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Curricular proposal to address diversity in mathematics class: A design on sequences and patterns

Ingrid Janeth Jácome Anaya [1,2], Sandra Evely Parada Rico [1*], Jorge Enrique Fiallo Leal [1]

[1] Universidad Industrial de Santander, Bucaramanga, COLOMBIA
[2] Universidad de Los Lagos, Osorno, CHILE

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Abstract

There is international emphasis on the right that all individuals should have to comprehensive education with learning opportunities tailored to their educational needs, and Colombia is no exception. Thus, the work reported here aims to (a) propose a curricular structure that allows addressing diversity in mathematics class, enabling flexibility and adaptation according to students' particularities and (b) construct didactic designs of mathematics adjusted to a flexible and adaptable curricular structure, addressing diversity in the mathematics classroom in Colombia. This article partially addresses these objectives by exploring the question: What conceptual elements need to be considered to construct didactic designs of mathematics that address diversity in the classroom? Consequently, the study presents elements of a curricular proposal based on universal design for learning (UDL) to address diversity in mathematics classes. This is exemplified through a didactic design created for the study of sequences and patterns, promoting, in basic and middle education, the development of algebraic thinking through activities involving generalization and the study of patterns.

Keywords: algebraic thinking, attention to diversity, didactic design, UDL

INTRODUCTION

The convention on the rights of persons with disabilities and its optional protocol, established by the European Union in 2006, defines people with disabilities as 'those who have long-term physical, mental, intellectual, or sensory impairments, which in interaction with various barriers may hinder their full and effective participation in society on an equal basis with others' (United Nations [UN], 2006, p. 4). Thus, in discussions about international educational public policies, educational inclusion is outlined as an environment aiming to provide 'equivalent learning opportunities, regardless of their social and cultural backgrounds and their differences in skills and abilities' (UNESCO, 2007, p. 4).

Thus, Political Constitution of Colombia (1991, Art. 67) establishes education as 'a right of the individual… whose responsible entities are state, society, and the family.' Educational institutions must ensure educational inclusion as a fundamental right, starting from resolution 2565 of 2003 (Colombia Resolution, 2003). However, according to Saenz (2012), 'this right is still violated in many educational institutions' (p. 193), as mentioned by Claro (2007), which could stem from 'inadequate resources, ranging from infrastructure, specialized professional support, to a flexible curriculum' (p. 181) and from the lack of adequate preparation among teachers, as noted by Padilla (2011).

In Colombia, teachers have guiding documents such as

(a) Curriculum guidelines and basic competency standards (MEN, 1998, 2006) and

(b) universal design for learning (UDL) (Pastor et al., 2014).

The former defines the competencies to be promoted in different grade levels within each component of mathematical thinking, which are standardized and do not cater to the specific characteristics of students. On the other hand, UDL sets out very general guidelines that need to be rethought for mathematics, as mentioned by

✉ jacomeaij@gmail.com ✉ sanevepa@uis.edu.co **(*Correspondence)** ✉ jfiallo@uis.edu.co

Contribution to the literature

- This study introduces a curriculum proposal based on UDL, aiming to promote mathematical activity for all students while addressing specific learning characteristics.
- Additionally, it showcases a curriculum design example that utilizes the conceptual framework supporting the proposal, addressing a mathematical concept with four levels of depth.
- Therefore, we consider the article's contribution to be primarily theoretical because it presents the conceptualization of a curricular proposal of which a didactic design is shown as an example.

González (2005). These guidelines require redefinition to ensure the participation of all students.

Bruno and Noda (2010) illustrate that in mathematics education, studies predominantly associate a clinical component with specific tasks related to a particular subject matter. Specifically, in the development of algebraic thinking, Romero et al. (2018) conducted a study presenting a teaching situation for students with intellectual disabilities regarding the concept of equivalence, considering the principles of "early algebra".

On the other hand, the development of algebraic thinking and its introduction into educational systems has been emphasized in several countries (Bojorque & Gonzales, 2021; Radford, 2014) including Colombia (the study's contextual country), considering it crucial for problem-solving not only in the sciences and mathematics themselves but also in everyday life (MEN, 2006). This matter, which for teachers and researchers at higher levels should include tasks that establish relationships between quantities, generalization, problem-solving, modelling, justification, proof, prediction, analysis, and expression of regularities to better prepare students (Becker & Rivera, 2008; Bojorque & Gonzales, 2021; Carpenter et al., 2005; Kieran, 2004; Radford, 2014).

In this regard, Dreyfus (1991) suggests that at an early age, promoting variational thinking through the study of patterns present in sequences and progressions, whether numerical and/or figural, should be encouraged. However, this content is not explicitly included in the school curriculum of countries such as Colombia.

In this regard, authors such as Aké et al. (2021) emphasize the importance of characterizing the use, development, and scope of these approaches in people with disabilities. Additionally, the use of tasks that promote mathematical activity in individuals with or without disabilities is highlighted, as this knowledge 'will generate a frame of reference for the functionality of mathematical knowledge in different groups and situations' (López-Mojica et al., 2017, p. 42).

In accordance with the above and being aware of both the importance of addressing diversity in mathematics class and the study of algebra from an early age, this article presents the conceptualization of a curricular proposal to address diversity in mathematics class and exhibits a didactic design incorporating its elements. The didactic design deals with sequences and patterns within the context of the 2020 Tokyo Olympics. This work acknowledges that each student has different learning characteristics and with this consideration seeks to

(1) propose a curricular structure that allows addressing diversity in mathematics class, enabling flexibility and adaptation according to students' particularities and

(2) construct didactic designs of mathematics adjusted to a flexible and adaptable curricular structure, addressing diversity in the classroom.

Understanding didactic design as a structured plan comprising a series of activities and resources for teaching and learning a specific topic (Amaro, 2011; Berger & Kam, 1996). The project from which the reported results emerge is a structured plan guiding the teaching and learning of mathematics. It offers a series of specific activities with their respective theoretical foundations and resources aimed at promoting mathematical activity in the classroom and addressing diversity.

THEORETICAL & CONCEPTUAL ASPECTS

One of the challenges faced by mathematics teachers when regulations are established to address diversity in the classroom is the lack of sufficient training to do so. Furthermore, upon reflection on the resources available, they encounter standardized and inflexible curriculum guidelines.

Considering the challenges, the ongoing research from which this article originates is focused on providing guidance for teachers to approach the mathematical objects of study for each grade group with all students according to their individual characteristics and capabilities. Hence, the main theoretical foundations include UDL, curriculum guidelines, and basic mathematics standards in Colombia, which advocate that all Colombian citizens can and should be mathematically competent to socially engage according to their needs.

