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Diagnostic thinking of in-service teachers in inclusive mathematics education: A video vignette-based study

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Abstract

Diagnostic thinking is understood as a central component of teachers' diagnostic competence, encompassing processes of perception and interpretation that shape instructional decision-making. This study investigates primary and special education teachers' diagnostic thinking when perceiving and interpreting students' solution processes in real-world problems in inclusive classroom settings. Data were collected through semi-structured interviews using video vignettes of students working on such problems. The findings reveal both shared and distinct patterns in diagnostic thinking. While both groups attend to number range and operational understanding, primary education teachers show a more differentiated diagnostic repertoire. Special education teachers focus more strongly on foundational competences, reflecting their professional emphasis on basic learning prerequisites. These findings may indicate the importance of mathematics-specific pedagogical content knowledge in shaping diagnostic thinking and suggest that professional background influences how teachers perceive and interpret students' mathematical thinking.

Keywords: diagnostic competence, diagnostic thinking, inclusive education, video vignettes

INTRODUCTION

Teachers' diagnostic competence is a central component of teacher professionalism, shaping how teachers perceive classroom situations, interpret students' learning processes, and make instructional decisions (Blömeke et al., 2015). These processes can be described as teachers' diagnostic thinking (Loibl et al., 2020). In mathematics education, diagnostic thinking is essential because effective teaching depends not only on recognizing correctness but also understanding students' thinking, difficulties, and appropriate forms of support. This enables differentiated instruction that meets students' individual learning needs (Leuders et al., 2018). These processes are closely tied to specific mathematical content domains.

Students' mathematical thinking becomes particularly visible in real-world problem solving. When translating contextual situations into mathematical representations (Blum & Leiß, 2007), students reveal how they connect ideas, interpret information, and select strategies. Such tasks require both subject-specific and interdisciplinary skills (e.g., reading skills or measurement skills) (e.g., Blum et al., 2007; Geisen, 2021) and pose substantial challenges for students and teachers alike. In inclusive classrooms, these challenges are amplified by heterogeneous student populations, including learners with diverse needs and abilities (Booth & Ainscow, 2011). Despite their curricular relevance, real-world problems remain underrepresented in classroom practice due to their complexity (Blum & Borromeo Ferri, 2009; Blum et al., 2007; Cabassut & Ferrando, 2017).

In Germany, these challenges are embedded in inclusive settings characterized by collaboration between primary and special education teachers from distinct educational pathways, traditionally preparing them for different school types and student populations. Primary education teachers typically have more extensive training in mathematics and mathematics didactics, whereas special education teachers often focus on individual learning needs and support (Geisen, 2021, 2023). This diversity is compounded by widespread out-of-field teaching (Bosse & Törner, 2013), resulting in a

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Contribution to the literature

- This study explores diagnostic thinking as a key component of diagnostic competence in a specific mathematical context by analyzing which content-related aspects primary and special education teachers perceive, how they interpret them, and which remain unconsidered in inclusive mathematics classrooms.
- Findings indicate that teachers differ in the scope and depth of their interpretations, and systematically vary in how strongly different mathematical domains (e.g., measurement) are taken into account, suggesting domain- and profession-specific patterns of diagnostic thinking.
- This study contributes methodologically by illustrating the iterative development process related to the authentic, realistic video vignettes that were used as prompts to reveal teachers' diagnostic thinking.

range of expertise, from content knowledge to diagnostic and pedagogical skills (Knigge & Kollosche, 2019). These differences may influence how teachers interpret students' solution processes in complex situations such as real-world problems.

However, little is known about how teachers with different professional backgrounds perceive and interpret students' solution processes in such contexts. Addressing this gap, the present study investigates differences between primary and special education teachers' diagnostic thinking in the context of real-world problems, aiming to better understand the knowledge and expertise underlying their interpretations.

