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Decision-making in contexts of risk and uncertainty: An instrument for secondary education

Andrea Stephanie Vergara-Gómez [1*], Carolina Andrea Henríquez-Rivas [1]

[1] Faculty of Basic Sciences, Center for Research in Mathematics and Statistics Education (CIEMAE), Universidad Católica del Maule, Talca, CHILE

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Abstract

Probability and statistics help us make well-informed decisions in context of risk and uncertainty. These contexts, which are inherent to daily life, range from games of chance to 21st century current events. However, in the field of mathematics education, research focused specifically on how young people make decisions is nascent. Through an exploratory and descriptive design in six stages, we present the formulation and content validation process of forced-response questionnaire that aims to explore how students in secondary education make decisions in contexts of risk and uncertainty, considering six everyday situations. The process of validation included the participation of six experts in this area, who assessed the content of the instrument in question. The results of the content validity coefficient show an adequate level of agreement among the expert judges, allowing for a validated instrument whose use in the secondary school classroom can contribute to the recognition and characterization of patterns of reasoning for decision-making.

Keywords: probability and statistics, secondary education, questionnaire, daily life, content validation, exploratory design

INTRODUCTION

The majority of human activities require systematic decision-making processes. According to van de Bles et al. (2019), the knowledge decisions are based on is always permeated by different types and degrees of uncertainty. The importance of decision-making in contexts of uncertainty is evidenced by the variety of disciplines that study it, which include social, cognitive, and scientific approaches to the topic (e.g., Costa et al., 2022; Leuker et al., 2018; Moallemi et al., 2023). The relevance and timeliness of the study of this area is such that there already exists a wide range of related research based on 21st century issues, including climate change, forest fires, sustainable construction, and water usage, among others (e.g., Cadena et al., 2020; Larson et al., 2015; Samuel et al., 2024). However, studies focused on which of these contexts are most relevant at the school level remain scarce. Therefore, the present study is centered on demonstrating the process of design and validation of an instrument that allows for the exploration of how secondary education students make decisions in certain contexts of risk and uncertainty. In this sense, *contexts of certainty* allude to the impossibility of predicting a specific result; meanwhile, *contexts of risk* allow us to quantify the possibilities of loss or danger (Arkes et al., 2016; Gigerenzer & Gaissmaier, 2011; Martignon, 2019).

Considering the above, the importance of well-informed decision-making based on data as an area of study is clear. Indeed, this topic has reached the level of school mathematics, which can be observed, among other factors, in the fact that international standards indicate that the responsibility of school mathematics is to prepare future citizens to confront problems, decisions, and challenges of everyday life (Organization for Economic Cooperation and Development [OECD], 2010, 2019). Likewise, current research describes the purpose of school statistics as preparing citizens capable of real-life decision-making based on data (Batanero, 2020; Shaughnessy, 2019). From this perspective, it is key that the organization of learning be oriented toward data-based tasks that entail decision-making and can be

✉ avergarag@ucm.cl **(*Correspondence)** ✉ chenriquezr@ucm.cl

Contribution to the literature

- This study presents an issue that has remained underexplored in the field of education. Specifically, it presents the design and validation process of a decision-making instrument based on six contexts of risk and uncertainty directed at secondary education students.
- A practical contribution of this study is the decision-making instrument itself–which is underpinned by the analysis of historical-epistemological and conceptual-theoretical factors and a scientific literature review along with a robust methodological design–for direct utilization or future adaptations.
- The authors propose six reference contexts and two problem types (risk and uncertainty), which can be used for the design tasks that stimulate decision-making in contexts of risk and uncertainty and their subsequent implementation in teaching and learning situations, curricular development or adjustment, and future research.

easily understood by students (e.g., Eichler & Vogel, 2016). Meanwhile, the importance of the teaching of probability has been recognized for decades in terms of its role in understanding uncertainty based on historical, cultural, social, and political contextualization (Greer & Mukhopadhyay, 2005). This point of view has persisted over time, and indeed, several current studies highlight the need to foster a foundation of statistical and probabilistic skills for decision-making at various educational levels (Malaspina & Malaspina, 2020; McNicholas & Marcella, 2022; Vergara-Gómez, 2024). In this vein, the work of Ramírez-Contreras et al. (2023) stands out, as it concludes that although the use of technological simulations positively influences the development of probabilistic intuition for decision-making, much remains to be studied and undertaken in this field. Thus, a study focused on decision-making in real-life contexts in the field of mathematics education can be viewed as a contribution to the discipline.

Another important factor in the study of decision-making processes is the understanding of risk. For educational purposes, the role played by risk perception, specifically in the development of probabilistic skills, has been studied previously (Batanero et al., 2016; Borovcnik & Kapadia, 2018; Martignon & Laskey, 2019). In general, in mathematics education, decision-making in contexts of risk or uncertainty has been researched primarily within the domain of statistics and probability, with a focus on the development of probabilistic, heuristic, and inferential reasoning (Borovcnik, 2011; Engel et al., 2019; Martignon & Laskey, 2019; Martignon et al., 2022).

Specifically, there are numerous studies on the teaching and learning of risk (e.g., Amaya-Gómez et al., 2023; Aven & Kessenich, 2020; Cebesoy, 2021). However, these are mainly formulated from the perspective of environmental education or the teaching of engineering. Few studies situate decision-making in the context of risk as a skill that should be fostered in children and young people through mathematics and statistics education (e.g., Garcia-Retamero et al., 2019; Martignon, 2019). That being said, whether decision-making is addressed in contexts of uncertainty or of risk, the incorporation of authentic contexts from everyday life is essential to promote the meaning-making on the part of the student. Indeed, from the beginning of educational approaches to statistics, the role of context in the development of reasoning from data has been underscored (Gal, 2002; Moore, 1991). Therefore, it must be stressed that the relationship between contextual knowledge and statistics knowledge has been defined as indispensable and reciprocal in this context (Cobb & Moore, 1997; Wild & Pfannkuch, 1999).

Based on the above, in terms of decision-making in authentic contexts of risk and uncertainty, although a wide range of studies exist that address the importance of context for teaching statistics and probability, research is still at a fledgling level regarding which contexts are most relevant for approaching decision-making with risk or uncertainty at the school level and the extent of their use in students' interpretation of data. Similarly, in the reviewed literature, no validated instruments were identified, specifically designed to study decision-making processes in contexts of risk or uncertainty in secondary education, nor at other levels of schooling. Thus, a study that considers authentic contexts and that provides a validated instrument for classroom use can contribute to the discipline, to teachers, and to the community of researchers in this area alike. For these reasons, it is evident that more in-depth research on contextualized decision-making is needed. Specifically, the objective of the present study is to design and validate a questionnaire that contributes to the understanding of how secondary education students address decision-making scenarios in realistic contexts of risk and uncertainty.

