MODESTUM
EURASIA Journal of Mathematics, Science and Technology Education, 2026, 22(3), em2795
ISSN:1305-8223 (online)
OPEN ACCESS
Research Paper
https://doi.org/10.29333/ejmste/18070

# Effectiveness of online cognitive apprenticeship in developing mathematical prowess among academically outstanding secondary school students

Sabah Abdalla Elsayed [1], Mamdouh Mosaad Helali [2*]

[1] Department of Education, Dhofar University, Salalah, OMAN
[2] The National Research Center for Giftedness and Creativity, King Faisal University, Al-Ahsa 31982, SAUDI ARABIA

Received 18 October 2025 ▪ Accepted 10 February 2026

**Abstract**
The study aimed to identify the effectiveness of cognitive apprenticeship (CA) via the Internet in developing mathematical prowess (MP) among academically outstanding (AC) high school students. The research sample consisted of 32 first-year secondary students at the 10th of Ramadan School, Suez Governorate, Egypt, who exceled academically in the middle school certificate. The sample was randomly divided into two equal groups, each consisting of 16 students. The control group studied in the usual way, while the experimental group studied using the CA strategy via the Internet. The quasi-experimental design was used, and the research tools (mathematical prowess test & productive desire scale) applied pre- and post-intervention to the research groups. The most important results concluded the effectiveness of CA via the Internet in developing MP among AC high school students. The study recommended providing technical support and training for teachers to employ CA in teaching mathematics to students in general and CA students in particular.

**Keywords:** cognitive apprenticeship, Internet, mathematical prowess, high school, academically outstanding

## INTRODUCTION

Outstanding students are a national treasure in countries around the world, and if their talents are nurtured and invested in, it contributes to the progress and elevation of society. The community plays a fundamental role in discovering and nurturing these students, as they are the hope and future of the nation. Attention should be given to using modern teaching strategies to nurture them and develop their mathematical thinking. Those interested in the education of gifted and high-achieving students believe that this group possesses high mental abilities and strong motivation to learn, which necessitates the allocation of curricula and the creation of special teaching programs and strategies that consider these abilities and needs (Saada, 2009). Many studies have focused on the activities provided to gifted and high-achieving students in mathematics, such as Dial's (2011) study and Al-Hadabi et al.'s (2013). Other studies have focused on researching the effectiveness of training gifted students in problem-solving (Shweihi & Al-Shahri, 2016). Research has also examined the impact of the problem-posing approach on the mathematical ability of gifted students (Kesan et al., 2010), the effectiveness of training gifted students in problem-posing and the correlation between problem-solving and creativity in mathematics among gifted students (Sriraman et al., 2013).

The mathematics curriculum can develop a diverse range of learning outcomes for academically outstanding (AC) students. Among these outcomes is mathematical prowess (MP), which is regarded as a primary objective of mathematics education and serves as a framework for the curriculum, particularly for AC students, by emphasizing the components of MP. The elements of MP include conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition (Regan, 2012). It is also "a contemporary approach to developing mathematics education and learning", and it is linked to three main dimensions:


✉ selsayed@du.edu.om ✉ mhelali@kfu.edu.sa **(*Correspondence)**

**Contribution to the literature**

- The results showed the effectiveness of the online cognitive training strategy in developing mathematical proficiency among AC first-year high school students.
- The study used a quasi-experimental design, and the research tools (mathematical prowess test & productive desire scale) were applied before and after the intervention on the research groups.
- The study recommended providing technical support and training for teachers to employ this strategy in teaching mathematics.

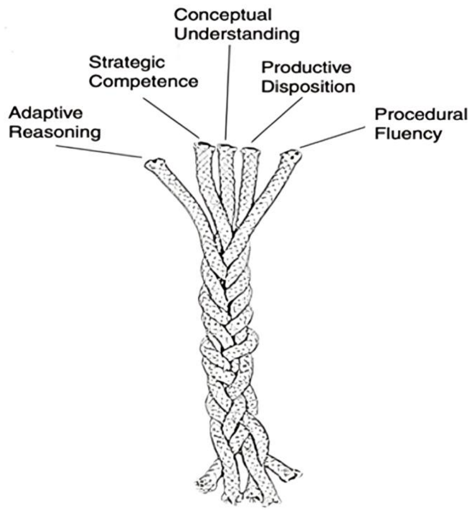


**Figure 1.** The 5 strands of MP (NRC, 2001)

(1) the scientific content in relation to each other and its importance to learners,

(2) teachers' proficiency in handling scientific content, and

(3) components of athletic prowess, and how to develop and measure them in learners (Philipp, 2010).

