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# Difficulties in non-constant acceleration: Treatment of algebraic and graphical representations

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**Abstract**

In university physics courses, instructors use graphical and algebraic representations in kinematics involving non-constant acceleration. However, students face difficulties when working within these representations, which affects their understanding of the physical phenomenon. This study applies to the theoretical framework of semiotic representations and a phenomenographic approach to analyze how 467 engineering students interpret and extract information from these types of representations. We designed six open-ended questions, three involving algebraic representations and three involving graphical representations, to explore difficulties related to treatment within the same representation. The results reveal that, in algebraic representations, students confuse physical concepts in kinematics with mathematical functions, using rote-learning strategies without understanding their physical meaning. In graphical representations, greater difficulties related to conceptual confusion were observed when recognizing and processing information than in algebraic representations. The findings suggest the need to develop instructional materials that promote a deeper understanding of different representations and kinematic concepts.

**Keywords:** semiotic representations, non-constant acceleration, phenomenography, physics education research

## INTRODUCTION

Research in physics education continually faces the challenge of transforming traditional teaching into a process that enables students not only to acquire knowledge but also to develop scientific skills and critical thinking, emphasizing a deep understanding of physics principles beyond the mere application of formulas (Viennot, 2001, 2014). This approach contrasts with traditional practices, in which problems are solved mechanically, without prior conceptual analysis (Guisasola & Zuza, 2024; Zuza et al., 2016).

One of the obstacles to learning physics, particularly in the study of motion, is the difficulty of connecting graphical and algebraic representations with the underlying physical concepts. Students' ideas and reasoning patterns regarding fundamental concepts in this area reveal systematic errors in interpretation, conceptual confusion, and even an inability to relate different types of graphical representations (Beichner, 1996; Brasell & Rowe, 1993; McDermott et al., 1987; Mokros & Tinker, 1987). Other studies indicate that students experience difficulties with qualitative and quantitative definitions of velocity and acceleration, as well as with understanding the relationship between these concepts (Reif & Allen, 1992; Shaffer & McDermott, 2005).

On the other hand, findings suggest that students' ability to solve problems is affected by different forms of representation, especially in contexts involving algebraic relationships. Students tend to experience greater difficulty when interpreting these representations in physics contexts and, although they often adopt an algorithmic approach, they struggle to transfer

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**Contribution to the literature**

- University instructors use graphical and algebraic representations to teach kinematics involving non-constant acceleration; however, students experience difficulties conceptualizing this non-uniform change through these representations.
- It is essential for students to interpret each mathematical symbol in terms of its physical meaning to ensure that mathematical operations remain consistent with the underlying physical theory, thereby strengthening their understanding of physics.
- The transition between physical and mathematical reasoning, particularly in kinematics involving non-constant acceleration, requires treatment within the same semiotic representation, whether graphical or algebraic, which represents a significant challenge for students.

mathematical knowledge to physical situations (Guisasola et al., 2023b; Nixon et al., 2016). The observed difficulties are mainly conceptual and representational in nature and may influence students' responses (De Cock, 2012).

Due to the close relationship between physics and mathematics, both disciplines can be understood as systems for representing abstract objects, using tools such as graphical representations and algebraic expressions to establish relationships among concepts and draw conclusions (Duval, 2006; Pospiech et al., 2019). Furthermore, in physics education, algebraic representations are essential for describing functional dependencies, particularly in advanced university courses (Ivanjek et al., 2016; Planinic et al., 2012, 2013; Pospiech & Karam, 2023; Pospiech et al., 2019).

To understand the meaning of physical functions in university contexts, it is important to distinguish between the implicit use of functions, the explicit formulation of functional dependencies, and the formal mathematical definition of a function (Leinhardt et al., 1990). This distinction is particularly relevant because the use of mathematical formalism in physics differs fundamentally from its use in mathematics (Heck & van Buuren, 2019). However, this difference is not always evident to students, who frequently encounter difficulties in this area.

Despite advances in research on kinematics, few studies have conducted an in-depth analysis of the difficulties university students face when working with graphical and algebraic representations in situations involving non-constant acceleration. Although instructors frequently use these representations to facilitate understanding of this topic, students often struggle to conceptualize non-uniform change because equations describing motion with variable acceleration are typically non-linear. Consequently, understanding these representations requires an advanced command of differential and integral calculus, which further complicates the physical interpretation of mathematical solutions (Dufresne et al., 1997; Pospiech et al., 2019).

This study differs from previous research on students' understanding of kinematics in physics courses by focusing specifically on situations involving motion with non-constant acceleration. In particular, it examines the difficulties students encounter when interpreting algebraic representations and, separately, graphical representations involved in the analysis of this type of motion. Moreover, several studies in physics education have identified difficulties associated with managing semiotic representations in similar contexts (Campos et al., 2022; Fredlund et al., 2014).

The aim of this study is to investigate the difficulties students experience in identifying the physical meaning and characteristics of motion with non-constant acceleration through graphical and algebraic semiotic representations. Specifically, we seek to relate difficulties in interpreting these representations to difficulties in understanding the underlying physical concepts at the university level. Accordingly, our research question is:

What obstacles and reasoning patterns emerge when second-year engineering students solve kinematics problems involving variable acceleration using graphical and algebraic semiotic representations as the primary analytical tool?

The present study focuses on students' difficulties when performing treatment within algebraic representations on the one hand and graphical representations on the other while analyzing motion with non-constant acceleration in advanced mechanics courses for science and engineering students.

## THEORETICAL BACKGROUND

This section focuses on three main issues:

(1) theoretical perspectives on registers of semiotic representation in physics and mathematics,

(2) treatment within algebraic semiotic representations, and

(3) treatment within graphical semiotic representations in university physics.

### Registers of Semiotic Representation in Physics and Mathematics

In this study, we analyze the difficulties students face when understanding algebraic and graphical representations of motion with non-constant acceleration using the theory of registers of semiotic representation (Duval, 2006). This theory considers graphical and algebraic representations as tools that use a set of symbols, rules, and associations to represent different abstract phenomena. It allows us to investigate how these representations function as complex systems of signs that influence knowledge acquisition and understanding of representational systems (Duval, 2020).

This theoretical framework allows us to explore the cognitive characteristics of different representational systems used in university physics education, particularly the relationships among the mathematical functions of position, velocity, and acceleration as functions of time, as well as the relationships among position – time, velocity – time, and acceleration – time graphs for this type of motion. In this context, both functions and graphs are considered mathematical objects that are not directly accessible through perception (Duval, 2006), which may complicate students' understanding of physics and increase the difficulties they experience when attempting to recognize the same mathematical object across different forms of representation. This increases cognitive complexity because it requires coordination among multiple semiotic systems (Duval, 2017).

Since students must be able to work within the same representation, whether algebraic or graphical, to understand physical phenomena involving non-constant acceleration, they need not only a sound understanding of the relevant physical concepts but also sufficient mathematical skills to construct appropriate solutions.

Within the interaction between physics and mathematics in university contexts, it is essential that students interpret each mathematical symbol in terms of its corresponding physical meaning. This ensures coherence between mathematical operations and physical theory and contributes to a deeper understanding of physics.