Article 46 of the Colombian Law (1994) stipulates that "education for people with physical, sensory, mental, cognitive, emotional, or exceptional abilities is an integral part of the public service" (p. 12). Therefore,

Table 1. Principles & guidelines of UDL

Principles	Guidelines
I. Providing multiple means of representation	1. Providing different options for perception. 2. Providing multiple options for language, mathematical expressions, & symbols. 3. Providing options for understanding.
II. Providing multiple means of action & expression	4. Providing options for physical interaction. 5. Providing options for expression and communication. 6. Providing options for executive functions.
III. Providing multiple means of engagement	7. Providing options for capturing interest. 8. Providing options for sustaining effort and persistence. 9. Providing options for self-regulation.

ensuring access to education for everyone is crucial to promoting the participation of all individuals and contributing to the construction of society.

Colombian laws 361 (Colombian Law, 1997), 1346 (Colombian Law, 2009), 1618 (Colombian Law, 2013), and Decree 1421 (2017) aim to guide inclusive education, defining it, as follows:

> A permanent process that recognizes, values, and responds appropriately to the diversity of characteristics, interests, possibilities, and expectations of children, adolescents, young people, and adults. Its objective is to promote their development, learning, and participation, alongside peers of the same age, in a common learning environment, without any discrimination or exclusion. This approach ensures, within the framework of human rights, the necessary supports and reasonable adjustments required in their educational process through practices, policies, and cultures that eliminate existing barriers in the educational environment (Decree 1421, 2017, p. 5).

According to Decree 1421 (2017), guidance is provided regarding reasonable adjustments that should be made at different levels of school education to address diversity. These adaptations should consider the specific needs and characteristics of students. These adjustments are guided by UDL and the individual reasonable adjustment plan (PIAR). UDL is a model that combines an inclusive approach to teaching with proposals for its practical application. PIAR, on the other hand, is a tool that allows the curriculum to be contrasted with the student's characteristics and needs, aiming to define reasonable adjustments and pedagogical supports that enable the student's participation in the classroom.

Meaning of Universal Learning Design in Proposal

UDL is based on the paradigm of universal design and is defined as

> "a pedagogical proposal that facilitates a curricular design accommodating all students through objectives, methods, materials, supports, and evaluations formulated based on their capacities and realities. It allows the teacher to transform the classroom and pedagogical practice and facilitates the assessment and monitoring of learning" (Decree 1421, 2017, p. 4).

According to Pastor et al. (2014), it should be understood under three fundamental principles. These are summarized in **Table 1**, given the document's length limitations.

The ongoing research acknowledges that each student has different abilities and learning paces. Therefore, it is essential to enable differentiated activities to accommodate that each student can learn and progress at their own pace and with their abilities. Velasco (2022) alludes to the fact that the principles and guidelines of UDL are generally proposed for teaching any subject area, hence a reinterpretation of these principles is required for the specific area of mathematics. In the proposal presented here, each of the principles and guidelines of UDL are articulated with the mathematical processes and thinking defined by MEN (1998, 2006) for the teaching and learning of mathematics in Colombia. Upon this conceptual articulation, the purposes to be achieved regarding mathematical objects are defined with four levels of depth, to approach them according to the characteristics of the students (as shown in **Table 2** of the first section of results).

Conceptualization of Variational Thinking for Didactic Design

The development of variational thinking and the study of algebraic systems should begin at an early age (MEN, 1998; Vasco, 2002), and can be done through experiences involving pattern analysis and generalization, referring to the notion of indeterminacy (D'Amore et al., 2007; Radford, 2010; Sibgatullin et al., 2022; Vergel, 2014; Zapatera, 2018).

Algebraic reasoning involves representing, generalizing, and formalizing mathematical patterns and regularities (Godino & Font, 2003). It 'should encompass the development of ways of thinking such as analyzing relationships between quantities, identifying structures, studying change, generalizing, problem-solving, modeling, justification, proof, and prediction' (Kieran, 2004, p. 49). Similarly, Dreyfus (1991) mentions

Table 2. Characteristics of depth designs 1, 2, 3, & 4

Depth design 1 (L1)	Depth design 2 (L2)	Depth design 3 (L3)	Depth design 4 (L4)
Provides multiple representations of mathematical object of study, especially concrete representations that highlight attributes of numbers & shapes. Problem situations at this level feature simple instructions, minimal text, & greater visual or auditory content, offering multiple means of action & expression, such as the use of keywords through alternative text (images, tables, bits of information, video, photography, physical or digital materials, puppets, etc.), aiming to activate students' auditory, visual, and/or tactile perception. Constant teacher support is required, helping students make connections & repeatedly reminding them of information. This level's design allows for different forms of engagement, involving collaborative work with peers & continuous sharing of their progress with group, achieved by addressing everyday situations & needs.	Prioritizes problem-solving activities involving interpretation of information presented verbally, numerically, or graphically, utilizing visual, auditory, and/or concrete materials. Design offers multiple means of action and expression by using culturally significant situations for students. Problem situations at this level contain straightforward instructions with moderate text, requiring connection of information to understand & solve a situation. Design provides various forms of action & expression, encouraging oral, gestural, pictorial expressions, among other production possibilities, involving teacher mediation to assess achievement of intended purpose. In addition to collaborative work, this level requires gradually granting autonomy to strengthen knowledge acquired in solving everyday situations. To achieve this, teacher may provide support with materials or concrete representations (counters, abacus, software, etc.), gradually withdrawing them as student progresses.	Prioritizes problem-solving activities involving the abstraction of information presented verbally, numerically, graphically, or in tables, utilizing various and diverse technologies (such as interactive virtual environments or software) that allow manipulation, diverse feedback, and problem-solving strategies. Activities are mostly designed for students to build numerical or algebraic expressions to model a problem situation within context, facilitating the development of abstract mathematical processes and precise mathematical language. The design offers various means of action and expression by using mathematical language aligned with the object of study, where the teacher mediates with the student to approach mathematical objects according to the established purpose, encouraging discussion about situations within mathematical and everyday contexts.	Prioritizes activities centered on problem-solving, deduction, and proposing mathematical conjectures using formal mathematical language, incorporating diverse and varied technologies (such as interactive virtual environments or software) to enable visualization, varied feedback, argumentation, and the development of problem-solving strategies. The activities are primarily designed for students to model situations from context (mathematical and non-mathematical), justifying and arguing their procedures and deductions using the formal mathematical language specific to the object of study. Similarly, in this design, the teacher assumes the role of a mediator, assisting the student in approaching and delving deeper into their objects of study based on their interest, motivation, and creativity, encouraging them to discuss and present their progress.

that generalization is 'deriving or inducing from particulars, identifying common points, and broadening the scope of validity' (p. 35). In this sense, Radford (2008) asserts that generalizing patterns involves:

(a) accounting for a common characteristic,

(b) generalizing the common characteristic across all terms, and

(c) determining a rule that allows finding any term.

