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Effects of Polya Questioning Instruction for Geometry Reasoning in Junior High School

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In teaching geometry, most instructors opt for direct demonstration with detailed explanations; however, under this kind of instruction students face considerable difficulties in the development of the reasoning skills required to deal with problems of a geometric nature. This study adopted a nonequivalent pretest-postest quasi-experimental design employing Polya's approach of four-stage problem solving using question prompts in conjunction with multimedia demonstration. Two classes of grade 7 students were randomly selected as the experimental group receiving instruction based on Polya questioning and two others were selected as the control group receiving instruction based on direct presentation. Our results revealed that the posttest performance in geometry reasoning of students receiving instruction based on Polya questioning was superior to that of students receiving direct presentation. In addition, students receiving instruction based on Polya questioning expressed a stronger sense of participation in the course than did students receiving direct presentation.

Keywords: geometry reasoning, problem solving, questioning, multimedia learning, course feeling

INTRODUCTION

Geometry instruction and multimedia learning

More than two thousand years ago, the King of Egypt asked Euclid: "Could you make geometry easier to learn?" To this day, this remains a common question among teachers and students throughout the world. The study of geometry is among the most challenging subjects in the field of mathematics. In November 2003, the Ministry of Education in Taiwan outlined a mathematical field guide for the curriculum of grades 1-9, dividing the content of mathematical learning into the following: numbers and quantity, connections, algebra, statistics and probability, and geometry. Within the field of geometry, the focus is on plane geometry (Ministry of Education, 2003). A huge difference exists in the instruction methods used to

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State of the literature

- Numerous studies have demonstrated the effectiveness of visual thinking in geometry instruction.In the process of problem solving when the problem solving steps which Polya suggested are carried out successfully and efficiently, the students' problem solving skills and achievements improve significantly.
- Questioning prompts have been proven an effective means of scaffolding the high-level thought processes of students in a range of disciplines.
- Polya questioning instruction is a teacher-centered method meant to prompt students to elucidate their problem-solving ideas as well as the graphical relationships observed in the lesson.

Contribution of this paper to the literature

- Conducting questioning prompts with the four stages of problem solving provides students with greater opportunity for reflection, which leaves them more time to focus on problem-solving activities.
- Students receiving Polya questioning instruction had more opportunities to solve problems on their own and therefore demonstrated a stronger willingness to participate in the lessons.
- This study presents a framework of Polya questioning instruction based on four stages of problem solving and the theory of questioning prompts, tailored specifically for the instruction of geometry reasoning.

develop reasoning skills required for plane geometry in junior high school and the experimental manipulation and intuition employed in elementary schools. Many students find it difficult to progress beyond the stage of "seeing is believing" due to obstacles in using more formal reasoning skills (Duval, 2006). Helping students to apply reasoning skills to problems of geometry requires that instructors develop the means to enhance the effectiveness of geometry instruction.

Numerous studies have demonstrated the effectiveness of visual thinking in geometry instruction. Fuys and Geddes (1984) found that most students usually begin with visual thinking when learning new concepts of geometry. Hoffer (1977) claimed that a significant interaction exists between the learning of geometry concepts and improvements in visual perception, and therefore enhancing visual perception can facilitate the learning of geometry concepts. Computer-generated images can stimulate visualization, making it a beneficial tool for instruction (Bishop, 1989). Clements and Battista (1992) also recommended the adoption of computer simulations to assist in learning geometry. Many studies have demonstrated that multimedia presentations can improve the learning performance of students beyond what are possible using traditional methods (Liao, 2007). The appropriate use of multimedia can create scenarios in which students are prompted to manipulate geometric figures, thereby promoting the development of mental imagery to function as a scaffold for the learning and development of geometry concepts (Yuan, Lee & Huang, 2007).

Inappropriate teaching methods using multimedia or electronic materials can cause perceptual overload resulting in a failure to grasp course content. Courses should be based upon the theoretical principles associated with multimedia learning to maximize learning effectiveness (Mayer, 2009). Mayer pointed out that a good multimedia-assisted learning system can help learners to develop three important forms of information processing: (1) Selection: Learners select text or images to establish a database of text memory and image memory. (2) Organization: During the processing of information, learners organize text and images in their short-term memory into a coherent entity referred to as a "situational model", either verbal or pictorial. (3) Integration: Verbal and pictorial situational models are then linked with prior knowledge stored in the long-term memory. This overall process is outlined in Fig. 1.