The term "MP" emerged in the early 21st century after the National Research Council (NRC) in the United States provided an assessment of the state of mathematics education and identified the primary goal of teaching it. It also identified five dimensions: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition (NRC, 2001) (**Figure 1**).

MP is defined as the ability to harness and utilize experience in forming mathematical knowledge and applying it to solve problems and build new knowledge so that the student can perform mathematical operations that enable them to acquire dimensions of MP (Obeida, 2017). Awofala (2017) underscore the connection between MP and the mastery of mathematical knowledge. It is an indicator of the student's excellent understanding of mathematics and is reflected in the selection of appropriate problem-solving strategies and mathematical thinking during practice. It consists of five interconnected dimensions. Groth (2017) emphasizes that the role of teachers is to help students learn mathematics efficiently, focusing on understanding the fundamentals of the subject and the importance of developing memorization and recall skills and executing algorithms so that education becomes more effective and contributes to the development of their MP. Saputro et al. (2021) view MP as the competence and ability in mathematics to explore, predict, and think logically about a problem to solve non-routine problems, communicate, and connect mathematical ideas with other intellectual activities. These are the outcomes achieved after learning mathematics, meaning the possession of mathematical skill, competence, and knowledge. They also indicated that MP enhances a student's confidence when using their mathematical knowledge to solve everyday problems, enabling them to infer new and advanced mathematical ideas and details and expand their creative scope.

Modern and diverse teaching methods have emerged to develop students' MP, among which the cognitive apprenticeship (CA) approach is one of the most prominent. The CA method relies on multiple strategies, such as modeling, scaffolding, training, reflection, and exploration, which contribute to enhancing students' understanding and effectively developing their skills, in line with the goals of competency and capability development. CA is a metacognitive educational approach and learning model that helps students acquire skills through real practices, which represent the intended goals via activities and social interactions similar to those found in learning trades and scientific professions. The strategies of the CA model help achieve diverse learning outcomes under the umbrella of the targeted subject matter and its characteristics. The results of the studies indicated that CA is an innovative approach that focuses on student understanding and ensures the possession of the intended knowledge (Al-Dmour, 2010; Austin, 2009; Ding, 2008; Kang, 2003; Poitras & Poitras, 2011). CA contributes to solidifying the contents of the curricula in the student's mind through reflection, clarification, training, and motivating their drive toward the learning process and thinking skills (Abu-Hadra, 2008; Yarkandi, 2010).

The CA strategy aims to help learners acquire various cognitive skills through active social participation and applicability in solving real problems, placing them in realistic situations that make them positively socially interactive (Rashed et al., 2018). The CA strategy relies on six stages (Brill et al., 2011; Ghefaili, 2003):

1. **Modeling:** This phase involves presenting and representing the ideas and information contained in the lesson, which are necessary for performing an educational task presented by the teacher. The learner observes the practical application of this task through the teacher's performance and how it is executed with proficiency. During training, they are encouraged to take notes and are assisted in performing their work in collaborative groups.
2. **Training:** In this stage, the teacher provides a set of educational activities that enhance the learner's level of understanding and knowledge. Then, the teacher or a qualified student aids their peers in groups with the aim of imparting the skills and abilities that enable them to continue learning independently.
3. **Interpretation:** where the learner explains the reasons for choosing the steps they used to reach their results, to deepen their understanding of the idea and concept.
4. **Expression:** This step means the learner's ability to express the results achieved either verbally or in writing, and the teacher's role at this stage is to guide and correct incorrect concepts and practices.
5. **Reflection:** Here, the teacher asks students to review their solutions after completing the educational task and compare the steps and stages of the solution with their peers. This section is where the teacher can distinguish good practices from others.
6. **Exploration:** In this phase, students learn to set different tasks from the previous ones, work on facing them, and seek knowledge independently. Consequently, the teacher concludes this phase by discovering the relationship that connects the educational tasks.

