Content, Interaction, or Both? Synthesizing Two German Traditions in a Video Study on Learning to Explain in Mathematics Classroom Microcultures

Susanne Prediger & Kirstin Erath
TU Dortmund University, GERMANY

Received 27 September 2013; accepted 14 May 2014

How do students learn to explain? We take this exemplary research question for presenting two antagonist traditions in German mathematics education research and their synthesis in an ongoing video study. These two traditions are (1) the German Didaktik approach that can be characterized by its epistemologically sensitive analyses and specifications of mathematical contents, and (2) the interactionist approach. By presenting the theoretical framework and some empirical insights, the article shows that learning to explain can be conceptualized as increasingly participating in navigating practices through different epistemic fields.

Keywords: Mathematics Education, Interactionism, Didaktik approach, Explaining Practices, Video Study

INTRODUCTION

Although the mathematical practice of explaining is listed in many mathematics curricula as an important process-oriented competence (e.g. MSW NRW, 2011; CCSSI, 2010), little is known about how explaining really takes place in classrooms and how students acquire this competence. In order to explore this question in a qualitative video study, we synthesize two antagonist traditions in German mathematics education research, namely:

(1) the German Didaktik approach (following Kirsch, 1978; Vollrath, 1978, 2001; Winter, 1983) that can be characterized by its epistemologically sensitive analyses and specifications of mathematical contents; and

(2) the interactionist approach that carefully studies classroom microcultures with respect to the negotiation of meanings and the established practices and norms (following Bauersfeld, 1988; Voigt, 1994, 1998).

We first sketch specific parts of these two traditional approaches, then synthesize selected aspects into a framework for a video study in classroom microcultures. The third section presents the research design and the next section provides some insights into the empirical analysis. The article shows that by overcoming traditional gaps between antagonist approaches without neglecting their specific line of reasoning, a rich framework can be built for grasping classroom practices with respect to a complex question.

Correspondence to: Susanne Prediger,
Faculty of Mathematics, TU Dortmund University, D-44221 Dortmund, Germany
E-mail: prediger@math.uni-dortmund.de
doi: 10.12973/eurasia.2014.1085a

ISSN: 1305-8223

State of the literature

- Curricula all over the world list the mathematical practice of explaining as an important process-oriented competence. However, few empirical insights exist into how explaining really takes place in classrooms and how students acquire this competence.
- The German Didaktik approach did important foundational work in specifying different logical levels and epistemic modes that students need to acquire in order to be able to explain mathematical contents.
- The interactionist approach developed the constructs of sociomathematical norms and practices in order to be able to describe how students learn to explain mathematical contents.

Contribution of this paper to the literature

- We integrate aspects of the Didaktik approach into interactionist classroom studies because a purpose-oriented synthesis of selected aspects of the approaches is promising.
- The conceptualization of explaining practices as navigating practices through the epistemic matrix allows capturing the wide spectrum of explaining practices in its mathematical core.
- The data analysis offers insights into the contingency of explaining practices as well as first tendencies about apparent consistency within one of the investigated microcultures.

TWO TRADITIONS WITH DIFFERENT FOCI

Content as a focus of an epistemologically sensitive Didaktik approach

Background

In many European countries, didactics is conceptualized as the educational sub-discipline that 'offers a rationale and a conceptual structure for interpreting, understanding, classifying, and framing educational purposes and practices' (Klette, 2009, p. 102). We refer to the didactics of these countries as the European Didaktik traditions. Internationally, the German Didaktik tradition has been highly valued for the normative framing and its consequent focus on instructional contents (Westbury, Hopmann & Riquarts, 2000; Klette, 2009). The German Didaktik tradition in general education was mainly shaped by Klafki's (1958) focus on the potential of a content for contributing to general educational aims, namely to Bildung.[1]

The Didaktik approach in mathematics education has adopted this normative framing by Bildung and substantiated it for mathematics by deep reflections on different mathematical contents and their epistemological background (starting with Felix Klein, e.g. Klein, 1924). A didactical analysis in this (sometimes so-called Stoffdidaktik) approach does not only analyse, but also restructures contents for elaborating the underlying ideas and epistemological specificities. This is paradigmatically represented in the work of Kirsch (1978), Vollrath (1978), or Winter (1983), to name only a few in the main years between the 1960s and 1980s. In contrast to Griesel's (1974, p. 118) claim that the method of Didaktik was mathematical analysis, the work of Kirsch, Vollrath, Winter, and their students goes beyond a purely pedagogical or purely mathematical analysis in several ways and still forms the fundamentals of the German teacher education in the field of Fachdidaktik (subject didactics), with a strong and traditional focus of what is now internationally called 'pedagogical content knowledge' (Shulman, 1986).

However, from the 1980s and 1990s on, the Stoffdidaktik approach has often been criticized for its mainly prescriptive perspective without theorizing or empirically investigating student thinking and classroom realities in a descriptive perspective (e.g. Sträßer, 1996). Although still very present in teacher education, the emergence of complementary research paradigms resulted in a decreasing influence of Stoffdidaktik in the German mathematics education research landscape.

Learning to explain as research issue in the Didaktik approach

For concretizing the German Didaktik approach, we name some major contributions to the exemplary research question: how do students learn to explain. It is typical for the approach that the question was mainly treated in a prescriptive way: which aspects would students need to acquire in order to be able to explain mathematical contents? The careful analysis was on the one hand logically informed and distinguished conceptual knowledge (concepts and theorems) from procedural knowledge (mathematical procedures and rules, cf. Hiebert, 1986), and on the other hand adopted epistemologically sensitive perspectives for carefully specifying different epistemic modes (by epistemic mode, we mean a specific way of knowing or of getting to know).

[1] The specificity of this German tradition is underlined by the fact that the German words Didaktik and Bildung do not have suitable equivalents in English. In contrast, the English use of 'didactic method' is pejorative for a strongly teacher-oriented pedagogy which is not meant here.

For mathematical concepts, Winter's (1983) treatise on six ways of concept formation was pathbreaking in that he first pointed out the need for diverse epistemic modes for characterizing concepts, namely exemplification (giving examples and counterexamples), explicit formulation (in a definition), or the functional epistemic mode of giving purposes (when a concept is directly tied to the situation in which it is needed). The focus on purposes has grown in many authors' emphasis on the genetic approach (e.g. Wagenschein, 1968), including also outside Germany (in the Netherlands by Freudenthal, 1983; in France by Brousseau, 1997). Next to these epistemic modes gathered by Winter, the focus on so-called 'Grundvorstellungen' is specific for the German Didaktik approach (Oehl, 1962; vom Hofe, 1998): Grundvorstellungen (meaning giving mental models) are those cognitive constructs that offer interpretations of mathematical concepts by connecting them to everyday contexts.

Vollrath (2001) has pointed out that understanding mathematical contents always comprises the interplay between several epistemic modes, not only for concepts, but also for theorems and procedures, and he identifies a similar list of epistemic modes: naming, exemplification, explicit formulation, meanings and purposes. From this perspective, being able to explain is to be understood as knowing several epistemic modes for a mathematical topic and mastering the connection between these modes. The relevant epistemic modes will be further explained below.

Although not being empirically underpinned, this epistemologically sensitive foundational work of specifying different logical levels and epistemic modes has substantially informed the design of German teaching materials and textbooks that try to cover the multifaceted aspects of concepts, theorems and mathematical procedures and to connect different epistemic modes.