Although university instructors commonly employ multiple representations to facilitate understanding of kinematic concepts, particularly in the study of motion with non-constant acceleration, students often encounter difficulties when working within the same register. This process requires not only an adequate command of the treatment rules associated with each register, whether algebraic or graphical, but also the ability to establish meaningful connections between mathematical representations and the corresponding physical concepts and laws.

Difficulties in understanding treatment within a given representation and in converting between different representations arise from the need to develop two cognitive abilities: Distinguishing the represented object from its representation and recognizing the same object across multiple representations (Duval, 2006, 2017). This phenomenon, originally identified in mathematics (Duval, 1998), has also been documented in physics education research (Campos et al., 2020, 2022, 2023; Guisasola et al., 2023b; Pospiech et al., 2019).

Although the use of multiple representations can enrich conceptual understanding, it may also present important challenges. In particular, equations describing motion with variable acceleration are typically non-linear and therefore require an advanced understanding of differential and integral calculus, which can increase the difficulties students experience in these contexts (Ivanjek et al., 2016; Planinic et al., 2012, 2013; Susac et al., 2018).

## Treatment of Algebraic Semiotic Representations in University Physics

The theory of semiotic representations that underpins this study also addresses the cognitive activities associated with understanding within the same representation, referred to as treatment (Duval, 2006). In treatment processes, transformations occur within the same semiotic system and depend exclusively on the symbols and rules of that register.

For example, an algebraic representation may involve a sequence of operations that include testing, explanation, heuristic exploration, calculation, or equation solving. The nature of these processes varies according to the register being used (Duval, 2020). Numerical and symbolic registers are generally more powerful and efficient than multifunctional registers, but they are also cognitively more demanding and therefore more difficult for many students to understand (Duval, 2017, 2020). This perspective has been used to explain difficulties related to recognition and conversion processes in mathematics (McGee & Martinez-Planell, 2014) and physics education (Mora, 2020).

However, in advanced university physics courses, the use of algebraic representations sometimes overshadows the understanding of fundamental physical processes. For example, in the context of motion with non-constant acceleration addressed in this study, the need to differentiate or integrate time-dependent functions continues to generate learning difficulties at all educational levels (Christensen & Thompson, 2012; Pospiech & Karam, 2023; Pospiech et al., 2019; Reif & Allen, 1992).

Despite these important contributions, physics employs mathematics for purposes that differ from those of mathematics itself, and these differences are often overlooked in physics instruction (Redish & Kuo, 2015; Taşar, 2010). One reason may be that expert physicists implicitly draw upon their physics knowledge when interpreting and applying mathematical descriptions and therefore overlook nuances that are not obvious to students (Pospiech & Karam, 2023).

In summary, the reviewed literature allows us to situate this study within the framework of the theory of semiotic representations, focusing specifically on treatment within the algebraic register. This treatment is understood as a set of transformations among symbolic expressions that require substantial cognitive effort from students in order to assign physical meaning to mathematical expressions. These theoretical contributions provide the basis for analyzing how students understand and operate with algebraic functions of position, velocity, and acceleration in the context of one-dimensional motion with non-constant acceleration, as well as the difficulties they encounter when connecting mathematical language with the corresponding physical concepts.

These symbolic transformations require not only mathematical skills, such as differentiation and integration, but also the ability to preserve and assign physical meaning to the expressions involved.

Finally, this theoretical framework provides the basis for investigating how students perform treatment within algebraic representations and which conceptual or semiotic obstacles emerge as they attempt to construct meaning from mathematical representations of motion with non-constant acceleration.

### Treatment of Graphical Semiotic Representations in University Physics

The earliest studies on graph comprehension in physics identified students' difficulties in connecting graphs with physical concepts, focusing on the interpretation and construction of graphs, particularly in kinematics involving constant acceleration (Beichner, 1990, 1996; Brasell & Rowe, 1993; McDermott et al., 1987; Mokros & Tinker, 1987). These studies stimulated a substantial body of subsequent research in this area (Hale, 2000; Ivanjek et al., 2016, 2023; Planinic et al., 2013; Pospiech et al., 2019; Wemyss & van Kampen, 2013).

Subsequent studies confirmed that students experience difficulties understanding graphs related to these topics, particularly confusion among position, velocity, and acceleration, as well as difficulties interpreting slopes and areas under curves (Araujo et al., 2008; Beichner, 1996; Dominguez et al., 2017; Zavala et al., 2017). Furthermore, the context in which derivative and antiderivative graphs are taught influences students' comprehension and performance (Barniol et al., 2024).

Recent studies have shown that students continue to experience difficulties interpreting graphs of motion with constant acceleration, particularly regarding direction of motion, slope interpretation, and area-under-the-curve reasoning. Students also frequently apply inappropriate formulas when solving kinematics problems (Ivanjek et al., 2016; Planinic et al., 2013; Wemyss & van Kampen, 2013). Moreover, students' problem-solving performance may vary depending on the form of representation employed (Ceuppens et al., 2019; De Cock, 2012; Hale, 2000; Ivanjek et al., 2023).

Students often experience greater difficulty interpreting representations involving algebraic relationships, demonstrating challenges in transferring mathematical knowledge to physical contexts (Dufresne et al., 1997; Geyer & Kuske-Janßen, 2019; Heck & van Buuren, 2019; Larkin, 2014; Leinhardt et al., 1990).

Although the relationship between graph-related mathematical knowledge and its application in physics remains an active area of research, students generally experience greater difficulty solving physics tasks than equivalent mathematics tasks (Planinic et al., 2012, 2013), particularly when applying concepts such as area under the curve (Ivanjek et al., 2016; Susac et al., 2018). These findings suggest that mathematical knowledge alone is insufficient for solving physics problems (Planinic et al., 2019; Pospiech et al., 2019).

These difficulties become even more evident when interpreting graphical representations associated with motion involving non-constant acceleration, a particularly challenging topic due to the complexity of the acceleration concept itself (Shodiqin & Taqwa, 2021).

The literature reviewed indicates that treatment within graphical representations constitutes a significant challenge for students. Although previous research has thoroughly documented difficulties related to position, velocity, and acceleration graphs in constant-acceleration contexts, little attention has been devoted to situations involving non-constant acceleration.

Furthermore, the studies that provide the theoretical foundation for this work suggest that these difficulties become more pronounced when students must perform treatment within the same graphical semiotic representation. For example, students may be required to move from a position – time graph to a velocity – time graph or an acceleration – time graph.

This type of treatment requires students to recognize derivative and area relationships while simultaneously assigning physical meaning to graphical elements. In university mechanics courses, algebraic and graphical representations are often used together, with graphical representations serving as a critical bridge between iconic and symbolic forms of representation. However, students frequently encounter serious difficulties conceptualizing non-uniform change because this process requires not only graphical treatment but also the coordination of multiple representations (Urrutia et al., 2024).

Such coordination demands a rigorous and coherent relationship between mathematical and physical elements, demonstrating that mathematical knowledge alone is insufficient for an adequate interpretation of physical phenomena. Within this framework, treatment within algebraic and graphical representations is understood as a complex cognitive activity that challenges students to operate within a single semiotic register while maintaining a connection to the physical meaning of motion.

