



## Developing Creativity of Schoolchildren through the Course “Developmental Mathematics”

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### ABSTRACT

The relevance of the present study is due to the importance of developing creativity which can be achieved through a variety of school subjects including mathematics. In the article the potential of extended (supplementary) mathematical education (in primary and secondary schools) is highlighted. The main objective of this study is to examine and evaluate the contents, practices and methods that are currently employed in extended education. The main empirical method of this study is modeling of the modular system of lessons (the course) that offers a variety of assignments including non-standard tasks, puzzles and problems; tasks and topics from academic Olympiads and other mathematical competitions; creative tasks, practical assignments and experiments with mathematical materials ("empirical" mathematics); team and individual competitions and organization of home readings on a specific subject. The article describes the author's methodology. The main feature of the developed course is the inclusion of various organizational forms and diverse materials aimed at sustaining schoolchildren's interest towards mathematics, enabling them to deal with advanced level mathematical problems and developing their curiosity and creativity. The course "Developmental Mathematics" that is presented in this article has been empirically tested since 2008.

**Keywords:** extended mathematical education; continuous creativity development; organization of extended mathematical education; primary and secondary schoolchildren.

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### **State of the literature**

- Introduction of new generation standards in the system of general education changes educational paradigm: there is a transition from the overwhelming substantial line to development of pupil's identity by means of a subject. Nowadays this transition is poorly reflected in methodological researches on mathematical education.
- The literature on theory and technique of training mathematics poorly investigated the questions of continuous transition from the initial education to secondary education, in particular in additional mathematical education.
- The vast majority of researches on additional mathematical education is directed on work with mathematically gifted schoolchildren, who are interested in the subject. It practically excludes conversation on mass development of learners by means of additional mathematical education.

### **Contribution of this paper to the literature**

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## INTRODUCTION

Among all the objectives determined by the Concept of Development of Mathematical Education in the Russian Federation (CDMERF) the modernization of educational programs and their contents is seen as a priority along with enabling teachers to design and implement their own modules, courses and programs aimed at promoting mathematical thinking (CDMERF, 2013). This is necessary to facilitate the formation of schoolchildren's identity at different levels (at the subject, personal, and meta-subject levels) by means of mathematics. Modern educational scholarship acknowledges the importance of creating specific conditions to help schoolchildren become active learners – autonomous, creative, ambitious people capable of identifying and solving different problems in both academic and every day contexts (Leikin & Pitta-Pantazi, 2013; Sriraman et al., 2014). According to a number of scholars (Gabdrakhmanova et al., 2015; Valeeva & Shakirova, 2015; Parfilova & Kalimullin, 2013) all this promotes further successful adaptation and development of high school graduates when they enter university.

For the most part this can be effectively achieved through designing and implementing a complex methodological strategy that encompasses strong general (compulsory) mathematic education and the system of extended education that involves supplementary lessons and workshops, mathematical competitions, school mathematical (summer and winter) camps (Gorev, 2012; Masalimova, Usak & Shaidullina, 2016; Valeeva & Kalimullin, 2014). The compulsory and supplementary components need to complement each other; they need to have a unified system of objectives set towards encouraging schoolchildren to develop mathematical curiosity and passion for the subject, creative and independent thinking and to broaden their mathematical outlook. These objectives underline the improvement of school mathematical education (Kennedy & Smolinsky, 2016; Kubiato, Usak & Masalimova, 2016).

However, it is not the purpose of this study to address all of the aforementioned strategic components. We aim to focus only on the main component – the development of a special educational course aimed at strengthening both general and specialized mathematical education of schoolchildren. Such special courses are run at institutions of both compulsory and extended mathematical education and most of these courses are aimed at the 8-11<sup>th</sup> grade schoolchildren. Hardly any of them involve primary school children. However, in order for one to become a creative person one's creative thinking needs to be purposefully and continuously developed (on all educational levels) (Zinovkina, 2008). Thus there are some gaps in school education that inhibit the process of continuous creativity development (Tabach & Friedlander, 2013). The purpose of this study is to attempt to bridge one of the gap by introducing the course "Developmental Mathematics" designed for the 3-6<sup>th</sup> grades.

