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Critical thinking skills in mathematical proof tasks in the context of quality education: Case study

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Abstract

The ability to make judgments is one of the outcomes of quality education, which can be developed through critical thinking in the mathematics learning process, especially in solving proof tasks. However, the performance scores observed in the basic school centralized exam in mathematics are low. Therefore, the study aimed to investigate the performance of 9th grade students in solving proof tasks and the importance of critical thinking skills. The findings confirm that the proof tasks make up 19% of the possible number of points that can be obtained, significantly affecting the overall evaluation.

Keywords: critical thinking skills, mathematics, proof tasks, quality education

INTRODUCTION

In Latvia, like the countries of the European Union (EU), significant attention is paid to improving and ensuring the quality of education aiming at the holistic development of each student. Quality education is defined in the Latvian education law as the educational process, content, environment, and management that ensure everyone the right to inclusive education and the opportunity to achieve high-quality results according to the goals set by society (European Education Area, n. d.; Grozījumi Izglītības likumā, 2021; Izglītības likumā nostiprinātas tiesības, 2021; Publications Office of the European Latvia's Union, 2024). ssustainable development strategy until 2030 also requires a paradigm shift in education. The economic market in Latvia is rapidly developing and changing, and the need for knowledgeable specialists who can solve problems, make decisions, and respond to changes is growing (Sustainable Development Strategy of Latvia, n. d.). Nowadays, every student is supported to learn the basics of knowledge and acquire transversal skills, including critical thinking and problem-solving, creativity and entrepreneurship, self-directed learning, civic participation, and digital skills. For this process to be effective, a change of attitudes and ways of thinking is required (Skola 2030a, n. d.; Skola 2030b, n. d.; Valsts izglitibas satura centrs, n. d.).

Although mathematics is one of the subjects that plays a significant role in line with these strategic goals, in Latvia the average student score in the centralized exam in mathematics continues to decline, reaching the lowest levels in a decade. Overall, student results are concerning, as 826 out of 17,970 students failed to meet the minimum threshold of 10% or score 8 points. That is 5% of the total number of students and indicates that many students lack basic skills in ss (Valsts izglitibas satura centrs, n. d.; Valsts pārbaudes darbi 2022./2023. m.g., 2023).

However, the performance results in mathematics are declining not only in Latvia but also in the EU-statistics show that 30% of students in the EU do not even reach the minimum level of knowledge in 2022. The top performance rates have also decreased over the past, reaching 7.9% in 2022. In the context of EU member states, the highest performance rates in mathematics range from 15.4% in the Netherlands to 2.0% in Greece, while in Latvia it is just over 6% (Report of PISA 2022, 2024).

Proof Tasks in Mathematics For Developing Critical Thinking Skills

The basic education standard states that in the process of acquiring mathematics, a student learns to use mathematical tools in various situations, processes data, uses the properties of figures, sees relationships between

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Contribution to the literature

- The article contributes to the current literature by highlighting the critical role of proof tasks in developing students' critical thinking skills within the context of Latvia's educational system.
- It underscores the declining performance in mathematics exams and the need for a paradigm shift in teaching methodologies. The study's uniqueness lies in its focus on the integration of proof tasks within mathematics education to enhance reasoning and problem-solving abilities. It is important for teacher training, curriculum development, and addressing students' attitudes towards mathematics.
- This research provides valuable insights into improving educational practices and policies to achieve better learning outcomes.

quantities, makes general judgments, models mathematically, chooses appropriate approaches and solutions in various problem situations, is aware of the need for evidence, and forms well-grounded judgments (Ministru kabineta noteikumi Nr. 747, 2018).

In achieving these goals, proof tasks in mathematics are relevant. Proof is defined as a mathematical argument and a logical justification of a related sequence of statements for or against a mathematical claim that uses previously known and accepted statements and forms of reasoning. There are different types of proofs, such as direct proof, proof by contradiction, and the principle of mathematical induction. The specifics of a mathematical task determine the role of proof in the solution and the methods we are allowed to use in mathematical proof (Hangil, 2022; Ndemo et al., 2018; Stylianides et al., 2023).