THEORETICAL FRAMEWORK

The beginnings of the study of decision-making in contexts of risk or uncertainty date back to the social sciences and economics of the early 20th century (Knight, 1921). The literature on this topic reports that in the course of its study, the nature of available information must be discerned, as it is presented in different ways. A key difference regarding said information consists in the fact that, in decision-making under risk, information about probability distribution is available or at least

possible to infer; by contrast, in decision-making under uncertainty, such information is not afforded (Arend, 2024; Mousavi & Gigerenzer, 2014). The present study differentiates between decision-making under risk and uncertainty in educational scenarios, highlighting the importance of authentic contexts (real or realistic) in which a given situation to be resolved is framed within mathematics teaching and learning processes.

Uncertainty and Risk

The notion of uncertainty conjures different meanings depending on the language and the discipline in question. For example, in Spanish, the terms *incerteza* and *incertidumbre* exist (both may refer directly to *uncertainty*, but the former refers to it explicitly, while the latter encompasses wider senses of doubt), while in English both concepts are referred to using the same word: *uncertainty*. All of these words (in both Spanish and English) share the Latin root *certus*, the past participle of *cernere*, which means "to sift, discern, or decide." According to Choi (1993), the close connection and shared origin among these words supports the intuitive understanding that just as uncertainty is a state that calls for decision-making, decision-making itself would be a trivial exercise in the absence of uncertainty. Similarly, Arend (2024) illustrates that while consensus does not exist regarding the definition of uncertainty, it is generally conceived of as a property that depends on a system formed by the decider and their environment. This research assumes a more classical definition (Arend, 2024), such as that characterized by situations in which the decider does not possess sufficient information to predict the results of a decision, nor can they assign, a priori, probabilities to the set of possible events in question.

Meanwhile, one of the first formal definitions of *risk*, as the expected value of loss, was coined by Wald (1939) and published in *Annals of Mathematical Statistics*. According to Levy (2006), at the everyday level, there is ambiguity regarding the concept of risk, and neither its definition nor its quantification are simple or direct, even in specialized disciplines such as economics and finance. Currently, risk is understood as a probability measure associated with the consequences of an uncertain future event, and therefore it involves two main components—an undesirable outcome and the probability of said outcome occurring (Yoe, 2019). In a similar vein, Borovcnik (2020) explains that risk is intrinsically related to probability, and its interpretation depends on the meaning of probability that is used. According to this author, risk can also be linked to the measurement of an impact (cost, damage, or compensation). In this sense, risk can be understood from two perspectives: it can be either the probability of an adverse event, without considering the characterization of the associated impact, or only the characterization of the impact, without considering the associated probability (Borovcnik, 2015, 2020). These differences between uncertainty and risk are often overlooked in school curricula, hence the importance of considering a conceptual basis for distinguishing these two notions when designing or proposing learning experiences that promote decision-making.

Distinction Between Uncertainty and Risk in Teaching and Learning Processes

Mousavi and Gigerenzer (2014) affirm that decisions in contexts of uncertainty are not the same as decisions in contexts of risk. To explain this difference in simple terms, the authors point out that when playing roulette in a casino, we would be in a situation of risk, but in real life, we primarily find ourselves in situations of uncertainty. They state the in situations of uncertainty, it is not possible to calculate the probabilities of all cases, and therefore, decision-making processes cannot be completely deductive or completely inductive. Rather, these situations tend to be resolved through heuristic strategies that are specifically devised to make decisions consciously or unconsciously. Thus, the immediate decisions associated with situations of uncertainty are not always compatible with the probabilistic calculations associated with risk.

The above has direct implications for the teaching and learning processes developed at the school level. One the one hand, uncertainty refers to the impossibility of foreseeing a specific outcome and to the lack of knowledge of the probabilities of the occurrence of possible subsequent events, so it is more natural for intuitive methods or heuristic strategies to emerge for decision-making (Arkes et al., 2016; Gigerenzer & Gaissmaier, 2011). Indeed, recent studies have demonstrated the value of intuition as an inherent element of decision-making under uncertainty, especially in learning situations (Lappas et al., 2023; Ramírez-Contreras et al., 2023). On the other hand, risk allows for the measurement or quantification of the possibilities of loss, damage, or danger, which involves a greater expectation and understanding of using probabilistic modeling in these types of situations (Borovcnik, 2016). Likewise, authors including Martignon (2019) propose working at the level of school mathematics on specific competencies associated with risk that entail the deployment of fundamental probabilistic knowledge. It follows that when presenting learning activities to students, situations of decision-making under risk offer greater coherence than situations of decision-making under uncertainty to activate, in particular, the use of probability distribution functions and, in general, probabilistic knowledge. Similarly, when working with situations of uncertainty, it can be expected that students will develop intuitive or heuristic decision-making strategies.

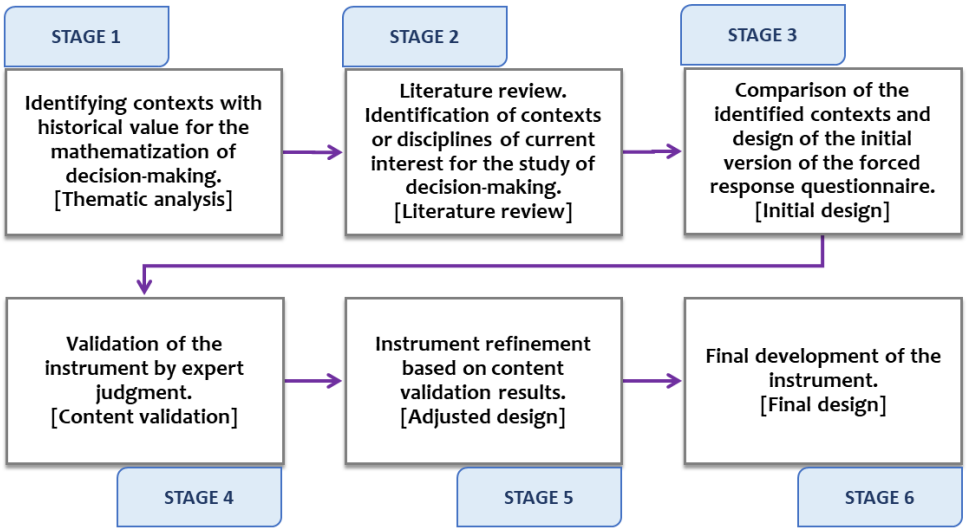

Figure 1. Diagram of the stages that organize the research methodology (Source: Authors' own elaboration)