Moreover, when courses of such kind are being designed the greatest attention is given to developing their contents but less attention is paid to their organization – teaching methods and practices hardly ever vary since these courses are usually developed with a specific group of schoolchildren in mind – those who are highly motivated and have solid basic preparation. In other words educators care more about making their courses intellectually challenging and less about making them engaging. This in essence goes against the main objectives of mathematical education indicated above as it doesn't allow equal opportunities for all schoolchildren to fully develop (Sriraman et al., 2011). The methodology of the course "Developmental Mathematics", which is given in this article, aims to create equal opportunities for all schoolchildren to develop a genuine interest towards mathematics and to facilitate their creative development regardless of their basic preparation and motivation level.

## MATERIAL AND METHODS

### **Research methods**

The following methods have been used in our research: the analysis of psycho-pedagogical and mathematical-methodological literature, analysis and generalization of materials derived from math teachers sharing their experience and from reflecting upon our own

experience, evaluation of educational outcomes, method of mental experiment, prediction, systematization and generalization of facts and concepts, modeling, method of expert evaluation, design of educational methodological materials for extended mathematical education, diagnostic methods, pedagogical experiment.

### **Experimental process**

Generalization and approbation of the results were carried through:

- teaching the course "Developmental Mathematics" to primary and secondary school schoolchildren (3–6 grades) using the materials developed by the author (Gorev & Utemov, 2014). The course has been taught since 2008 at the Lyceum No. 21 – a general education institution in Kirov (with more than 200 schoolchildren getting involved every year) and since 2012 at other educational institutions of Kirov and the Kirov region and a number of schools in other Russian regions (involving around 1000 schoolchildren annually):

- *teaching an advanced distance course "The theory and methodology of extended mathematical education for schoolchildren under the Federal State Educational Standards and the Professional Teacher Standard" (108 academic hours). 229 teachers (secondary mathematics teachers and primary school teachers) have completed the course by attending the Interregional Center of Educational Innovations (Kirov) since December, 2012;*

- *presenting findings (from different stages) and giving speeches at various academic conferences and seminars (including international events), publishing academic articles in peer-reviewed journals.*

### **Research Stages**

The research has been conducted in three stages

- The first stage involved: analyzing the state of the art in theory and practice of teaching mathematical subjects within the system of extended education; studying and evaluating relevant psycho-pedagogical and methodological mathematical literature, observing teachers and analyzing their experiences in order to continuously improve mathematical education.
- At the second stage we developed various methodological approaches to ensure that our modular course "The Developmental Mathematics" could be effectively implemented. We selected materials and used them for designing ten schoolchildren's activity books and one teachers' guide book (Gorev & Utemov, 2014). The course model has been discussed and systematically improved through collecting feedback from teachers (primary school teachers and secondary school mathematics teachers) who participated in our distance course and through presenting reports at academic conferences and seminars.
- The third stage has begun at the same time as the second one. We together with other teachers have been teaching and systematically improving the course "Developmental Mathematics".