The concept of mathematical proof is central to mathematics learning developing by and communicating mathematical knowledge, thus promoting students' understanding and reasoning skills essential for the development of deep learning when new knowledge is accepted because it makes mathematical sense, not based on what the teacher said or what was read in a textbook (Ennis, 2018; Rocha, 2019; Stylianides et al., 2023). Proof tasks in mathematics develop students' critical thinking skills, and conversely, through critical thinking, students learn to solve proof and higher-difficulty tasks. The basis of critical thinking are cognitive skills like interpretation, analysis, evaluation, inference, explanation, and self-regulation (Facione, 2015). From an educational perspective, critical thinking is characterized by purposeful judgment and the use of cognitive strategies and skills that increase the reliability of the results obtained in the thinking process. Students improve their critical capability, reasoning, evaluation, and justification skills that help them identify and analyze arguments, draw conclusions, evaluate the consequences of decisions, generate questions and evidence-based arguments, think logically, generalize, and interpret different sources of information. However, research suggests that proof tasks are not being used appropriately in mathematics teaching at all levels of education (Geiger et al., 2023; Heard et al., 2020; Rocha, 2019; Sommerhoff et al., 2021).

Some of the reasons mentioned in studies on why students lack the necessary skills in solving proof problems are their incompetence in creating deductive arguments and beginning the proving process. Also, the comprehensive curriculum in basic education does not contain enough proof tasks and they are more focused on memorization, as well as teachers rarely offer novel proof tasks that could motivate the learners. Other reasons cited are a lack of positive attitudes towards critical thinking and intellectual curiosity, caused by the students' misunderstanding of the need to confirm something obvious and already known to be true. Students still find it difficult to prove, and teachers tend to fail in their mission to help them formulate their arguments (Collier, 2018; Facione, 2015; Hangil, 2022; Ndemo et al., 2018; Rocha, 2019; Stylianides et al., 2023).

Mathematical reasoning and proof skills are considered resource-based cognitive skills and teaching approaches are proposed for the development of these skills: a sequential approach that focuses on each resource separately and a simultaneous approach that supports multiple resources at the same time (Sommerhoff et al., 2021). The studies emphasize that proofs in mathematics differ from proofs in school mathematics and insufficient attention is paid to the communication or explanation function of proofs in the pedagogical approaches used by teachers, because of which students do not understand the meaning of proofs. It is necessary to consider the age of the students, for example, in the initial stage, students' experience could begin with justifications based on examples and gradually move to more general and deductive arguments. To improve critical thinking abilities, students need to gain experience with various proof functions like investigating and taking and defending a position. The most relevant in school mathematics is the explanation function, which allows for a deeper understanding of concepts and subject-matter knowledge (Bieda, 2010; Ennis, 2018; Rocha, 2019).

The ability to form reasoned judgments is essential, which in modern society is considered one of the means of personal self-realization. In recent decades, in the Latvian educational space, much attention has been paid to critical thinking as a transversal skill by emphasizing the evaluation of information, conceptualization, and judgment-making, which is aimed at cultivating reasoned thinking. However, it is believed that the effective development of these skills is still considered problematic (Gubenko et al., 2018; Rubene et al., 2018; Valsts izglitibas satura centrs, n. d.). This could explain the low results observed in the centralized basic school exam in mathematics, particularly in solving proof tasks, which indicate students' inability to reason, as well as their reluctance to solve such tasks that is justified by a lack of experience and understanding of how to do it.

Therefore, the study aims to investigate the results of the performance of 9th grade students in solving proof tasks in the centralized mathematics exam and the importance of critical thinking skills in this process, obtaining answers to the research questions:

- 1. What was the proportion of proof tasks in the centralized basic school exam in mathematics,
- 2. How did students perform in solving proof tasks and did the achieved result affect the final assessment.