Interaction as a focus of the interactionist approach

Background

In the early 1980s, Heinrich Bauersfeld founded a new approach that was explicitly constituted as antagonist to the Stoffdidaktik approach with its exclusive focus on what should be learned. His students summarize: "the unsatisfying results of the reforms in the 1960s and 1970s raised the interest in understanding the inherent laws of everyday school situations. Understanding mathematics classrooms seemed to be of higher priority than changing them" (Krummheuer & Voigt, 1991, p. 13). As a consequence, the research group developed a strong research approach called social interactionism (e.g. Bauersfeld, 1988), and reconstructed subtle mechanisms for how typical interaction patterns between teacher and students shape the learning opportunities in mathematics classrooms. Inspired by microsociological theories such as symbolic interactionism (Blumer, 1969) and ethnomethodology (Mehan, 1979), the interactionist approach conceptualized mathematical classroom activities in a social dimension, as being constituted interactively. Learning was now conceptualized as an increasing participation in the culture of the mathematics classroom (Bauersfeld, 1988; see Sierpinska & Lerman, 1996, for an overview of the social dimension of learning and Bauersfeld's influence). Various classroom studies shed light on the mechanisms that allowed or hindered mathematics learning in the classroom interaction (see Krummheuer & Voigt, 1991, and Jungwirth & Krummheuer, 2006, for overviews of the wide research tradition which is only very partially reflected here).

The introduced social dimension has intensively influenced other research groups outside Germany, especially Cobb and Yackel who complemented their constructivist approach by Bauersfeld's interactionist approach (cf. Cobb & Bauersfeld, 1995; Bauersfeld, 2012, retrospectively reports on the fruitful collaboration). One central construct that was invented in this German–American collaboration for describing classroom microcultures was the construct of sociomathematical norms, defined as "criteria of values with regard to mathematical activities" (Voigt, 1994, p. 105). Similar to general social norms (about social roles and distribution of responsibilities), sociomathematical norms are conceptualized as being successively and often implicitly established in the interaction "when the teacher's expectations and the students' own aims are becoming compatible" (ibid, p. 107). Early examples given for sociomathematical norms were "normative understandings of what counts as mathematically different, mathematically sophisticated, mathematically efficient, and mathematically elegant in a classroom" (Yackel & Cobb, 1996, p. 461).

As the theoretical constructs of social and sociomathematical norms mainly target the metamathematical issues of a classroom microculture but not the mathematical learning contents, Cobb (1998) adopted a third component in order to be able to characterize the concrete 'doing culture' in classroom microcultures, namely the construct of mathematical practices. This construct allows us to "talk explicitly about collective mathematical development" (Cobb, 1998, p. 34) and to reconstruct, ethnomethodologically, the practices in concrete interactions. Practices are defined as "ways of acting that have emerged ... it makes it possible to characterize mathematics as a complex human activity and in that it brings meaning to the fore by eschewing a focus on socially accepted ways of behaving" (Cobb, Stephan, McClain & Gravemeijer, 2001, p. 120). The construct of practices is widely used in many disciplines (with slightly different meanings), and even raised a discourse about a 'practice turn' in social sciences (cf. Reckwitz, 2003, for a thorough discussion of different practice approaches in sociology).

The interactionist constructs of sociomathematical norms and practices were widely adopted in many studies since they allow characterizing how the participation in the classroom microculture successively changes and how it is implicitly regulated (e.g. Yackel, Rasmussen & King, 2000; McClain & Cobb, 2001; Mottier Lopez & Allal, 2007). Some of these contributions were embedded into design projects, aiming at changing classroom practices (e.g. Cobb et al., 2001; Yackel et al., 2000; McClain & Cobb, 2001), whereas the German group continued a strong research tradition focusing on descriptive classroom studies, also with a focus on the negotiation of meanings and other topics (cf. Jungwirth & Krummheuer, 2006, for an overview of more current developments). Since the interactionist perspective nowadays has many facets, we must emphasize that we only use one out of many different interactionist approaches.

Learning to explain as research issue in the interactionist approach

The briefly sketched interactionist approach can be illustrated by its contributions to the exemplary research question: how do students learn to explain? In the interactionist approach, explaining is conceptualized as a mathematical practice being established in a classroom microculture and regulated by specific sociomathematical norms that are constituted in the classroom interaction (Yackel, 2004). Learning to explain in the interactionist approach means to successively engage in the explaining practices of the classroom microculture. In contrast to the Didaktik approach, researchers in the interactionist approach mainly adopt a descriptive perspective, less on what students should learn but more on more on how explanations are carried out in the classroom microculture and by which subtle interactional mechanisms (or sometimes explicit formulations) students learn to successively participate in these explaining practices (Yackel & Cobb, 1996; Yackel, 2004; and Yackel et al., 2000, for explaining in undergraduate courses).

The constructs of microculture, norms and practices allow shifting from evaluating students' utterances as valid/invalid explanations to those matching/mismatching the classroom microculture's norms and practices (Yackel, 2004, p. 3). Whereas several European Didaktik approaches start from the assumption that there is a universal normative way of explaining well in mathematics, the interactionist approach in this line emphasized the relativity to each classroom and reconstructed it for some examples without yet having specified the range of contingency (Yackel & Cobb, 1996; Mottier Lopez & Allal, 2007).

Shortcomings of both traditional approaches

Bauersfeld (1979) introduced the distinction between the matter meant (the intended curriculum), the matter taught (the implemented curriculum) and the matter learned (the attained curriculum). He criticized the limits of the Didaktik approach with its exclusive focus on the matter meant (and at that time empirical research being restricted to assessing individual achievement, i.e. the matter learned). Instead, Bauersfeld pointed out that the research community can only make sense of the condition how the matter learned can really be attained when also the matter taught is systematically considered. For this purpose, he developed the interactionist approach that allowed careful accounts of real learning opportunities as they appeared in the classrooms (matter taught). In a first approach, the matter taught might be easy to reconstruct by considering the teaching materials: 'Has the median been taught, yes or no?' However, if the matter meant is a discourse practice such as ex-

Explanans in epistemic modes / Explanandum in logical levels	Labelling & naming	Explicit formulation	Exemplification	Meaning & connection	Purpose	Evaluation
Conceptual levels						
Concepts						
Propositions						
Semiotic representations						
Models						
Procedural levels						
Procedures						
Concrete solutions						
Conventional rules						

epistemic field of an explanation

Figure 1. Epistemic matrix for distinguishing explanans and explanandum in explaining practices

plaining, this 'matter' is constituted in the interaction. As a consequence, the classroom interaction must be investigated for exploring the matter taught.

On the other hand, the interactionist approach also has its shortcomings, even within a purely descriptive perspective. (1) The notions of norms and practices alone cannot grasp the mathematical core of the explaining practices. That is why some researchers criticize the interactionist approach as being not sufficiently focused on mathematics. (2) The methodological framework is not specific enough to give an epistemologically sensitive account of the differences between the teacher's expectations and student's first realizations. This refers back to shortcoming (1), but raises not only mathematical but also epistemological concerns, focusing on the learning content and the specific ways of knowing and getting to know.

As the shortcomings in both approaches are located in different areas and the potentials of both can be considered complementary, a purpose-oriented synthesis of selected aspects of the approaches seemed to be promising (see Prediger, Bikner-Ahsbahs & Arzarello, 2008, for the methodological concerns of synthesizing theoretical approaches). Therefore, we followed Klette's (2009) claim to integrate aspects of the Didaktik approach into classroom studies, here on the basis of a classroom video study that follows the interactionist approach in an epistemologically sensitive way by which the content is grasped.

LEARNING TO EXPLAIN AS INCREASINGLY PARTICIPATING IN NAVIGATING PRACTICES THROUGH DIFFERENT EPISTEMIC FIELDS: SYNTHESIZING ANTAGONIST TRADITIONAL APPROACHES

Although everybody has an informal ad hoc understanding of explaining, linguists emphasize that it is not easy to give a sharp definition and distinguish it from arguing, elucidating and other discourse practices (Morek, 2012, p. 27ff.). Explaining always aims at building and connecting knowledge in a systematic, structured way by linking an explanandum (i.e. the issue that needs to be explained) to an explanans (i.e. by which the issue is explained). This structure is not restricted to explaining-why (as often mentioned in scientific contexts), but also includes explaining-what and explaining-how. In everyday communication, where usually one knowledgeable person explains to a less knowledgeable person, the recipient orientation is crucial for evaluating an explanation. However, in classroom interaction, this asymmetry of knowledge is often inverted as the person asking for an explanation (mostly the teacher) usually knows the answer better than the explaining person. That is why the recipient orientation does not apply and is substituted by sociomathematical norms of what counts as a good explanation. Due to the topic- and situation-specificity of explaining, we follow Morek's (2012, p. 40) suggestion to conceptualize explaining by a whole spectrum of explaining practices being established in interactions. This conceptualization is in line with the interactionist perspective on explaining as mathematical practice being relative to each classroom microculture. We specifically rely on Morek's (2012) idea of orchestrated explaining, in which an explanation is completed by several speakers in the interaction.