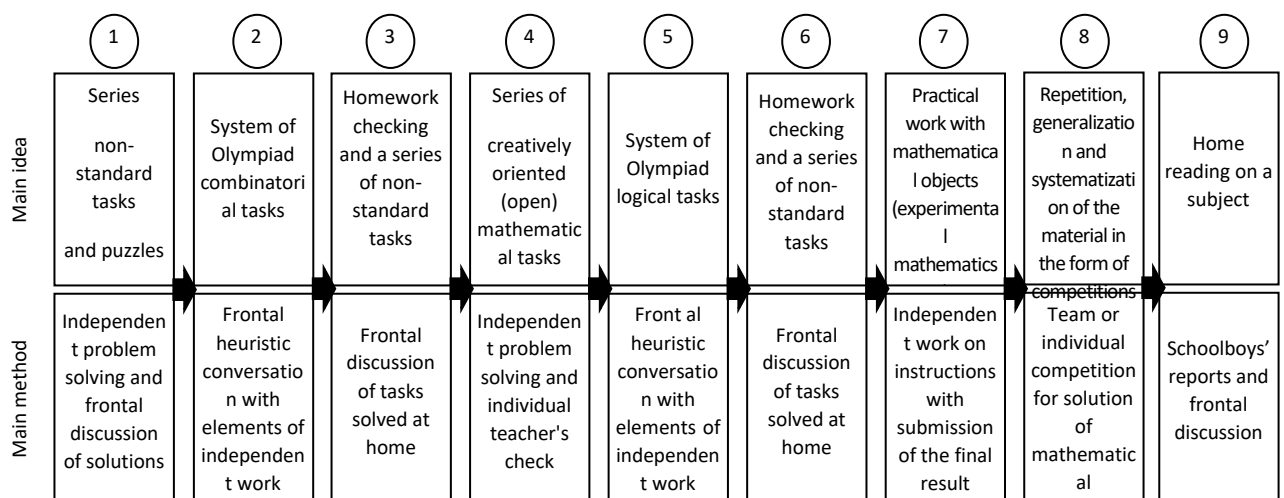
## RESULTS

Below we describe the structure of the developed course model, the organizational forms of the course and other important aspects of the course “Developmental Mathematics”. The course has been carried out once a week in accordance with the new generation educational standards. When working on the course structure we were keeping in mind the recommendations given in the book "Leningrad Mathematical Clubs" (Genkin & Itenberg & Fomin, 1994):

- When working with primary school children it is counterproductive to teach them the same topic over a long period of time; it is recommended to offer a range of activities even within one lesson;
- It is important to provide schoolchildren with opportunities to revise; some of the topics can be reviewed through working on sample tasks taken from mathematical Olympiads and other academic competitions;
- It is important to use non-standard and "sport-like" forms of teaching.

Having considered these recommendations we developed our course "Developmental Mathematics". The course has modular structure; each module contains various organization forms that involve working on non-standard tasks that help creative and independent thinking (practicum, experiments, competitions, group and individual projects, independent reading on a subject). The course attempts to help schoolchildren develop an interest towards mathematics and achieve personal fulfillment.

The course consists of 16 modules – four modules for 3, 4, 5, 6 grades. Each module has the same structure which is presented in **Figure 1**.



**Figure 1.** The structure of each module of the course “Developmental Mathematics”

Here is an example how the structure is used in the module 5.1 (the first module for the 5<sup>th</sup> grade schoolchildren):

- *Series of non-standard tasks (Puzzle "A many-faced balloon")*
- *System of combinatorial Olympiad tasks on the topic "Let's Try All Options"*
- *Homework checking and a series of non-standard tasks*
- *Series of creative (open) mathematical problems*
- *System of Olympiad logic problems on the topic "Tabular Logic"*
- *Homework checking and a series of non-standard tasks*
- *Practical work "Designing using identical shapes"*
- *Revision, generalization and systematization through the competition "Mathematical Merry-go-round"*
- *Home reading "Compute like a computer"*

Three out of nine lessons in this module use sets of non-standard tasks that are specifically designed to maximize children's creativity development. Each set consists of six tasks among which are arithmetic, combinatorial, logic and geometry problems. Tasks are selected from the popular books of problems of non-standard mathematical problems (Galkin, 1996; Gorbachev, 2004; Kozlova, 2006; Spivak, 2004; Sharygin, 2002; Yaschenko, 2009). A selection of these tasks is presented in our book (Gorev & Utemov, 2014).

**The first** lesson of each module begins with this set of tasks. The majority of them can be solved mentally and presented orally. Answers are written in a special printed notebook designed to speed up task processing. The main objective in this is to develop higher thinking of schoolchildren, to introduce them to mathematical creative thinking without putting too much stress on formalities (such as writing down tasks in a proper form). During a lesson the emphasis is placed on the question "Why have you solved the task this way?" (Karagöz-Akar, 2016). The following set of tasks is an example of what can be offered to schoolchildren.

- The father is 4 times older than his son. In 20 years time the father will be 2 times older than his son. How old is the father?
- The teacher gave a difficult problem at a lesson. The number of boys who solved it was equal to the number of girls, who could not solve it. Which is greater: the number of schoolchildren in the class who solved the problem or the number of girls in the class?
- How many zeros are there in the number that comes from multiplying all numbers from 1 to 50?

- Vintik and Shpuntik (*Russian folklore characters*) made a parallelepiped by putting two identical cubes together. The surface area of the parallelepiped is equal to  $90 \text{ cm}^2$ . What is the volume of this parallelepiped?
- A horse can eat a haystack in two days, a cow – in three days, a sheep – in 6 days. How long would it take if a horse, a cow and a sheep eat the same haystack together?
- One day three hunters met and decided to cook porridge together. The first hunter provided two mugs of grain, the second hunter provided one mug of grain but the third hunter had no grain. Instead he offered 5 bullets as a payment. They all ate equal portions of porridge. How should the bullets be divided?