Finally, the present theoretical framework provides the basis for analyzing the difficulties students encounter when performing treatment within each register and the conceptual obstacles that emerge as university students attempt to construct meaning around one-dimensional motion with non-constant acceleration.

## METHODOLOGY

The objective of this study is to investigate the difficulties university students experience when identifying the physical meaning of one-dimensional motion with non-constant acceleration, specifically when working with its semiotic representations, both algebraic and graphical. The study seeks to analyze how these difficulties, associated with recognizing and processing information within a single type of representation, relate to students' conceptual understanding of the physical phenomenon.

As stated in the Introduction, our research question focuses on investigating the obstacles and reasoning patterns displayed by second-year engineering students when solving kinematics problems involving non-constant acceleration that require treatment of information within a single representation, whether graphical or algebraic.

To address this objective, an open-ended questionnaire consisting of six questions was designed. Three questions focused on treatment within the same algebraic semiotic representation, whereas the other three focused on treatment within the same graphical semiotic representation. The six questions were developed in accordance with the textbook used in the course (Beer et al., 2017), with a focus on motion involving non-constant acceleration and alignment with the course learning objectives. Each question presented a statement accompanied by an initial algebraic or graphical representation and required students to respond using a corresponding final representation while maintaining the same type of register. In addition, each item included a space in which students were asked to justify their procedures and responses.

The collected data were analyzed using a phenomenographic approach (Marton, 1981), which considers the different ways in which individuals interpret and understand their experiences of reality.

According to this approach, each structure, concept, and line of reasoning constitutes a descriptive category that can be collectively identified (Marton, 1981). These categories reflect a form of collective understanding, in which a particular category may be expressed through different forms of reasoning. Within this framework, data collection and analysis focus on students' explanations (Guisasola et al., 2023a).

This method enabled the development of descriptive categories and an outcome space illustrating the relationships among them. Each category focuses on critical aspects of understanding a specific phenomenon (Guisasola et al., 2023a), and categories are related in a hierarchical and inclusive manner. This structure supports the development of increasingly sophisticated and complex categories while ensuring internal consistency. In this study, we identified descriptive categories that explain students' abilities to recognize algebraic representations on the one hand and graphical representations on the other in the context of motion with non-constant acceleration.

To achieve a more detailed understanding of students' reasoning when responding to the open-ended questionnaire, interviews were conducted using the think-aloud methodology (van Someren et al., 1994). This approach allowed researchers to explore students' thought processes while they reviewed and commented on their responses, which were audio-recorded. The resulting transcripts facilitated a deeper analysis of the cognitive strategies employed by participants. Students selected for the interviews were purposefully chosen because they provided detailed explanations that offered a clear view of their reasoning processes.

The combined use of open-ended questionnaires and interviews provided a more comprehensive understanding of the difficulties students encounter when working with semiotic representations in university physics. While the questionnaires offered an initial identification of problematic areas, the interviews enabled a deeper examination of the underlying cognitive processes (van Someren et al., 1994). Together, these methods facilitated the identification of reasoning patterns and enriched the analysis of students' justifications. This mixed-methods approach also enabled data triangulation, thereby strengthening the validity of the findings and providing a more holistic understanding of students' cognitive difficulties.

### Research Context

This study involved 467 students enrolled in a calculus-based mechanics course at a private Chilean university with three campuses located in different regions of the country. The course was offered during the third semester of the engineering program and was taken by students who had previously completed two physics courses (introduction to physics and experimental physics) and two mathematics courses (introduction to mathematics and differential calculus).

Participants were distributed across several course sections, each comprising between 35 and 45 students.

Although all sections followed the same schedule and pedagogical approach, each was taught by a different instructor. Within this context, 106 students responded to question 1 (Q1), 107 students responded to question 2 (Q2) and question 4 (Q4), 119 students responded to question 3 (Q3) and question 5 (Q5), and 135 students responded to question 6 (Q6). Therefore, the total number of unique participants who completed the questionnaire was 467.

The first unit of the mechanics course addressed motion in one, two, and three dimensions, including concepts such as displacement, velocity, and acceleration under both constant and non-constant conditions. The unit lasted four weeks and included three lectures per week as well as ninety minutes of practical workshops. Instruction was primarily delivered through traditional lecture-based teaching complemented by active-learning strategies implemented during workshop sessions. Each week, students complete online formative assessments that provide immediate feedback. At the end of the unit, students took a final examination consisting of problems similar to those presented at the end of each textbook chapter. Data collection was conducted during an in-person class session after four weeks of instruction and following completion of the final examination.

## Instruments

To address the research question, a questionnaire consisting of six open-ended questions was designed: Three questions focused on algebraic semiotic representations and three focused on graphical representations of motion with non-constant acceleration.

The questionnaire was initially validated through expert review. This validation process aimed to ensure both the content validity and the representational coherence of the instrument. Four independent evaluators participated in this process: Two specialists in university physics education and two researchers in physics education, all of whom had experience in the use of algebraic and graphical semiotic representations for this type of motion.

Each evaluator received the complete questionnaire together with a table of specifications and was asked to analyze the correspondence between each item and the learning objectives related to interpreting and extracting information through treatment within algebraic and graphical representations. In addition, they evaluated the coherence between the initial and final representations, whether algebraic-to-algebraic or graphical-to-graphical, thereby ensuring the internal consistency of the instrument.

The experts evaluated four fundamental criteria:

(a) clarity of wording,

(b) appropriateness of the physical context,

(c) representational coherence, and

(d) conceptual validity.

Each item was rated using a five-point Likert scale, and qualitative observations were collected to support refinement of the instrument.

Although quantitative content validity indices were not calculated, the expert review process involved a detailed qualitative analysis based on the table of specifications, focusing on content alignment, representational coherence, and internal consistency. This process led to several refinements of the instrument. Following the review, minor modifications were made to the wording and graphical format of several questions in order to improve readability and ensure consistency between the initial and final representations.

This validation procedure ensured that the instrument was appropriate not only for assessing students' understanding within each semiotic register but also for maintaining theoretical coherence with the framework of semiotic representations and the phenomenographic methodology that guided this study.

Additionally, inter-rater reliability was evaluated using Cohen's kappa coefficient ($\kappa = 0.95$), indicating a high level of agreement among evaluators.

Once the questionnaire had been revised, an external validation was conducted with a small sample of students ($N = 30$) who completed the instrument. This stage examined whether students adequately understood the purpose of each question and confirmed that the vast majority experienced no difficulties in understanding the statements. Based on this analysis, a small number of final wording adjustments were introduced, resulting in the final version of the questionnaire used in this study.

Before the questionnaire was administered, students signed an informed consent form approved by the Bioethics and Biosecurity Committee of the Pontificia Universidad Católica de Valparaíso (code BIOPUCV-H 647-2023). Participants received a detailed description of the study, including the measures adopted to protect their privacy and personal data. Furthermore, they were informed that no academic penalties would result from incorrect responses and were encouraged to complete the questionnaire in its entirety in order to provide a comprehensive picture of their reasoning processes and difficulties.