The Didaktik approach offers the conceptual framework for clarifying the addressed spectrum of practices in detail. From an epistemological perspective, explainning practices can be distinguished by different logical levels and epistemic modes. Starting from a matrix of modes and facets originally formulated for design purposes (Barzel, Leuders, Prediger & Hußmann, 2013), we therefore developed the epistemic matrix in Figure 1.

The rows in the epistemic matrix distinguish the explanandum on different logical levels. The four conceptual levels are:

- --concepts--, i.e. categories such as 'average' or 'function';
- --propositions--, i.e. mathematical patterns, statements or theorems;
- --semiotic representations--, e.g. verbal or graphic realization of a mathematical topic;
- --models--, i.e. addressing the relation between reality and mathematical objects/statements (e.g. 'what can an average say about data?').
- The three procedural levels are:
- --procedures--, such as general way of calculating the average;
- --conventional rules--, e.g. 'brackets first'; and
- --concrete solutions--, such as individual ways of solving a concrete task.

The columns of the epistemic matrix address the explanans in different epistemic modes (as introduced above):

- The epistemic mode ||labelling & naming|| is the only one which can be addressed by a single word, for example by naming an applied --procedure-- or just saying a number as --concrete solution-- for an arithmetic problem.
- The mode ||explicit formulation|| is the linguistically most elaborate way of treating an explanandum since it includes definitions and formulating patterns and theorems.
- The mode ||exemplification|| is addressed by giving examples and counterexamples.
- The mode ||meaning & connection|| comprises all aspects of an explanandum that bridge to another level or mode, for example pre-existing

knowledge (in the case of meanings/Grundvorstellungen), arguments, reasons.

- The mode ||purpose|| belongs to a pragmatic, functional approach of explaining an explanandum by its inner mathematical or everyday functions, for example 'by an average, we can get a rough idea of the whole group'.
- The mode ||evaluation|| is an important part in the context of presenting solutions in class. Although only rarely directly addressed as nans, it is therefore a crucial epistemic mode. Here not only aspects such as right or wrong are brought up but also, for example, constraints of application.
- The additional mode ||subjective experience|| (that is omitted in this article) is characterized by the very personal experiences and feelings towards a mathematical topic. It is therefore rarely observable.

In terms of the empirical approach presented below, each explanation that is demanded or given in a classroom interaction can be characterized by its so-called epistemic field, that is, the combination of addressed logical level and epistemic mode. For example, if a teacher wants students to explain the average by giving a definition, this addresses the epistemic field --concepts-- ||explicit formulation||. If he asks for an example for the calculation of the average, it is --general procedure-- ||exemplification||. This structure allows us to compare explaining practices for different mathematical topics.

Since good explanations often comprise several epistemic fields, we conceptualize the explaining practices in classroom microcultures as navigating practices through different epistemic fields.

As learning to explain in the interactionist approach means to successively engage in the explaining practices of the classroom microculture, it means increasingly participating in navigating practices through different epistemic fields. Whereas preceding empirical studies explored the interactionist mechanisms for how practices and norms can be established in principle (e.g., Yackel & Cobb, 1996; Yackel et al., 2000), our current study intends to specify the explaining practices by systematically taking into account the epistemic fields as a core of the content.

Altogether, the interactionist approach with its focus on the matter taught and the interpretative sequence analysis of classroom interaction together with the matrix of epistemic fields derived from the Didaktik approach provide the framework for reconstructing the practices and norms for explaining and the processes of increasing participation, that is, learning to explain. We therefore operationalize the wide question 'How do students learn to explain' into the following concrete research questions for reconstructing the practices and norms for 'explaining' as a matter taught:

Q1. Which epistemic fields are usually addressed in explaining practices? (specification)

Q2. How do the explaining practices differ between microcultures with respect to the navigation between epistemic fields? (contingency)

Q3. How far are the explaining practices consistent within a microculture with respect to the epistemic fields? (consistency)

The last question in particular is necessary for justifying the application of the theoretical construct 'microculture' for the content 'explaining': only if practices are established in a classroom with a certain consistency over time does it make sense to address the 'microculture of explaining'.

Although not explicitly addressed in this article (cf. Heller, 2014, instead), the analysis will also shed light on the interactive processes of how students successively engage in the explaining practices.

RESEARCH DESIGN AND METHODS OF THE VIDEO STUDY

The video study has been conducted within the interdisciplinary project INTERPASS[2] that generally investigates interactive procedures of establishing discourse practices, comparatively for mathematics and German language classrooms, for gaining deeper insights into processes of successful and unsuccessful participation in classroom interaction. Here, we only focus on those aspects of the research design that are relevant for the research questions on explaining in mathematics classrooms identified above.

Methods for data collection and sampling

Data corpus. Video data were gathered in five different mathematics classrooms. As the classroom microculture becomes most explicit while being newly established, the first set of videos was recorded in the first eight lessons of Grade 5 classes when the 10–11-year-old children entered the secondary school and first met their new teachers. The long-term processes were captured by a second set of videos with four lessons each, six months later. Altogether, 12 x 5 lessons (of 45 minutes each) were videotaped, each with four cameras and two additional microphones. The data corpus also included students' and teachers' written products and classroom materials.

[2] The project 'INTERPASS – Interactive procedures of establishing matches and divergences for linguistic and microcultural practices in German language and mathematics classrooms' is conducted by the authors together with Anna-Marietha Vogler and the linguists Uta Quasthoff and Vivien Heller. It is funded by the German ministry BMBF (Grant 01JC1112).

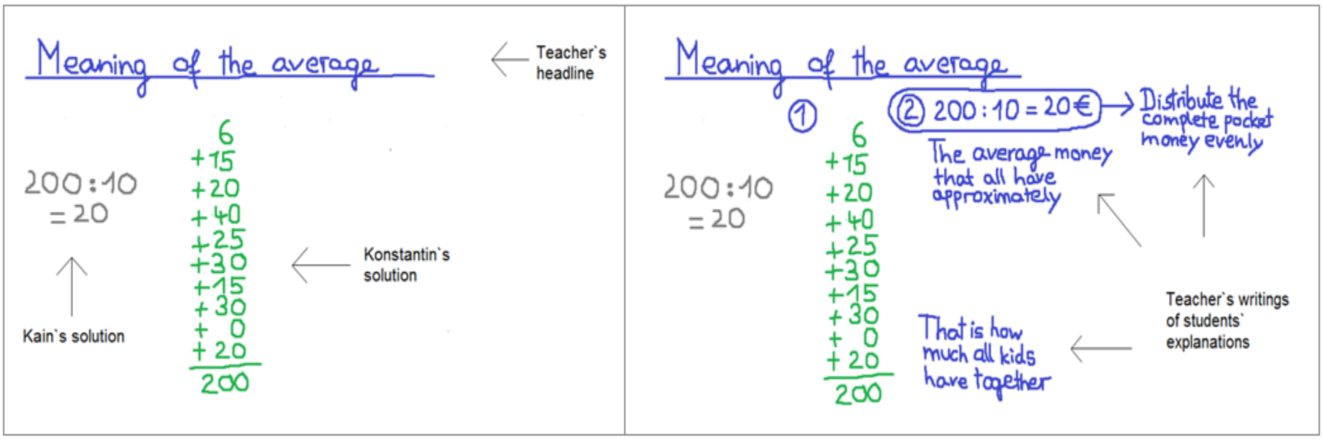

Figure 2. Starting point (left) and end point (right) on the blackboard in Episode 1 (translated)

Sampling. The five classes with n=147 students were sampled systematically with respect to school type (grammar schools and comprehensive schools) and socio-economic background of the school area (low SES versus high SES) for covering heterogeneity.