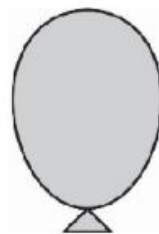
Afterwards children are presented with a puzzle (Barr, 1987; Mochalov, 1996; Hardy, 1998; Gardner, 1999 and al.). They can either receive a ready-made puzzle or be asked to make their own combinatorial or topological puzzles. Jerome Seymour Bruner, an American psychologist and educator, a distinguished researcher in the field of cognitive processes argued that puzzles can be a very powerful creativity developing tool (Bruner, 1977). The following puzzle "A many-faced balloon" was first published in our teacher guide book (Gorev and Utemov, 2015).

*Task 1.* Using 11 pieces of the puzzle (**Figure 2**) try putting together a balloon (as shown in **Figure 3**). All the pieces must be used; they mustn't overlap and no empty (not filled with the pieces) places are allowed.

*Task 2.* "A many-faced balloon" is a fascinating game. We can put together a variety of amazing shapes using the pieces of the puzzle. Try playing around with it.



**Figure 2.** Puzzle pieces



**Figure 3.** The result of the puzzle

In **the second** lesson children are introduced to the exhaustive search method as one of the key methods for solving mathematical problems. It is important to discuss all classroom tasks with schoolchildren showing them how to search for possible solutions. It is recommended to provide 1 or 2 tasks for homework. For example, the following set of tasks can be offered:

- There are cookies, jam, candies, halvah and chocolate in the buffet. The boy wants to buy three different things. What sets of sweets can he get? Write down all the options.

- Three knights – Ilya Muromets, Dobrynya Nikitich and Alyosha Popovic (*Russian folklore characters*) to protect their homeland from invasion had to cut off 13 heads of Zmey Gorynych (*many-headed evil character*). The greatest number of the heads was cut off by Ilya Muromets and the smallest number – by Alyosha Popovic. How many heads were cut off by each of the three knights?
- How to divide a square into as many parts as possible by using three straight lines?
- One day Wise Man, Sly Fellow and Liar met. It is known that the Wise Man always tells the truth, Liar always lies, and Sly Fellow tells the truth whenever he is told the truth, and lies whenever he is lied to, in cases when he has to be the first to speak, he lies. They all had a conversation: One of them said to another: "You are Sly Fellow" and that person replied: "This is a lie, you are Sly Fellow". – The third person objected: "Both of you lie, I am Sly Fellow!" Can you tell who is Wise Man, Sly Fellow and Liar?
- Twelve people are carrying 12 loafs of bread. Every man is carrying two loafs, every woman – a half of a loaf, and every child – a quarter of a loaf. All 12 people are involved in this. How many men, women and children are there?
- 30 students from five courses took part in compiling 40 tasks. All the students came up with the same number of tasks as their fellow students (from the same course) and at the same time all the students came up with a different number of tasks as the students from other courses. How many people came up with only one task?
- A tetramino is a polygon which is made of 4 equal squares. A pentamino is made of 5 squares. How many various a) tetraminos; b) pentaminos can you draw?
- A little squirrel collected 21 nuts and divided them into small groups so that the number of nuts in each group is consecutive to the number of nuts in the next group. Please, write down all the possible solutions.
- Over five years a student takes 31 examinations. Every year students have to take more exams than in the previous year. In the fifth (final) year students take three times more examinations than in the first year. How many examinations do students take in the first year?
- List all possible sets of four natural numbers that give 15 in summation.

On similar lessons of other modules of the course "Developmental Mathematics" schoolboys are offered to study the following subjects: "Transfusions", "Effect plus or minus one", "Reverse analysis", "Combinations counting rules", "evenness", "Dirichlet's Principle". "Rule of the final number", "Estimation + example", "Invariants and coloring", "Cuttings", "Games", "Geometry".



**The third** lesson is devoted to checking homework. After that 1 or 2 tasks that require the exhaustive search method are given to children. If there is still enough time the lesson can be finished with a new portion of non-standard tasks.

**The fourth** lesson aims to popularize scientific work. This lesson is devoted to open and partially-open problems (Bahar & Maker, 2015; Gin & Barkan, 2014). None of the problems offered in this lesson have the right answer (Gorev & Utemov, 2011). These problems can have several answers that satisfy all the conditions. These problems are collectively discussed in the course of the lesson (a brainstorming session). More problems of this type are given to be solved individually at home. One of them is recommended to be solved within the following week.