FROM THEORETICAL AND EMPIRICAL PERSPECTIVES TO THE RESEARCH QUESTION

Diagnostic Thinking as a Core Component of Teachers' Diagnostic Competence

Competence is defined as an individual's available or acquired cognitive abilities and skills for solving problems, along with the motivational and affective dispositions needed to apply these solutions successfully and responsibly across a range of situations (Weinert, 2001, pp. 27-28; see also Blömeke et al., 2015). This definition highlights the contextual and domain-specific nature of competences (Koeppen et al., 2008).

In line with this understanding, Blömeke et al. (2015) conceptualize competence along a horizontal continuum, linking dispositions (e. g. competences and beliefs), situation-specific skills such as perception, interpretation, and decision-making, and observable performance. Teachers' dispositions include subject matter knowledge (SMK), which refers to expertise in mathematics itself (e.g., concepts, procedures, structures), pedagogical content knowledge (PCK), which involves knowledge of how to teach mathematical content (e.g., typical student errors, didactic strategies, representations) (Ball et al., 2008; Selling et al., 2016), and beliefs regarding the structure of mathematics and the acquisition of mathematical knowledge (Grigutsch et al., 1998; Viholainen et al., 2014). These dispositions shape teachers' situation-specific skills, including perception, interpretation, and decision-making (Goodwin, 1994; Sherin, 2007; Sherin et al., 2011), which ultimately guide their actions in classroom situations. This perspective overcomes the dichotomy between disposition and performance by emphasizing the connecting processes (Kaiser et al., 2017). Competence also unfolds along a vertical continuum of performance levels and developmental stages, reflecting that individuals may show higher proficiency in some areas while being less developed in others (Blömeke et al., 2015, p. 7).

Building on this competence perspective, diagnostic competence has been specified within the DiaCoM framework (Loibl et al., 2020), a process-oriented model that describes how it becomes visible in teachers' judgments and decisions. Originally introduced by Weinert et al. (1990), the concept of diagnostic competence is largely used synonymously with assessment competence in German educational research. The framework highlights how instructional actions emerge from the interaction of situation characteristics, personal characteristics, diagnostic thinking, and diagnostic behavior (Loibl et al., 2020). For the purposes of this study, which examines teachers' perception and interpretation when students solve real-world problems, teachers' diagnostic thinking is taken as the analytical focus. As the analysis of teachers' diagnostic thinking is intended to provide insights into their cognitive dispositions, partial aspects of the component of personal characteristics are also central. The central components of the DiaCoM model are outlined below to clarify diagnostic thinking and its relation to teachers' dispositions.

Situation characteristics refer to contextual features that define a diagnostic situation, such as task characteristics, student responses, and contextual cues. They constitute the main source of information for diagnosis and shape the cognitive demands placed on teachers. Personal characteristics comprise both stable individual dispositions and situationally variable factors that influence diagnosis (Loibl. et al., 2020). This includes domain-specific knowledge (e.g., PCK or SMK), motivational orientations, beliefs, and self-regulatory capacities (see also Geisen, 2021; Hoth et al., 2016).

Within the DiaCoM model, diagnostic thinking includes processes such as perceiving, interpreting, and decision-making. Diagnostic thinking is conceptualized as an information-processing sequence shaped by both personal and situation characteristics and refers to the internal cognitive processes underlying teachers' interpretations of diagnostic situations, whereas diagnostic behavior denotes the observable outcomes, such as actions to support learning.

This framework provides the basis for investigating how teachers perceive and interpret students' processes when solving real-world problems, offering a valuable lens for analyzing their cognitive processes.