METHODS OF DATA ANALYSIS

For this article, the analysis of selected parts of the video data was conducted in five steps:

Step 1. Data preparation and transcription. The video data were prepared in Transana (a network-based qualitative video analysis software) by indexing all episodes of whole-class interaction in which a common explaining practice was conducted as a discourse unit of more than a sentence (altogether approximately 200 episodes) and by transcribing those indexed episodes that raise interesting phenomena on matches and mismatches (so far about 50 episodes; more will follow).

Step 2. Sequential analysis. The sequential analysis of the transcribed episodes is conducted in order to detect IRE-sequences (initiation – response – evaluation) that often characterize polyadic teacher–student interaction (cf. Mehan, 1979), also in mathematics classrooms (Mottier Lopez & Allal, 2007). Although the pattern is often criticized for reducing students' space for participation (e.g. Mehan, 1979) from a prescriptive perspective, it is still relevant for descriptive and analytic purposes. Each move of the IRE-sequence (or of the multiturn sequence IRRRE or IRERE, if appearing) is analysed with respect to the epistemic matrix:

I: In which epistemic fields does the initiating move locate the explanation (usually by the teacher, in rare cases by a student)?

R: In which epistemic fields do the students answer?

E: How are these answers evaluated as matching or mismatching by the teacher?

Step 3. Reconstruction of the navigation pathway in the epistemic matrix. The succeeding sequences belonging to one episode are condensed to a navigation pathway identifying the course through the epistemic fields in the matrix. This allows one condensed picture for an episode such as that in Figure 3.

Step 4. Reconstruction of the practices and norms. As an abstraction of the concrete pathway, the sequential analysis of an episode is condensed. By systematic comparison with other navigation pathways, we reconstruct (as underlying categories) the enacted or established practices and/or norms in the interaction.

Step 5. Comparative analysis by contrasting episodes. For investigating consistency and contingency of practices and norms within and between classroom microcultures, several episodes are systematically contrasted.

SELECTED INSIGHTS INTO THE EMPIRICAL ANALYSIS

Contingency of explaining practices

For answering research questions Q1 and Q2, we briefly present the analysis of two episodes on similar explaining situations in the first two weeks of grade 5: Episode 1 on calculating the average, Episode 2 on rounding.

Episode 1: Explaining the procedure of calculating the average

The following episode on explaining the calculation of the average shows how the teacher's and students' orchestrated explaining practice can be described as navigating through different epistemic fields that takes place in IRE-sequences.

Table 1. Transcript of Episode 1 – first part (translated and simplified)

1 TE	What is HANDIER in the way Konstantin wrote it than in Kain's way? I KNOW a small step is missing here. But what is handier here in THIS version than in THIS one. […]	*Initiation*: --concrete solutions/procedures-- ‖purpose/evaluation‖
2 eva	no IDEA;	
3 TE	hm_hm; it's not yet CLEAR what they are doing and WHY.	*Initiation*: --procedures-- ‖meaning & connection‖
6 nah	With Kain's way, have to calculate it all in your HEAD, and Konstantin has a handier way because he calculated all one BELOW the other, that was EASIER	*Response:* --concrete solutions-- ‖purpose/evaluation‖
7 eri	summed up written.	
8 TE	EXACTLY. […]	*Evaluation*: explicit mark of match
	As a START, you can say what these two hundred here, what else do these two HUNDRED indicate?	Initiation: --concrete solutions/procedures-- ‖meaning & connection‖
10 TE	Then it's possibly going to be clearer soon for Evan, WHAT one has to do. WHAT else do these two hundred indicate? [4.5 sec break] Lilja. [4.7 sec break] What does one KNOW, what do these two HUNDRED indicate?	Initiation: --concrete solutions / procedures-- ‖meaning & connection‖
11 lil	How much MONEY this is all calculated together.	*Response*: --concrete solutions-- ‖meaning & connection‖
12 TE	*[writes on the blackboard 'That is how much all kids have together', 16.0 sec break]* You see, THIS is very important at the first step that you have THIS value. […]	*Evaluation*: explicit mark of match emphasis by blackboard
16 TE	[…] I just asked Lilja what these two HUNDRED mean; she described it COMPLETELY correct. Now, what do these TWENTY Euro tell us that come out here and that also got out here at Kain's? *[4.0 sec break]* Two hundred Euro, all kids have together who told their pocket money. Now, what do these TWENTY Euro tell us? [*7.0 sec break*] Dilay, Eric.	*Initiation*: --concrete solutions/procedures-- ‖meaning & connection‖
17 dil	It's *the [expressed in wrong grammatical gender: das Durchschnitt]* AVERAGE of the money.	*Response:* --concepts-- ‖naming & labelling‖
18 TE	Hm, hm, the *[expressed in correct grammatical gender: der Durchschnitt]* AVERAGE of the money. Eric.	*Evaluation*: implicit mark of mismatch
19 eri	It's the average of the POCKET money that all have APPROXIMATELY.	*Response:* --concepts/concrete solutions-- ‖naming & labelling / meaning & connection‖
20 TE	*[writes on the blackboard 'The average money that all have approximately']* […]	*Evaluation*: implicit mark of match

Preceding the episode, the class has worked on the following task: 'Calculate the average pocket money: 15 € / 15 € / 20 € / 40 € / 15 € / 30 € / 15 € / 30 € / 0 € / 20 €'. At the request of the teacher Mr. Schrödinger (TE in the transcript), Kain has written his solution on the blackboard and afterwards Konstantin adds his calculation in order to make it more explicit (see left side of Figure 2). The transcript (which was translated from German and simplified, see Table 1) starts after he went back to his place.

The teacher initiates an explanation for the procedure of calculating the average on the logical level of --concrete solution-- in the epistemic modes ‖purpose‖ and ‖evaluation‖ (#1). Although the --concrete solutions-- are explicitly addressed, he implicitly already treats the concrete solutions also on the level of --procedures--, as indicated by the heading 'Meaning of the average' (see Figure 2). Thus he addresses two levels at the same time (that is why the utterances appear in two epistemic fields in Figure 3). Nahema's response (#6, complemented by Eric in #7) picks up one of the expected epistemic fields and receives a positive evaluation by the teacher (#8). The interaction shows a classical IRE-sequence in a case without mismatches.

In the IRE-sequence, the teacher navigates more explicitly towards the epistemic mode mentioned in the heading ‖meaning & connection‖: in turn #8/10 the teacher stays on the logical level of --concrete solutions/procedures-- but shifts to the epistemic mode ‖meaning & connection‖. Lilja follows this switching of modes and gives a response (#11) that is evaluated explicitly as matching by the teacher by means of reformulation by writing on the blackboard (#12).

In the following (partly) non-printed turns #12–16, the teacher mentions that Konstantin's solution is incomplete. Rather than evaluating the solution as false, he establishes it to be the first step, which should be followed by the second step 200 : 10 (Kain's solution). By this, he establishes (or reinforces) the sociomathematical norm that all partial solutions are welcome in the classroom.