After administration of the open-ended questionnaire, twelve interviews were conducted with students purposefully selected from each region of the country. Priority was given to students who were able to clearly and coherently explain their strategies and justifications while reviewing their own responses aloud, without requiring intervention from the researchers.

The interviews focused on the six questionnaire items and were intended to provide a deeper understanding of students' thinking processes (van Someren et al., 1994). All interviews were audio-recorded and transcribed for analysis. This procedure enabled direct access to students' cognitive processes, facilitating the identification of conceptual difficulties and misunderstandings as well as a deeper understanding of the cognitive barriers associated with treatment within both graphical and algebraic representations of motion with non-constant acceleration.

### Data Collection and Analysis

Students' responses to the questionnaire were analyzed using a phenomenographic approach based on Marton's (1981) framework of explanatory categories and following several stages of analysis (Guisasola et al., 2023a).

In the first stage, one of the researchers transcribed and reviewed all responses, identifying recurring patterns and difficulties that led to the development of preliminary categories. In the second stage, two researchers independently analyzed a random sample of 50 questionnaires, identifying emerging categories before comparing and discussing their findings to reach agreement. In the third stage, the remaining responses were reviewed to validate the explanatory categories, resolve discrepancies, and ensure the comprehensiveness of the analysis.

To evaluate the reliability of the analysis, κ was used to measure inter-rater agreement. A κ value of 0.95 was obtained, indicating a high level of reliability, as values above 0.80 are generally considered evidence of strong agreement (Banerjee et al., 1999). This result supports the validity of the analysis and suggests that the explanatory categories adequately reflect students' conceptual difficulties.

Despite the rigor of the study, several limitations should be considered when interpreting the findings. First, the sample was limited to engineering students from a private Chilean university, which restricts the generalizability of the results. Second, the interviews were conducted with a purposive sample of students, which may have introduced selection bias. Third, the phenomenographic analysis was based on only six questionnaire items and therefore does not encompass all possible difficulties associated with the use of algebraic and graphical representations in kinematics involving non-constant acceleration.

Furthermore, although the reliability analysis revealed a high level of agreement among evaluators (κ = 0.95), this indicator does not guarantee the complete identification of all cognitive difficulties. Consequently, additional factors not considered in this study may also influence students' performance and reasoning.

Despite these limitations, the findings provide valuable insights into the difficulties students encounter when working with algebraic and graphical semiotic representations in kinematics involving non-constant acceleration. Moreover, the results offer a methodological framework that may support the identification of similar difficulties in other areas of university physics.

## RESULTS

This section presents the findings derived from the phenomenographic analysis of students' reasoning in response to the six open-ended questions designed to explore treatment within the same semiotic representation. First, the results related to treatment within algebraic representations (Q1-Q3) are presented. Next, the results related to treatment within graphical representations (Q4-Q6) are discussed. Finally, the findings from both sets of questions are considered together in order to identify common patterns and differences across representational registers.

### Treatment of Algebraic Semiotic Representations

The first set of questions in the open-ended questionnaire on motion with variable acceleration (Q1, Q2, and Q3, presented in **Table 1**) addresses three situations involving bodies moving in a straight line with variable acceleration. Each question presents an initial algebraic semiotic representation.

To answer these questions, students were required to complete the corresponding section using algebraic representations, thus working within the same representational register.

Q1 presents an initial algebraic representation of position as a function of time. Students were asked to indicate the initial position, initial velocity, and initial acceleration using algebraic representations. Q2 presents the position function of a cyclist moving in a straight line with non-constant acceleration. Students were required to determine the instant at which the cyclist returns to the origin, again responding using algebraic representations. Finally, Q3 presents a velocity function as a function of time and asks students to determine the instant at which acceleration reaches its minimum value using algebraic semiotic representations. The common objective of these three questions was to evaluate students' ability to interpret, discriminate, and extract information from position, velocity, and acceleration functions of time in situations involving one-dimensional motion with variable acceleration.

In addition, these questions made it possible to assess students' ability to correctly apply differentiation procedures and to interpret the resulting expressions within the physical context of motion with non-constant acceleration.

**Table 1.** Open-ended questionnaire items (Q1, Q2, and Q3) involving motion with non-constant acceleration: Treatment within algebraic semiotic representations

| Question | To complete the task, students were required to: | Algebraic representation of the task |
|---|---|---|
| Q1. A physics student is moving on a skateboard in a straight line, defined by the following equation *x(t)*, where $x$ is expressed in meters and $t$ in seconds. $x(t) = 2t^3 - 15t^2 + 24t + 4m$. a) Indicate the value of the initial position b) Indicate the value of the initial velocity c) Indicate the value of the initial acceleration | (a) Recognize that $t = 0$ represents the initial instant and substitute this value into the position function *x(t)* to determine the skater's initial position. (b) Differentiate the position function *x(t)* to obtain the velocity function and substitute $t = 0$ s to determine the skater's initial velocity. (c) Differentiate the velocity function to obtain the acceleration function and substitute $t = 0$ s to determine the skater's initial acceleration. | $x(t) = 2t^3 - 15t^2 + 24t + 4m$ $x_{(t=0s)} = 4m$ $v(t) = \frac{dx}{dt}$ $v(t) = 6t^2 - 30t + 24\frac{m}{s}$ $v_{(t=0s)} = 24\ m/s$ $a(t) = \frac{dv}{dt}$ $a(t) = 12t - 30\ m/s^2$ $a_{(t=0s)} = -30\ m/s^2$ |
| Q2. For a cyclist moving in a straight line, it is determined that their position as a function of time is defined by the equation: $x(t) = 6t^2 - t^3m$, where $t$ is expressed in seconds and $x$ in meters. a) Determine the moment when the cyclist returns to the origin of coordinates b) What statement could help justify your answer? | Differentiate the position function with respect to time to obtain the velocity function. Then, set the velocity equal to zero ($v = 0$ *m/s*) and solve the resulting equation to determine the two possible time values. Finally, identify which non-zero time corresponds to the instant at which the cyclist returns to its origin. | $x(t) = 6t^2 - t^3m$ $v(t) = \frac{dx}{dt} = 12t - 3t^2\frac{m}{s}$ $12t - 3t^2 = 0$ $(t = 0s\ y\ t = 4s)$ |
| Q3. For a skater moving in a straight line, it is determined that her velocity as a function of time is defined by the equation: $v(t) = 36t^2 - 36t + 2$, where $t$ is expressed in seconds and $v$ in meters/second. a) Determine the instant when the value of the skater's acceleration is minimum b) What statement might help justify your answer? | Determine the instant at which the skater's acceleration reaches its minimum value. To do so, students were required to differentiate the velocity function to obtain the acceleration function and then determine the time corresponding to the minimum value of acceleration. | $v(t) = 36t^2 - 36t + 2$ $a(t) = \frac{dv}{dt} = 72t - 36$ $a(t) = 0 = 72t - 36 = 0$ $t = 0.5\ s$ |

**Table 2.** Descriptive categories identified through the phenomenographic analysis of treatment within algebraic semiotic representations

| Category | Category description | Q1 | Q2 | Q3 |
|---|---|---|---|---|
| A | Correctly interprets and extracts information from algebraic representations | 52% | 54% | 55% |
| B | Applies mathematical procedures without physical interpretation | 16% | - | - |
| C | Misinterprets the physical meaning of algebraic representations | 23% | 21% | 27% |
| E | Demonstrates difficulties with basic mathematical procedures | 3% | 17% | 11% |
| F | No response | 6% | 8% | 7% |

**Table 1** presents the three algebraic questions together with the tasks students were required to perform and the expected algebraic representations used in the questionnaire.