Kuhn (2012) pointed out that CA involves the four fundamental principles of the learning process, where content focuses on the transfer of authentic knowledge, including book content and domain knowledge, processing strategies, control strategies, and learning strategies. As for methods, they require providing students with the opportunity for exploration, observation, and creativity, under the supervision of teachers and with feedback, while supporting the gradual transfer of control over the learning process to the students. Sequencing is one of the fundamental principles that clarifies that education should be presented in a constructive manner, allowing the learner to build on multiple skills efficiently through diversifying problem-solving situations, developing learning to achieve comprehensive understanding, and then focusing on details.

Yarkandi (2010) identified that the four aspects of the CA strategy aim to make the teacher's thinking processes visible and enable the teacher to employ CA methods to effectively guide student learning. It also aims to enhance collaboration among students in groups for knowledge production, while promoting the idea that students learn from one another instead of depending solely on the teacher, ensuring that everyone actively participates in the learning process and contributes to better interaction and the development of their thinking skills.

MP refers to a high level of competence and skill in performing mathematical activities, and it is considered an indicator of excellence, as it requires the ability to solve problems creatively, critical thinking, planning and organization, and applying mathematical knowledge in various situations. CA via the Internet is considered one of the educational methods that rely on continuous interaction between the teacher and the learner, encouraging dialog and personal guidance with the aim of enhancing students' skills and developing their abilities individually and effectively. To the best of the researchers' knowledge, there are not enough studies that clarify the effectiveness of online CA in developing MP among AC students. Therefore, the current study aims to identify the effectiveness of web-based CA in developing MP skills among AC first-year secondary school students.

## LITERATURE

Several studies have addressed MP as one of the most important goals of teaching mathematics; the study by Hutajulu et al. (2019) found that the use of the multiple representation method led to an improvement in MP and mathematical disposition among 11$^{th}$ grade students compared to traditional methods. The study by Putri et al. (2020) found that the cooperative inquiry strategy effectively contributed to the development of MP among eighth-grade students. Jawad's (2021) study indicated that the problem tree strategy was more effective than the innovative matrix strategy in improving MP, as problem analysis allowed for a deeper exploration of concepts. As for the study by Hilal (2020), it showed that using the "GeoGebra" program as a proposed educational strategy led to the development of all dimensions of MP with a significant level of impact. Al-Badri's (2021) study confirmed that the Schwartz model was effective in improving achievement and developing MP among fourth-grade science students, while Al-Khatib's (2021) study showed that robotics mathematics unit based on the STEM approach enhances the mathematical abilities of high school students. The study by Al-Juhani (2020) demonstrated the effectiveness of the IDEAL model in developing the dimensions of MP for third-grade preparatory students, while Ahmed's (2021) study concluded that the "Jigsaw2" strategy is effective in improving mathematics achievement, developing MP, and increasing productive desire among elementary school students, with a positive relationship between achievement and the components of MP.

While other studies have addressed the CA strategy, Lee's (2019) study concluded that it helps in acquiring thinking skills and developing learners' cognitive abilities through engaging in educational activities that enable them to solve complex and ambiguous problems. Dickey (2008) found that the CA strategy increases learners' motivation to learn, provides immediate feedback to learners to process learning experiences, and enhances their skills and abilities in self-learning and problem-solving. Saadati et al. (2015) reported that the application of web-based and computer-based technologies provides additional advantages to the CA strategy in teaching mathematics and supports students' learning of mathematics. The study by Wu et al. (2016) on the effectiveness of the CA strategy in improving problem-solving skills, especially when relying on domain and collaborative knowledge. A study by Jianqi (2021) showed that computer-assisted virtual CA effectively influences the development of teaching skills and the design of integrated educational systems. The study by Sai et al. (2012) confirmed that online CA strengthens the scientific discussion skills of fifth-grade students, especially when discussing everyday life topics. Studies by Liu (2005) and Saadati (2014) have shown that the CA strategy improves the performance of both teachers and students, along with increasing their motivation and problem-solving skills. The study by Rittideche (2012) showed that students who learned through the CA model demonstrated a higher ability than their peers to perform projects and execute tasks. The study by Yusepa et al. (2018) confirmed that this CA strategy enhances abstract thinking in mathematics and that students who studied using it outperformed their peers in mathematical knowledge and abstract thinking.