Research on Teachers' Diagnostic Competence in Mathematics

The research on teachers' diagnostic competence is a broad field. It has evolved from a primary focus on measuring diagnostic accuracy toward a more process-oriented perspective (Sommerhoff et al., 2022), reflecting an expansion from a narrow to a broader conceptual understanding of diagnostic competence (Herppich et al., 2018). While diagnostic accuracy refers to the extent to which teachers' judgments correspond to a reference standard (Südkamp et al., 2012; Urhahne & Wijnia, 2021), this broader perspective emphasizes the cognitive processes underlying diagnostic activity. These processes involve perceiving and interpreting relevant classroom situations as well as decision-making, which ultimately influences instructional actions (Loibl et al., 2020). Teachers incorporate their perceptions and interpretations into their decision-making (Dröse & Prediger, 2023; Sherin et al., 2011).

On the one hand, diagnostic practice involves domain-general components (Blömeke et al., 2015; Sherin et al., 2011). On the other hand, it is strongly domain-specific, as its development is closely tied to particular content areas (Hoppe et al., 2020; Prediger, 2010). Given a content-dependent, subject-specific understanding, there is a need for studies that explicitly examine the role of mathematical content in diagnostic processes. Previous studies in mathematics education often implicitly work with mathematical content but do not explicitly integrate it into their analyses (Stahnke et al., 2016).

Accordingly, further research should investigate diagnostic competence in specific mathematical contexts and examine which content-related aspects teachers notice, how they interpret them, and which aspects remain unconsidered (Dröse & Prediger, 2023), in order to further develop teacher education with regard to a process orientation and a stronger content focus. These observations directly motivate the present study's focus on real-world problems as a domain-specific context for investigating diagnostic thinking of primary and special education teachers.

Conclusion and Research Question

Given the need to conceptualize diagnostic thinking as embedded in content-specific contexts and to move beyond implicit treatments of mathematical content in research, real-world problems represent particularly complex instructional contexts in which students' mathematical thinking becomes visible through their solution processes.

In inclusive primary classrooms in Germany, differences in teacher education between primary and special education teachers (Geisen, 2021, 2023; Knigge & Kollosche, 2019), as well as the prevalence of out-of-field teaching (Bosse & Törner, 2013), may shape how teachers attend to and interpret students' mathematical thinking. Against this background, it remains an open question how teachers with different professional backgrounds (e.g., PCK; Ball et al., 2008; Selling et al., 2016) engage with content-specific diagnostic demands in such complex situations.

Grounded in the DiaCoM framework (Loibl et al., 2020), this study examines primary and special education teachers' diagnostic thinking by focusing on how they perceive and interpret students' difficulties in authentic, partner-based real-world problem-solving situations (see **Figure 1**).

This perspective is particularly relevant for teacher education, as it provides insights into how diagnostic competence can be fostered in inclusive mathematics classrooms.

The following research question guide the study: How do primary and special education teachers differ in their perceptions and interpretations of students' difficulties when solving real-world problems in inclusive settings?

METHODS OF INVESTIGATING TEACHERS' DIAGNOSTIC THINKING

Study Design

This study examined differences between primary and special education teachers by focusing on their diagnostic thinking in situations where students solve real-world problems, which serve as both the content and situational framework. Data were collected through qualitative, guided, semi-structured video-cued interviews (Blömeke, 2013, pp. 34-35), using video vignettes as context-sensitive prompts, and treating teachers as experts in their professional practice. This approach provides insight into the cognitive and affective-motivational processes underlying teachers' decisions (Blömeke et al., 2015). The interviews focused on typical everyday situations in mathematics education and were structured around four components (Geisen, 2021, p. 132-134):