According to multimedia learning theory, employing two discrete information processing units during information reception can help to improve learning performance. Both types of working memory are limited in their capacity; however, multimedia systems can be used to leverage the working memory through the processes of selection, organization, and integration.

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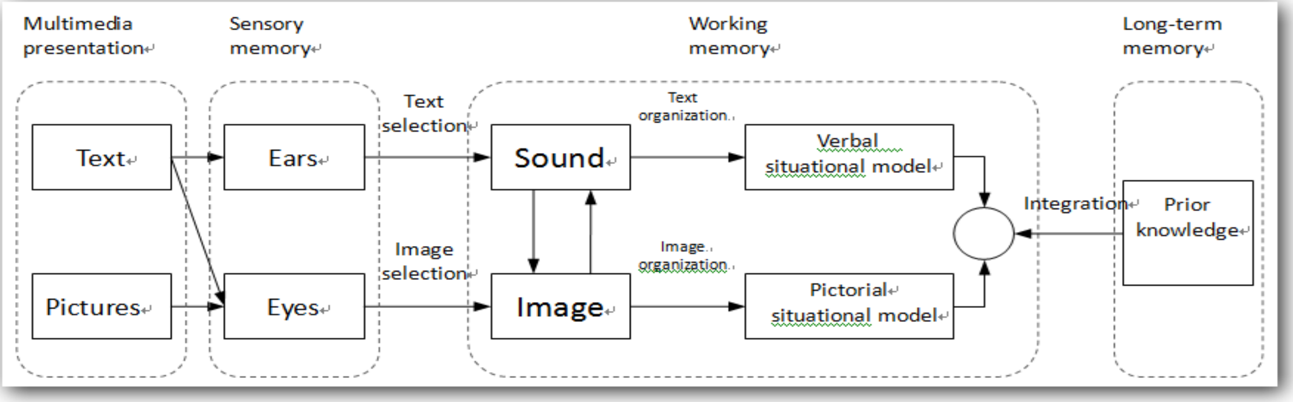

Figure 1. Cognitive model of multimedia learning
Source of data: modified from Multimedia Learning (2nd ed.) (p.61), by R. E. Mayer, 2009, New York: Cambridge University Press.

The main stumbling block in the design of teaching materials is the limited capacity of working memory. Ill-conceived teaching materials, excessive complexity in explanations, and instruction delivered without motivation can all result in cognitive overload among learners (Mayer, 2009). To avoid these issues, Mayer proposed twelve design principles for multimedia instruction. Course design can be facilitated by adhering to the principles of coherence, signaling, redundancy, spatial contiguity, and temporal contiguity. Complexity can be dealt with by adhering to the principles of segmenting, pre-training, and modality. Finally, motivation can be enhanced by incorporating multimedia, personalizing lessons, employing voice control, and providing images.

This study adopted the principles of multimedia learning (Mayer, 2009) in the design of instructional materials. We also employed the AMA (Activate Mind Attention) presentation system to facilitate the assessment of important information by students. The AMA presentation system is outlined in the following paragraph.

Introduction of AMA

AMA (Activate Mind Attention) is a well-known software program commonly used in mathematics instruction. It provides an environment for media design and demonstration developed on the PowerPoint platform (Chen 2008; Lee & Chen, in press). It can be downloaded for free at the following website: http://ama.nctu.edu.tw/index.php. The core functions of this software include structural cloning method (SCM) and trigger-based animation (TA). SCM was originally developed to overcome the problem of positioning during the design of teaching materials; however, it can also be used for the creation of figures, including landscapes, complex symmetrical compositions, and light spot series. TA uses objects as buttons to control a range of dynamic animations. TA can help the presenter to demonstrate digital content, hold the attention of the audience, guide cognitive processes in observers, and reduce cognitive load. A number of features can be demonstrated in teaching materials based on TA: (1) Triggering attention: triggering buttons can be used to highlight primary information and eliminate extraneous information. (2) Segmentation: TA allows the segmentation of teaching materials in accordance with information capacity and the relationship between previous messages and those to come. (3) Combination: Segmented messages can be grouped or ungrouped using a range of triggers. Messages can be included in different groups, which are controlled by different triggers. The main objective is to guide the learning experience and reduce cognitive load. (4) Flexible triggering: Messages can be triggered in a preset sequence, a selected sequence, or randomly. (5) Flow: The speed with which information is presented is controlled entirely by the presenter. (6) Interactivity: Instructors and students can interact through the presentation of teaching materials. (7) Adaptive teaching: The presenter controls the content as well as the sequence of the information presented, in accordance with the progress of the lesson.