MATERIALS AND METHODS

Context of the Research

The state examination system in Latvia is managed by the National Education Centre, which is directly subordinate to the Minister of Education and Science. State examinations in grades 3, 6, and 9 of general basic education include tests, exams, and centralized examinations and are designed to determine the level of knowledge and skills of students. At the end of grade 9, students must take centralized exams in Latvian, a foreign language, and mathematics evaluated by lecturers and teaching staff of higher education institutions prepared by the National Education Centre (National Centre for Education Republic of Latvia, 2020).

In Latvia, in the 2022/23 academic year, the centralized exam in mathematics for 9th grade students was held for the first time as the content and structure of the exam have been changed. The students' performance was evaluated centrally and expressed in percentage terms. To receive a certificate for the results of the centralized exam, the student had to obtain at least 10% of the maximum possible points in the overall assessment (Valsts pārbaudes darbi 2022./2023. m.g., 2023).

Research Methods

The study analyses the results of the basic school centralized exam in mathematics for the 2022/23 academic year. The data is available in the report published by the National Centre for Education on state examination performance results, which describes data on student achievements and task performance. 17,970 9th grade students took the centralized mathematics exam, of which 17,144 passed successfully, obtaining at

least 10% of the maximum possible points. 826 students, which makes up 5% of the total number of 9th grade students, did not pass the exam (Valsts pārbaudes darbi 2022./2023. m.g., 2023).

The study is a descriptive case study and uses a quantitative research design. It is based on secondary data analysis following a question-driven approach and was conducted to describe the current situation in the centralized exam in mathematics for 9th grade students to determine the extent to which mathematical proof problem-solving skills have been mastered. The study, which is based on the use of anonymous secondary data, did not require ethical approval.

To evaluate students' performance results in solving proof tasks additional data was requested from the National Centre for Education. The research sample was 600 exam papers of 9th grade students, who were selected according to the stratified random sampling principle and divided into three groups:

- (1) 200 exam papers scored 35% and less of the maximum possible points,
- (2) 200 exam papers scored 36%-65% points, and
- (3) 200 exam papers scored above 65% in the centralized exam in mathematics.

RESULTS

Description of the Content of the Centralized Exam in Mathematics

The centralized exam in mathematics consists of two parts. The total score for the centralized exam in mathematics is 80 points, which corresponds to 100%. The first part, "knowledge, understanding and skills" includes 27 tasks assessing students' knowledge, understanding, and skills in algebra and geometry. These tasks require mainly simple operations, recognizing symbols, knowing properties and characteristics, and solving algorithms. The time given is 105 minutes and the maximum points available are 60.

The second part, "solving complex problems" includes 5 tasks containing complex problems requiring skills to prove, argue, and justify solutions. The time given is 75 minutes and the maximum points available are 20.

Evaluating the proportion of proof tasks in the centralized mathematics exam, it was revealed that in its first part, they make up 4% of the total number of tasks (1 out of 27) and 8% of the possible points (5 out of 60). In the second part, there are 3 proof tasks out of 5, which make up 60%, and 50% of the possible points (10 out of 20). Thus, in total, proof tasks make up 13% of all exam tasks, which is 4 tasks out of 32. The maximum number of points that can be obtained is 15 out of 80, which is 19% of the total points in the exam (see **Table 1**).

Table 1. Proportion of proof tasks in the centralized exam in mathematics				
Frequency of proof tasks in both parts of the exam	Proof tasks in total	Number of points available		
1 st part-40%	13%	19%		
2 nd part-60%				

Table 2. The percentage of ninth-grade students who completed the proof tasks

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1 st part of the exam: "Knowledge, understanding, and skills"	2 nd part of the exam: "Solving complex problems"	
Task 25.1-47.51%	Task 30–27.94%	
Task 25.2-43.71%	Task 31–21.34%	
	Task 32–19.83%	

Table 3. Performance results in solving proof tasks in exam papers of ninth grade students			
Students' performance	Average percentage of completion of task 25	Average percentage of completion of tasks 30-	
results in percentages (%)	of part 1 of the exam per student (%)	32 of part 2 of the exam per student (%)	
1 st group: 1-35	14	5	
2 nd group: 36-65	44	13	
3 rd group: 66-100	78	45	

The study also analyzed the evaluation results of the centralized exam in mathematics in solving proof tasks for 9th grade students (see **Table 2**).