Lessons aimed at popularizing scientific work are the most unpredictable type of lessons in our course. Not only do they present mathematical problems but they also introduce the main ideas and methods of scientific work and creative thinking (for example, the Theory of Inventive Problem Solving (TRIZ)). Schoolchildren learn how to use: the trial and error method, the morphological analysis method, the reverse method, the method of resolving contradictions, the concept of an ideal answer, the method of systematic thinking, etc. The teaching methodology developed by us along with sample problems and tasks adapted for young age can be found in our courses published as: "Flying to creativity horizons" (Gorev & Utemov, 2012), "Magic dreams of the Small Owl" (Gorev & Utemov, 2012), "Traveling to the country of creativity" (Gorev & Utemov, 2013), "Entering the world of creativity" (Gorev & Utemov, 2013), "Creative walks under the stars" (Gorev & Utemov, 2014), "The fascinating voyage of the Small Owl" (Gorev & Utemov, 2015), "Significant events in the life of the Small Owl" (Gorev & Utemov, 2016). All in all there are 14 courses published to date.

Below we provide a number of examples of partially open mathematical tasks. All the tasks were tested with 8-12 years old schoolchildren in the course of academic Olympiads for younger schoolchildren "Small Owl" in 2008-2016.

- *Cards Exercises.* Using all the five cards (provided below), make exercises that give a) 20; b) 14. What other examples can be made using these cards?



- *A fresh look at numbers.* Look at the images of numbers below. The number 1 has one angle, the number 2 - two angles and the number 3 - three angles. Think of how to write/draw numbers 4, 8 and 0 using the same principle. Come up with your own way(s) of writing/drawing numbers and explain it (them).

1 2 3

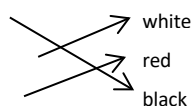
- *Time and laces.* Imagine you have two fast-burning cords and they are different in length. Each of them burns out precisely in one hour but they burn unevenly: some parts of these cords burn faster than others. Using only these two uneven cords and a lighter please time when exactly 45 minutes will pass.
- *Measuring liquids in meters.* We are used to measuring length in meters, weight – in kilograms, volume – in liters. But maybe it would be more convenient to measure volume in meters and length in kilograms? Think of ways to measure volume of liquids in meters and length in kilograms.
- *Scales out of the kettle.* The Small Owl and his friends went hiking. They stopped near a lake and decided to pick up berries. Is it possible to measure the weight of picked-up berries? Friends only have a kettle with them and a piece of soap in its original package. Think of a way to weigh the picked-up berries using available means.

In **the fifth** lesson ("Tabular Logic") children get introduced to a specific type of logic problems. In this lesson it is important to demonstrate how to search for multiple solutions highlighting at the same time how to solve problems using tables. The following task is offered to children:

*Task.* Three girlfriends Ira White, Nadia Red and Olya Black have met today. One of them is wearing a black dress, another is wearing a red dress and the third girl is wearing a white dress. The girl in white told Olya Black: "We should change our dresses because the colors of our clothes do not correspond to our surnames". Who is wearing which dress?

*The first way to solve this task (reasoning):* From the conversation it is clear that Ira White isn't wearing white, Olya Black isn't wearing black and Nadia Red isn't wearing red. Thus Olya Black is wearing either white or red. But the girl in white speaks to Olya Black about their dresses therefore Olya Black isn't wearing white. It means that she is wearing red. From this follows that Ira White is in black and Nadia Red is in white.

*The second way to solve this task (graphic):* Let's write down the surnames of the girls and the colors of their dresses. As Olya Black can be neither in black nor in white, she is the one wearing red. To indicate this let's put an arrow (shown below). Ira White cannot be in white, therefore she is in black. Let's put another arrow. It is obvious now that Nadia Red is wearing white.



*The third way to solve this task (creating a table):* As you have already noticed we could not tell the colors of the girls' dresses straight away. First, we had to think of colors they were not wearing. This can be easily indicated in a table.