Studies on Context in Statistics Education

Some studies in the field of statistics education have shown that knowledge of context contributes to the development of inferential reasoning and informal statistical inference (Makar & Ben-Zvi, 2011; Makar & Confrey, 2005; Makar & Rubin, 2018). In this respect, its study reinforces the understanding of statistical modeling (Pfannkuch et al., 2018) and promotes the construction of a socially critical outlook regarding data (Ubilla & Gorgorió, 2023). Within this area, some research indicates that students may have difficulties with the use and interpretation of contexts for data analysis (Rodríguez-Muñiz et al., 2022; Wroughton et al., 2013). Specifically, in terms of probabilistic reasoning, knowledge of context becomes especially important in situations that call for interpretation and generation of judgments for contextualized decision-making (Gal, 2005).

Research carried out by Elbehary (2021) examines the characteristics of probabilistic reasoning exhibited by pre-service mathematics teachers when analyzing real-life contexts of uncertainty, such as environmental problems, school experiences, gender issues, life expectancy, and family preferences, among others. The present study adopts this approach and underscores the need to explore the probabilistic and statistical thinking that arises in real situations of uncertainty and risk, as educational experiences in probability or statistics at the school level are rarely based on a critical examination of data in authentic contexts (Brückler & Šipuš, 2023; Yilmaz et al., 2023).

Given the above, we consider the conceptual distinction between risk and uncertainty, together with the importance of context in learning statistics and probability, to represent indispensable elements for the creation or selection of situations that effectively promote decision-making in secondary education and, consequently, to define a coherent theoretical basis for structuring the design process of the instrument that we present in this study.

MATERIALS AND METHODS

The present study proposes a research process based on the identification of contexts linked to decision-making for the design and validation of an instrument that can be utilized in secondary education, with the purpose of understanding how students confront decision-making situations in contexts of risk and uncertainty. To this end, an exploratory and descriptive design methodology was selected (Vasilachis de Galdino, 2006).

The use of this type of design is well-suited to processes of construction and validation of instruments whose aim is to the understand new links or theoretical relationships (e.g., Turra et al., 2022). The present study in particular demonstrates the design and content validation process of the *Instrument for understanding decision-making in contexts of risk and uncertainty among secondary education students*. Specifically, the validation of the instrument was carried out through content analysis by expert judges (Escobar-Pérez & Cuervo-Martínez, 2008), which allowed for its internal consistency to be determined. The complete process of design and validation of the instrument included the stages displayed in **Figure 1**.

Stage 1. Historical-Epistemological Study Using Thematic Analysis

In the first stage, a historical-epistemological study was undertaken using the technique of hybrid thematic analysis (Boyatzis, 1998). In this stage, a coding procedure was used based on the topics and contexts related to the study. The researchers coded the data blindly and independently and later met to collaboratively review the work carried out, this with the purpose of ensuring consistency in the interpretation

and coding of the information (Creswell & Creswell, 2018). The coding and deriving themes of contexts was performed using ATLAS.ti 8 software.

Stage 2. Systematic Literature Review

In the second stage, a systematic review was carried out (Mian et al., 2005) using the Scopus database in order to identify the areas or disciplines in which decision-making under risk and uncertainty has been researched for the purpose of teaching or learning, along with the most frequent contexts therein. The Scopus database was selected because it includes more than 25,000 journals at the international level, ensures peer review and disciplinary relevance, and provides impact factors which, like those of WoS, allow journals to be ranked in quartiles. Moreover, while this study entails a broad review without restrictions to particular areas of knowledge, it should be noted that journals specialized in mathematics education originate principally from the Scopus database (Gaona & Arévalo-Meneses, 2024). Likewise, in general, the journals found in WoS are typically also found in Scopus.

Stage 3. Comparison and Construction of the Initial Version of the Instrument

In the third stage, the findings obtained in the first two stages were compared, searching for possible convergences that would allow for the definition of a framework for designing the situations and questions of the instrument. Based on the results of this process, a forced-response questionnaire (Bartram, 2007) was designed, consisting of 24 questions and six reference contexts. Each reference context grouped four items with their respective alternatives. The alternatives, in this stage, were constructed considering four response options, whose purpose was not to assess students' mathematics or statistics knowledge, but rather to identify whether their choices were inclined more toward the intuitive or toward the normative or deliberate.

Stage 4. Content Validation by Expert Judgment

In the fourth stage, the instrument underwent validation by expert judgment, (Escobar-Pérez & Cuervo-Martínez, 2008), using the content validation coefficient (CVC) as an analysis technique, because, unlike other techniques, it allows the importance of each item to be evaluated independently and its use is especially recommended when there are 3 or more judges, ideally 5 to 10 (Hernández-Nieto, 2002). To accomplish this, an international group of six expert judges was consulted, all of whom hold doctoral degrees in mathematics, mathematics didactics, or mathematics education, and whose lines of research correspond to the research objective of the present study. All of the judges recruited are researchers in processes of learning and teaching mathematics, both in secondary and tertiary education. **Table 1** shows the characteristics of the expert judges.

Table 1. Distribution of expert judges on doctoral degrees

	Area of doctoral study		
	Mathematics	Mathematics didactics	Mathematics education
Number of judges	2	1	3

Table 2. Phases of the hybrid thematic analysis

Phase of analysis	Type of analysis
I. Preliminary review (primitive coding)	Inductive
II. Theme and code development (thematic coding)	Inductive and deductive
III. Theme evaluation	Deductive

Stage 5 and Stage 6. Refinement and Final Version of the Instrument

In the fifth stage, the questionnaire was revised and refined according to the results of the content validity analysis and the qualitative suggestions provided by the expert judges. This resulted in an initial proposal of the instrument, which retains the six reference contexts but is reduced to 20 items. Finally, in the sixth stage, the final and complete version of the validated instrument is presented.