AMA does not require coding knowledge, making it easy for anyone to use in the creation and presentation of sophisticated teaching materials.

Mathematics problem solving and learning by questioning prompts

Teaching strategies are another important factor affecting learning performance (Lee & Chen, 2008; Lee & Chen, 2009). Problem-solving strategies make up a large proportion of the studies on mathematics and science education (Crippen & Earl, 2007; Mayer, 1992). Dufresne, Gerace and Leonard (1997) modeled problem-solving processes according to three types of knowledge: (1) conceptual knowledge, (2) operational or procedural knowledge, and (3) problem-state knowledge. In this model, the conceptual knowledge of experts is gathered and arranged hierarchically to form a strong bilateral link with the other two types of knowledge. Conceptual knowledge is generally difficult for novice learners to grasp, as evidenced by the fact that learners tend to arrange it chronologically, resulting in linkages with the two other kinds of knowledge in a weak, misconstrued, or uni-lateral manner.

Schoenfeld (1992) proposed a number of factors that influence the problem-solving ability of students. The resulting framework provides four components used to explain the important aspects of problem solving: (1) Resources: facts, definitions, procedures, rules, and intuitive understanding; (2) Heuristics: problem-solving strategies and techniques; (3) Control: the methods used to monitor one's own problem-solving processes. Likewise, one may observe partial results to determine the next step in the problem-solving sequence, thereby making use of the available resources and strategies; (4) Beliefs: beliefs regarding the nature of mathematics and mathematics-related tasks. Garofalo and Lester (1985) confirmed the existence of four stages in the problem-solving process and explained the metacognitive behaviors involved in performing mathematical tasks, including orientation, organization, execution, and verification. These are quite similar to the four problem-solving steps introduced by Polya (1957): (1) Understanding the problem; (2) Developing a plan; (3) Executing the plan; (4) Examination and review.

In the process of problem solving when the problem solving steps which Polya suggested are carried out successfully and efficiently, the students' problem solving skills and achievements improve significantly. For example, Karatas and Baki (2013) created a problem solving based learning environment to enhance students' problem skills according to Polya's problem solving phases. While experimental group students received problem solving based learning environment, control group students have continued their present program. The findings illustrated that the experimental group students' success in problem solving activities has increased while the control group students' success has not change significantly. Chang, Sung, and Lin (2006) proposed a computer-assisted system named MathCAL, the design of which was based on the Polya's four problem solving steps. 130 fifth-grade students completed a range of elementary school mathematical problems. The results showed that MathCAL was effective in improving the performance of students with lower problem solving ability.

Huang, Liu, and Chang (2012) also developed a computer-assisted mathematical problem solving system in the form of an online instruction website designed according to Polya's problem solving stages. This system was used to help low-achieving second- and third-graders in mathematics with word-based addition and substraction questions in Taiwan. The results indicated that the computer-assisted mathematical problem solving system can serve effectively as a tool for teachers engaged in remedial education. Therefore, our study combined Polya's four problem-solving steps with questioning by the instructor to guide students through the process of geometric reasoning and then examined learning performance.

Problem solving requires that students execute high-level reasoning skills in conjunction with intensive learning support, such as demonstration, modeling, and scaffolding (Johnassen, 1997, 1999). Questioning prompts have been proven an effective means of scaffolding the high-level thought processes of students in a range of disciplines (Scardamalia, Bereiter, McLean, Swallow & Woodruff, 1989; Scardamalia, Bereiter & Steinbach, 1984). These prompts also encourage students to engage in self-explanation (Chi, Lewis, Peimann & Glaser, 1989), self-questioning (King, 1991, 1992), and self-monitoring and self-reflection (Lin, 2001). These activities can help learners refine their thinking, make inferences, and most importantly, monitor and assess their own learning processes (Lin, Hmelo, Kinzer & Secules, 1999). Questioning prompts include procedural prompts, elaboration prompts, and reflection prompts. Different types of prompts are suitable for different cognitive and metacognitive purposes, such as writing (Scardamalia, Bereiter & Steinbach, 1984) or problem solving (King, 1991). Procedural prompts have been successfully used to help learners learn cognitive strategies in specific fields (Rosenshine, Meister & Chapman, 1996). Elaboration prompts remind learners to express their thoughts clearly through explanations. Reflection prompts encourage reflection at the level of metacognition to cover some situations that students often ignore (Davis & Linn, 2000).