**Table 1.** The initial view of the table before we solve the problem

Problems' types	white	red	black
Ira White	-		
Nadya Red		-	
Olya Black			-

Ira White is not in white, Olya Black is not in black and Nadya Red is not in red. We have crossed out the three sections in the table. The girl in white speaks to Olya Black which means that Olya Black is not in white – we are crossing out the first box in the third line. Now look at the first column of the table. Only the second box is empty. It means that Nadya Red is wearing white. Let's put "+" here or some other symbol to distinguish this box from others. Now let's look at the third line of the table. The second box is empty here. Let's put "+" here as well.

**Table 2.** Working with the table to solve the problem

Problems' types	white	red	black
Ira White	-		
Nadya Red	+	-	
Olya Black	-	+	-

Now we know that Olya Black is in red. Therefore, Ira White is in black. Let's put "+" in the third box of the first line.

*Answer.* Ira White is in black, Nadya Red is in white and Olya Black is in red.

The main goal of this lesson is to teach schoolchildren how to make logical connections when dealing with their tasks – this helps children's mathematical development as well as boosts their creativity. 1 or 2 tasks of this type are given as homework.

**The sixth** lesson begins with checking homework and then more complex tasks are considered. The lesson is ended with a series of non-standard tasks.

Each module covers two main topics (one combinatorial and one related to logics). Topics are selected purposefully to encourage the development of schoolchildren in two fundamentally important directions of mathematical thinking (combinatorial and logic) – this boosts creativity development (Antonijević, 2016).

**The seventh** lesson is practical – children learn how to independently construct real objects using written instructions. Here we offer an example “Constructing with T” from the book “Visual Geometry” (Sharygin and Erganzhiyeva, 2001).

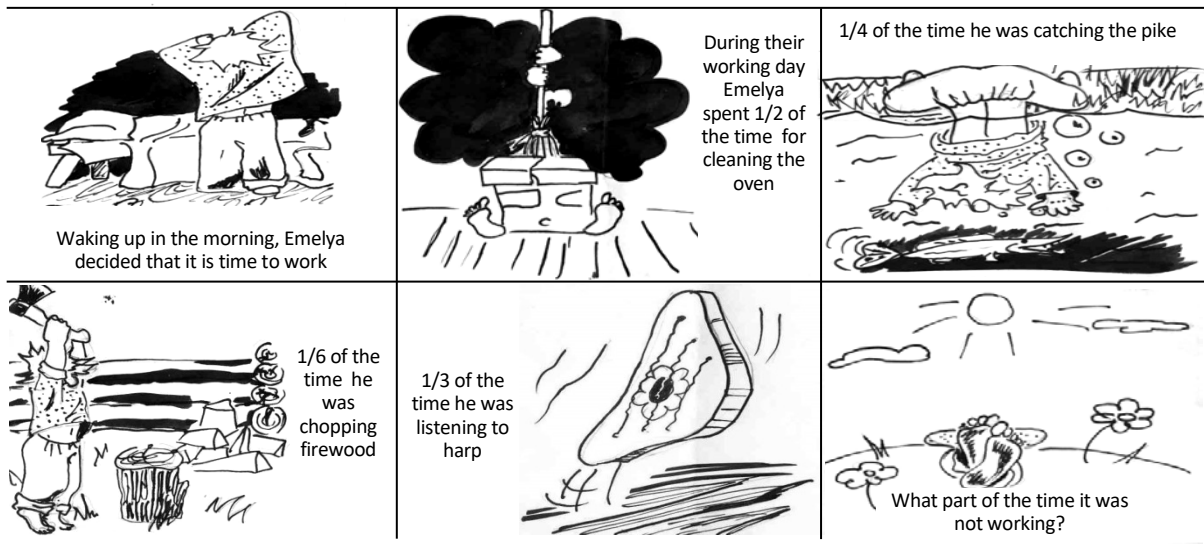
We classify such kind of lessons as lessons of so-called empirical mathematics – they are based on the activity-based approach. This approach corresponds directly to the approach of systematic activities. In our course both arithmetic and geometrical materials can be used. Both arithmetic and geometrical tasks require schoolchildren to formulate a hypothesis that comes from using incomplete induction of exhaustive search. In additions it is appropriate to let children produce their own puzzles and tasks (combinatory or topological in nature) at this stage.

**The eighth** lesson is a game-competition in which children can participate either individually or in a team. Through taking part in this game children revise in a playful way and develop their interest towards mathematics further.