RESULTS

Below, the results are presented according to the six stages described in the previous section.

Stage 1. Historical-Epistemological Study Using Thematic Analysis

Hybrid thematic analysis, as utilized in the present study, is carried out based on an emblematic 19th century work written by the French mathematician Cournot (1843). This work, entitled *Exposition de la théorie des chances et des probabilités*, studies how to confront uncertainty, risk, and error in distinct real-life contexts using probability. The present analysis includes both deductive and inductive approaches (see **Table 2**). Phase 1 is inductive in nature, consisting of an initial exploration and primitive coding based on the work's textual corpus, without considering a priori categories. Phase 2 is both inductive and deductive, as the codes from the first phase are organized using the conceptual framework. Finally, phase 3 is deductive, reviewing the entire process of coding considering the themes that have emerged in the previous stage.

The results obtained through this analysis included the initial identification of 45 intelligent codes or code groups, of which eight referred to contexts situated within the problems addressed in the study. Based on

Table 3. Codes referring to contexts and their rootedness in the thematic analysis of the work

Code	Rootedness
Games and betting	93
Juries and jurisprudence	175
Demographics	71
Astronomy	84
Annuities and insurance	91
Economics and finance	117
Oceanography	3
Health	10

these intelligent codes and using the functions of ATLAS.ti to construct relationships, search for patterns, and make classifications based on networks (Hwang, 2008), the thematization was finalized, grouping and obtaining eight codes associated with contexts. These contexts are specifically related to 19th century problems, which arose around uncertainty and risk, whose study and corresponding scientific interest was only just beginning at that time. **Table 3** presents these eight codes together with their respective frequency, or rootedness.

As displayed in **Table 3**, the highest level of rootedness is manifested in the themes under the codes "Juries and jurisprudence" and "Economics and finance." This could be explained by two factors, respectively:

(1) Cournot's mentor was Siméon-Denis Poisson, who proposed the law of large numbers in his study of probability in legal trials, using large quantities of historical data from the court systems of France and Belguim and

(2) Cournot was a mathematician and economist, being the first to propose the use of mathematical functions to describe demand, supply, and price, among other economic variables.

Other themes that stand out in the analysis include gambling and betting, as well as the study of annuities and insurance, whose mathematical problematization can be traced back to the 17th century.

Stage 2. Systematic Literature Review

To complement the identification of contexts, a systematic review of existing research on the topic was carried out. The search utilized Boolean connectives and the English keywords "decision-making" and ("risk" or "uncertainty") in the title of a work. Thus, the first inclusion criterion encompassed research whose primary focus was decision-making under uncertainty or risk. In addition, the terms "teach*" and "learn*" were incorporated in the search for article titles, abstracts, or keywords. The latter were included with the aim of identifying research within the scope of teaching and learning, in a broad sense. Through the advanced search, the following formula was utilized: ((TITLE ("decision-making") AND TITLE (uncertainty) OR TITLE (risk)) AND TITLE-ABS-KEY (teach*) OR TITLE-ABS-KEY (learn*)).

Regarding exclusion criteria, studies not published in English or Spanish were discarded in order to facilitate the reading of abstracts. Additionally, research in the area of formal mathematics was excluded, along with thematic areas related to the natural sciences, such as biochemistry, genetics and molecular biology, chemistry, earth and planetary sciences, physics, and astronomy. Research in the veterinary field was also discarded from this study. Meanwhile, it should be noted that research was not excluded based on document type or year of publication. Once the aforementioned filters were applied, 435 documents remained, at which point further exclusion criteria were considered. Specifically, all document titles and abstracts were read to eliminate those studies not addressing the authentic participation of people as informants, or those that generated data from cognitive simulation processes or from experiments with animals. Following the application of these additional criteria, 236 publications were left to be reviewed, corresponding to the period 1990-2024.

Some of the research reviewed stands out for its educational implications, an important example being the article "Decision making and learning while taking sequential risks" (Pleskac, 2008) published in *Journal of Experimental Psychology: Learning Memory and Cognition*. This article addresses participants' adaptive Bayesian learning processes when they need to make decisions in contexts of risk, offering specific recommendations to better understand and evaluate risk-taking behavior. Likewise, in the article "Navigating uncertainty: The role of mood and confidence in decision-making flexibility and performance" (Lavín et al., 2024), published in the journal *Behavioral Sciences*, the authors recognize the importance of more in-depth study of people's state of mind, emotions, and learning when making decisions in contexts of uncertainty. The authors point to involving processes of flexibilization in decision-making based in a metacognitive dimension. In general, this type of research demonstrates the need to study decision-making processes using explicitly educational approaches.

It should be noted that this review is intended to be representative of the types of contexts identified; emphasis is placed on the percentage distribution of the primary context focused on in the studies selected. The findings of this literature review are shown in **Table 4**, in which the research contexts are identified along with whether or not they have educational purposes according to the studies reviewed.

As exhibited by **Table 4**, in contrast to what was identified in the historical-epistemological study, research on decision-making in field of jurisprudence is scarce. On the other hand, finance and economics are

Table 4. Percentage distribution of the contexts identified in the literature review

Context	With school or educational purposes	Outside of school or educational purposes	Total
Actuarial and security	0%	2%	2%
Natural and social disasters	0%	19%	19%
Finance and economics	0%	11%	11%
Organizational or business management	2%	4%	6%
Transportation and traffic	0%	4%	4%
Games	3%	0%	3%
Health	0%	16%	16%
Environment	0%	6%	6%
Meteorology	0%	3%	3%
Education	5%	10%	15%
Politics	0%	2%	2%
Law	0%	1%	1%
Other (with less than 1% each)	0%	12%	12%

Table 5. Organization of instrument items based on context and problem type

Type of context	Problems related to uncertainty	Problems related to risk	Total items
Juries & jurisprudence	3	1, 2, & 4	4
Games & betting		5, 6, 7, & 8	4
Insurance	9 & 10	11, 12	4
Health		13, 14, 15, & 16	4
Climate	18 & 19	17 & 20	4
Finance	22 & 24	21 & 23	4
Total items	7	17	24

present as topics of interest both in the 19th century (**Table 3**) and in contemporary scientific production (**Table 4**). The results of the review reveal that decision-making has been of interest to distinctive and wide-ranging areas of knowledge in the 21st century, from organizational or business management to the study of disasters.