It is crucial that geometry teachers learn how to provide hints through questioning prompts, in order to guide the thought processes of students and to help them build knowledge-linking connections by themselves. This study combined the four stages of problem solving with questioning prompts as a framework from which to develop the reasoning required to solve geometry-related problems, as outlined in Appendix 1.

Polya questioning instruction is a teacher-centered method meant to prompt students to elucidate their problem-solving ideas as well as the graphical relationships observed in the lesson. The process of verbal guidance can enhance learning motivation and activate thinking, leading to the development of stronger conceptual links and deeper understanding. In this study, the process of guiding students included the following: (1) providing motivation, understanding the question, understanding the requirements to deal with this question, understanding the graphic relationships, discovering the correlation between figures; (2) segmenting the questions, carefully exploring the information, and emphasizing the linkage in information.

The purpose of this paper

Based on aforementioned motivation and background, the main questions in this study are as follows:

1. Is there an interaction between teaching strategy (Polya questioning instruction vs. direct presentation instruction) and prior knowledge (high vs. low) as evidenced by performance in learning geometry concepts?
2. Is there an interaction between teaching strategy (Polya questioning instruction vs. direct presentation instruction) and prior knowledge (high vs. low) manifested in evaluation of the geometry course by students?

METHODOLOGY

Procedure

This study adopted a non-equivalent group pretest and posttest experimental design. Teaching materials suitable for the teaching of geometry teaching were designed using presentation software in combination with AMA. A two-way factorial design was adopted to investigate the impact of Polya questioning instruction versus direct presentation instruction as well as prior knowledge (high vs. low) on the effectiveness, delayed effectiveness, and opinions of students regarding the course.

Pretests were administered to all students at the beginning of the teaching experiment. Polya questioning instruction was used with the experimental group and direct presentation instruction was used with the control group. Following the completion of classes, both groups completed post-tests and questionnaires about the class. Participants in both groups filled out delayed post-tests one month later.

Participants

Convenience sampling was used in the selection of four grade 7 classes from a junior high school in Miaoli County, Taiwan. Two of the classes were randomly selected as the experimental group and the other two classes were designated the control group. An independent sample t-test was used to obtain the average score of three regular mathematic assessments in the first semester as well as the pretest scores of both groups. The results showed no significant difference in scores between the experimental and control groups. These two groups are therefore deemed to have equivalent prior knowledge of mathematics.

To understand the impact of prior knowledge on teaching strategy, the research participants were categorized into a group with high prior knowledge (scores in the top 50%) and low prior knowledge (scores in the bottom 50%), according to the mean of average scores of the three regular assessments.

Instruments

Teaching materials

This study developed the teaching materials used in the experiment using PowerPoint 2003 with AMA plug-ins. The design of the curriculum was meant to address misconceptions of students as well as their possible responses. This enabled the instructor to adjust the content dynamically according to the reaction and answers of students during the courses. The aspects of geometric reasoning targeted in this experiment included the following: (1) understanding and defining points, lines, and angles in geometric figures, (2) understanding the basic properties of angles, (3) solving problems involving the summing of interior angles, the summing of exterior angles, and the theorem of polygons, and (4) compiling geometric properties as a basic step in geometric reasoning.

The scope of the course was the same for both groups; however, there was a slight difference in the teaching strategies, as shown in the Appendix 2. The teaching materials used in both groups were designed in accordance with the principles of multimedia learning. In addition, three senior mathematic teachers and two experts in mathematic education provided suggestions for modification. Pre-courses were conducted with 51 students in two grade 7 classes in the same school as the participants in the experiment, and reactions to the course were collected as the basis for modification and adjustment of teaching materials.

Geometry reasoning pretest

The purpose of this test was to gauge the knowledge of students prior to the experiment. Because the research participants were students in grade 7, the influences of pre-class tutoring or pre-learning was minimized. A Cronbach α of 0.879 verified the internal consistency among question items. The test was also reviewed and corrected by two professors in relevant fields and three senior mathematic teachers in junior high schools, thereby ensuring good expert content validity.