In the first part of the exam, the only proof task (25) where students had to prove the similarity of triangles was divided into two parts (25.1 and 25.2). Task 25.1 was solved by less than half of the students (47.51%), while task 25.2 by 43.71%.

In the second part of the exam, the results were even lower:

- (1) task 30, where students had to prove the equality of the sides of a trapezoid, was solved by 27.94%,
- (2) task 31, which required justifying the data obtained in a quadratic function, was solved by 21.34%, and
- (3) task 32, involving a real-life situation where trigonometric functions had to be used and the results justified, was solved by only 19.83% of the students (Valsts pārbaudes darbi 2022./2023. m.g., 2023).

Analysis of 9th Grade Students' Performance Results in Solving Proof Tasks

Based on additional secondary data requested from the National Centre for Education, students' performance in solving proof tasks and their impact on the final evaluation of the centralized exam in mathematics were evaluated. Following the developed methodology, the research sample was separated into three groups.

200 exam papers of 9th grade students from the first group, whose overall score of the centralized exam in mathematics reached 35% or less, in the 25th task of the first part of the exam scored only 0.6875 points out of 5 points on average (14%). The evaluation criteria for this task were the ability to supplement the text and find connections to prove the similarity of triangles, write

statements, and calculate the similarity coefficient and the unknown side of the triangle using a proportion. In turn, in the 30th-32nd tasks of the second part of the exam, the respondents of this group scored an average of 0.5375 points out of 10 possible points per student, which means that only 5% solved the task. In these tasks, students had to be able to create well-founded and logically connected statements, observe the given conditions, calculate the necessary quantities, organize the solution, and justify the answer given by linking the calculations to the geometric model.

The second group contained 200 exam papers of 9th grade students whose overall scores ranged from 36% to 65% and showed better performance indicators in solving proof tasks. The students of this group obtained an average of 2.1975 points out of 5 possible points in the 25th task of the first part of the exam, which indicates that the necessary skills have been mastered by 44%. Unfortunately, the percentage of solving tasks 30-32 of the second part of the exam is not high–only 13% (1.29 points out of 10 possible were obtained for each student). These results indicate the inability of students to complete complex tasks that require critical thinking skills - the ability to evaluate and analyze, as well as to create a sequential and logical record.

In turn, the results obtained in the performance of proof tasks for 200 exam papers of 9th grade students, whose scores were above 65%, were much higher-the performance of task 25 of the first part of the exam reached 78% (on average, each student obtained 3.9 points out of 5) and the achievements in the performance of the tasks 30-32 of the second part of the exam are also higher-the percentage of performance of these tasks is 45% (on average, each student obtained 4.4475 points out of 10).

To make the results for all three respondent groups more transparent, they are presented in **Table 3**.

DISCUSSION

By analyzing the exam papers of students, the data was obtained about the number of students who completed proof tasks, the percentage of these tasks completed, the proportion of proof tasks in relation to the total number of tasks, and how the points scored on proof tasks affected students' performance results in total.

The results of the study confirm that the proportion of proof tasks in the centralized exam in mathematics makes up 12.5% (4 tasks out of 32) and 19% regarding the maximum number of points that can be obtained for proof tasks (15 out of 80 possible for the entire exam). Although the number of tasks is not large concerning the total number of tasks, they significantly affect the overall evaluation of the centralized exam. This is especially true for the tasks of part 2 of the exam, in which the number of points to be obtained is proportionally high enough (12.5%) to the total number of points (10 out of 80) and the completion of the tasks requires good basic knowledge and reasoning skills to obtain a high score. However, the number of tasks in the centralized exam in mathematics probably is not motivating for teachers to focus more on solving proof tasks, as the required minimum percentage score of 10% can be achieved without solving proof tasks.