## STUDY PROBLEM

Despite the significant importance of teaching strategies in enhancing critical thinking among AC students through learning mathematics, it has been observed that teachers do not sufficiently rely on modern teaching strategies when teaching mathematics to AC students. Current educational practices lack the use of interactive methods and educational innovations, such as CA and active learning, which enhance the effectiveness of developing MP, fostering creative thinking skills, and problem-solving. As a result, opportunities for developing students' intellectual and innovative abilities remain limited, necessitating the need to focus on enhancing the use of modern teaching strategies that align with the aspirations of outstanding students and contribute to the deeper development of their skills.

To understand the reality of mathematics teachers' use of modern teaching strategies, semi-structured interviews were conducted with 12 mathematics supervisors from the Educational Guidance and Supervision Office in Suez Governorate, Arab Republic of Egypt. The interview addressed two main questions:

1. To what extent do secondary school mathematics teachers in gifted classes implement modern teaching strategies?
2. What is the degree of activation of high school mathematics teachers in gifted classes for the strategy of CA via the Internet?

The interview sample confirmed that mathematics teachers do not use the Internet in teaching at schools and that 90% of the teachers are not familiar with the CA strategy via the Internet. Only a small percentage of teachers actively incorporate MP activities into their test questions. The interview sample also reported that through monitoring teachers in schools, they found that more than 70% of teachers do not prioritize activating higher order thinking skills questions with students during teaching because it requires more time and effort from the teacher.

The research problem became clear in the weak use of modern teaching strategies by teachers in classrooms, especially with AC students, which affects the development of their skills in learning mathematics. The main research question is: How effective is the CA strategy delivered via the Internet in teaching mathematics and developing MP among AC first-year secondary students?

The sub-questions of the research:

1. What is the effectiveness of the CA strategy via the Internet in developing MP for AC first-year secondary students?
2. What is the effectiveness of the CA strategy via the Internet in developing productive desires for AC first-year secondary students?

### Hypotheses

1. There is a statistically significant difference at the 0.05 significance level between the mean ranks of students in the experimental group and those in the control group after applying for the proficiency test, favoring the academically superior experimental group.
2. There is a statistically significant difference at the 0.05 significance level between the mean ranks of the experimental group students and the mean ranks of the control group students in the post-application of the productive desire scale, in favor of the academically superior experimental group.

### Research Importance

The research results contribute to improving the quality of education and encouraging students to learn in the field of mathematics, especially for AC students.

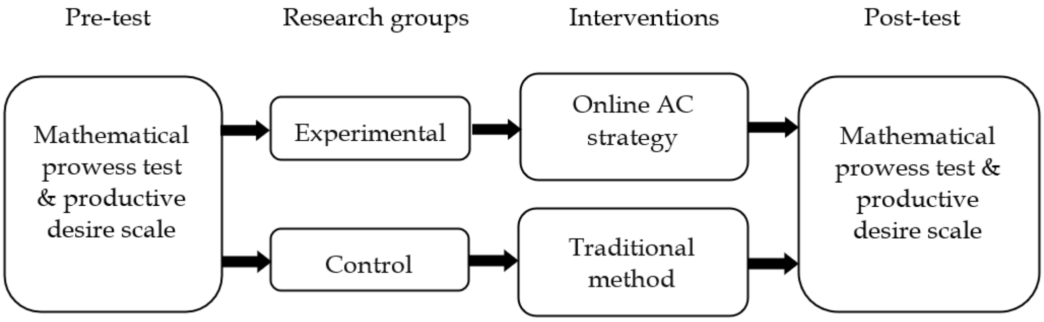


**Figure 2.** Quasi-experimental research design (Source: Authors' own elaboration)

The research highlights the importance of the CA strategy via the Internet in providing an interactive environment that transforms the learner into a central figure in the learning process. It works to increase motivation and participation levels and provides rich educational resources, which positively reflects on their academic performance in mathematics. The research also facilitates the work of teachers by providing procedural steps for modern strategies and techniques, which contribute to monitoring and developing mathematical skills and improving the academic achievement of high-achieving secondary school students.

## Research Boundaries

The subject boundaries included the "similarity" unit of the first-year secondary mathematics curriculum in the first semester and the dimensions of MP (conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition). The research was conducted in the first semester of the 2024-2025 academic year at the 10th of Ramadan Secondary School in the Suez Governorate. The program duration was two months (20/10-19/12/2024), with two sessions per week, each session lasting 45 minutes. The human subjects were limited to a purposive sample of AC first-year secondary students, with an academic average of 90% or higher in middle stage. Ethical approval for the application on living organisms was obtained from the Ethics Committee of Suez Canal University (ETH-APPR-23233-2-3-2023). Written consent was obtained from all participating students to join the program voluntarily without obligation, with everyone being given the freedom to withdraw from the program at any time, based on their personal desire. During the implementation, the Helsinki regulations for application on living beings were adhered to.