Stage 3. Comparison and Construction of the Initial Version of the Instrument

In this stage, the characteristics of the contexts defined based on the historical-epistemological analysis were compared with the descriptions, explicit or implicit, of the contexts identified in the literature review in order to construct the initial version of the instrument. This resulted in the recognition of six common contexts: economy (or finance), juries and jurisprudence, climate (or meteorology), health, insurance, and games and betting. The common contexts identified and the types of phenomena presented by the studies in the literature review in relation to said contexts lead to the creation of a forced-response instrument with 24 items. Each item addresses a contextualized problem or situation, presenting four alternatives described in terms of a possible decision; two alternatives correspond to preferences based more on probabilistic reasoning, while the other two correspond to more intuitive or heuristic decisions. The alternatives are not intended to define correct or incorrect responses in terms of mathematics, but rather to lead the student to declare an inclination, preference, or opinion regarding the problem. The organization of the items of the instrument, designed according to type of context and type of problem (uncertainty; risk), is presented in **Table 5**.

As an example, some of the items designed at this stage are presented below (see **Table 6**). Specifically, the wording of two items is presented; one item is associated with decision-making under risk in the context of jury and jurisprudence (item 2)–this was ultimately eliminated from the final version–while the other is associated with decision-making under uncertainty in the context of climate (item 18). The latter passed the various stages of the content validation process.

Stage 4. Content Validation by Expert Judgment

In this stage, the initial instrument design from Stage 3 is subject to content validation involving the judgment of six experts in the field. In addition to the instrument, each judge received a cover letter with the research objective and an analysis protocol containing instructions for rating each item on a scale of 1 to 4 for the criteria of clarity, relevance, and importance. More specifically, the protocol asks judges to evaluate whether each item's wording and question are clearly comprehensible (clarity), whether the specific content of the item is relevant to the purpose of the study (relevance), and whether the situation posed in general related to the research in an important way (importance). The participation of the experts was voluntary, and data confidentiality was maintained throughout the process.

Table 6. Examples of items designed in the preliminary version of the instrument

Item	Type of context	Associated notion	Wording
2	Juries and jurisprudence	Risk	*The criminal court of a European country uses a mixed system comprised of six common citizens, selected randomly, and three professional judges. Both the judges and the citizen jury deliberate together and vote on the defendant's guilt. In order to reach a guilty verdict, a two-thirds majority is required. If this majority is not reached, the defendant is declared innocent.* ***If it depended on you, what decision would you make to analyze the effectiveness of this system?***
18	Climate	Uncertainty	*An agronomist owns a roofed structure for drying walnuts. This year, he hopes to offer his walnut production to a new company and finalize a lucrative deal. In mid-autumn, with the first rains, he identifies several leaks in the roof. If the walnut harvest is damaged by the resulting humidity, the agronomist will not be able to close the business agreement. On the news, they announce that a big storm is coming, with a forecast of four consecutive days of heavy rain. However, one of the workers on the farm tells the agronomist that he knows how to read the weather very well and that it will not rain on the first day reported by the news forecast. He recommends using this day to replace the roof, which will fully fix the humidity problem but implies removing the existing roof completely.* ***What would you decided to do if you were the agronomist?***

Table 7. CVCic results according to the scores given by all judges for each item

No	Criteria			Average	CVCic	Qualification
	Clarity	Relevance	Importance			
1	20	22	23	10.83	.90275634	Excellent
2	15	17	19	8.5	.70831190	Deficient
3	21	23	24	11.33	.94442301	Excellent
4	24	23	24	11.83	.98608968	Excellent
5	21	23	23	11.17	.93053412	Excellent
6	20	22	22	10.67	.88886746	Good
7	19	22	22	10.5	.87497857	Good
8	21	23	23	11.17	.93053412	Excellent
9	23	24	24	11.83	.98608968	Excellent
10	23	24	24	11.83	.98608968	Excellent
11	23	24	24	11.83	.98608968	Excellent
12	23	24	24	11.83	.98608968	Excellent
13	20	23	23	11	.91664523	Excellent
14	22	24	24	11.33	.97220079	Excellent
15	21	21	20	10.33	.86108968	Good
16	20	23	23	11	.91664523	Excellent
17	24	24	24	12	.99997857	Excellent
18	23	24	22	11.5	.95831190	Excellent
19	23	24	22	11.5	.95831190	Excellent
20	21	23	24	11.33	.94442301	Excellent
21	20	19	19	9.67	.80553412	Good
22	24	24	24	12	.99997857	Excellent
23	24	24	24	12	.99997857	Excellent
24	24	24	24	12	.99997857	Excellent

The context types and their justification are also presented in the protocol. Hernández-Nieto's (2002) CVC is used, which requires a score equal to or greater than 0.8 to accept the item. The procedure utilized considers the three criteria together, to evaluate the CVCic for each item, which corresponds to the relative proportion in terms of the maximum value of the scale of the average of the scores among judges for each item, corrected for random agreement. Given the above, the maximum value per item is presumed to be V max = 12 and the probability that two judges randomly assign equal point values $P_{ei} = 2.143 \cdot 10^{-5}$. The results obtained for the categorization of the item based on validity and agreement, following the expert judgment, are shown in **Table 7**. The final column shows the condition of each item according to the score obtained from the values assigned by the judges.

As shown in **Table 7**, only item 2 is rated with the condition of "deficient," while items 6, 7, 15, and 21 are rated as "good." All other items are rated as "excellent," reflecting the instrument's potential for adjustment.

Additionally, the experts qualitatively reviewed the items, identifying the following:

Table 8. Final organization of the instrument by .tem and CVCtotal according to context

Type of context	Problems related to uncertainty	Problems related to risk	CVCtotal
Juries and jurisprudence	2	1 & 3	.94442
Games and betting		4, 5, & 6	.91664
Insurance	7 & 8	9 & 10	.98608
Health		11, 12, & 13	.93516
Climate	15 & 16	14 & 17	.96525
Finance	18 & 20	19	.99997
Total	7	13	

4. In the game "find the ring," the contestant is offered three upside-down cups, one of which contains the ring. The game host hides a very precious ring under one of the cups but doesn't show which one. He then gives the contestant the option to choose a cup. If they guess correctly, they keep the ring. The contestant chooses a cup, but before revealing its contents, the host selects another cup, showing that it is empty. Then, they offer the contestant the option of making their first choice or switching to the other cup, which is still upside down. ***What would you do in the place of the contestant?***

A. I would stay with the first cup chosen, because changing cups could bring me bad luck.
B. I would stay with the first cup chosen, because the two cups that remain upside-down have the same probability of containing the ring.
C. I would change my choice, because changing cups increases my chances of winning.
D. I would change my choice, because changing cups could bring me good luck.