Taking into account the aspects highlighted by Dreyfus (1991) and Radford (2008), the design mentioned in this article is constructed with the aim of not only promoting the development of algebraic reasoning but also to promote activities that encourage students to: explore, model, predict, discuss, argue, verify ideas, observe, and describe patterns, relationships, and mathematical properties (Blanton & Kaput, 2005; Radford, 2010). This design also considers elements such as analyzing and finding the nearest term, the nth term, the inverse process, describing strategies used, and errors associated with incorrect strategies (Aké, 2022; Rivera, 2013; Zapatera & Callejo, 2018).

METHODOLOGICAL ASPECTS

Below, the methodological process of the research is described, which aimed to

(a) propose a curricular structure that allows addressing diversity in mathematics class, enabling flexibility and adaptation according to students' particularities and

(b) construct didactic designs of mathematics adjusted to a flexible and adaptable curricular structure, addressing diversity in the classroom.

Each objective was achieved in two different stages:

1. **Stage 1.** Conceptualization of a flexible and adaptable curricular structure tailored to students' characteristics.
2. **Stage 2.** Construction of didactic designs (here we present one as an example on sequences and patterns aimed at children between nine and 11 years old).

Below is a brief description of the results in each stage.

Stage 1. Conceptualization of a Curricular Structure

In this stage, it was first necessary to conduct a literature review (for over a year), which revealed that in mathematics education, research has been developed in an atomized manner (teaching a specific topic for a specific condition, for example: Development of geometric thinking applied to children with down syndrome). The identification of this phenomenon motivated us to consider an approach to the school curriculum that truly makes possible the implementation of UDL in inclusive classrooms. Subsequently, a curricular proposal was postulated with the aim of addressing diversity in the mathematics class. This proposal is based on two aspects:

1. Flexibility, which pertains to adaptability in response to the cultural and social diversity of students. Therefore, the curriculum should be open to review, modification, and continuous updates (MEN, 2013).
2. Adaptability, which implies that education should be compatible with the requirements, interests, and specific conditions of all children in society (Köster, 2016).

Given the above, an initial literature review was conducted concerning diversity in education, revealing the existence of various conditions, abilities, and characteristics, each with a broad spectrum of differentiation. For instance, there are syndromes or perceptual and sensory difficulties classified as severe, profound, moderate, or mild. Consequently, considering the principles and guidelines of UDL (Pastor et al., 2014; Velasco, 2022), the project determined to approach mathematical objects of study using four levels of depth as described in **Table 2**.

The results of this stage constitute one of the research outputs and serve as a theoretical contribution to mathematics education.

Stage 2. Construction of Didactic Design

For this, the methodological process proposed by Díaz et al. (1984) was followed, who established the following steps:

1. *Preliminary analysis*

The study focused on fourth and fifth grades of primary education in Colombia for which two standards of variational and algebraic analytical thinking were selected:

a. Predict patterns of variation in a numerical, geometric, or graphic sequence.
b. Represent and relate numerical patterns with tables and verbal rules (MEN, 2006, p. 83).

2. *Didactic analysis*

The didactic analysis of the notion of sequences in primary education from different theoretical and methodological perspectives allowed the definition of a context to approach the notion of sequence and propose guiding questions for the study. Thus, it brought attention to an ongoing event at that time. Between July and August 2021, the Tokyo 2020 Olympic Games (postponed to 2021 due to COVID-19) were taking place. National and international news outlets (newspapers, TV, social media, among others) were broadcasting competition results, engaging people to analyze their country's performance compared to others. The event raised questions such as how many sports disciplines participate in the Olympic Games? What is measured in each sport? What mathematics do we find in the context of the Olympic Games? Among other inquiries, which were intended to be problematized in the design.

3. *Proposal of the didactic design*

The proposed didactic design consists of

(a) curriculum framework,
(b) student worksheet, and
(c) guidelines for the teacher, described, as follows:

(a) The curriculum framework poses the question: 'How to organize synchronized swimming competencies?' and presents a table that shows the purposes of activities at each depth level, linked with mathematical processes, addressing the guidelines from MEN (1998), and the insights provided by various authors on their development (Fiallo & Parada, 2018; Fiallo et al., 2021; Parada et al., 2023).

(b) The student worksheet for each depth level is divided into four stages:

1. *First stage:* Introduction of a mathematical concept from the context using dynamic activities for assessment and connection with prior knowledge.
2. *Second stage:* Emerging mathematical concepts are built from the situation planned in the first stage. Here, the aim is to construct or deduce properties, relationships, representations, and connections of the mathematical object of study with the context.
3. *Third stage:* Space, where students put into practice and reinforce the constructed knowledge through dynamic activities, exercises, applications, problem-solving, games, projects, etc.
4. *Fourth stage:* Time to assess students' performances, acknowledging differences in learning rhythms and styles. This can be done through the development of challenging activities, dynamic tasks, or problems that allow for the assessment of learning.

Table 3. Curriculum grid of purposes & descriptors at each level of depth

Depth level	Purpose	Descriptors	Purpose	Descriptors
Question	L1		L2	
How to organize synchronized swimming competencies?	Variational thinking, algebraic, and analytic systems: Recognize patterns of variation in a numerical sequence when forming paired and trio competitions in synchronized swimming.	Modelling: Recognize the verbal correspondence rule that relates the terms of numerical sequences for pairs and trios in synchronized swimming competitions.	Variational thinking, algebraic, and analytic systems: Interpret patterns of variation in a numerical sequence when forming competitions for pairs and trios in synchronized swimming.	Modelling: Recognize the verbal correspondence rule that relates numerical and figurative terms of pairs and trios in synchronized swimming competitions based on the correspondence rule.
	L3		L4	
	Variational thinking, algebraic, and analytic systems: Predict and use patterns of variation in a numerical and figurative sequence when shaping competitions of pairs, trios, and quartets in synchronized swimming.	Modelling: Construct the symbolic correspondence rule that relates the terms of numerical sequences from pairs, trios, and quartets' competitions in synchronized swimming. Build the verbal correspondence rule and predict the terms of a figurative sequence from pairs, trios, and quartets' competitions in synchronized swimming.	Variational thinking, algebraic, and analytic systems: Predict, represent, and relate variation patterns in a numerical and figurative sequence when organizing competitions involving pairs, trios, quartets, and mixed teams in synchronized swimming.	Modelling: Predict, schematize, and transform the terms of a sequence according to the given variation patterns in numerical and figurative sequences of pairs, trios, quartets, and mixed teams in synchronized swimming.

(c) Teacher guidelines: Considering that achieving the objectives depends on the teacher's interventions, a document was created with didactic, theoretical, and methodological recommendations to implement the design. This aims to guide the way to direct and flexibilize its development according to the students' characteristics.

As results of this article, the design on sequences and patterns aimed at elementary school students is presented in which the theoretical elements of the proposed curriculum are identified along with an example of how the design was constructed. This aims to motivate teachers to contribute to the process of reflecting on mathematical classroom activities.

DIDACTIC DESIGN ON SEQUENCES ADRESSING UDL

This section presents designs that exemplify the proposal described earlier, focusing on sequences and patterns from the perspective of early algebra for primary education students. Due to space limitations, teacher guidelines are omitted here.