When students' written reasoning was analyzed (Guisasola et al., 2023a), responses were classified into five explanatory categories (A, B, C, E, and F), as shown in **Table 2**, together with the number of responses and the percentage of students corresponding to each category.

The descriptive categories presented in **Table 2** emerged from the phenomenographic analysis of students' responses related to treatment within algebraic semiotic representations. Across the three questions analyzed (Q1, Q2, and Q3), slightly more than half of the students (52%-55%) were able to correctly interpret and extract information from the algebraic expressions (category A), indicating a basic level of understanding.

However, a substantial proportion of students experienced difficulties. Between 21% and 27% incorrectly interpreted the physical meaning of the expressions (category C), while 16% applied mathematical procedures such as differentiation in a mechanical manner without conceptual understanding (category B). Weaknesses in basic mathematical skills were also observed (category E), particularly in Q2 (17%)

and Q3 (11%), and between 6% and 8% of students did not provide a response (category F).

These results reveal different levels of understanding and highlight the need to strengthen the connection between mathematical procedures and physical interpretation in the teaching of kinematics involving non-constant acceleration.

Analysis of the interview transcripts further corroborated these findings. Although some students were unable to identify the physical meaning of the algebraic representations and extract information from the functions, they nevertheless attempted to determine the instant at which the particle returned to the origin. For example, one student stated:

> "I think that to calculate velocity, it has to be differentiated [pointing to the position – time function]." (Q1, category B)

In other cases, students appeared to automate the use of differentiation procedures without understanding the physical meaning of the algebraic representations involved. An example of this type of reasoning is:

> "It is the double derivative, and then I set it equal to zero ..." (Q1, category C)

Another student explained:

> "First I calculate the inflection points here [pointing to the *x(t)* function]. If I differentiate it twice and set it equal to zero, then zero equals twelve minus six *t*. I move the twelve and divide by six, which gives me two. Time is two. Then those points are infinity and two ... or two and infinity. Then I am going to substitute one and three for time." (Q2, category C)

Additional statements included:

> "Because it is the initial position, we place it at zero. We could isolate t if we had the velocity." (Q2, category E)

Another student in the same category stated:

> "Since the acceleration is zero, if we have the velocity equation, we have to differentiate it to obtain the result." (Q3, category E)

**Table 3** presents examples of descriptive categories together with illustrative evidence obtained from the think-aloud interviews (van Someren et al., 1994). These examples correspond to situations in which students worked within the same algebraic representation, starting from an algebraic representation in the question statement and producing an algebraic representation in their response.

**Table 3.** Examples of descriptive categories and illustrative evidence from interviews related to treatment within algebraic semiotic representations

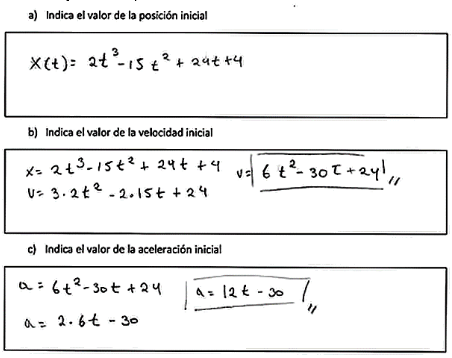
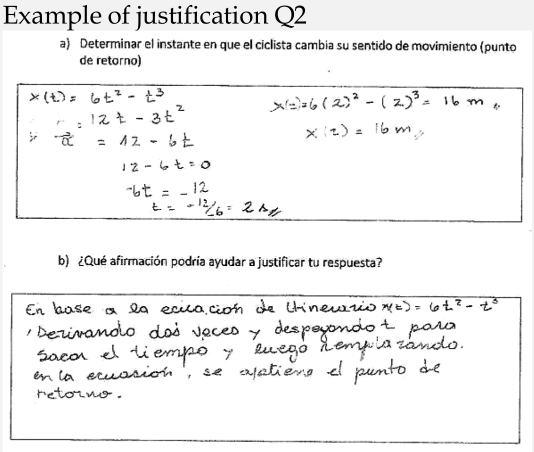
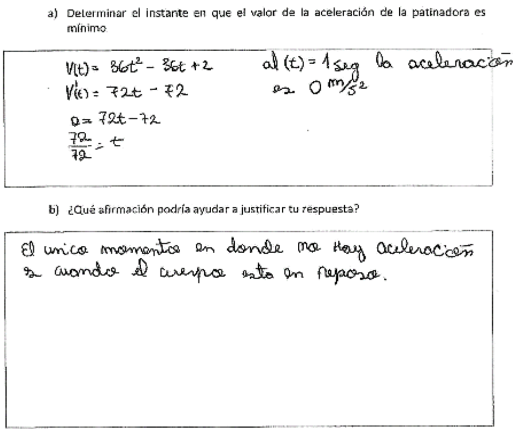
| Example of categories | Illustrative evidence |
|---|---|
| Example of justification Q1<br><br> | In Q1, 23% of the students, although they did not achieve the intended objective, differentiated the position function to obtain velocity and acceleration functions without correctly identifying the initial position, velocity, or acceleration: "It seems to me that in order to calculate the velocity, you have to differentiate that" [pointing to the position–time function] (category B). |
| Example of justification Q2<br><br> | Analysis of the interview transcripts revealed that some students experienced difficulties with mathematical procedures such as differentiation, substitution, and algebraic manipulation. One student commented: "Based on the position equation, differentiating twice and solving for time, and then substituting it into the equation, the return point is obtained." In Q2, 27% of the students exhibited confusion when distinguishing between the position function, *x(t)*, and its derivative (category C). |

**Table 3 (Continued).**

| Example of categories | Illustrative evidence |
|---|---|
| Example of justification Q3<br> | Analysis of the interview transcripts revealed that a considerable proportion of students who answered Q3 tended to automatically differentiate functions of position or velocity with respect to time and then set the resulting expression equal to zero. One representative statement was: "The only time when there is no acceleration is when the body is at rest." In addition, 11% of the responses indicated difficulties with basic mathematical procedures required in the course, such as differentiation and algebraic manipulation (category E). |

These results suggest a persistent confusion among the physical concepts of position, velocity, and acceleration, as well as the use of learning strategies based primarily on memorization rather than on a deep understanding of the physical meaning of algebraic functions. In particular, students tended to assume that differentiation must automatically be applied to obtain velocity or acceleration without understanding the physical concepts underlying these operations.

Although the students who completed the questionnaire had previously passed two physics courses and two mathematics courses, these findings highlight the need to strengthen both mathematical skills and conceptual understanding so that students can appropriately interpret and apply algebraic representations in situations involving motion with non-constant acceleration.