## Terms

CA: Researchers define CA as a strategy for designing active online learning via the Internet, which helps learners generate knowledge, construct meanings, and express them individually or collectively through mental and practical models and evidence. This is achieved through intensive training, mental modeling, reflection, expression, inquiry, and discovery.

MP: The NRC of the United States defines it as one of the outcomes of learning mathematics, consisting of five essential dimensions: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition, indicating successful learning of mathematics (NRC, 2001, p. 115).

Outstanding students: Al-Surour (2010) defines an outstanding student as one whose academic achievement is significantly higher than most of their peers, with an academic performance rate exceeding 90%.

AC student in the current study is defined as the student who demonstrates clear academic excellence in the first year of secondary school in the public schools of Suez Governorate, with an academic average exceeding 90%.

## Method and Experimental Design

The research relied on the quasi-experimental design of experimental research, based on the design of two groups, one of which studies the "similarity" unit from the first secondary school mathematics textbook in the first semester using the CA strategy via the Internet, while the other control group studies the same unit in the usual way. The research tools were applied to both groups pre- and post-test. The duration of implementing the program was five teaching sessions, each lasting 45 minutes. **Figure 2** illustrates the experimental design of the study.

The research population consisted of all students (105) in the gifted classrooms, first-year secondary school at the 10th of Ramadan School, Suez Governorate, Egypt. The criteria for student excellence were based on achieving a grade point average of 90% or higher in middle stage. The research sample was randomly selected, consisting of 32 students. It was divided into two groups: one experimental group of 16 students, studied the similarity unit through CA via the Internet, and control group of 16 students, studied the similarity unit using the traditional method.

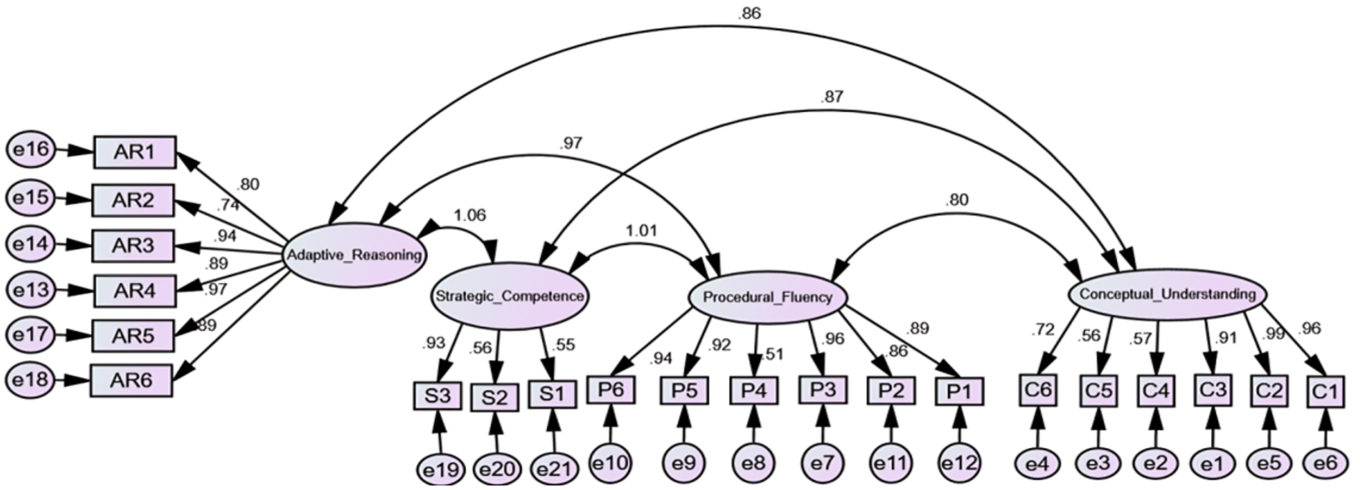


**Figure 3.** The results of the confirmatory factor analysis reveal the relationship between the test questions and their corresponding dimensions, as well as the strength of their loadings on these dimensions (Source: Authors' own elaboration, using AMOS 26)