Curriculum Framework

Considering the context (at the time, the design was current and of great interest to the global community), the guiding question is posed: 'How to organize synchronized swimming competencies?' Framed within the standard "predicting patterns of variation in a numerical, geometric, or graphic sequence, and Representing and relating numerical patterns with tables and verbal rules" (MEN, 2006, p. 86). To achieve this, learning purposes are proposed for each depth level (**Table 2**) with their respective descriptors associated with all mathematical processes outlined by MEN (2006) (**Table 3**). As an example, it presents the descriptors of the modeling process, as defined in MEN (2006, p. 53), which involves "detecting recurring schemes in everyday, scientific, and mathematical situations to mentally reconstruct them".

The curriculum framework establishes two fundamental relationships:

(a) vertical coherence and

(b) horizontal coherence.

Vertical coherence is evident when comparing a design with one that could be developed before or after it, so that their sequence allows for comprehensive coverage of the curriculum. Horizontal coherence involves the treatment of the same mathematical object of study with four levels of depth, ensuring that resources exist in the classroom that enable students, regardless of their characteristics, to approach the curricular content alongside their peers but with differentiated purposes in each of the mathematical processes outlined by MEN (2006).

Student Worksheet

Considering the purposes and descriptors shown in the curriculum framework (**Table 3**) and the foundations of the curricular proposal, four work guides were designed (one for each level of depth). Below is the

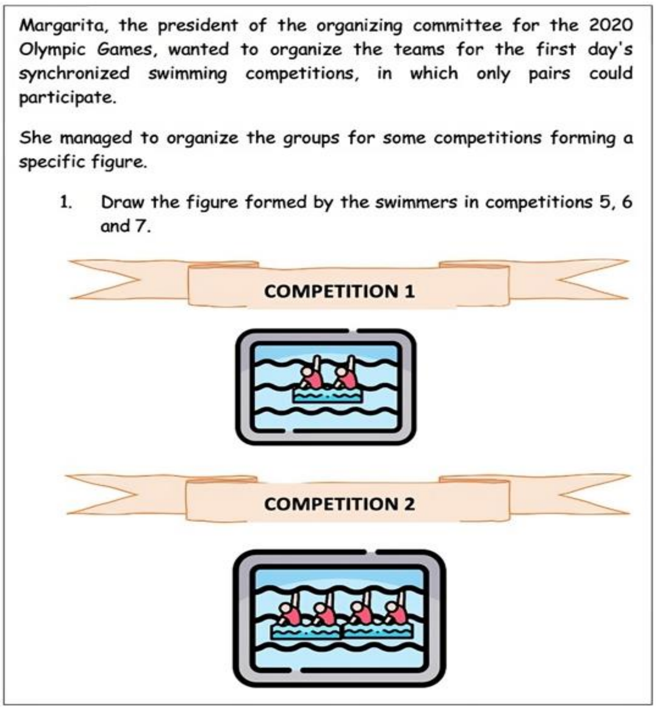

Figure 1. Problem situation L3 (Source: Authors' own elaboration)

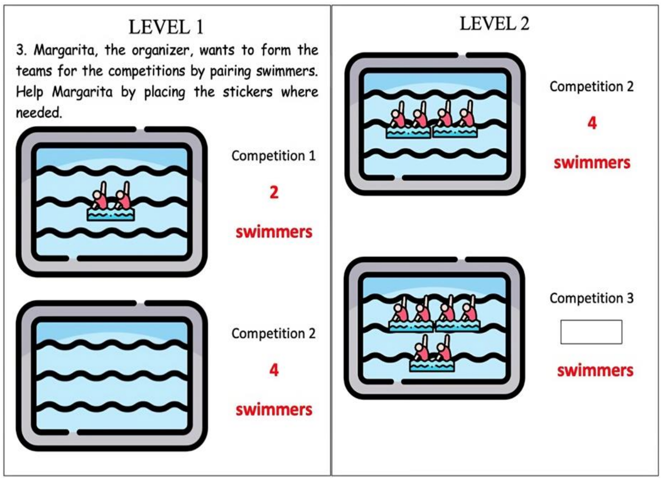

Figure 2. Problem situation L1 & L2 (Source: Authors' own elaboration)

description of the student worksheet for each of the stages, where the differences in the mathematical activity expected to be promoted in each proposed level–level 1 (L1), level 2 (L2), level 3 (L3), and level 4 (L4) –are explained in more detail.

First stage

The design begins with a short text related to synchronized swimming, accompanied by a video aimed at familiarizing the students with the context of the Olympic Games and swimming. In L1 and L2, questions are posed to guide the understanding of the situation. The teacher is also encouraged to initiate a brainstorming session for students to discuss their comprehension of the text. Afterwards, the task involves organizing the swimming competition teams in pairs' mode, considering the information provided in the initial competitions (**Figure 1**).

In the first two levels (L1 and L2), only the numerical pattern is addressed. In L1, the use of concrete materials (stickers of swimmer pairs) is encouraged to complete the sequence by sticking the corresponding pairs based on the total number of swimmers participating in the first five competitions. In L2, only the figures formed in the first five competitions and the number of swimmers in competition 1 and competition 2 are shown. Students are expected to fill in the blank spaces with the number corresponding to the quantity of swimmers participating in competitions 3, 4, and 5.

In L3 and L4 (**Figure 1**), both the numerical and figurative sequences are worked on. They are given the figurative information from the first five competitions to predict the terms of sequence 6 and sequence 7 (Blanton & Kaput, 2005; Kieran, 2004; Radford, 2010). In L4, they are asked to predict a greater number of terms compared to L3.

In subsequent activities, questions are posed to guide students in identifying the variation pattern, considering the cognitive abilities defined in each level of depth. At the end of each stage, discussion of results is encouraged to promote communicative skills such as explanation, justification, and argumentation.

Subsequently, the task requests organizing synchronized swimming competitions based on the information provided in the initial competitions, encouraging the exploration and observation of patterns, as suggested by Blanton and Kaput (2005) and Radford (2010) to promote the development of algebraic thinking.

In L1, the use of concrete materials is encouraged to organize the competition groups, providing the total number of swimmers participating so they can stick to stickers representing that quantity (**Figure 2**). In L2, the number of swimmers is depicted figuratively, with the idea that they count the total number of swimmers and represent them numerically. In L3 and L4, only the figurative sequence is shown, expecting them to predict the figure and the total number of swimmers participating in subsequent competitions (**Figure 1**). This variety of representations and the use of concrete materials *provide multiple options for language, mathematical expressions, and symbols, offering diverse ways to understand the information*.