Figure 2. Wording and alternatives of item 4 in the final version of the instrument (Source: Authors' own elaboration)

(a) ambiguities in the wording of the problems that could confuse students or cause deviation from the item's purpose;

(b) characteristics of item content that could be culturally unfamiliar, complex, or of little significance for students;

(c) inappropriate levels of semantic or syntactical difficulty within an item considering the target population (14- to 15-year-olds);

(d) redundancies or inconsistencies in an item's response alternatives.

These observations and recommendations were likewise considered in order to adjust the instrument, which is illustrated in the following stage.

Stage 5. Refinement of the Instrument

In this stage, the content validity results from the previous stage are evaluated in order to adjust the instrument. As can be observed in **Table 7**, a high number of items were qualified as "Excellent," with the exception of items 2, 6, 7, 15, and 21. To decide which items to eliminate, in addition to considering the quantitative CVCic results, the expert judges' comments on each item were also analyzed. In this manner, the decision was made to eliminate items 2, 7, 14, and 15; meanwhile, item 6 was maintained, but with the respective modifications recommended by the expert judges, especially in the wording of both the main problem and the response alternatives.

Subsequently, all items were reviewed again with the aim of incorporating new adjustments where necessary based on the experts' qualitative observations. The resulting instrument includes 20 items, which are organized with a new sequential numbering, as illustrated in **Table 8**. Of the 20 items, seven correspond to problems related to decision-making under uncertainty, and 13 to problems related to decision-making under risk. For each of the contexts, a total CVC above 0.91 was identified, and for both problem types a CVC greater than 0.95.

It should be noted that, based on the content validation, both the wording of the items and their alternatives were prepared to facilitate response classification. In the case of items related to risk, the alternatives allow for the identification of risk propensity or aversion (Lucarelli et al., 2021). In the case of items related to uncertainty, the alternatives allow for distinction between intuitive and deliberate (or normative) judgments (Kruglanski & Gigerenzer, 2018). The latter refers to decisions made based on probabilistic reasoning. As an example, two items are presented in **Figure 2** and **Figure 3** respectively, from the final version of the instrument, one associated with risk and the other with uncertainty, along with the dichotomization of their alternatives as described above.

Figure 2 shows item 4 of the final version of the instrument, which corresponds to item 5 of the initial version presented to the expert judges. This item is associated with uncertainty. Among the alternatives, A and D relate to non-deliberate decisions guided by intuition, while B and C represent decisions based on probabilistic arguments. While the wording of case B does imply an equiprobability bias, it likewise refers to probabilistic reasoning. In this sense, it is important to reiterate that the design of the alternatives and their dichotomization were not undertaken with the intention of separating correct and incorrect answers, but rather to identify tendencies in the types of decisions made based on the criteria indicated.

19. In recent years, various cryptocurrencies have attracted investors, but at the same time they have generated various financial controversies. An emerging cryptocurrency has shown rapid growth in its currency value over the past year, promising high rates of return to its investors. On the other hand, a different cryptocurrency that has been on the market for over a decade has shown low volatility its in values in the past few years and moderate rates of return. ***If you had the opportunity to make a short-term investment in crotypcurrencies, which one would you invest your capital in?***

A. *In the emerging cryptocurrency, because the promise of a greater rate of return allows for greater gains in the short term.*
B. *In the cryptocurrency with the longer history on the market, because its low volatility indicates a less risky investment.*
C. *In the emerging cryptocurrency, because it is worth taking advantage of its rising currency values, which might be short-lived.*
D. *In the cryptocurrency with the longer history on the market, because moderate returns ensure a more secure and reliable investment.*

Figure 3. Wording and alternatives of item 19 in the final version of the instrument (Source: Authors' own elaboration)

Table 9. Dimension 1 items

No	Wording	Notion
1	*Some countries use jury systems composed of common citizens to make decisions in trials. In juries with 12 people, it is most typical to require unanimous agreement to reach a verdict (all 12 people must agree). Only in certain exceptional cases are majority agreements (at least 10 of 12 must agree) or simple majority agreements (at least 7 of 12 must agree) accepted.* ***If it were up to you, which of these jury systems would you choose?***	Risk
2	*In some countries, courts are formed only be professional judges to reach verdicts; in most cases, a three-judge system is used. In a nation that utilizes this system, a news story recently came out involving a man convicted in a crime who spent several years in prison despite being innocent. Imagine that you are an attorney who has to represent a client before the same court.* ***Which of the following decisions would you make regarding the case?***	Uncertainty
3	*In German regional courts, which decide on serious criminal cases a panel of three professional judges can reach a verdict by simple majority (two out of three votes). In criminal trials in Scotland, a 15-member citizen jury system is used, also requiring a simple majority (eight out of 15 votes) to reach a verdict.* ***Which of these two systems do you think is more reliable?***	Risk

Item 19 of the final version of the instrument (see **Figure 3**) corresponds to item 23 in the initial version presented to the judges. This item is associated with risk. Among the alternatives, while options B and D reflect more moderate choices in terms of taking potential risks, options A and C represent a greater propensity for risk-taking when the reward could be greater or more immediate.

It should be noted that the classification of items in contexts of risk or uncertainty does not correspond to explicit references to these notions in the wording of the items or their alternatives, but rather to the epistemic distinction stated in the conceptual framework.

Stage 6. Final Version of the Instrument

In this stage, the final version of the instrument is presented, obtained as a result of all of the preceding stages of the research process, which include thematic analysis, literature review, expert validation of the initial version of the instrument, and the reorganization and adjustments informed by the content validation. This allows for the formulation of a definitive version of the instrument based on dimensions underpinned by six contexts, with 20 items related to decision-making situations involving uncertainty–when it is not possible to make probabilistic estimations given the conditions or information available–or decision-making situations involving risk–when it *is* possible to make probabilistic predictions in some measure based in the information available. The final version of the instrument is exhibited below.

Dimension 1. Context of jury and jurisprudence

This dimension relates to the ways in which a jury can be formed in terms of the number of members, the level of professionalization of the members, and the types of agreements required to reach a verdict in a case, specifically considering how these elements can affect the trust or confidence we place in the decisions of a given jury (see **Table 9**).