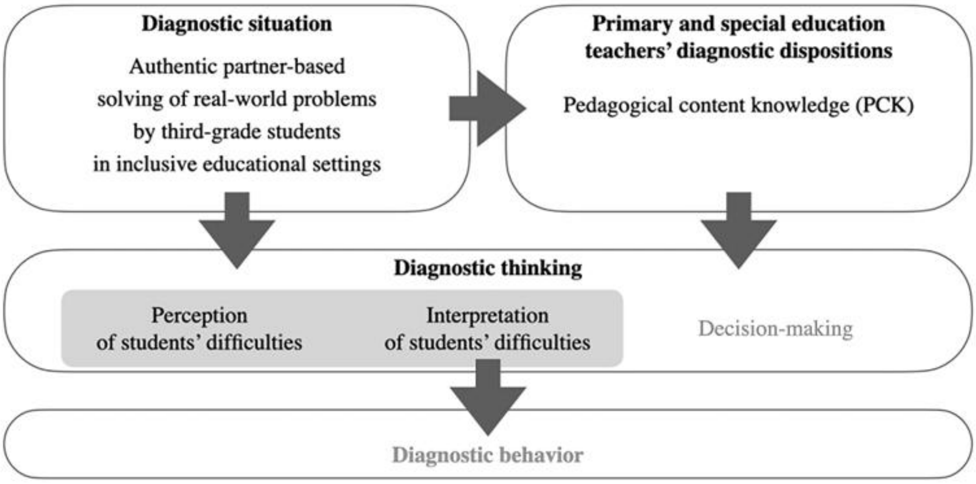


Figure 1. Analytical framework of the study based on the DiaCoM model (Loibl et al., 2020) and its operationalization for analyzing teachers' diagnostic thinking in authentic, partner-based real-world problem-solving situations in inclusive primary classrooms (Source: Author's own elaboration)

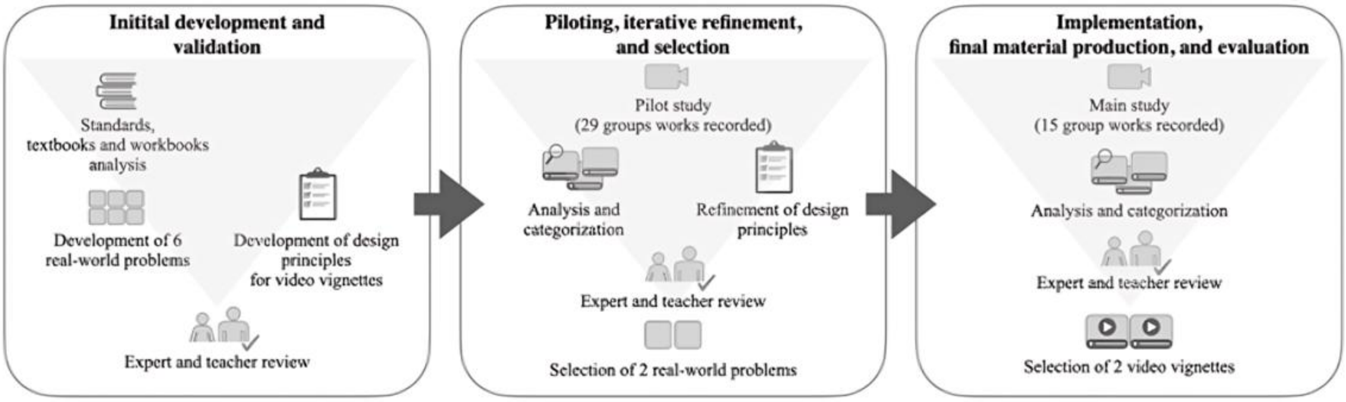


Figure 2. Course of the study with a focus on the development of real-world problems and video vignettes (Source: Author's own elaboration)

- Component 1: Teachers engaged with real-world problems to demonstrate their own approaches.
- Component 2: Teachers analyzed these problems from a subject-didactic perspective.
- Component 3: Teachers observed and analyzed students' work processes via video vignettes and derived diagnostic judgments.
- Component 4: Teachers anticipated interventions to support learning based on their analyses.