The results of the study indicate that in the context of Latvian basic education, serious consideration should be given to improving students' skills in mathematics, especially in solving proof problems. This is in line with the main goal of educational reforms, which emphasize competency-based education to develop students' ability to think critically, solve various problems, and think more deeply and broadly across all disciplines. The ability to form judgments plays a particularly important role in the learning process, which can be developed through critical thinking, and this method is offered in competency education as one of the socialization tools (Rubene et al., 2018; Skola 2030a, n. d.; Sustainable Development Strategy of Latvia, n. d.; Valsts izglitibas satura centrs, n. d.).

It is essential to recognize that school proofs are different from mathematical proofs and that students need to be encouraged to think critically. In turn, critical thinking skills to analyze, evaluate, synthesize, and conclude are closely related to the ability to solve proof tasks. Proof tasks play a role in understanding mathematical concepts and communicating knowledge, as well as connecting them to real-life situations, which are essential in the deep learning process (Ennis, 2018; Facione, 2015; Hangil, 2022; Heard et al., 2020; Rocha, 2019; Stylianides et al., 2023).

The study highlights pedagogical implications for how teachers can practically integrate proof tasks into their daily teaching process. This demands a meaningful and gradual approach to teaching from primary school onwards, developing students' ability to answer questions, promoting discussions and reasoned thinking by using visual materials, and applying regularities. It is essential to teach students a clear algorithm for solving proofs and to foster an understanding that proofs are based on regularities and are applicable in other subjects. Such an approach not only develops logical thinking but also promotes a deeper understanding of the learning content and its connection to the real world. The support of teachers is essential, as they help students to develop the ability to justify their ideas and judgments.

There are also some limitations of the study. Since the study is a case study that analyses the performance of the 2022/2023 centralized exam in mathematics, it cannot be attributed to the results over a longer period. Similarly, the exam results included in the study may have been influenced by various factors that may have lowered performance, e.g., stress, anxiety, feeling unwell, etc. Thus, it cannot be attributed solely to the inability to apply critical thinking skills when solving proof tasks.

CONCLUSIONS

Based on the theoretical and empirical findings of the study, the following conclusions can be drawn:

- 1. The study reveals that students' ability to solve proof problems significantly influences their achievements in exams and develops critical thinking, analytical, and argumentative skills. Teachers need to select tasks that can improve critical thinking skills enhance students' abilities to solve proof tasks and develop skills in evaluation, analysis, and structured thinking, which is essential in solving proof problems.
- 2. However, there are several obstacles to developing these skills, for example, there is a lack of clear methodology for teachers and insufficient time for discussions and conversations in class, which limit students' opportunities to express and justify their thoughts.
- 3. Since the proof tasks in the national mathematics exam make up 19% of the possible number of points that can be obtained it significantly affects the evaluation of the centralized exam in mathematics.
- 4. The 9th grade students with the highest scores on the proof tasks were those whose overall exam evaluation was high, ranging from 66-100%. Conversely, students whose performance results were rated below 35% did not score enough points in proof tasks.
- 5. Therefore, it is essential to develop ageappropriate methodological recommendations for teaching proof problems as well as ensure targeted teacher training. Future curriculum development should provide greater flexibility

and time resources so that students can fully acquire proof skills through conversations, discussions, and analysis of practical examples.

6. Research outcomes should contribute to the broadening of theoretical and practical knowledge, as well as teaching and learning practices. Implementing quality education and improving learning outcomes requires highly qualified educators and education policy support, as well as efficient and sustainable investment in education.

To achieve the goals of quality education future research could focus on improving teaching practices and making proof tasks more engaging and effective for students by developing curricula and designing interventions to develop critical thinking skills, enhancing appropriate instructional methods, and supporting students' positive dispositions towards proof tasks.

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Ethical statement: The authors stated that the study uses secondary data that is fully anonymized and does not contain any identifiable personal information. As no new data is being collected directly from human subjects, and there is no intervention or interaction with individuals, the study does not fall under the category of human subjects research requiring ethical approval.

Declaration of interest: No conflict of interest is declared by the authors.

Data sharing statement: Data supporting the findings and conclusions are available upon request from the corresponding author.

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