Consequently, it becomes evident that students experience difficulties when performing treatment within the same algebraic semiotic representation. The identified difficulties reveal problems distinguishing, for example, between instantaneous position and the position function of a body moving with non-constant acceleration within the same algebraic framework. Furthermore, a significant proportion of students performed differentiation procedures without understanding the underlying physical meaning of velocity and acceleration functions.

These findings underscore the importance of strengthening not only mathematical skills but also students' conceptual understanding of physical quantities so that they can effectively operate within the same algebraic semiotic representation. Instruction should support students in associating each mathematical symbol with its corresponding physical meaning and in applying this understanding to solve physics problems, particularly those involving motion with non-constant acceleration.

## Treatment of Graphical Semiotic Representations

The second set of questions in the open-ended questionnaire on motion with variable acceleration (Q4, Q5, and Q6, presented in **Table 4**) addresses three situations involving bodies moving in a straight line

**Table 4.** Illustration of the open-ended questionnaire (Q4, Q5, and Q6) for motion with non-constant acceleration: Treatment within graph representations

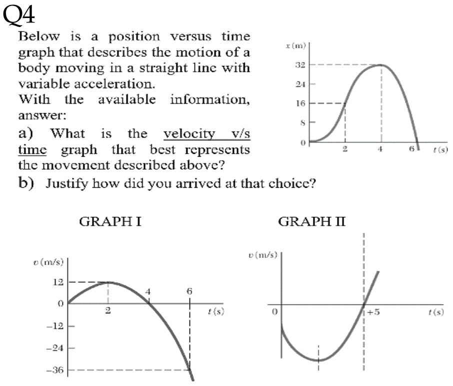
| Question | To complete the task, students were required to: | Graphical interpretation of the task |
|---|---|---|
| Q4<br> | (a) Identify the key features of a position-time graph for motion with non-constant acceleration, recognizing the relationship between slope and velocity, the maximum velocity at $t$ = 2 $s$, and the change in slope before and after $t$ = 4 $s$.<br>(b) Justify the selection of one of the two graphs. The correct answer is graph I. | The position-time graph shows that the particle starts from rest and accelerates in the positive direction until $t$ = 2 $s$, where its velocity reaches a maximum. Between $t$ = 2 $s$ and $t$ = 4 $s$, the particle continues moving in the positive direction while slowing down. At $t$ = 4 $s$, the velocity becomes zero and the particle reaches its maximum position. Beyond this point, the velocity becomes negative and the particle accelerates in a negative direction. |

**Table 4 (Continued).**

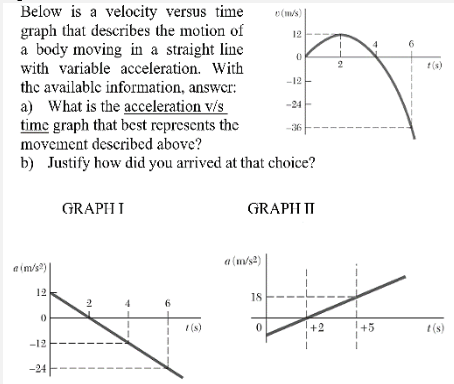
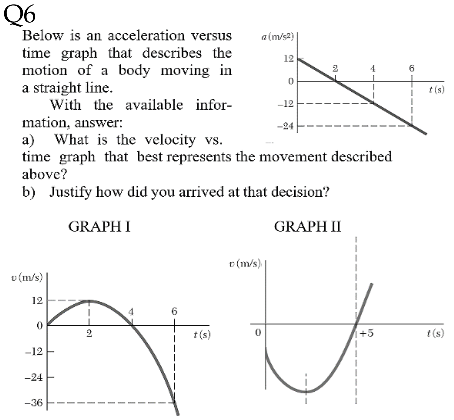
| Question | To complete the task, students were required to: | Graphical interpretation of the task |
|---|---|---|
| Q5   | (a) Identify the key features of a velocity-time graph for motion with non-constant acceleration, recognizing that the slope represents acceleration, that acceleration is zero at $t = 2\ s$, and that the body undergoes deceleration before accelerating in the negative direction. (b) Justify the selection of one of the two graphs. The correct answer is graph II. | The velocity-time graph shows that the body starts with a decreasing positive acceleration. At $t = 2\ s$, the acceleration is zero and the velocity reaches its maximum value. After this instant, the acceleration becomes negative and the body moves in a negative direction with increasing velocity. |
| Q6   | (a) Identify the key features of an acceleration-time graph for motion with non-constant acceleration, recognizing the relationships among acceleration, velocity, and the slope of the velocity–time graph, the zero acceleration at $t = 2\ s$, and the subsequent acceleration in the negative direction. (b) Justify the selection of one of the two graphs. The correct answer is graph I. | The acceleration-time graph shows that the body starts with a decreasing positive acceleration. At $t = 2\ s$, the acceleration is zero and the velocity reaches its maximum value. After this instant, the acceleration becomes negative and the body accelerates in a negative direction. |

**Table 5.** Descriptive categories identified through the phenomenographic analysis of treatment within graphical semiotic representations

| Category | Category description | Q4 | Q5 | Q6 |
|---|---|---|---|---|
| A | Correctly interprets and extracts information from graphical representations | 25% | 35% | 37% |
| B | Applies mathematical procedures without physical interpretation | 21% | 23% | 18% |
| C | Misinterprets the physical meaning of graphical representations | 37% | 32% | 29% |
| F | No response | 17% | 10% | 16% |

with variable acceleration, where both the initial and final representations are graphical.

In Q4, students are first presented with a position – time graph together with two possible velocity – time graphs from which they must select the one that best represents the motion and justify their choice. In Q5, students are presented with a velocity – time graph and are required to select the appropriate acceleration – time graph from the available options, providing justification for their decision. Finally, Q6 presents an acceleration – time graph describing the motion of a body moving in a straight line with non-constant acceleration. Students must choose between two possible velocity – time graphs and justify their selection.

The common objective of these three questions was to evaluate students' ability to interpret, discriminate, and extract information from position – time, velocity – time, and acceleration – time graphs in situations involving one-dimensional motion with variable acceleration.

In addition, these questions enabled the assessment of students' ability to correctly apply derivative and integral relationships represented graphically and to interpret the resulting information within the physical context of motion with non-constant acceleration.

**Table 4** presents the three graphical questions together with the tasks students were required to perform and the graphical representations used in the questionnaire.

When students' written reasoning was analyzed (Guisasola et al., 2023a), responses were classified into four explanatory categories (A, B, C, and F), as shown in **Table 5**, together with the number of responses and the percentage of students who responded to the open-ended questionnaire.

**Table 5** presents the descriptive categories obtained from the phenomenographic analysis of treatment within graphical representations in the open-ended questionnaire. In these questions, both the initial representations provided in the statements and the final representations selected by students were graphical semiotic representations.

Each question assessed students' ability to interpret and extract information through treatment within the same representation, as well as their ability to establish relationships among graphs.

For Q4, the initial representation corresponded to a position – time graph. From this representation, students had to choose between two velocity – time graphs with similar characteristics.

**Table 5** shows that, in this case, only 25% of students correctly justified their choice between the two graphical options, achieving the objective of interpreting and extracting information from a graphical representation related to motion with variable acceleration.