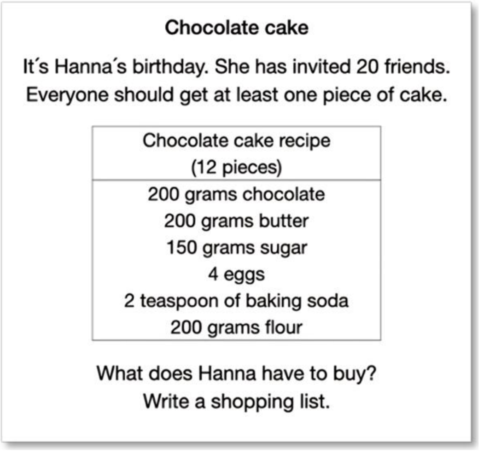
The study focuses on component 3, addressing teachers' diagnostic thinking when observing and interpreting students' solving processes. Although the interview design also included component 4, which elicits teachers' decision-making regarding instructional actions, this aspect is not analyzed in the present study. Two real-world problems with corresponding video vignettes were used, resulting in two parallel interview settings, with each teacher analyzing one problem. Additionally, a questionnaire was administered to collect background information on participants (Geisen, 2021, p. 115).

Materials: Real-World Problems and Video Vignettes

To elicit teachers' perception and interpretation, the study employed real-world problems and video vignettes, which were systematically developed to ensure both didactic quality and alignment with the research aims (see **Figure 2**).

Initial development and validation

Six real-world problems addressing functional relations in authentic situations were generated based on curricula, educational standards, and commonly used textbooks and workbooks for primary and special schools. The problems were reviewed by teachers and experts to ensure their relevance and feasibility. At the same time, design principles for the video vignettes were established, including group composition, group size, teacher interventions, offers of support, and camera positioning.



Chocolate cake

It´s Hanna´s birthday. She has invited 20 friends. Everyone should get at least one piece of cake.

Chocolate cake recipe (12 pieces)
200 grams chocolate 200 grams butter 150 grams sugar 4 eggs 2 teaspoon of baking soda 200 grams flour

What does Hanna have to buy? Write a shopping list.

Figure 3. Chocolate cake problem (based on Geisen, 2021, p. 118)

Piloting, iterative refinement, and selection

The six real-world problems and the initial design principles were piloted with 29 groups of third-grade students. Students worked on the problems, with optional support provided through interventions or materials. Two special education teachers additionally evaluated the problems with regard to their suitability for use in mathematics education. Based on the analysis of the pilot data, anticipated students' difficulties, and expert discussions, two real-world problems were selected. These selections aligned with the special education teachers' evaluations. The design principles for the video vignettes were iteratively refined accordingly.

Implementation, final material production, and evaluation

In the main study, 15 further groups of third-grade students were videotaped while working on the selected real-world problems. The resulting video data were analyzed and categorized according to subject matter and didactical criteria (e.g., types of difficulties, alignment with mathematics education literature). Based on this analysis and expert-teacher consensus, two video vignettes were selected for use in the interviews. These vignettes were chosen to illustrate students' observable, analyzable problem-solving processes and to serve as a basis for eliciting teachers' interpretations.

Illustrative example: Chocolate cake problem

The chocolate cake problem is an open-ended real-world problem designed to stimulate professional discussion (see **Figure 3**). Students are asked to determine ingredient quantities and compile a shopping list, while all quantities are given in grams (g), following common German recipe conventions. Following Blum and Leiß (2007), the modelling process for solving such problems involves

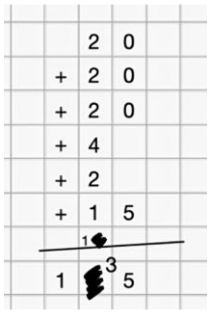


Figure 4. Written calculation of S1 (based on Geisen, 2021, p. 126)

- understanding the problem context (e.g., number of guests, including the host herself),
- structuring and simplifying the situation, considering alternative solution strategies (e.g., portioning or scaling),
- performing calculations (e.g., unit conversion), and
- and interpreting and validating the results (e.g., checking totals and plausibility).