## Research Tools

### *Mathematical prowess test*

The researchers prepared for the mathematical prowess test, and the dimensions of MP were determined by reviewing previous studies. It was found that measuring the first four dimensions of MP (conceptual understanding, procedural fluency, strategic competence, and adaptive reasoning) is done through preparing a test that measures the cognitive and skill aspects of students. As for the fifth dimension (productive disposition), it is measured through a scale of the students' affective aspect. To construct the mathematical prowess test, the researchers benefited from the studies of Hassan (2016), Hamid et al. (2020), Al-Dhani (2017), and Obeida (2017). The test was presented to a group of 10 judges in the field of mathematics education to gather their opinions and observations and to make the necessary adjustments. The scale was modified based on the judges' feedback. The percentage of agreement among the judges on the test items was 80%. The validity of the test was verified by applying it to a pilot sample of 20 students outside the study sample. Confirmatory factor analysis was conducted using the SPSS Amos program to verify the validity of the dimensions related to the mathematical prowess test, verify the internal consistency validity, and ensure that the test dimensions contribute to the structural framework of the mathematical prowess test. **Figure 3** illustrates the results of the confirmatory factor analysis for the mathematical prowess test:

**Figure 3** shows that the values of the standardized measurement weight ranged between 0.51 and 0.99; all these values are greater than 0.50, indicating that the test items are saturated with their dimensions, which indicates that the test has a high degree of internal consistency validity. Reliability was calculated by determining the reliability of each of the four dimensions and the overall test. The values of Cronbach's alpha reliability coefficient for the four dimensions ranged between 0.86 and 0.72, and for the entire test, it was 0.84, which is an acceptable rate. After verifying the validity and reliability of the test, the time required to administer the test was calculated using the formula for calculating the average test duration. The time taken was measured from when the first student finished answering to when the last student finished answering, and the average between them was calculated, which was 60 minutes. The final valid version of the test consisted of 21 items.

### *Productive desire scale*

The researchers prepared the productive desire scale for mathematics after reviewing previous studies, such as Al-Masrouh (2012), Awofala (2017), and Al-Omari (2019). Thirty statements were formulated for the productive desire scale for AC first-year secondary school students. The scale was presented to 10 specialized judges in psychology and sports to ensure its suitability for AC first-year secondary students. The modifications were made based on the agreement of 80% of the judges. Cronbach's alpha reliability coefficient for the scale was 0.93, which is an acceptable value for reliability.

### *Teaching with the CA strategy via the Internet*

Because there was no existing website that effectively met the objectives of this research, a new website was created to support the research using Google Classroom, following a review of educational website design models available online. After determining the content of the similarity unit, a plan for the progression of the unit topics was created, which the teacher follows. Some CA models were utilized to enhance learning and teaching (Brandt et al., 1993; Collins et al., 1989; Farmer et al., 1992). Steps for the CA strategy via the Internet were proposed, which are as follows:

**Table 1.** Mann-Whitney U test of the difference between the mean score of the control group and the experimental group in the MP and the productive desire scale pre-test

| | Group | N | Mean rank | Mann-Whitney U test | z | p |
|---|---|---|---|---|---|---|
| Mathematical prowess test | Control | 16 | 18.75 | 92 | 1.36 | 0.184 |
| | Experimental | 16 | 14.25 | | | |
| Productive desire scale | Control | 16 | 15.06 | 105 | 0.877 | 0.402 |
| | Experimental | 16 | 17.94 | | | |

1. **Modeling:** In this stage, the teacher presents an electronic lesson (video or presentation) through the website and discusses some activities with the students to clarify the lesson.
2. **Training:** The teacher asks the students to solve the lesson activities within the specified time, activating a timer to announce the end of each activity session. The teacher asks the students to extract the conclusions after discussing the activity collaboratively. The teacher moves among the students during the activity, commenting on their contributions, intervening to aid, encouraging role exchange among students, and ensuring everyone participates in the discussion forum. This process is done through student discussion forums on the website simultaneously while the students are in the classroom.
3. **Reflection:** The teacher guides students to reflect on their performance while monitoring the groups in the discussion forums during the activities.
4. **Clarification:** When the activities are completed, each group simultaneously presents the steps to solve the activities and posts the solution steps in the main forum. After each group presents its final solution, the teacher and students provide feedback. The teacher helps the students write the correct solution for each activity in clear and proper language and then assigns an individual task to ensure that everyone has understood the lesson.
5. **Discovery:** The teacher assigns students to work in groups asynchronously on a project that involves solving activities. The teacher explains the project idea to the students and then holds sessions for consultation and discussion about the type of presentation each group wishes to create (video, presentation, concept map). The teacher explains that each group must define their project and the tools they will use. In the end, each group presents a detailed report on their project, including the reason for its selection, the tools used, the method of work, and the project's name.