Then the teacher goes on with explaining the procedure by eliciting meanings for each step: the initiation of an explanation in the epistemic mode of ‖meaning & connection‖ is strengthened by reformulating Lilja's

Table 2. Transcript of Episode 1 – second part (translated and simplified)

22 TE	[…] By the way, what do we do by this DIVIDING computation; here two hundred Euro divided by TEN; and I guess, you can imagine this BETTER with sharing out candy, than with POCKET money. If you have TWO hundred candy and you DIVIDE them by TEN, what are you DOING then?	Initiation: --concepts-- ‖meaning & connection‖
…		
24 TE	And that's how you also made CLEAR the dividing in primary school.	Initiation: --concepts-- ‖meaning & connection‖
…		
27 tha	[…] the dividing by ten IS the THING that ten things share – so you DO have two hundred candy. And you DO put twenty for each one. Then you SPLIT the ten things – well in TEN	*Response:* --concepts-- ‖meaning & connection‖
28 TE	GROUPS? In ten PACKAGES?	
29 tha	Yes, EXACTLY.	
…		
31 dil	You maybe divide the AVERAGE.	*Response*: *unclear, possibly --concepts--* ‖naming & labelling‖
32 tha	Then you HAVE the average.	*Evaluation*: explicit mark of mismatch
33 TE	Yes, but in PRIMARY school you didn't work with average. But anyway you could DIVIDE. *[6.5 sec. break]* Insofar I think, arguing with the average is DIFFICULT, because you, don't you, you already DID it in primary school. Tilbe, Tilbe and then Uwe.	
34 til	Well, the AVERAGE is therefore, well, if you, if a WOMAN for example weighs fifty kilo; and the NINE others; one weighs NINETY kilo, one weighs FORTY kilo, one weighs forty-NINE kilo, and so on. Then you would calculate approximately and if the most are from forty-nine to sixty, then you would say the AVERAGE is fifty. *[2.5 sec. break]*	*Response*: --concepts-- ‖meaning & connection‖
...		
37 TE	[…] OKAY. You can envision it like that from the IMAGINATION. That what THASIM just said, that you split in such PACKAGES, this meets the meaning here even BETTER. In fact, you split EVERYTHING you have; you are MAKING ten packages out of it; and then you have twenty Euro in EACH package.	*Evaluation*: explicit mark of match (Thasin #27) implicit mark of match (Tilbe #34) implicit mark of mismatch (Dilay #31)

explanation (earlier in #11). In spite of this orientation to the intended epistemic field, Dilay's response (#17) does not match the expectations: her wrong grammatical gender for the German word for average (neutral 'das Durchschnitt' instead of masculine 'der Durchschnitt') is corrected simply by rephrasing it in the corrected gender. Secondly, Dilay explains the meaning of the average by the *word* average. She hence replies in the epistemic mode of ‖naming & labelling‖ the relevant concept. Dilay's epistemic mismatch is evaluated only implicitly, by not working with it in the further course (#20-end; see also Table 2). Eric (#19) also uses the word average but continues with 'what all have approximately' and thereby also addresses the mode ‖meaning & connection‖. The teacher evaluates Eric's answer as matching by writing it on the blackboard.

After discussing each step of the solution separately, the teacher focuses the meaning of division and navigates towards a more general logical level than --concrete solutions--, in this case the level --concepts-- (see Table 2).

The non-printed first response of a student (#25–26) is evaluated as a mismatch with regard to the content by the teacher. Thasin's response, however, takes on the example from the teacher's initiating move as well as the epistemic field (#27). The teacher helps him in finding a suitable word for his explanation but does not give a direct evaluation until two other students (Dilay, Tilbe) gave their answers.

Dilay's reply (#31) does not address the demanded epistemic field and is also mathematically wrong. It is evaluated explicitly as mismatching by the teacher (#33) and corrected (unasked) by Thasin (#32). The next response by Tilbe (#34), still referring to the initiating move in turn #22, addresses the epistemic field --concepts-- ‖meaning & connection‖ but deals with a different concept. Instead of talking about the basic meaning of division, Tilbe refers to a second meaning of the average, namely balancing. Like in Thasin's case, the teacher does not give a direct response and first waits for other students' contributions.

The evaluation given by the teacher (#37) is modelled more like a summary and, in this way, the evaluation of the different answers is determined by the way it is picked up in this summary. Hence, Thasin's response is evaluated explicitly as matching whereas the other responses are only evaluated implicitly, matching (Tilbe) respectively mismatching (Dilay).

The condensation of these sequential analyses into the navigation pathway in Figure 3 shows that the teacher navigates through several epistemic modes for the logical levels --concrete solutions-- and --procedures--. In several IRERE-sequences, these kind of interactions establish a common explaining practice for general procedures by referring to meanings for single steps and (intermediate) results. These single steps are extracted from different individual solutions and combined in a complete exemplary way. In contrast, an explicit formulation of the general procedure does not occur in this episode.

Although the comparison of this episode with other episodes in the same classroom of Mr. Schrödinger is still ongoing, we find a certain tendency to consider this orchestrated and understanding-oriented explaining practice for complete procedures with a high degree of participation by many students as typical for this classroom with this teacher. However, the reconstruction of Episode 2 will show that other classrooms have established contrasting practices.

Episode 2: Explaining the rounding procedure

Episode 2 also deals with explaining a procedure but, in comparison to Episode 1, other epistemic fields are addressed and a quite different navigation pathway is established. The transcript in Table 3 starts after the teacher, Mr. Maler, wrote the task 63 ≈ on the blackboard, the first rounding task in the new class (rounding having been learnt in primary school before).

The teacher initiates an explanation for the procedure of rounding on the logical level of --concrete solutions-- and asks for why or how, that is, ||naming & labelling|| and either ||explicit formulation|| or ||meaning & connection|| (#1). When Kostas only addresses the mode ||naming & labelling|| (#2) the teacher does not give a direct evaluation but establishes a necessity of further explanation by asking other students (partly non-printed turns #3–6). In contrast to the shared responsibility that was established in Episode 1, the teacher personalizes the necessity of explaining to Kostas in the next turn: the teacher's evaluation is at the same time an initiating move (#7), this time only addressing the mode ||meaning & connection|| but not explicitly excluding one of the other modes from the first initiating move (#1). Kostas assumes the responsibility of explaining and starts with ||naming & labelling|| the used --concept-- of rounding on tens (#8). For this concept of rounding on tens, Kostas shifts to the mode ||meaning & connection|| (#12). With the notion 'nearer number', he activates a mental representation on the number line and hence explains the core meaning of rounding.

In #14, the teacher evaluates Kostas' explanation as partly correct ('I think I already understood some parts of what you wanted to explain') but marks very explicitly that a transformation of the explanation is necessary ('I filtered out'). Still, Kostas is the only one in charge when the teacher asks him to improve his explanation by pointing to the tens (--concepts-- ||naming & labeling||). The student response (#15) is evaluated as matching by the teacher (#16).

In the non-printed turns of the transcript (#17–19), the class is asked to describe the changes of the ones and tens. Afterwards the teacher makes explicit that a good explanation in this situation in his view should have been carried out in the field --conventional rules-- ||explicit formulation|| (#20, see Table 4).

With his initiating move (#20) the teacher not only makes his expectations explicit but also makes clear again that Kostas's utterances (#8/12) were not adequate. Furthermore, he navigates from the --concrete

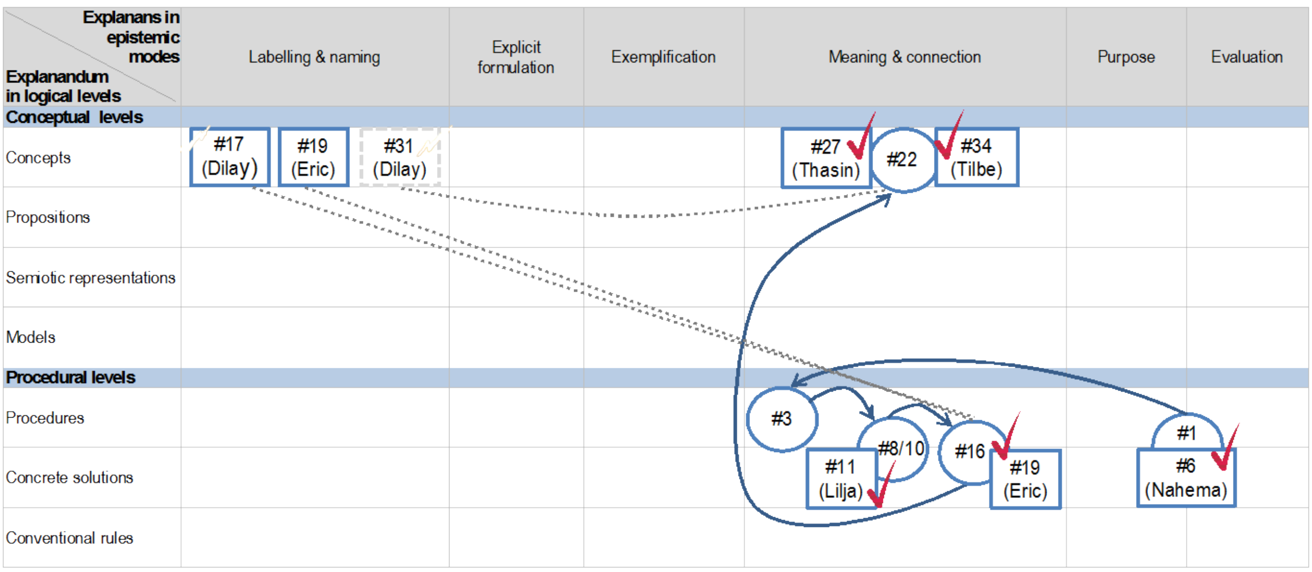

Figure 3. Navigation pathway in the epistemic matrix for Episode 1

Table 3. Transcript of Episode 2 – first part (translated and simplified)