Subsequently, in L1 and L2, tasks such as "color the number of swimmers participating in competition 3 ..." and "what operation should you use to find the total number of swimmers in each competition? Explain to Margarita what procedure she should follow with an example." In L3 and L4, completing tabular information and predicting the nearby terms in the numerical and figurative sequences are proposed, allowing students to reinforce their initial observations by identifying the

Table 4. Tabular information L1

Competition	Operation	Total swimmers
1	2	2
2	2+2	4
3	2+2+2	6
4	2+2+2+2	8
5	__+__+__+__+__	__
6	__+__+__+__+__+__	__
7	__+__+__+__+__+__+__	__
8	__+__+__+__+__+__+__+__	__

common characteristic in sequence terms, which is part of generalization process (Dreyfus, 1991; Radford, 2008).

In summary, during the first stage, students are expected to predict the terms of the numerical and figurative sequence, and describe the pattern according to the depth design, following suggestions provided by Blanton and Kaput (2005) and regarding the use of tasks that promote development of algebraic thinking.

Second stage

The proposed activities continue the situation from stage 1, aiming for students to represent the total number of swimmers using operations. Additionally, they are encouraged to generalize both the numerical pattern and the figurative pattern in L3 and L4. In L3, students are asked to fill in tabular information, where they need to write the operation and the total number of swimmers participating in the first ten competitions, accompanied by questions aimed at writing the numerical pattern in a general form. While it is true that both repeated addition and multiplication can be used to find the total number of swimmers, in L3 and L4, with the question "if I only know the competition number, how can I find the total number of swimmers participating? Explain your answer," the aim is to generate a discussion about the use of both addition and multiplication.

Later, the focus shifts to predicting the patterns formed in competitions 8, 9, and 10 based on the figural sequence. Through questions like "what should we do to the pattern formed in competition 7 to obtain the patterns formed in competition 8?" and the instruction "explain to Margarita how to find the pattern formed by the swimmers in competition n", students are expected to identify the generalized pattern of variation.

In L1 and L2, explanatory information is provided about the operation that could be used to obtain the total number of swimmers, and this process is guided using the tabular information shown in **Table 4**, where only repeated addition is expected to be used as the operation to find the total number of swimmers.

In the following activities, the student is expected to predict the number of swimmers participating in subsequent competitions based on the recognition of the correspondence rule and its relation to the operation used to find these terms. This involves posing questions such as "which operation do we use to find the total

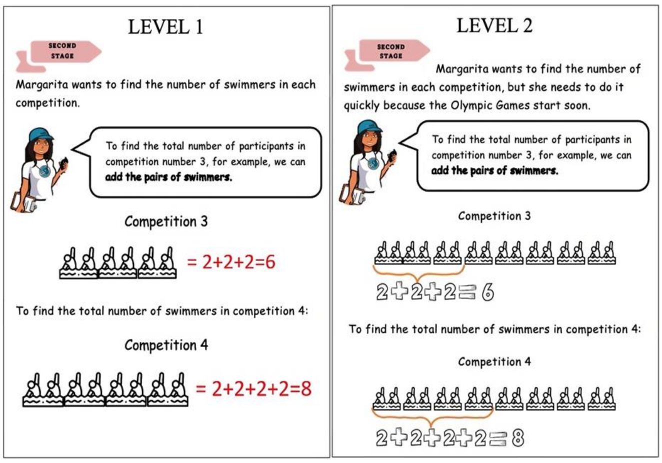

Figure 3. Information for second stage L1 & L2 (Source: Authors' own elaboration)

number of swimmers?" and "how many swimmers participate in competition 9 and competition 10 …?" This helps them identify the variation pattern verbally, textually, symbolically, among other methods.

Finishing the second stage, students are expected to represent in a general way both the total number of swimmers participating in the swimming competitions and the pattern formed, in the case of L3 and L4. To this end, in L1 and L2, questions like "explain to Margarita the procedure she should use to calculate the total number of swimmers participating in any competition" are proposed. In L3 and L4, questions such as "how can we find the total number of swimmers participating in competition n? Why? ... Explain to Margarita the way to find the pattern formed by the swimmers in competition n" are presented, allowing them to predict the terms of the sequences and, in turn, validate conjectures found.

In the second stage, students are expected to find the terms of both the numerical sequence (L1, L2, L3, and L4) and the figural sequence (L3 and L4) and determine a rule that allows them to find any term of the sequence. This aligns with the recommendations proposed by Dreyfus (1991) and Radford (2008) to promote the generalization process, which fosters algebraic and variational reasoning in students from an early age.

For this purpose, in L1 and L2, verbal, iconic, and algebraic information is provided to guide them in the search for the rule that allows them to find the terms of the numerical sequence, *thus offering different options for perception and multiple choices for understanding* (**Figure 3**).

In L1 and L2, tasks like "explain to Margarita the procedure she should use to calculate the total number of swimmers participating in any competition" and in L3 and L4 "explain to Margarita the way to find the figure formed by the swimmers in competition n" create spaces, where students can use the records that they consider relevant. These tasks aim to enable multiple response formats from them.

Additionally, in L3 and L4, questions and tasks are posed with the purpose of having students use the identified pattern to perform the inverse process (Aké, 2022; Rivera, 2013; Zapatera & Callejo, 2018).

In L3, the question asked is "can a team of 35 swimmers participate in a double's competition? Why? Discuss the results with your peers."

In L4, they are required to complete a table, where only the total number of swimmers is given, and they must state whether these groups of swimmers can participate in a doubles competition and specify in which competition.

Additionally, the question "in the previous table, were there any groups that could not compete in the doubles category? Which groups? Why?" is posed, aiming for students to validate their conjectures, encouraging them to assess their learning process, *thus promoting options for executive functions.*

At this stage, students are asked to explain their procedures, giving them the option to use different representations (numeric, symbolic, verbal, or graphical) to determine the rule that allows them to find the total number of swimmers for any competition and the pattern they form (Dreyfus, 1991; Radford, 2008). The third moment concludes by enabling the discussion of results, *providing varied forms of interaction, communication, and expression.*

Third stage

In the third stage, a sequence of trios is presented within the same context. Students are expected to construct the rules of correspondence using different representations. In L1, emphasis is placed on predicting the near and far terms of the numerical sequence and identifying the general pattern of variation in various representations, guiding the process with questions such as "what operation is being used to find the total number of swimmers in each competition?" and "How many times should I add three in competition 2?"

In L2, in addition to working on the numerical sequence, the figural sequence is introduced (**Figure 4**), but the identification of this pattern is guided. When asked about the figure formed in competition number four, three answer options are given. In L3, they are expected to draw the corresponding figure.

Afterward, questions are posed for the students to identify both the numerical and figural patterns and represent them in a general form. In L1 and L2, only repeated addition is used to find the total number of swimmers.

In L3, the use of multiplication is additionally explored, along with its relation to repeated addition, through questions such as "how to find the total number of swimmers participating in a competition, knowing the total number of swimmers in the previous competition? How to determine the total number of swimmers participating in a competition, knowing only the competition number?" They are asked to validate the identified pattern of variation.