Twenty-one percent of students used mathematical tools without assigning them physical meaning when choosing between the two graphs presented. This was reflected in the analysis of the interviews conducted using the think-aloud technique (van Someren et al., 1994), as illustrated by the following statement:

> "When differentiating a function, we obtain velocity, and when differentiating, we substitute the obtained value." (Q4, category B)

Another student stated:

> "Because in the position – time graph, you can see that acceleration is decreasing; therefore, velocity also decreases, as can be observed in graph 1. That is why I chose it."

For Q5, another student stated:

> "The velocity – time graph is the derivative of the position – time graph. Then, we apply the integral to the velocity – time graph and obtain the position – time graph." (Q5, category C)

In Q5, the highest proportion of students misinterpreted the physical meaning of the graph. This can be observed in the interview data, where one student commented:

> "I think I would choose number one, since it increases its velocity and then decreases, so I can conclude this by looking at the increase in meters over a period of time." (Q5, category C)

It is also important to note that 17% of students who answered Q4 either could not provide a justification for their choice or did not answer the question. This is a concerning finding, considering that these students had previously passed two physics courses and two mathematics courses.

**Table 6** presents examples of descriptive categories and illustrative evidence from the interviews regarding treatment within graphical semiotic representations.

From the examples shown in **Table 6**, it is possible to identify several difficulties experienced by university students when working within graphical semiotic representations. Although some students were able to adequately interpret the graphs, most experienced difficulties connecting visual information with its physical meaning. For example, when moving from a position – time graph to a velocity – time graph, even when students selected the correct option, their justifications did not always demonstrate an adequate understanding of motion with variable acceleration.

In Q5, the results show that students tended to confuse physical concepts and misinterpret them. For example, during the interview, one student stated:

> "Since it is variable acceleration with respect to time, for velocity to decrease, acceleration must become smaller and smaller with each second that elapses."

In this question, students had to relate a velocity – time graph to one of two available acceleration – time graphs and justify their choice. However, 10% of students indicated that they could not answer the question or did not know how to justify their selection.

For Q6, only 37% of students achieved the objective, while 18% used mathematical tools without assigning them physical meaning. For example, one student indicated that

> "one parabola is positive and the other parabola is negative,"

while another explained:

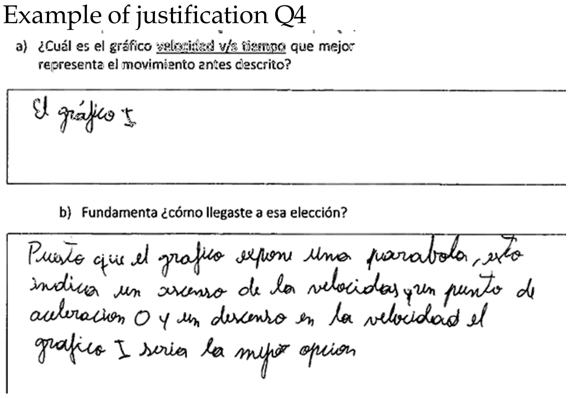
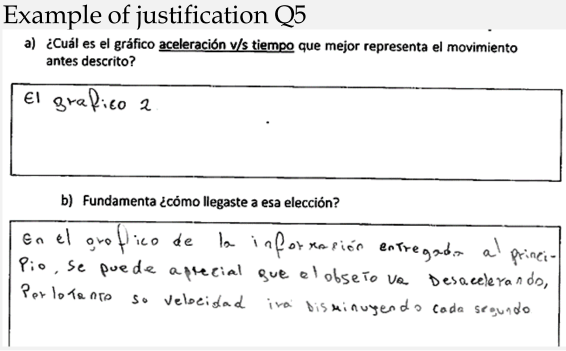
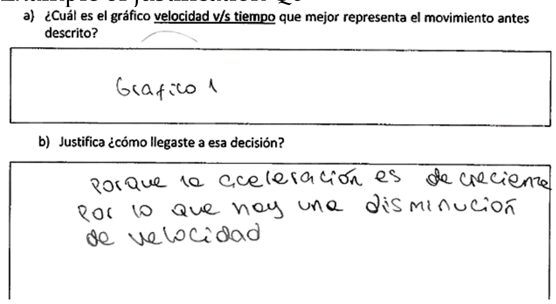
> "I choose one because the acceleration is a decreasing line, and therefore the other graph should also be decreasing."

A considerable proportion of students (29%, category C) confused the concepts of acceleration and deceleration with those of positive and negative acceleration. This was reflected in the interviews, for example, when one student stated:

> "It is because the acceleration is slowing down, going down through the negative values; therefore, velocity will continue to decrease through the negative values, faster and faster."

These results show that students experience significant difficulties when interpreting graphical representations of motion with non-constant acceleration. In particular, they often confuse the visual shape of the graph with the physical behavior of the body and have difficulty distinguishing between

**Table 6.** Examples of descriptive categories and illustrative evidence from interviews related to treatment within graphical semiotic representations

| Example of categories | Illustrative evidence |
|---|---|
| Example of justification Q4<br><br>a) ¿Cuál es el gráfico velocidad v/s tiempo que mejor representa el movimiento antes descrito?<br>El gráfico I<br>b) Fundamenta ¿cómo llegaste a esa elección?<br>Puesto que el grafico expone una parabola, esto indica un ascenso de la velocidad, un punto de aceleración 0 y un descenso en la velocidad el grafico I sería la mejor opción | In Q4, 35% of the students correctly interpreted the velocity-time graph, recognizing that the slope is zero at $t = 2\ s$ and that the body decelerates before accelerating in the negative direction (category A). |
| Example of justification Q5<br><br>a) ¿Cuál es el gráfico aceleración v/s tiempo que mejor representa el movimiento antes descrito?<br>El grafico 2<br>b) Fundamenta ¿cómo llegaste a esa elección?<br>En el grafico de la información entregada al principio, se puede apreciar que el objeto va desacelerando, por lo tanto su velocidad irá disminuyendo cada segundo | In Q5, 32% of the students misinterpreted the relationship between velocity and acceleration when selecting an acceleration-time graph from a given velocity-time graph. One student explained: "*Since it is a variable acceleration with respect to time, so that the velocity decreases, then the acceleration must be less and less for each second that has elapsed*" (category C). |
| Example of justification Q6<br><br>a) ¿Cuál es el gráfico velocidad v/s tiempo que mejor representa el movimiento antes descrito?<br>Grafico 1<br>b) Justifica ¿cómo llegaste a esa decisión?<br>Porque la aceleración es decreciente por lo que hay una disminución de velocidad | In Q6, 29% of the students misinterpreted the relationship between acceleration and velocity when selecting a velocity-time graph from a given acceleration-time graph. A common difficulty was confusing deceleration with negative acceleration. One student explained: "*It is because the acceleration is slowing down, going down through the negatives, therefore, the velocity will continue to go down through the negatives and faster and faster*" (category C). |

velocity, acceleration, deceleration, and direction of motion.