1 TE	[…] And now, YOU tell me *[3.5 sec. break]* a NUMBER that could fit on the other side; and OPTIMALLY, you also tell me why or HOW. […] Kostas.	*Initiation*: --concrete solutions/*procedures*-- ‖ naming & labelling/explicit formulation/meaning & connection ‖
2 kos	SIXTY.	*Response:* --concrete solutions-- ‖ naming & labelling ‖
3 TE	SIXTY. Can we WRITE this down?	*Initiation*: --concrete solutions/*procedures*-- ‖ evaluation ‖
…		
5 TE	NO?	
…		
7 TE	Hm, LOOK. I'm WRITING it down, Kostas, and now YOU convince us, why the sixty can stand there and why this is CORRECT.	*Initiation*: --concrete solutions/*procedures*-- ‖ meaning & connection ‖
8 kos	°hhh *[articulated clearing his throat]* Well, if you are rounding DOWN the sixty-three on TENS; then it comes, it gets, there must be ALWAYS a zero at the end, it MUST be.	*Response:* --concepts/concrete solutions-- ‖ naming & labelling/explicit formulation ‖
9 TE	hm_hm,	
10 kos	when you are rounding.	
11 TE	On TENS yes.	
12 kos	And then there, if you take AWAY the three and shift the ZERO to it. So, you could DO that, but actually it's WRONG. You just have to round down and nea.. nearest number with a ZERO you have to write there.	*Response:* --concepts/concrete solutions-- ‖ meaning & connection/explicit formulation ‖
…		
14 TE	KAY, I think I already UNDERSTOOD SOME parts, of what you wanted to explain; so FIRST of all I filtered OUT, you rounded on TENS; what does that mean HERE. if you are rounding on TENS, what ARE the TENS here actually? Can you show that simply once in the front, Kostas? I am not completely sure, if you DID round on tens.	*Evaluation*: mark of partial match mark of partial mismatch *Initiation:* --concepts-- ‖ naming & labelling ‖
15 kos	Well, THIS. [*points to the 6 in 63 and 60*]	*Response:* --concepts-- ‖ naming & labelling ‖
16 TE	THESE are the tens; OKAY; OKAY; these are the tens; WELL. […]	*Evaluation*: explicit mark of match

Table 4. Transcript of Episode 2 – second part (translated and simplified)

20 TE	[…] and you already implied WHY; but do any of you know a RULE, HOW one has to proceed here, and when one here, when the ten stays the SAME? In this case, and the place BEHIND, which is rounded, goes to ZERO? Ha; *[4.5 sec. break]* Katja.	*Initiation:* --conventional rules-- ‖ explicit formulation ‖
21 kat	With zero one two three FOUR you are rounding down and with five six seven eight NINE you are rounding (up). *[3.5 sec. break]*	*Response:* --conventional rules-- ‖ explicit formulation ‖
22 TE	Did EVERYBODY understand that?	*Evaluation* implicit mark of match
23 cla	*[affirms in chorus]*	
…		
32 TE	[…] in MATHEMATICS, you are probably going to find OUT, you always TRY as much as possible to express the things precise, brief and lean. […]	

solutions-- to the more general level, in this case --conventional rules--. Katja's response (#21) follows to the navigated field and the teacher evaluates this as matching by emphasizing its importance (#22/23) and without asking for further clarification, for example to which place in the place-value system she is referring.

The importance of giving an ‖explicit formulation‖ of --conventional rules-- is strengthened in the non-printed turns #24–31. The episode ends with an explicit verbalization of the underlying sociomathematical norm (#32): the teacher characterizes mathematical formulations as precise, brief and lean. Given that this statement occurs in the environment of explanations, it can be assumed that the teacher also applies these attributes to good explanations. Since these criteria match very well to the characterization of mathematical theorems, definitions and rules, it is to be expected that explaining practices in this microculture are closely linked to using the levels of --propositions-- and --conventional rules--

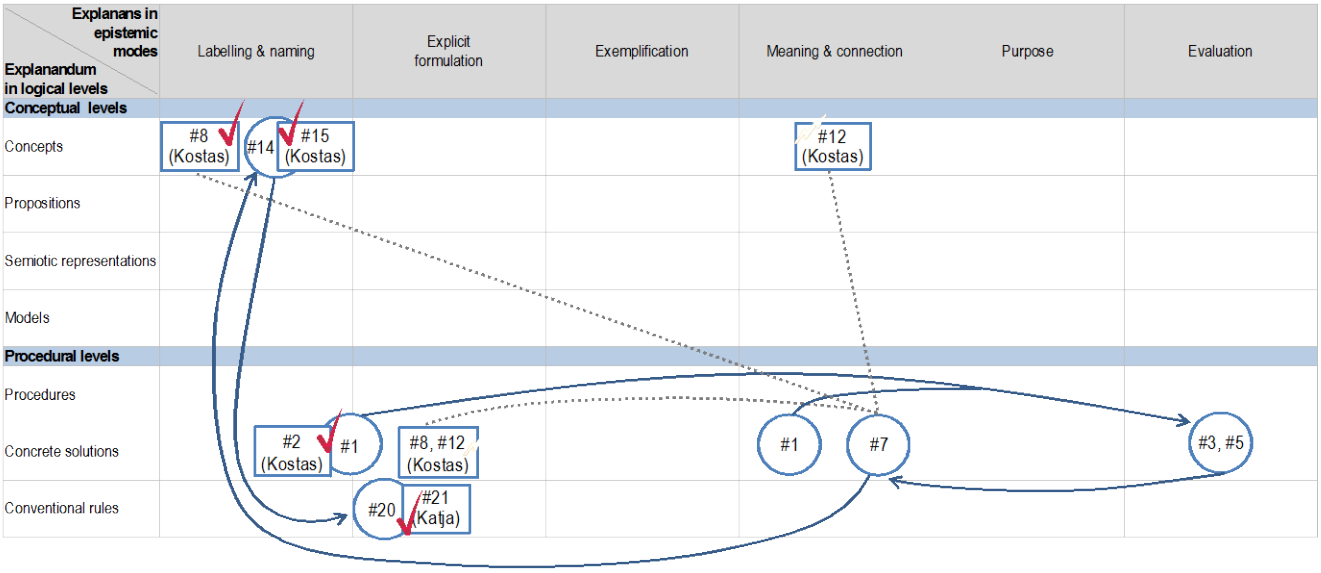

Figure 4. Navigation pathway in the epistemic matrix for Episode 2

in the mode of ||explicit formulation||, whereas ||meaning & connections|| is not addressed.

The condensation of these sequential analyses in a navigation pathway in Figure 4 shows that the teacher navigates through various epistemic modes and logical levels, partly highly structured in small steps. The comparison of this sequence to other sequences allows reconstruction of a common explaining practice for procedures by referring to ||explicit formulations|| of –conventional rules--; the underlying norm is explicitly formulated in #32. Kostas, who explained by giving the ||meaning|| of --concepts--, is rejected as not understandable (#14).