Learning achievement test

The purpose of this test was to gauge the knowledge of students after the experiment. The same achievement test was used for both the posttest and delayed posttest, conducted one month after the implementation of posttest. The total score of this test was 55, divided into two sections: 5 calculation questions worth five points each and five reasoning questions worth 6 points each. The reliability coefficient of the test was 0.916, indicating good internal reliability. In addition, four mathematics instructors with over 10 years of teaching experience and two members of the mathematics counseling group of Miaoli County attested to the validity of the test. The difficulty coefficients of this test fell between 0.44 and 0.55, and the discrimination coefficients were between 0.73 and 0.99, both therefore in compliance with the standard.

Course evaluation questionaire

Following the completion of the course, a questionaire was administered to gauge the impression of participants regarding the course content and learning process. This study modified the scale developed by Tai-Yee Zhuo et al. (2011). The reliability coffecient of this questionaire was 0.79, which falls within the standardly acceptable range. The five dimensions of this questionaire included participation willingness, degree of difficulty, mental effort, degree of understanding, and invested effort.

Data Analysis

Two-way factorial analysis of covariance was adopted to analyze the interaction between prior knowledge and teaching strategy on posttest performance, delayed posttest performance, and impressions of the course using the pretest score as the covariance, prior knowledge (high vs. low) and teaching strategy (Polya questioning instruction vs. direct presentation instruction) as independent variables, and posttest scores, delayed posttest, and course evaluation as dependent variables. One important assumption for the ANCOVA was checked before the analysis proceeded. The test for homogeneity of regression coefficients of the covariate for different levels of prior knowledge and types of teaching strategy was not significant. Therefore, it would be appropriate to conduct the two-way analysis of covariance. If the interaction between prior knowledge and teaching strategy is significant, then the simple main effects should be examined for both prior knowledge and teaching strategy. If not, then the main effects for each factor (prior knowledge and teaching strategy) would be conducted respectively.

RESULTS

Analysis on learning achievements

Posttest analysis

As shown in Table 1, the adjusted mean (31.74) of the posttest performance in geometric reasoning following Polya questioning instrtuction was higher than that of direct presentation instruction (M=21.43). The adjusted mean (35.85) posttest performance of the group with high prior knowledge was higher than that of the group with low prior knowledge (M=16.57).

Table 1. Summary of adjusted means of posttest performance according to teaching strategies and prior knowledge

Teaching strategy / Prior knowledge		Polya questioning (58)	Direct presentation (61)	Sum (119)
High level (61)	**(M)**	41.14	31.06	35.85
	(SD)	12.4552	16.473	15.437
	(N)	29	32	61
Low level (58)	**(M)**	22.34	10.79	16.57
	(SD)	16.821	12.653	15.861
	(N)	29	29	58
Sum (119)	**(M)**	31.74	21.43	26.45
	(SD)	17.464	17.863	18.341
	(N)	58	61	119

Table 2. Summary of adjusted means of posttest performance with prior knowledge and teaching strategy

Teaching strategy / Prior knowledge		Polya questioning (58)	Direct presentation (61)	Sum (119)
High level (61)	**(M)**	38.03	22.91	30.10
	(SD)	12.571	15.350	15.922
	(N)	29	32	61
Low level (58)	**(M)**	12.24	7.45	9.84
	(SD)	13.848	12.443	13.270
	(N)	29	29	58
Sum (119)	**(M)**	25.14	15.56	20.23
	(SD)	18.468	15.956	17.814
	(N)	58	61	119

The tests for homogeneity in the regression coefficients of the covariate for different teaching strategies and levels of prior knowledge were not significant (F=1.706, p=.194), suggesting that a common regression coefficient was appropriate for the covariance portion of the analysis. After excluding the impact of the pretest, the interaction between the posttest performances using different teaching strategies and prior knowledge of geometry reasoning did not reach significance (F=.157, p=.692). By observing the main effect of each factor, the main effect of teaching strategy reached significance (F=22.539, p<.001, η^2=.165), indicating that the posttest performance following Poyla questioning instruction was superior to direct presentation instruction. The main effect of prior knowledge also reached signficance (F=4.265, p=.041, η^2=.036), indicating that the posttest performance of students with high prior knowledge was better than that of students with low prior knowledge.

Analysis of delayed posttest

As shown in Table 2, the adjusted mean (25.14) of the delayed posttest performance following Polya questioning instruction was higher than that of direct presentation instruction (M=15.56), and the adjusted mean (30.10) of posttest performance of the group with high prior knowledge was higher than that of the group with low prior knowledge (M=9.84).