Dimension 2. Context of games and betting

This dimension relates to games of chance, which use some type of randomizing artifact and involve a placing bet, in some manner, on one or more results. In this context, potential winnings or rewards exist, hence the interest in anticipating probabilities of success or winning and, eventually, the minimum or maximum number of games required to achieve some degree of confidence in the probabilistic estimates in question (see **Table 10**).

Table 10. Dimension 2 items

No	Wording	Notion
4	*In the game "find the ring," the contestant is offered three upside-down cups, one of which contains the ring. The game host hides a very precious ring under one of the cups but doesn't show which one. He then gives the contestant the option to choose a cup. If they guess correctly, they keep the ring. The contestant chooses a cup, but before revealing its contents, the host selects another cup, showing that it is empty. Then, they offer the contestant the option of making their first choice or switching to the other cup, which is still upside down.* ***What would you do in the place of the contestant?***	Risk
5	*The host of the game from above (4) increases the difficulty. Now he places 12 cups upside-down on the table and hides a ring under one of them, without showing which. He allows the contestant to choose a cup, and from that point he repeats the same procedure explained in the previous question. Each time the contestant chooses a cup, the host lifts a different cup to show its empty contents and offers the contestant the possibility of staying with their choice or changing it, in the latter case allowing them to choose any of the other cups that are still upside-down. The discarded cups are removed from the table. The game host does this until the player finds the ring or loses the game.* ***What would you do in the place of the contestant?***	Risk
6	*In a game of two balanced dice, you win double your bet if a double-six is rolled, but you lose your entire bet otherwise. The player decides how much to bet.* ***What you do if you were the player?***	Risk

Table 11. Dimension 3 items

No	Wording	Notion
7	*People often want to protect their property against catastrophes such as fire, natural disasters, theft, and others. One way of doing so is to take out insurance to cover the potential financial losses generated in the event of a catastrophe. Imagine that you want to purchase insurance for the total loss of a vehicle, which provides compensation in case your vehicle is considered a total loss due to an accident.* ***What would you do to decide whether the price of the insurance is fair?***	Uncertainty
8	*Actuaries are professionals specializing in the study and management of financial risk in different areas, including insurance, pensions, and investment. A group of actuaries is studying the possibility of offering health insurance to cover the costs of diagnosis and treatment of a new disease in your country. To calculate the value of the coverage and the monthly premiums, they are going to consider the age and medical history of the applicant.* ***What other variable do you think should be considered when calculating the insurance premiums in question?***	Uncertainty
9	*A residential neighborhood has a low reported burglary rate: 1 in 50 homes has suffered some type of burglary in the past year. However, one of the homes has already been burglarized three times in the past year, and the owner is considering the option of purchasing an expensive, full-coverage burglary insurance.* ***What decision do you think the homeowner should make?***	Risk
10	*A local business is insured against fire in a neighborhood where the annual risk of such catastrophes is less than 5%. However, during the past year, several serious fires have occurred in close succession in the area, the causes of which remain under investigation. At the end of the year, the insurer will readjust the insurance value.* ***What type of adjustment do you think the insurer will make?***	Risk

Dimension 3. Insurance context

This dimension is related to the study and evaluation of the insurance business (life, health or catastrophe, theft, etc.), which includes its basic functioning and modeling. In this context, it is important to become aware of the uncertainty of catastrophic events and estimate the fair or fitting value of an insurance premium based on the probability of risk and the magnitude of the potential loss (see **Table 11**).

Dimension 4. Health context

This dimension relates to the criteria that guide medical and public health decisions about diagnosing, preventing, or providing treatment for diseases or medical conditions, with the objective of mitigating possible risks and optimally managing the effects or consequences (adverse or beneficial) of these decisions for the various stakeholders involved (see **Table 12**).

Dimension 5. Climate context

This dimension relates to the tools available to understand the behavior of the climate at a specific location and time and using said information to predict weather conditions with some margin of probability and error. This context affects decision-making in important areas including agriculture, aviation, navigation, and public administration, as its study allows for the anticipation of dangerous natural phenomena that threaten the population and evaluating risks and appropriate measures (see **Table 13**).

Dimension 6. Finance context

This dimension is related to the strategies and methods that we use to manage economic resources. In this context, the aim is to understand how and why people, businesses, and governments generate, utilize, invest, and manage resources to make financial decisions, assessing the balance between risks and

Table 12. Dimension 4 items

No	Wording	Notion
11	*In a region of South America, three out of every 100 people have a bacterial disease whose treatment is very expensive. A laboratory is developing a new exam to diagnose this illness and conducting some experimental tests to evaluate its effectiveness. At the end of the study, the effectiveness of the exam is reported to be 60%. A person is then given the new diagnostic exam, and they test positive for the disease.* ***If you were the doctor in charge, what decision would you make about this scenario?***	Risk
12	*In Study A, blood samples are taken from 100 patients over 18 years of age, selected randomly, to study their cholesterol levels. In Study B, the same procedure is carried out, but with only 20 patients. Meanwhile, it is known that 25% of the general population over the age of 18 has high cholesterol. One of the studies shows that 50% of the patients in the sample have high cholesterol, but it is unknown which study.* ***If you were the doctor in charge, what decision would you make regarding this situation?***	Risk
13	*An international study has reported that people who smoke are two times more likely to develop heart disease than non-smokers. Two groups are randomly selected: one with 30 non-smokers and the other with 30 smokers. Five participants from the smokers group end up being diagnosed with heart disease.* ***What do you think might happen in the non-smokers group?***	Risk