The accompanying video vignette depicted two students, S1 and S2, working on the task. Neither student read the problem statement aloud in full. S1 initially attempted mental calculations but inconsistently applied units, replacing grams with the German currency euros (€), and experienced difficulties with mental addition (e.g., 66 + 15). This prompted a switch to written calculations (see **Figure 4**), where S1 aligned the summands vertically and solved the task step by step, occasionally repeating partial sums and showing hesitation in calculations beyond ten.

Meanwhile, S2 remained engaged and provided occasional support, for instance by repeating intermediate results or guiding S1's calculations. In the end, S1 recorded "three tens and one hundred," resulting in a final value of 135 €. This was interpreted as 135 grams per ingredient, which S1 critically evaluated, noting that 135 grams of sugar would likely be excessive.

Teachers could perceive and interpret several aspects while observing this process. Regarding problem understanding, incomplete reading may lead to misunderstandings of the task, highlighting the relevance of text comprehension (e.g., Blum & Borromeo Ferri, 2009). Inconsistent handling of units may reflect associations with a shopping context, simplification strategies, or a need to support students' conceptual understanding of measurement. Difficulties with mental addition of two-digit numbers, particularly in maintaining intermediate sums in working memory, may also become visible. Challenges in written addition, such as digit alignment, may indicate difficulties with place value understanding or written strategies. Hesitations and errors during ten-transition within the number range up to 20 may further indicate difficulties with basic number composition.

Table 1. Teachers perceived and interpreted arithmetical and measurement aspects with quantities in brackets (based on Geisen, 2021, p. 228)

Domain	Perceived and interpreted difficulties of primary school teachers (n)	Perceived and interpreted difficulties of special education teachers (n)
Arithmetic	Number sense (1) Understanding of number space (6) Understanding of place value (1) Understanding of operations (6) Written strategies concerning calculation (1) Mental strategies concerning calculation (4) Numerical calculation (1)	Understanding of number space (2) Understanding of operations (3)
Measurement	Concept of measurement (5) Understanding of measurement (1) Calculation with measures (1)	Concept of measurement (1)

Sample

The sample comprised 24 participants (22 female and 2 male), including 16 primary school teachers and eight special education teachers, aged 28-60 years. 15 participants had studied mathematics, while nine taught mathematics out-of-field (one primary and eight special education) (Bosse & Törner, 2013; Geisen, 2021, 2023). Mathematics teaching experience ranged from less than 3 to over 25 years. Participants held various professional roles, including 13 classroom teachers, 4 school administrators, and 3 professional development instructors in mathematics. The heterogeneous sample was appropriate for the study's exploratory qualitative approach (Geisen, 2021, p. 137, 2023).

A total of 24 guided, semi-structured interviews were conducted, video-recorded, and transcribed (30-60 minutes each). The interviews were divided into two parallel environments, each corresponding to one real-world problem and its video vignette (12 teachers each: eight primary and four special education). The sample size was deemed sufficient, as no new themes emerged from additional interviews, consistent with data saturation principles (Hennink et al., 2017).

Methods of Data Analysis

A content-structuring qualitative content analysis (Kuckartz, 2019) was used to analyze how teachers perceive and interpret students' difficulties in solving real-world problems. The analysis combined deductive and inductive category development. Categories were developed and validated through interpretative consensus, recognizing that analysts' knowledge and experience shape the construction process (Kuckartz & Rädiker, 2023). To ensure reliability and intersubjective consistency, transcript excerpts were jointly coded and category definitions iteratively refined by experienced colleagues.

The category system was designed to capture multiple facets of teachers' diagnostic thinking (Geisen, 2021), with the present paper focusing specifically on content-related difficulties, as this emphasis emerged from the interview data.

FINDINGS

This section presents differences in primary and special education teachers' perceptions and interpretations of students' emerging content-related difficulties, illustrated through the chocolate cake problem and the corresponding video. The perceived and interpreted difficulties are summarized in **Table 1**, with the numbers in parentheses indicating how often each aspect was mentioned.