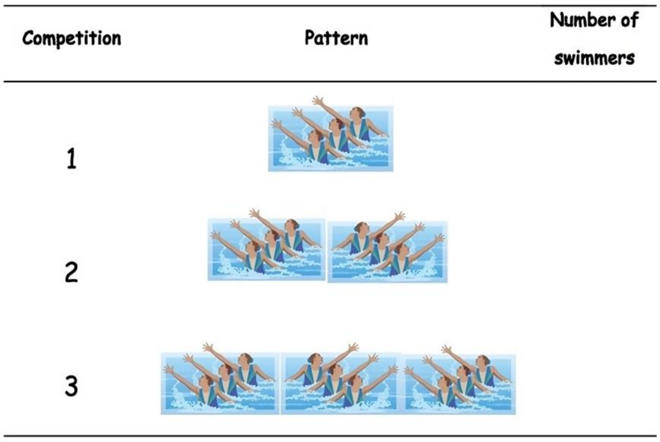

Figure 4. Trios sequence L2 & L3 (Source: Authors' own elaboration)

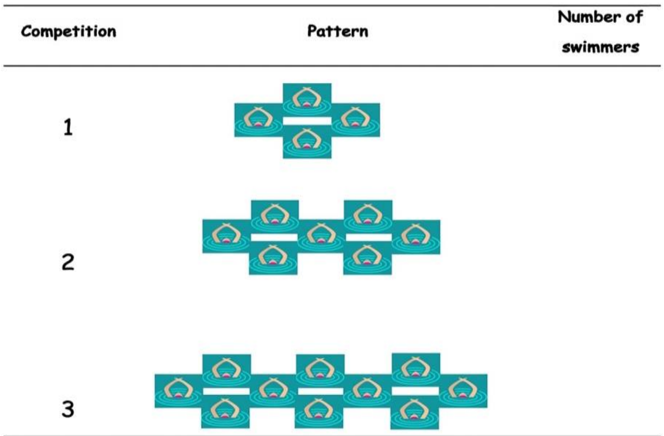

Figure 5. Quartet sequence L4 (Source: Authors' own elaboration)

In L4, the level of difficulty increases as both the figural and numerical patterns differ (**Figure 5**). This level introduces quartets and trios, where initially, a group of four swimmers is formed, and then trios are added (mixed modality). It's worth noting that the figure's composition can also be seen as starting with a single swimmer to whom trios are added. However, it's important to guide students to identify quartets, a group formed by four swimmers, to introduce the term 'mixed modality'. Students are expected to predict both the total number of swimmers and the figure formed in competitions 4, 5, 6, and 7.

Having identified the variation pattern, similar questions are posed to those in L3, expecting students to generalize both the numerical and figural patterns. At this stage, students will identify the common characteristic in the competitions formed by trios (L1, L2, and L3) and mixed (L4). They will generalize the common characteristic in the numerical and figural sequences, ultimately establishing a rule to find any term in the sequences (Dreyfus, 1991; Radford, 2008).

In L1, instructions are provided such as "write the operation needed to determine the total number of swimmers participating in competition number 12", "write the total number of swimmers participating in the specified competitions …", "explain to Margarita the procedure for calculating the total number of swimmers participating in any competition and in L2", "how many swimmers participate in competitions number six, seven, and eight? Why?", "if I want to find the number of swimmers in the trio competition, what should I do?", and "to draw the figure formed by the swimmers in the trio competition, what should I do?" This aims to promote pattern analysis, finding the near term, and the nth term (Aké, 2022; Rivera, 2013; Zapatera & Callejo, 2018).

In L3 and L4, the question is asked: "How to find the total number of swimmers in a competition? Explain your answer … How to find the total number of swimmers in competition *n* knowing only the number of the competition? Explain to Margarita how to find the figure formed by the swimmers in competition *n*", where it is expected that students generalize the pattern of variation. For L4, the reverse process is proposed, discussing the strategies used, as well as errors related to incorrect strategies (Aké, 2022; Rivera, 2013; Zapatera & Callejo, 2018).

When requesting explanations and justifications *options for communication and expression are provided*, as well as the management of executive brain functions and the development of problem-solving strategies such as verifying and validating conjectures.

Fourth stage

In all four levels, tabular information is presented with a different announcement (**Figure 6**). In L1 and L2, the focus is on pairs and trios with less information. In L3, quartets are introduced, and in L4, mixed groups are added. The main objective of the following activities is for students to conjecture and validate the general characteristics of the sequences previously worked on to establish the terms of the sequences shown in the announcement. Additionally, they are expected to identify which countries can participate in the competitions according to established criteria (avoiding excess swimmers and those who can participate in more than one category).

The tasks outlined in the fourth stage aim to help students consolidate their knowledge through exploration, modeling, prediction, argumentation of patterns, relationships, and properties of the sequences studied (Blanton & Kaput, 2005; Radford, 2010).

In L1, the use of stickers containing the country's flag, name, and the number of swimmers is encouraged to gather tabular information, *providing various options for perception and physical interaction*.

COUNTRY	TOTAL NUMBER OF SWIMMERS
Russia	15
United States	18
Canada	12
France	8
Japan	21
China	16
Spain	28
Colombia	24
Ukraine	14

Figure 6. L3: Tabular information stage 4 (Source: Authors' own elaboration)

In L3 and L4, the scenario is presented, where a country participates in all modalities without any swimmers left out, asking about the characteristics of this quantity. This allows them to assess their learning, recognize successful strategies, and identify errors, thereby enabling the *management of executive brain functions*.

CONCLUSIONS

In response to the question "what conceptual elements need to be considered to construct didactic designs of mathematics that address diversity in the classroom?", we present the following reflections:

- We present a curriculum proposal that aims to promote attention to diversity in mathematics class, proposing a flexible and adaptable curriculum that addresses the particularities of students through the study of the same mathematical object with four levels of depth, addressing the principles and guidelines of UDL among which the multiple forms of representation, expression, and interaction are highlighted. This responds to the concerns raised by authors such as Aké et al. (2021), who emphasize the importance of characterizing the use, development, and scope of these representations in people with disabilities.
- The curriculum proposal presented also addresses the suggestions of Giberti et al. (2023), who discuss the need to promote argumentation among students by developing effective tools to deepen understanding of mathematical concepts and address problem situations more effectively.