The comparison of Episode 1 and Episode 2 gives first answers to the research questions Q1 and Q2: explaining practices can be compared across different topics by means of the epistemic matrix. IRE-sequences can be reconstructed with respect to the epistemic fields, being addressed usually by the teacher's initiations and students' responses. The evaluation of matches/mismatches offers insights into what counts as good explanation with respect to the navigation through the epistemic fields. The different sequences in Episodes 1 and 2 showed that explaining practices can be established on a wide range between implicit and explicit evaluations and even, but rarely, explicit formulations of sociomathematical norms.

However, the sociomathematical norms of what counts as good explanation differ from classroom to classroom. The comparison allowed reconstruction of a large contingency even for explanations on the same logical level: both episodes deal with the explanation of a general procedure starting from one concrete task. In the classroom of Mr. Schrödinger, especially the mode ||meaning & connection|| on the levels --concrete solutions--, --procedures-- and --concepts-- is established as matching for explanations, whereas the level --conventional rules-- is not addressed. In the classroom microculture of Mr Maler, the mode ||explicit formulation|| of --conventional rules-- is established as adequate for explaining, whereas references to meanings of concepts are implicitly or explicitly rejected. Therefore both microcultures differ with respect to the preferred established levels and modes, and also with respect to shared responsibilities for orchestrated explaining versus single responsibility. These first empirical findings could be confirmed in the analysis of other episodes, not only for general procedures, but also for other logical levels and epistemic modes (cf. Erath & Prediger, 2014, for a further analysis of the classroom microcultures introduced here).

Consistency of explaining practices

For research question Q3, the consistency of practices within a microculture was investigated with respect to the epistemic fields. More precisely, we asked whether the practices and norms established for each classroom microculture within 12 observed hours in the two sets with six months distance have common patterns or whether they differ incoherently.

Although the analysis of the complete data is still ongoing, a case study on the classroom of Mr Maler (from Episode 2) has already been accomplished that shows indeed an amazing consistency: in this classroom microculture, the level --conventional rules-- is consistently established as the most-often matching one. Without being able to account for the complete material here, we will give some examples from further episodes in which again the mode ||explicit formulation|| on the level --conventional rules-- is established as matching for explanations.

Table 5. Transcript of Episode 3 (translated and simplified)

7 tab	SIX thousand;	*Response:* --concrete solutions-- ||naming & labelling||
8 TE	GOOD. but now my QUESTION is, HOW did you arrive at this six thousand? Since we also want the RULE.	*Evaluation:* implicit mark of match *Initiation*: --concrete solutions/conventional rules-- ||explicit formulation||
9 tab	Because from e:r.	
10 TE	..to be CLEAR.	*Initiation*: --conventional rules-- ||explicit formulation||
11 tab	Well up to five, well up to four, you have to round DOWN, and from five six seven eight nine you have to round UP.	*Response:* --conventional rules-- ||explicit formulation||
12 TE	EXACTLY. And which place value did you LOOK at? When you rounded on THOUSANDS?	*Evaluation*: explicit mark of match *Initiation*: --concepts-- ||naming & labelling||
13 tab	the SECOND.	*Response:* --concepts-- ||naming & labelling||
14 TE	EXACTLY; the SECOND; okay. […]	*Evaluation*: explicit mark of match

Table 6. Transcript of Episode 4 (translated and simplified)

19 TE	[…] and YOU explain to us, how you come about the solution, how you PROCEED; so I don't want you to just read OUT your results, but I would like, that you explain, HOW you arrived at the corresponding result.	*Initiation*: -- concrete solutions/conventional rules-- ||explicit formulation/meaning & connection||
…		
21 TE	Thus HOW you applied the rules, THAT you brought forward the first time we spoke about rounding. […]	

Episode 3: Explaining a result of a task

The following sequence (Table 5) is extracted from an episode of discussing a task on displaying high numbers and therefore rounding lengths of rivers. In the non-printed turns #1–6 Tabea first suggests a wrong solution followed by a short discussion in the class and Tabea finally saying the right solution (#7).

With his evaluation/initiating move (#8/10) the teacher links the explanation of the solution directly to an ||explicit formulation|| of the corresponding --conventional rule--. Tabea's formulation is evaluated positively and an explanation in another logical level is not demanded. The episode illustrates a very explicit and short (re-)establishment of the epistemic field --conventional rule-- ||explicit formulation|| as being adequate for explanations.

Episode 4: Good explanations in maths lessons

The last episode presented here is extracted from a homework discussion. In his initiating move, the teacher talks very explicitly about what counts as good explanation from his point of view in a situation like this (see Table 6).

Again, the teacher points out that not only the field --concrete solution-- ||naming & labelling|| is important but also how the solution was achieved. At the same time he specifies that the underlying --conventional rule-- should be used as point of reference, as repeatedly reconstructed in other episodes.

Summing up the search in many of Mr Maler's episodes (here exemplified for Episodes 2–4), we find a high consistency in the established explaining practices in this microculture. The second set of videos six months later does not only contribute to this picture of consistency again, but also shows an effect of students' long-term appropriation: most students now seem to behave according to the established practices with less explicit regulation. Kostas, who often tried to explain by giving meanings at the beginning of the year, seems to have become more silent.

In further case studies, these tendencies of long-term appropriation will be investigated more in depth and also for other classroom microcultures.

CONCLUSIONS

Explaining is an important process-oriented competency, but its acquisition is complicated. When students learn skills and concepts such as rounding, they usually get a very explicit instruction (or explicit learning opportunity) for what to do. In contrast, when students learn to explain, they are often obliged to infer what and how to do from quite subtle (often implicit) marks of matches and mismatches given as evaluation for explaining attempts. Even the most obvious rejection that could be reconstructed ('I think I already understood some parts') is still very implicit. The interactionist approach helps to reconstruct these subtle interactive mechanisms that show the difficulty of participating in explaining practices.

However, the explaining practices in classroom microcultures cannot only be captured by the interactive mechanisms of their establishment. Instead, they need to be analysed with respect to their epistemic qualities in order to give justice to the mathematical content and the content-specific ways of knowing by systematically developing a language for important epistemological distinctions. The reconstruction of explaining practices through systematically comparing navigating practices in the epistemic matrix offered the opportunity to capture the wide spectrum of explaining practices in its mathematical core. The data analysis offered insights into the contingency of explaining practices as well as first tendencies about apparent consistency within one of the investigated microcultures.

Especially the implicitness of all these interactive procedures for established explaining practices should have substantial consequences for preservice and in-service teacher education: when some important learning contents (such as explaining) are mainly established in the interactions, all teachers should systematically reflect on their importance and become aware of their implicit choices. This could enable teachers to manage the initiating practices more consciously.

Although this article could only give very limited empirical insights into a complex and ongoing video study, these first impressions already show why it is valuable to synthesize aspects of former antagonist theoretical approaches. This work may motivate other kinds of synthesis for other research questions beyond explaining.

REFERENCES

Barzel, B., Leuders, T., Prediger, S., & Hußmann, S. (2013). Designing tasks for engaging students in active knowledge organization. In A. Watson et al. (Eds.), *ICMI Study 22 on Task Design – Proceedings of Study Conference* (pp. 285–294). Oxford: ICMI.

Bauersfeld, H. (1979). Research related to the mathematical learning process. In International Commission on Mathematical Instruction (Ed.), *New trends in mathematics teaching* (pp. 199–213). Paris: UNESCO.

Bauersfeld, H. (1988). Interaction, construction and knowledge: Alternative perspectives for mathematics education. In D. A. Grouws, T. J. Cooney & D. Jones (Eds.), *Effective mathematics teaching* (pp. 27–46). Reston, VA: NCTM & Lawrence Erlbaum.

Bauersfeld, H. (2012). Interview, conducted by Alexander Karp. *International Journal for the History of Mathematics Education*, *7*(2), 67–92.

Blumer, H. (1969). *Symbolic interactionism*. Englewood Cliffs, NJ: Prentice-Hall.