The purpose of this lesson is to introduce children to competitive forms of activities – this is necessary to encourage children’s engagement with mathematics and boost their ambitions. It is better to organize this type of competitions to be relatively brief as young children get tired very quickly, even if they are playing games. We offer such games as "Mathematical brain ring", "Mathematical hockey", "Firefight", "Fishing party", etc.

Each module is ended with home reading which encourages children’s independent work. Materials are to some extent provided by the teacher. But children need to have an opportunity to be in charge of their workload and search for additional challenges independently using the Internet and other recourses.

Along with tasks taken from academic Olympiads children are offered entertaining materials as part of their homework: mathematical jokes, sophisms, ‘deceptive’ tasks, games, rebuses, puzzles, etc. For example, here is an example of a task presented as a comic strip (**figure 4**). Ten of these tasks we offer in the author's textbook "Lessons of the Developmental Mathematics. 5–6 grades: Problems of a mathematical section" (Gorev & Utemov, 2014).



**Figure 4.** The comic strip task

All in all, this modular mathematical course "Developmental Mathematics" takes 136 hours. Children's feedback mostly indicates that they enjoy taking this course. Moreover, our findings show that children who took this course often attain impressive results outside our course: they win various academic competitions (at different levels) and consistently demonstrate progress in mastering the subject.

### DISCUSSIONS

The important role of creativity in mathematics was emphasized by a number of distinguished mathematicians (Arnold, 2002; Singer, Sheffield & Leikin, 2017; Gnedenko, 1979; Courant and Robbins, 1967; Poincare, 1983; Hinchin, 1989). Moreover, such educational methodologists as Balk, 1969; Gusev, 2003; Episheva and Krupich, 1990; Polya, 1991; Shvartsburd, 1964 underline that it is very important to have an opportunity to be creative within mathematics lessons at secondary school and extended education institutions.

The analysis of psycho-pedagogical and methodological mathematical literature as well as our qualitative findings (derived from working with other mathematics teachers) show that creative growth of children is extremely important for their mathematical development. Extended mathematical education that aims to boost children's creativity also helps them become independent learners who are proactive and responsible, think outside the box and work effectively, develop solid problem-solving skills and an ability to be flexible and responsive to changing circumstances (Akkaya, 2016; Pino-Fan and al., 2015).

However, the theory and methodology of teaching mathematics haven't yet considered holistic methodological concepts that realize approaches of introducing schoolchildren to creative thinking in the context of extended mathematical education.

For example, in accordance with traditions only most capable and passionate schoolchildren are invited to the lessons of extended mathematical education (mathematical clubs). This fact defines the format of organization in such clubs. Balk (1956) gave his own example of organizing mathematical clubs by involving schoolchildren in presenting and discussing different ideas. Petrakov (1987) offered the following structure of a mathematical club: 1) one participant gives a 5–10 minute presentation about the history of mathematics (one particular stage or a breakthrough is considered); 2), the leader of the club (teacher) or another participant (schoolchild) delivers a new topic; 3) they all work on advanced level tasks; 4) they consider entertaining tasks; 5) they move on to problems and tasks that are approached in institutions of higher education (university level tasks); 6) in the end there is a Q&A session. Farkov (2008) has very similar views. Such forms of organization are appropriate for high school students who are highly motivated. Following the recommendations from the book "Leningrad Mathematical Clubs" (Genkin, Itenberg & Fomin, 1994) we have begun organizing extracurricular activities aimed specifically at schoolboys of younger age. Another factor that influenced our approach to organization was our intention to engage all schoolchildren without any selection. We organize our classes once a week. These classes aim to build a link between basic (compulsory) and extended (in traditional understanding) education of schoolchildren. In view of this the organization of these classes is very similar to the lessons of basic education but still it is different as we pursue different objectives.

## CONCLUSION

The course "Developmental Mathematics" for primary and secondary school children was first designed through considering different perspectives on the problem of continuous creativity development. The implementation of our course enabled us to reflect and improve our materials. As a result we developed our modular system of lessons that use a range of organizational forms and diverse materials. The outcomes of schoolchildren (they often win academic competitions at different levels) confirm that our course contributes to their success and helps them sustain their interest towards mathematics. All in all, the described system has been created to develop continuous extended mathematical education (specifically for 3–6 graders) in order to ensure children's smooth transition from primary to secondary school and to help them sustain their interest towards mathematics and to become creative and capable of thinking outside the box.

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