Table 13. Dimension 5 items

No	Wording	Notion
14	*An agronomist owns a roofed structure for drying walnuts. This year, he hopes to offer his walnut production to a new company and finalize a lucrative deal. In mid-autumn, with the first rain, he identifies several leaks in the roof. If the walnut harvest is damaged by the resulting humidity, the agronomist will not be able to close the business agreement. On the news, they announced that a big storm is coming, with a forecast of four consecutive days of heavy rain. However, one of the workers on the farm tells the agronomist that he knows how to read the weather very well and that it will not rain on the first day reported by the news forecast. He recommends using this day to replace the roof, which will fully fix the humidity problem but implies removing the existing roof completely.* ***What would you decided to do if you were the agronomist?***	Uncertainty
15	*The ultraviolet (UV) index is a standard measure of the intensity of the sun's ultraviolet radiation, which is expressed on a scale of 1 to 11+. UV radiation can cause severe sunburn and increase the risk of skin cancer. In midsummer, the weather forecast predicts that the UV index will be very high risk (8 to 10) for the next five days, and it recommends avoiding outdoor activities. A running enthusiast needs to train for a race that will take place in a week, and they are frustrated about not being able to go outside to practice. The first three days, the runner does not go out to train, although the UV index at the end of the day is only reported to be at a moderate level.* ***What would you decide to do in the place of the runner?***	Uncertainty
16	*In many tropical regions, like the Amazon, the daily probability of thunderstorms during the summer season can be very high, often exceeding 50%. Thunderstorms generate severe turbulence that can seriously damage aircraft and cause dangerous flight conditions. A prestigious commercial airline has frequent flights in the Amazon region and constantly monitors meteorological conditions to guarantee flight safety. For a specific flight, a 70% probability of thunderstorms is forecast for the route, so the airline offers passengers the choice of a 15-hour delay or an internal transfer to another airline to complete the original trip at one's own risk.* ***What would you decide to do if you were one of the passengers?***	Risk
17	*At the height of summer, you are preparing for your vacation: you will be visiting a beautiful city in southern Chile, where some friends live, for a week. The weather forecast for the city predicts a week with an average high of 12°C and an average low of 7°C, with a 60% probability of rain. Your friends, who are waiting for you there, tell you that the past ten days have been hot with clear skies. On this trip, there are luggage restrictions, and you can only bring a small bag or backpack, which means that it is necessary to be very selective about what you pack.* ***What decision would you make in this scenario?***	Risk

benefits. Moreover, this context encompasses both micro- and macroeconomic levels (see **Table 14**).

Content validation enables the piloting of the instrument with a sample of secondary education students, whether to explore other types of validation or to better understand how decision-making operates at this educational level. To achieve the latter, it is necessary to identify patterns or profiles in the context of decision-making through the dichotomization of alternatives, according to the problem type in each question. These profiles could be defined both at the level of each context and at a general level. The profiles refer to identifying, on the one hand, whether risk propensity or aversion predominates, and on the other, whether intuition or probabilistic reasoning predominates. It follows that the four possible response patterns include the following: risk propensity and intuitive decisions; risk propensity and normative decisions; risk aversion and intuitive decisions; risk aversion and normative decisions. The study of these profiles could contribute to improving or reorienting students' decision-making skills in contexts of risk or uncertainty.

Table 14. Dimension 6 items

No	Wording	Notion
18	*A group of five people want to set up a savings cooperative. Each member must contribute a monthly fee, in order to eventually raffle off the pool of funds to one of the participants at the end of each year. The cooperative continues this way until all members have received the year's savings at some point. Everyone is very enthusiastic about participating, but one of the members suggests using the pooled capital to invest, thus offering the possibility of increasing the fund amount.* ***What would you decide if you were part of this cooperative?***	Uncertainty
19	*In recent years, various cryptocurrencies have attracted investors, but at the same time they have generated various financial controversies. An emerging cryptocurrency has shown rapid growth in its currency value over the past year, promising high rates of return to its investors. On the other hand, a different cryptocurrency that has been on the market for over a decade has shown low volatility its in values in the past few years and moderate levels of return.* ***If you had the opportunity to make a short-term investment in crotypcurrencies, which one would you invest your capital in?***	Risk
20	*An investment portfolio is a set of financial assets that can include a variety of instruments, such as stocks, bonds, real estate, investments in other currencies, etc. The objective of an investment portfolio is to maximize profits and/or minimize risks. Imagine that you have capital available to make a long-term investment in a portfolio.* ***What factors would you prioritize when making a decision regarding this situation?***	Uncertainty

DISCUSSION AND CONCLUSIONS

Both decision-making under uncertainty and decision-making under risk require realistic contexts and scenarios that are conducive to students' meaning-making. To achieve this, it is necessary to adequately understand how young people confront situations that require decision-making and the nature of their preferences or opinions in such situations.

This study addressed the need to consider realistic and educationally relevant contexts to explore how secondary-level students approach decision-making under risk and uncertainty. The primary result is a validated instrument that allows students' preferences, reasoning strategies, and tendencies to be apprehended in said contexts. The questionnaire, entitled *Instrument for understanding decision-making in contexts of risk and uncertainty among secondary education students,* is a forced-response questionnaire that presents students with specific circumstances of decision-making under risk (Mousavi & Gigerenzer, 2014) and decision-making under uncertainty (Arend, 2024), organized according to six reference contexts, which in turn define the dimensions for analysis. These contexts were identified on a historical-epistemological basis and reaffirmed by a thorough literature review of current research in this area.

In a subsequent stage, as a result of the process of content validation through expert judgment, those items that did not manifest sufficient levels of internal consistency were eliminated, while the wording of others was adjusted in order to make them more comprehensible for students. Likewise, item alternatives were revised to ensure that they address, with greater explicitness, the difference between risk propensity and risk aversion (Lucarelli et al., 2021) and the difference between intuition and probabilistic reasoning (Kruglanski & Gigerenzer, 2018), depending on the type of situation expressed in the wording of the problem.

Meanwhile, regarding the limitations of the study, the extent and linguistic complexity of the problems that make up the items are identified, which involved greater time dedication from the judges (García Alarcón et al., 2017), who proposed several changes in an effort to ensure the accessibility of the students. These potential difficulties will be possible to assess once the instrument is empirically implemented. Notwithstanding, the analyses and stages of the process of design and validation of the instrument have been carried out in a rigorous and organized manner.

Finally, the improved final version of the instrument is presented, which is put forward as a tool with strong theoretical and methodological foundations that can contribute to further advancements in educational research on decision-making processes. This initial proposal could be used for exploratory data collection, or it could be subjected to further validation processes involving empirical implementations with students in the school system. One possible future use for this instrument is its application in the classroom in order to gain a more in-depth understanding of how real-life contexts can influence deliberation processes in decision-making. Likewise, the results derived from the application of the instrument could contribute to discussions on the complementarities and tensions between intuition and reason in decision-making processes (e.g., Lappas et al., 2023). The latter is especially important in light of the great need to incorporate decision-making skills in the school curriculum. From this perspective, the instrument can also be utilized as a support tool for teacher training on these topics and offer concrete input to processes of evaluation, development, and adjustment of the school curriculum.

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