## RESULTS

During the first week of the first semester of the 2024-2025 academic year, a pre-test was conducted for both the mathematical prowess test, and the productive desire scale on both the experimental and control groups to ensure equivalence between the two groups before the implementation of the teaching program. **Table 1** shows the results of MP and the productive desire scale pretests for both the experimental and control groups, indicating the equivalence of the two groups.

After that, the similarity unit from the mathematics book was taught to both groups. The teaching process for the experimental group was conducted using the teacher's guide prepared by the researchers according to the online CA strategy, while the control group was taught using the traditional method. The teaching continued for both groups for a duration of two months, with two sessions per week, each session lasting 45 minutes. After completing the teaching, the mathematical prowess test and the productive desire scale were administered again to all students, with the aim of measuring the impact of the online CA strategy on the development of MP skills and productive desire and verifying the effectiveness and impact resulting from the use of this strategy.

### Results of the First Question

The first hypothesis of the research was validated: "There is a statistically significant difference between the mean ranks of the experimental group students and the control group students in the post-application of the mathematical prowess test in favor of the academically superior experimental group students." Mann-Whitney test was used, where the average ranks of the experimental group students' scores and the average ranks of the control group students' scores were calculated. **Table 2** presents the value of U, the value of Z, and their statistical significance for the difference between the mean ranks of the experimental group students and the control group students in the post-application of the mathematical prowess test.

**Table 2** shows a statistically significant difference at the 0.01 level between the mean ranks of the experimental group students and the mean ranks of the control group students in the post-application of the mathematical prowess test, in favor of the academically superior students in the experimental group.

### Results of the Second Question

The second question addresses the effectiveness of the CA strategy delivered via the Internet in fostering productive desire among AC first-year secondary school students.

**Table 2.** Mann-Whitney U test of the difference between the mean score of the control group and the experimental group in the MP post-test

| Group | N | Mean rank | Mann-Whitney U test | z | p |
|---|---|---|---|---|---|
| Control | 16 | 8.81 | 5 | 4.65 | 0.000 |
| Experimental | 16 | 24.19 | | | |
| Total | 32 | | | | |

**Table 3.** Mann-Whitney U test of the difference between the mean score of the control group and the experimental group in the productive desire scale post-test

| Group | N | Mean rank | Mann-Whitney U test | z | p |
|---|---|---|---|---|---|
| Control | 16 | 8.50 | 0 | 4.83 | 0.000 |
| Experimental | 16 | 24.50 | | | |
| Total | 32 | | | | |

The second hypothesis of the research was verified: "There is a statistically significant difference between the mean ranks of the experimental group students and the mean ranks of the control group students in the post-application of the productive desire scale in favor of the AC experimental group students." Mann-Whitney test was used, where the mean ranks of the experimental group students' scores and the mean ranks of the control group students' scores were calculated. **Table 3** displays the value of U, the value of Z, and their statistical significance for the difference between the mean ranks of the experimental group students and the control group students in the post-application of the productive desire scale.

**Table 3** shows a statistically significant difference at the 0.01 level between the mean ranks of the experimental group students and the control group students in the post-application of the productive desire scale, in favor of the academically superior students of the experimental group.