- In this sense, the relevance of using tasks that promote mathematical activity is emphasized, both for people with disabilities and those without, since this approach, as mentioned by López-Mojica et al. (2017), "will allow for the generation of a framework of reference for the functional aspects of mathematical knowledge in different groups and situations" (p. 42).
- The curriculum proposal includes, among its components, guidelines for mathematics teachers aimed at enhancing their mathematical training and their training in addressing diversity.
- The presented design is an example of the curriculum proposal presented, aiming to provide conceptual tools to teachers to promote attention to diversity in mathematics class in Colombia, specifically focusing on the study of patterns and sequences.
- The design's activities offer various types of questions and request different response options to provide multiple avenues for language, mathematical and symbolic expressions, and diverse avenues for comprehension. As seen in the first moment, problems are introduced through written text, pictorial representations, tables, etc.
- It can be observed that activities focused on identifying the variation pattern and establishing generalizations foster the use of different options for perception and multiple choices for comprehension, executive functions, physical interaction, communication, and expression.
- The activities are aimed at generalizing common characteristics and determining a rule to find any term in sequences of trios (L1, L2, and L3) and mixed sequences (L4) offer various options for communication, expression, executive function management, physical interaction, communication, and expression.
- The matters relating to the validation and formulation of conjectures in terms of numerical sequences allow for various options for perception, physical interaction, communication, and expression.

With the proposed approach showcased here, diversity in the math class in Colombia is addressed by offering the study of the same mathematical object at varying levels of depth. This flexible and adaptable approach aims to make mathematics accessible to all students, regardless of their characteristics. This was achieved through the implementation of UDL and considering the epistemological and didactic aspects of the mathematical object in question.

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REFERENCES

Aké, L. (2022). Desarrollo del razonamiento algebraico en el bachillerato: Una ruta didáctica a través de la generalización de patrones [Development of algebraic reasoning in high school: A didactic route through the generalization of patterns]. In *Proceedings of the 7th Conference on Mathematics Education*.

Aké, L., Hernández, J., Ordaz, M., Larios, J., & Parada, S. (2021). Formación de profesores de matemáticas: Avances para promover aulas de matemáticas inclusivas [Mathematics teacher training: Advances to promote inclusive mathematics classrooms]. *Investigación e Innovación en Matemática Educativa* [*Research and Innovation in Educational Mathematics*], *6*, 1-21. https://doi.org/10.46618/iime.105

Amaro, R. (2011). La planificación didáctica y el diseño instruccional en ambientes virtuales [Didactic planning and instructional design in virtual environments]. *Investigación y Postgrado* [*Research and Postgraduate*], *26*(2), 129-160.

Becker, J., & Rivera, F. (2008). Generalization in algebra: The foundation of algebraic thinking and reasoning across the grades. *ZDM Mathematics Education*, *40*(1), 1. https://doi.org/10.1007/s11858-007-0068-6

Berger, C., & Kam, R. (1996). Definitions of instructional design. *Applied Research Laboratory, Penn State University*. http://www.umich.edu/~ed626/define.html

Blanton, M., & Kaput, J. (2005). Characterizing a classroom practice that promotes algebraic reasoning. *Journal for Research in Mathematics Education, 36*(5), 412-446.

Bojorque, G., & Gonzales, N. (2021). Patrones matemáticos en los niveles Inicial y Preparatoria: Análisis del currículo [Mathematical patterns at the initial and preparatory levels: Analysis of the curriculum]. *INNOVA Research Journal, 6*(1), 47-60. https://doi.org/10.33890/innova.v6.n1.2021.1433

Bruno, A., & Noda, A. (2010). Necesidades educativas especiales en matemáticas. El caso de personas con síndrome de down [Special educational needs in mathematics. The case of people with down syndrome]. In M. M. Moreno, A. Estrada, J. Carrillo, & T. A. Sierra (Eds.), *Investigación en educación matemática XIV* [*Research in mathematics education XIV*] (pp. 141-162). SEIEM.

Carpenter, T., Levi, L., Franke, M., & Zeringue, J. (2005). Algebra in elementary school: Developing relational thinking. *Zentralblatt für Didaktik der Mathematik* [*Central Journal for Mathematics Didactics*], *37*, 53-59. https://doi.org/10.1007/BF02655897

Claro, J. (2007). Estado y desafíos de la inclusión educativa en las Regiones Andina y Cono Sur [State and challenges of educational inclusion in the Andean and Southern Cone Regions]. *Revista Electrónica Iberoamericana sobre Calidad, Eficacia y Cambio en Educación* [*Ibero-American Electronic Magazine on Quality, Efficiency and Change in Education*], *5*(5e), 179-187. https://doi.org/10.15366/reice2007.5.5.025

Colombian Law. (1994). *Ley 115 de 1994 por la cual se expide la ley general de educación [Law 115 of 1994, which establishes the general education law].*

Colombian Law. (1997). *Ley 361 de 1997 por la cual se establecen mecanismos de integración social de las personas con limitación y se dictan otras disposiciones* [*Law 361 of 1997, which establishes mechanisms for the social integration of people with limitations and dictates other provisions*].

Colombian Law. (2009). *Ley 1346 de 2009 por medio de la cual se aprueba la "convención sobre los derechos de las personas con discapacidad"* [*Law 1346 of 2009 by which the "convention on the rights of people with disabilities" is approved*].

Colombian Law. (2013). *Ley 1618 de 2013 por medio de la cual se establecen las disposiciones para garantizar el pleno ejercicio de los derechos de las personas con discapacidad* [*Law 1618 of 2013 through which the provisions are established to guarantee the full exercise of the rights of people with disabilities*].

Colombia Resolution. (2003). *Colombia, Resolución 2565/2003 de 4 de Octubre* [*Colombia, Resolution 2565/2003 of October 4*]. Ministerio de Educación [Ministry of Education].

D'Amore, B., Radford, L., & Bagni, G. (2007). *Obstáculos epistemológicos y perspectiva socio-cultural de la matemática* [*Epistemological obstacles and socio-cultural perspective of mathematics*]. Universidad Nacional de Colombia.

Decree 1421. (2017). *Decreto 1421 de 2017 [Ministerio de Educación Nacional] por el cual se reglamenta en el marco de la educación inclusiva la atención educativa a la población con discapacidad* [*Decree 1421 of 2017 [Ministry of National Education] by which educational attention to the population with disabilities is regulated within the framework of inclusive education*].

Díaz, F., Lule, M., Pacheco, D., Rojas, S., & Saad, E. (1984). Metodología de diseño curricular para la enseñanza superior [Curricular design methodology for higher education]. *Perfiles Educativos* [*Educational Profiles*], *7*(26), 30-40.

Dreyfus, T. (1991). Advanced mathematical thinking processes. In D. Tall (Ed.), *Advanced mathematical thinking* (pp. 25-41). Kluwer. https://doi.org/10.1007/0-306-47203-1_2

Fiallo, J., & Parada, S. (2018). *Estudio dinámico del cambio y la variación. Curso de precálculo mediado por Geogebra [Dynamic study of change and variation. Pre-calculus course mediated by Geogebra].* Ediciones UIS.