Perceived and Interpreted Arithmetical Aspects

Across both groups, difficulties related to understanding number space and operations were most frequently identified. These two aspects emerged as central diagnostic indicators across both groups. A broader range of arithmetical aspects was referred to by primary school teachers, suggesting a more differentiated perception of students' arithmetical difficulties. The most frequently mentioned aspects by them were:

- understanding of number space,
- understanding of operations, and
- mental strategies concerning calculation.

This is illustrated by one primary school teacher:

> "[…] he can't decide, ehm, do I calculate this task in my head, do I calculate it half-written, or do I calculate it in writing […]. He seems to add numbers that just come to mind ehm, it's quite slow and incorrect, so he has to […] he has no solid understanding of numbers or quantities because he reads two hundred and writes down twenty, that is, it would first have to be checked to what extent he has any idea what two hundred is […]." (Quote of a primary school teacher)

This excerpt illustrates how teachers interpret students' difficulties in underlying mathematical concepts such as place value and calculation strategies. In the special education group, the focus was primarily on understanding number space and operations, while other arithmetical aspects were rarely addressed.

Perceived and Interpreted Measurement Aspects

Group differences were also apparent in reflections on measurement (see **Table 1**). Primary school teachers referred to a broader range of measurement-related aspects, particularly students' conceptual understanding of measurement, for example, by noting a lack of understanding of the magnitude of standard units, such as 100 or 1,000 grams. One primary school teacher stated:

> "The size of weight ultimately means that he has an idea of what a hundred grams or a thousand grams are. […]" (Quote of a primary school teacher)

The teacher stresses that solving the task requires not only numerical calculations but also an understanding of the magnitude of standard units.

In addition to unit magnitude, difficulties in students' understanding of measurement processes and calculations with measures were highlighted by primary school teachers:

> "I had the feeling that they hadn't even noticed Euro. They were just numbers." (Quote of a primary school teacher)

> "But he cannot calculate with the weights." (Quote of a primary school teacher)

These statements indicate that teachers attended to both students' linking of numbers to real-world referents and their ability to perform calculations with units. The first quote highlights a lack of contextual understanding, while the second identifies gaps in operational competence with measurement units.

In contrast, special education teachers referred to measurement aspects only rarely, with a single mention of students' conceptual understanding of measurement. Calculation with measures was mentioned infrequently in both groups.

DISCUSSION

A pattern of both shared and distinct interpretations emerges across the two teacher groups. The common focus on number range and operational understanding indicates a shared awareness of these fundamental arithmetic competences. These foundational competences appear to structure teachers' attention as they interpret students' solution processes, indicating a shared baseline of mathematics-related diagnostic sensitivity.

At the same time, differences between the groups become visible in the scope and depth of their interpretations. The broader range of aspects addressed by primary school teachers may point to a more elaborated use of mathematics-specific PCK, recognizing a wider array of arithmetical difficulties and more advanced considerations of measurement concepts. Their diagnostic reasoning suggests an ability to connect students' observable actions with underlying mathematical concepts, learning trajectories, and typical misconceptions. In contrast, the focus of special education teachers on foundational competences reflects a diagnostic orientation aligned with their professional emphasis on basic learning prerequisites. This may indicate a more selective activation of PCK, focused on identifying fundamental barriers to learning rather than on differentiating among more advanced mathematical aspects.

The relatively limited attention to measurement across both groups may indicate that this domain is not strongly integrated into teachers' diagnostic repertoires. Although measurement was an explicit component of the tasks and video vignettes, it appears to have played only a minor role in teachers' interpretations. This may suggest that measurement-related knowledge is less activated in the diagnostic interview situations or maybe less firmly anchored in teachers' PCK. At the same time, the design of the video vignette may have directed teachers' attention toward other aspects of students' work. From a content-specific perspective on diagnostic thinking, this finding highlights that diagnostic attention is not evenly distributed across mathematical domains, even when they are explicitly present in instructional material.