## DISCUSSION

The results of **Table 2** indicate the effectiveness of the CA strategy via the Internet in developing the MP of AC first-year secondary school students. This conclusion is because the strategy of CA via the Internet is based on the interaction between the teacher and the learner, allows for a significant amount of positive participation in the learning process, and makes the student responsible for the learning process. The learner's reflection on their learning process also involves working to improve their level of understanding and performance. The CA strategy offers opportunities for individual practice through worksheets and collective practice of various skills via the forums provided by the educational site, contributing to an increase in the MP of AC students. On the other hand, group discussions through forums and reviewing presentations have helped in exchanging knowledge, skills, and experiences among students, thereby enhancing their MP. Interactive online learning is also gaining wide popularity among students nowadays, as they find it enjoyable and engaging, and it helps them acquire more educational skills and considers individual differences. It can be said that knowledge apprenticeship via the Internet allows the student to integrate into the learning environment, which in turn helps achieve learning objectives and develops students' athletic prowess. The results of **Table 1** agreed with the findings of the studies by Al-Obaid (2021) and Al-Rifai (2019) regarding the effectiveness of CA in enhancing students' learning efficiency. It also aligns with the study by (Saadati et al., 2015; Sai et al., 2012; Wu et al. 2016; Yusepa et al., 2018), which concluded the effectiveness of electronic CA in developing problem-solving skills, self-regulation, and discussion skills, and differs from the study by Al-Jabr (2013), which concluded the ineffectiveness of the electronic CA model in developing the ability to solve mathematical problems.

**Table 3** shows that the CA strategy implemented online effectively fosters a productive desire for learning among AC first-year secondary school students. This is explained by the diversity of educational methods in CA through the Internet, which meets the needs of students and their different learning styles, thereby increasing their desire to learn mathematics. It also highlights the components of the CA strategy via the Internet, including self-directed learning, which helped students feel responsible for their own education and confidence in their abilities. CA includes a reflection phase that increases the interest of AC students. And their attention and thinking, and the sequence of activities that increase diversity, positive reinforcement, and feedback, which played an effective role in learning the subject and increasing productive desire, and what CA provided in terms of support methods through the training phase, which increased student satisfaction and enhanced their love for learning mathematics. This strategy also aligns with constructivist theory, which asserts that effective learning occurs when learners construct their own knowledge through interaction, thus explaining the improved outcomes of students who learned using this strategy. The results of **Table 3** are consistent with Al-Jabr's (2013) study, which concluded the effectiveness of the electronic CA model in enhancing motivation toward learning mathematics. They also align with

(Ahmed, 2021; Al-Badri, 2021; Helal, 2020; Ibn-Merdah & Suleiman, 2019; Jawad, 2021; Khattab, 2021; Putri et al., 2020), which emphasized the importance of applying diverse teaching strategies and recognizing their effectiveness on dimensions of MP, including productive desire.

## CONCLUSIONS

The study results indicate that AC students need a dynamic and renewed environment that keeps pace with modern educational developments. CA provides outstanding students with the opportunity to interact with rich and diverse knowledge sources, in addition to building a network of knowledge relationships with their peers and professors on a global level. The use of digital platforms enhances students' ability for self-directed learning and stimulates critical thinking and creativity.

Recommendations: The study recommends adopting innovative educational strategies, such as online CA, to achieve quality education and enhance students' academic performance. The study emphasizes the significance of engaging teachers in the creation and execution of contemporary educational strategies. Training teachers to use effective technological tools and implementing them in a way that captivates interest and motivates students to engage in learning is crucial. Organizing workshops and online discussion groups is crucial to fostering the exchange of ideas, encouraging initiative, and foster problem-solving among students. Educational institutions must embrace innovative teaching strategies and provide sufficient technical and educational support to enable exceptional students to reap the benefits of modern learning methods.

**Author contributions: MMH:** conceptualization, methodology, writing–original draft, writing–review & editing, and project administration & **SAE:** conceptualization, methodology, validation, investigation, data curation, visualization, writing–original draft, writing–review & editing, formal analysis, and resources. Both authors agreed with the results and conclusions.

**Funding:** This study was supported by the Deanship of Scientific Research, Vice Presidency for Graduate Studies and Scientific Research, King Faisal University, Saudi Arabia (grant number 253721).

**Acknowledgments:** The authors would like to thank the Deanship of Scientific Research at King Faisal University, Al-Ahsa 31982, for providing the research fund for publishing research grant (KFU253721).

**Ethical statement:** The authors stated that the study was approved by the Ethics Committee at Suez Canal University on 2 March 2023 with approval number ETH-APPR-23233-2-3-2023. Written informed consents were obtained from the participants.

**AI statement:** The authors stated that no AI tools were used in the writing, analysis, or preparation of this manuscript.

**Declaration of interest:** No conflict of interest is declared by the authors.

**Data sharing statement:** Data supporting the findings and conclusions are available upon request from the corresponding author.

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