Brousseau, G. (1997). *The theory of didactical situations in mathematics*. Dordrecht: Kluwer.

Cobb, P. (1998). Analyzing the mathematical learning of the classroom community: The case of statistical data analysis. In A. Olivier (Ed.), *Proceedings of the 22nd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 33–48). Stellenbosch: University of Stellenbosch / PME.

Cobb, P., & Bauersfeld, H. (Eds.) (1995). *The emergence of mathematical meaning: Interaction in classroom cultures*. Hillsdale, NJ: Lawrence Erlbaum.

Cobb, P., Stephan, M., McClain, K., & Gravemeijer, K. (2001). Participating in classroom mathematical practices. *The Journal of the Learning Science*, *10*, 113–163.

Common Core State Standards Initiative (CCSSI) (2010). Common Core State Standards for Mathematics. http://www.corestandards.org/assets/ CCSSI_ Math%20Standards.pdf. Accessed 5 June 2014.

Erath, K., & Prediger, S. (2014). Mathematical practices as underdetermined learning goals: The case of explaining diagrams. In S. Oesterle, P. Liljedahl, C. Nicol, & D. Allan (Eds.), *Proceedings of the Joint Meeting 3 - 17 of PME 38 and PME-NA 36* (Vol. 3, pp. 17-24). Vancouver, Canada: PME.

Freudenthal, H. (1983). *Didactical phenomenology of mathematical structures*. Dordrecht: Kluwer.

Griesel, H. (1974). Überlegungen zur Didaktik der Mathematik als Wissenschaft [Reflections on Didaktik of Mathematics as scientific discipline]. *Zentralblatt für Didaktik der Mathematik*, *6*, 115–119.

Heller, V. (2014, submitted). Academic discourse practices in action: Invoking discursive norms in mathematics and language lessons. Submitted to *Academic Discourse as Situated Practice*. Special Issue of *Linguistics & Education*.

Hiebert, J. (1986). *Conceptual and procedural knowledge: The case of mathematics*. Hillsdale, NJ: Erlbaum.

Jungwirth, H., & Krummheuer, G. (2006) (Eds.). *Der Blick nach innen: Aspekte der alltäglichen Lebenswelt Mathematikunterricht* [A lense on the inside: Aspects of the everyday mathematics classroom]. Münster: Waxmann.

Kirsch, A. (1978). Aspects of simplification in mathematics teaching. In H. Athen & H. Kunle (Eds.), *Proceedings of the Third International Congress on Mathematical Education, Karlsruhe 1976* (pp. 98–120). Karlsruhe: FIZ. (Reprinted in Westbury, Hopmann & Riquarts, 2000, pp. 267–284.)

Klafki, W. (1958). Didaktische Analyse als Kern der Unterrichtsvorbereitung. *Die Deutsche Schule*, *50*, 450-471. Reprinted in English: Klafki, W. (1995). Didactic analyses as the core of preparation for instruction. *Journal of Curriculum Studies*, *27*(1), 13–30.

Klein, F. (1924). *Elementarmathematik vom höheren Standpunkte* [Elementary mathematics from the higher point of view] (Vol. 1). Berlin: Springer.

Klette, K. (2009). Didactics meet classroom studies. In M. Meyer, M. Prenzel & S. Hellekamps (Eds.), *Perspektiven der Didaktik* (pp. 101–114). Wiesbaden: VS Verlag für Sozialwissenschaften.

Krummheuer, G., & Voigt, J. (1991). Interaktionsanalysen von Mathematikunterricht. Ein Überblick über einige Bielefelder Arbeiten [Interaction analysis of mathematics classrooms. A survey on some articles of the Bielefeld group]. In H. Maier & J. Voigt (Eds.), *Interpretative Unterrichtsforschung* (pp. 13–32). Köln: Aulis.

McClain, K., & Cobb, P. (2001). An analysis of development of sociomathematical norms in one first-grade classroom. *Journal for Research in Mathematics Education*, *32*(3), 236–266.

Mehan, H. (1979). *Learning lessons: Social organization in the classroom*. Cambridge, MA: Harvard University Press.

MSW NRW – Ministry for school and professinal development, Northrhine Westfalia (MSW NRW) (2011). *Kernlehrplan und Richtlinien für die Hauptschule in Nordrhein-Westfalen Mathematik* [Curriculum and syllabus for the lower streamed school in NRW mathematics]. Düsseldorf: MSW.

Morek, M. (2012). *Kinder erklären. Interaktionen in Familie und Unterricht im Vergleich* [Children explain. Comparing interctions in families and classoms]. Tübingen: Stauffenburg.

Mottier Lopez, L., & Allal, L. (2007). Sociomathematical norms and the regulation of problem solving in classroom microcultures. *International Journal of Educational Research*, *46*, 252-265.

Oehl, W. (1962). *Der Rechenunterricht in der Grundschule* [The primary classroom on calculating]. Hannover: Schroedel.

Prediger, S., Bikner-Ahsbahs, A., & Arzarello, F. (2008). Networking strategies and methods for connecting theoretical approaches. *ZDM – The International Journal on Mathematics Education*, *40*(2), 165–178.

Reckwitz, A. (2003). Grundelemente einer Theorie sozialer Praktiken. [Basic elements of a theory of social practices]. *Zeitschrift für Soziologie*, *32*(4), 282–301.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, *15*, 4–14.

Sierpinska, A., & Lerman, S. (1996). Epistemologies of mathematics and of mathematics education. In A. J. Bishop et al. (Eds.), *International handbook of mathematics education* (pp. 827–876). Dordrecht: Kluwer.

Sträßer, R. (1996). Stoffdidaktik und Ingénierie didactique – ein Vergleich [Comparing Stoffdidaktik and Ingénierie didactique]. In G. Kadunz et al. (Eds.), *Trends und Perspektiven* (pp. 369–376). Vienna: Hölder-Pichler-Tempsky.

Voigt, J. (1994). Entwicklung mathematischer Themen und Normen im Unterricht [The development of mathematical themes and norms in classrooms]. In H. Maier & J. Voigt (Eds.), *Verstehen und Verständnis* (pp. 77–111). Köln: Aulis.

Voigt, J. (1998). The culture of the mathematics classroom: Negotiating the mathematical meaning of empirical phenomena. In F. Seeger, J. Voigt & U. Waschescio (Eds.), *The culture of the mathematics classroom* (pp. 191–220). Cambridge: Cambridge University Press.

Vollrath, H.-J. (1978). Rettet die Ideen! [Save the big ideas!] *Der Mathematisch Naturwissenschaftliche Unterricht*, *31*, 449–455.

Vollrath, H.-J. (2001). *Grundlagen des Mathematikunterrichts in der Sekundarstufe* [Foundations of teaching mathematics in secondary schools]. Heidelberg: Spektrum.

vom Hofe, R. (1998). On the generation of basic ideas and individual images. In A. Sierpinska & J. Kilpatrick (Eds.), *Mathematics education as a research domain: A search for identity. An ICMI study* (Vol. 2, pp. 317–331). Dordrecht: Kluwer.

Wagenschein, M. (1968). *Verstehen lehren. Genetisch – Sokratisch – Exemplarisch* [Teaching understanding. Genetic – socratic – paradigmatic]. Weinheim: Beltz.

Westbury, I., Hopmann, S., & Riquarts, K. (Eds.) (2000). *Teaching as a reflective practice. The German Didaktik tradition.* London: Lawrence Erlbaum.

Winter, H. (1983). Über die Entfaltung begrifflichen Denkens [On the development of conceptual thinking]. *Journal für Mathematik-Didaktik*, *4*(3), 175–204.

Yackel, E. (2004). Theoretical perspectives for analyzing explanation, justification and argumentation in mathematics classrooms. *Journal of the Korea Society of Mathematical Education*, *8*(1), 1–18.

Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education*, *27*(4), 458–477.

Yackel, E., Rasmussen, C., & King, K. (2000). Social and sociomathematical norms in an advanced undergraduate mathematics course. *Journal of Mathematical Behaviour*, *19*, 275–287.

◈◇◈