Fiallo, J., Velasco, A., &Parada, S. (2021). Demonstration process skills: From explanation to validation in a precalculus laboratory course. *EURASIA Journal of Mathematics, Science and Technology Education, 17*(11), em2033. https://doi.org/10.29333/ejmste/11265

Giberti, C., Santi, G., & Spagnolo, C. (2023). The role of metaphors in interpreting students' difficulties in operating with percentages: A mixed method study based on large scale assessment. *European Journal of Science and Mathematics Education, 11*(2), 297-321. https://doi.org/10.30935/scimath/12642

Godino, J., & Font, V. (2003). *Razonamiento algebraico y su didáctica para maestros* [*Algebraic reasoning and its teaching for teachers*]. Universidad de Granada.

González, P. (2005). La respuesta educativa a la diversidad desde el enfoque de las escuelas inclusivas: Una propuesta de investigación [The educational response to diversity from the approach of inclusive schools: A research proposal]. *Revista de Psicodidáctica* [*Psychodidactics Magazine*], *10*(2), 97-109.

Kieran, C. (2004). Algebraic thinking in the early grades: What is it? *The Mathematics Educator, 18*(1), 139-151.

Köster, A. (2016). Educación asequible, accesible, aceptable y adaptable para los pueblos indígenas en México: Una revisión estadística [Affordable, accessible, acceptable and adaptable education for indigenous peoples in Mexico: A statistical review]. *Alteridad. Revista de Educación* [*Otherness. Education*

Magazine], *11*(1), 33-52. https://doi.org/10.17163/alt.v11n1.2016.03

López-Mojica, J., Méndez, C., Ávila, M., & Olvera, B. (2017). Matemática educativa y educación especial: Experiencias en investigación y del aula [Educational mathematics and special education: Research and classroom experiences]. *Investigación e Innovación en Matemática Educativa* [*Research and Innovation in Educational Mathematics*], *2*, 38-50.

MEN. (1998). *Lineamientos curriculares de matemáticas* [*Mathematics curricular guidelines*]. Ministerio de Educación Nacional [Ministry of National Education].

MEN. (2006). *Estándares básicos de competencia en lenguaje, matemáticas, ciencias y ciudadanas* [*Basic competency standards in language, mathematics, science and citizenship*]. Ministerio de Educación Nacional [Ministry of National Education].

MEN. (2013). *Competencias TIC para el desarrollo profesional docente* [*ICT skills for teacher professional development*]. Ministerio de Educación Nacional [Ministry of National Education].

Padilla, A. (2011). Inclusión educativa de personas con discapacidad [Educational inclusion of people with disabilities]. *Revista Colombiana de Psiquiatría* [*Colombian Journal of Psychiatry*], *40*(4), 670-699. https://doi.org/10.1016/S0034-7450(14)60157-8

Parada, S., Velasco, A., & Fiallo, J. (2023). Communication skills enabled in a pre-calculus course using dynamic geometry software. *EURASIA Journal of Mathematics, Science and Technology Education, 19*(3), em2235. https://doi.org/10.29333/ejmste/11265

Pastor, A., Sánchez, J., & Zubillaga, A. (2014). Diseño universal para el aprendizaje (DUA). Pautas para su introducción en el currículo [Universal design for learning (UDL). Guidelines for its introduction in the curriculum]. *DUALETIC*. http://www.educadua.es/doc/dua/dua_pautas_intro_cv.pdf

Political Constitution of Colombia. (1991). *Political Constitution of Colombia, Art. 67, 7 July 1991 (Colombia)*. https://www.constituteproject.org/constitution/Colombia_2015

Radford, L. (2008). Iconicity and contraction: A semiotic investigation of forms of algebraic generalizations of patterns in different contexts. *ZDM Mathematics Education, 40*(1), 83-96. https://doi.org/10.1007/s11858-007-0061-0

Radford, L. (2010). The eye as a theoretician: Seeing structures in generalizing activities. *For the Learning of Mathematics, 30*(2), 2-7.

Radford, L. (2014). The progressive development of early embodied algebraic thinking. *Mathematics Education Research Journal, 26*, 257-277. https://doi.org/10.1007/s13394-013-0087-2

Rivera, F. (2013). *Teaching and learning patterns in school mathematics*. Springer. https://doi.org/10.1007/978-94-007-2712-0

Romero, P., Carrillo, C., & López, J. (2018). La noción de equivalencia en alumnos con dispacacidad intelectual: Construcción de su pensamiento algebraico [The notion of equivalence in students with intellectual disabilities: Construction of their algebraic thinking]. *Acara Latinoamericana de Matemática Educativa* [*Latin American Acara of Educational Mathematics*], *31*(2), 1332-1337.

Saenz, L. (2012). Derecho a la educación inclusiva en el marco de las políticas públicas [Right to inclusive education within the framework of public policies]. *Revista de Derecho Principia IURIS* [*Principi IURIS Law Magazine*], *17*(17), 20-34.

Sibgatullin, I., Korzhuev, A., Khairullina, E., Sadykova, A., Baturina, R., & Chauzova, V. (2022). A systematic review on algebraic thinking in education. *EURASIA Journal of Mathematics, Science and Technology Education, 18*(1), em2065. https://doi.org/10.29333/ejmste/11486

UN. (2006). Convención sobre los derechos de las personas con discapacidad y protocolo facultative [Convention on the rights of persons with disabilities and optional protocol]. *United Nations*. https://www.un.org/disabilities/documents/convention/convoptprot-s.pdf

UNESCO. (2007). Taller internacional sobre inclusión educativa América Latina–Regiones cono sur y andina [International workshop on educational inclusion Latin America–Southern Cone and Andean Regions]. *UNESCO*.

Vasco, C. (2002). El pensamiento variacional, la modelación y las nuevas tecnologías [Variational thinking, modeling and new technologies]. In *Proceedings of the International Conference: Computational Technologies in the Mathematics Curriculum*.

Velasco, A. (2022). *Profesores de matemáticas en ejercicio que reflexionan sobre la atención a la diversidad en clase de matemáticas* [*Practicing mathematics teachers reflecting on attention to diversity in mathematics class*] [Master's thesis, Universidad Industrial de Santander].

Vergel, R. (2014). *Formas de pensamiento algebraico temprano en alumnos de cuarto y quinto grados de educación básica primaria (9-10 años)* [*Forms of early algebraic thinking in fourth and fifth grade students of primary basic education (9-10 years)*] [PhD thesis, Universidad Distrital Francisco José de Caldas].

Zapatera, A. (2018). Introducción del pensamiento algebraico mediante la generalización de patrones. Una secuencia de tareas para educación infantil y primaria [Introduction of algebraic thinking through pattern generalization. A sequence of tasks for early childhood and primary education]. *Números* [*Numbers*], *97*, 51-67.

Zapatera, A., & Callejo, M. (2018). El conocimiento matemático y la mirada profesional de estudiantes para maestro en el contexto de la generalización de patrones. Caracterización de perfiles [Mathematical knowledge and professional perspective of student teachers in the context of pattern generalization. Profile characterization]. *Revista Complutense de Educación* [*Complutense Education Magazine*], *29*(4), 1217-1235. https://doi.org/10.5209/RCED.55070