Limitations

The qualitative design, with a small, heterogeneous sample of 24 teachers, allows for in-depth exploration of diagnostic thinking but limits generalizability. In addition, the unequal distribution of primary (n = 16) and special education teachers (n = 8) constrains group comparability. The inclusion of out-of-field teachers may have influenced participants' perceptions and interpretations; however, this aspect was not systematically examined, leaving potential differences between in-field and out-of-field teachers unexplored.

While video vignettes provide a structured way to examine diagnostic thinking, they cannot fully capture the complexity of authentic classroom interactions, including simultaneous demands, student diversity, and unpredictable events. Although considerable effort was invested in task and vignette design, their influence on teachers' perceptions and interpretations cannot be entirely excluded.

The focus on two real-world problems and corresponding video vignettes further restricts the range of diagnostic situations considered. Moreover, relying solely on teachers' verbal reflections limits methodological triangulation (e.g., classroom observations), even though it ensures strong practice orientation. Finally, as the study was conducted within the German education system, transferability to other contexts may be limited.

Further research should address these limitations by using larger, more diverse samples, multiple data sources, and cross-national comparisons to deepen understanding of diagnostic thinking in inclusive mathematics education.

CONCLUSION

This study provides empirical insights into how primary and special education teachers engage in diagnostic thinking when interpreting students' solution processes in real-world mathematical problems. By focusing on perception and interpretation processes in a content-specific context, it extends existing research on diagnostic competence beyond an emphasis on judgment accuracy. The DiaCoM framework (Loibl et al., 2020) proved useful for capturing these perception and interpretation processes without reducing diagnostic competence to correctness of judgments.

The findings show both shared and distinct patterns in teachers' perception and interpretation of students' difficulties. In this respect, the study points to the central role of PCK in structuring diagnostic attention in mathematics classrooms, influencing how teachers interpret students' solution processes and which mathematical aspects become salient. Differences between primary and special education teachers indicate that diagnostic interpretations are not uniform but reflect different emphases in professional training and practice (Geisen, 2021, 2023; Knigge & Kollosche, 2019). Potential influencing factors, such as out-of-field teaching, were not examined. In addition, relatively weak emphasis on measurement in German teacher education, where it is often embedded within other domains, may further contribute to its marginal role in diagnostic thinking.

These findings have important implications for teacher education. They highlight the importance of strengthening mathematics-specific diagnostic thinking, particularly regarding measurement and modelling in real-world contexts. The study thus contributes to a more differentiated understanding of how diagnostic competence can be fostered in content-specific and inclusive learning environments. Learning opportunities that involve the analysis of authentic student work, such as transcripts or video vignettes, appear promising for fostering diagnostic thinking (Prediger & Zindel, 2017). Moreover, collaborative formats between primary and special education teachers may support the exchange of complementary perspectives and contribute to more differentiated interpretations of students' learning processes in inclusive settings. This could be implemented through joint professional development, co-planning, or structured case discussions. Overall, the findings point to the importance of strengthening diagnostic thinking in mathematics education to support more equitable and adaptive learning opportunities.

While the qualitative design and context-specific sample limit the generalizability of the findings, the study provides a foundation for future research. Future studies could extend the present focus on perception and interpretation by more systematically examining decision-making as a further component of diagnostic thinking and its relation to these processes. They could also investigate diagnostic thinking across larger, more diverse samples and examine how targeted interventions influence teachers' diagnostic thinking over time. In particular, analyzing how instructional decisions are derived from teachers' perceptions and interpretations may provide deeper insights into the coherence of diagnostic thinking processes.

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AI statement: The author stated that generative AI tools (e.g., ChatGPT by OpenAI) were used to check the English language clarity of the manuscript. No content generation was performed by AI.

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