## DISCUSSION

The results obtained in this study regarding students' treatment of algebraic representations indicate that slightly more than half of the students were able to recognize and appropriately interpret basic aspects of motion with non-constant acceleration (see **Table 2**, category A). However, a considerable proportion of the responses revealed difficulties in establishing connections between algebraic notation and the underlying physical concepts. In particular, 16% of the responses classified in category B for Q1 (see **Table 2**) reflected a predominantly mechanical use of mathematical tools. Similarly, approximately one quarter of the responses to Q1, Q2, and Q3 (**Table 2**, category C) revealed difficulties in interpreting the physical meaning of algebraic expressions.

A question-by-question analysis further indicates that many students experience difficulties distinguishing between instantaneous position and the position function (Q1), as well as understanding the relationships among position, velocity, and acceleration functions (Q2 and Q3). This lack of conceptual clarity suggests that, for a significant number of students, algebraic expressions do not function as meaningful representations of physical phenomena. Consequently, a substantial proportion of students tend to prioritize algebraic manipulation without consistently linking these operations to their physical meaning, even when they formally master the required mathematical procedures. Similar patterns have been reported in previous studies on motion with constant acceleration, where persistent confusions between concepts such as velocity and acceleration have been identified (Reif & Allen, 1992; Shaffer & McDermott, 2005).

With respect to treatment within graphical representations, fewer than 40% of the responses demonstrated an adequate interpretation of the graphs (see **Table 5**, category A: 25% for Q4, 35% for Q5, and 37% for Q6). These findings indicate that students face greater challenges when interpreting graphs relating position, velocity, and acceleration as functions of time than when working with algebraic representations. Although some students selected the correct graphs, their justifications frequently revealed only a limited understanding of the underlying physical meaning.

In particular, recurring confusions were observed between positive and negative acceleration and deceleration (Q6, **Table 5**, category B and category C), as well as difficulties in recognizing changes in the direction of motion (Q5, **Table 5**, category B and category C). These difficulties hinder the development of a coherent understanding of kinematics. Furthermore, many students experienced difficulties recognizing that non-constant acceleration gives rise to non-linear curves and that the slope of a velocity – time graph is not constant. These observations are consistent with previous studies that have identified graphical representations as a persistent source of difficulty, even in advanced courses (Ivanjek et al., 2016; Planinic et al., 2013).

The results allow for a deeper understanding of the difficulties university students experience when performing treatment within a single representation, whether algebraic or graphical, in contexts involving motion with non-constant acceleration. In particular, the findings indicate that students tend to perform algebraic treatments, such as differentiation, integration, or substitution, in an automatic manner without reflecting on the physical meaning of the expressions involved, thereby producing largely procedural responses. In contrast, treatment within the graphical register was associated with higher levels of difficulty, especially in identifying slopes and areas under curves as representations of velocity and displacement, respectively.

Overall, the results reveal a marked disconnection between representational processes and students' conceptual understanding of physical phenomena. As argued by Duval (2006, 2017), learning in the sciences requires the coordination of different semiotic registers and the ability to preserve the meaning of the represented object across transformations and conversions.

It should be acknowledged that this study has certain limitations, including the characteristics of the sample and the fact that it was conducted at a private Chilean university, which limits the generalizability of the findings. Nevertheless, because the characteristics of the participants are comparable to those of students enrolled in other science and engineering programs, the findings may be useful to instructors working in different educational contexts. Future studies could extend this research to other institutions and countries in order to examine the extent to which the identified difficulties are shared across different educational settings.

From a didactic perspective, these findings reinforce the need to reconsider the teaching of university-level kinematics through instructional approaches that explicitly integrate algebraic and graphical registers, avoiding the assumption that mastery of mathematical procedures alone is sufficient. It is essential that physics instruction in these contexts includes opportunities for reflection on the physical meaning of mathematical expressions (Redish & Kuo, 2015), fostering deep conceptual understanding rather than merely procedural knowledge.

In this regard, the implementation of research-based teaching – learning sequences (Guisasola et al., 2023a; Fredlund et al., 2014) may support students' ability to perform treatment within a single representation by strengthening their capacity to interpret, discriminate, and justify the physical meaning associated with each semiotic register examined in this study. Such approaches also create opportunities for the design of future instructional interventions adapted to the different levels of understanding identified in this research.

In conclusion, the present discussion suggests that students' difficulties are not exclusively attributable to shortcomings in mathematical proficiency, but rather to an epistemic disconnection between mathematical and physical forms of reasoning. Addressing this gap will require instructional approaches that promote semiotic reflection, the construction of shared meanings, and the continuous coordination of different representational registers. In this way, it may be possible to foster a form of kinematics learning that combines mathematical precision with a deep conceptual understanding of motion with non-constant acceleration.

## CONCLUSIONS

This study investigated the difficulties university students experience when working with algebraic and graphical semiotic representations in the context of one-dimensional motion with non-constant acceleration. Using the theoretical framework of registers of semiotic representation and a phenomenographic approach, the study identified a range of obstacles and reasoning patterns that emerge when students perform treatment within a single representational register.

The findings indicate that students encounter difficulties in both algebraic and graphical representations, although the nature of these difficulties differs across registers. In algebraic representations, many students tend to apply mathematical procedures such as differentiation, integration, and substitution in a mechanical manner, without adequately connecting these operations to the underlying physical concepts. In graphical representations, students experience greater difficulties interpreting slopes, areas under curves, and the relationships among position, velocity, and acceleration graphs, particularly in situations involving non-constant acceleration.

The results also reveal that students frequently confuse key kinematic concepts, including velocity, acceleration, deceleration, and direction of motion. These difficulties suggest the existence of a persistent disconnect between mathematical operations and their physical interpretation, even among students who have successfully completed previous mathematics and physics courses.

From a theoretical perspective, the findings support the relevance of the theory of registers of semiotic representation as a framework for understanding students' difficulties in physics. In particular, the results highlight the challenges students face when performing treatment within a single semiotic register while maintaining the physical meaning of the represented object.

From an educational perspective, the findings suggest the need for instructional approaches that explicitly promote the interpretation and physical meaning of algebraic and graphical representations, rather than focusing exclusively on procedural competence. Strengthening the connection between mathematical representations and physical concepts may contribute to a deeper understanding of kinematics and improve students' ability to reason about motion with non-constant acceleration.

Although the study was conducted within a specific educational context, the identified difficulties may be relevant to similar university physics courses in science and engineering programs. Future research could extend this analysis to other institutions and educational settings, as well as investigate instructional interventions designed to support students in developing a more integrated understanding of algebraic and graphical representations.

Overall, the findings contribute to the growing body of research on students' understanding of kinematics and provide evidence that difficulties associated with semiotic representations remain a significant challenge in university physics education. Addressing these difficulties requires instructional practices that foster conceptual understanding, semiotic reflection, and the meaningful coordination of multiple forms of representation.

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**Ethical statement:** This study was conducted in accordance with ethical standards for research involving human participants. Written informed consent was obtained from all participants. The study was approved by the Bioethics and Biosecurity Committee of the Pontificia Universidad Católica de Valparaíso (BIOPUCV-H 647-2023).

**AI statement:** Grammarly was used exclusively for language refinement, grammar and spelling correction, and word-choice suggestions. The tool did not contribute to the study design, data collection, data analysis, interpretation of results, or scientific conclusions. All content was developed and verified by the authors, who assume full responsibility for this publication.

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