Tests for homogeneity in the regression coefficients of the covariate for different teaching strategies and levels of prior knowledge were not significant (F=3.762, *p*=.055), suggesting that a common regression coefficient was appropriate for the covariance portion of the analysis. After excluding the impact of the pretest, no significant interaction was observed in the delayed posttest performance between different teaching strategies or prior knowledge (F=6.814, p=.010, η^2=.056). Therefore, the simple main effects were tested for each factor.

As shown in Table 3, among students with high prior knowledge, the delayed test performance of students receiving Polya questioning instruction (M=38.03) was superior to that of students receiving direct presentation instruction (M=22.91). However, among students with low prior knowledge, no significant difference was observed in delayed test performance between students receiving Polya questioning instruction (M=12.24) and those receiving direct presentation instruction (M=7.45).

Among students who received Poyla questioning instruction, those with high prior knowledge (M=38.03) had better delayed test performance than did those with low prior knowledge (M=12.24). However, among students who received direct presentation instruction, no significant difference was observed in the delayed test performances between students with high prior knowledge (M=22.91) and those with low prior knowledge (M=7.45).

Analysis of course evaluation questionnaire

The two-way factorial analysis of covariance was conducted using the pretest as the covariance, teaching strategy and prior knowledge as independent variables, and course evaluation as dependent variables. After excluding the impact of the pretest, the interactions between teaching strategy and prior knowledge did not reach significance in any of the dimensions of course evaluation (participation willingness F=.211, p=.647, degree of difficulty F=.021, p=.885, mental effort F=.040, p=.842, degree of understanding F=1.333, p=.251, and invested effort F=.120, p=.730). By observing the main effects of each factor, it was found that teaching strategy only reached significance with its main effect on participation willingness (F=10.472, p=.002), indicating that students who received Polya questioning instruction (M=5.47) were more willing to participate than those who received direct presentation instruction (M=4.51). Prior knowledge only reached significance with its main effect on degree of understanding (F=4.721, p=.032), indicating that students with high prior knowledge (M=5.57) had a better understanding of the material than did those with low prior knowledge (M=4.31).

DISCUSSION AND CONCLUSIONS

The purpose of this study was to evaluate the effectiveness of Polya questioning instruction and prior knowledge on posttests, delayed posttests, and students' impressions of the course in geometry reasoning. Students who received Polya

Table 3. Summary of two-way factorial analysis of covariance of prior knowledge and teaching strategy with respect to simple main effects of the delayed posttest

Source of variance	SS	df	MS	F	Sig	Posterior comparison
Prior knowledge						
with high prior knowledge	2968.290	1	2968.290	25.082	.000	Polya questioning＞Direct presentation
with low prior knowledge	231.168	1	231.168	2.129	.150	Polya questioning＝Direct presentation
Teaching strategy						
with Polya questioning instruction	2131.701	1	2131.701	17.175	.000	High prior knowledge > low prior knowledge
with direct presentation instruction	12.109	1	12.109	.119	.732	High prior knowledge = low prior knowledge

questioning instruction outperformed those who received direct presentation instruction, indicating that conducting questioning prompts with the four stages of problem solving provides students with greater opportunity for reflection (Lee & Chen, 2009), which leaves them more time to focus on problem-solving activities. In this manner, Polya questioning instruction provides an effective learning scaffold for the instruction of geometry. Students with high prior knowledge outperformed those with low prior knowledge in the posttest, indicating that teachers should consider prior knowledge in the design of teaching activities in geometry courses.

Among students with high prior knowledge, those who received Polya questioning instruction outperformed those who received direct presentation instruction in the delayed posttest; however, a significant difference was not observed among students with low prior knowledge. One reason may be that students with high prior knowledge have richer links with regard to mathematical knowledge, which causes them to receive more questioning prompts during Polya questioning instruction, compared to students receiving direct presentation instruction. Questioning prompts help students with high prior knowledge to extract more problem-solving clues for integration and reasoning skills. A lack of links in the mathematical knowledge of students with low prior knowledge prevents the integration of reasoning skills, even upon receipt of questioning prompts. As a result, no significant difference was observed between these two teaching strategies with regard to the delayed effects of instruction to promote geometry reasoning.

Among students receiving Polya questioning instruction, those with high prior knowledge outperformed those with low prior knowledge in the delayed posttest. However, among students who received direct presentation instruction, no significant difference was observed between students with high and low prior knowledge in the performances on the delayed posttest. It is possible that the hints provided by Polya questioning instruction helped the students with high prior knowledge to connect with this knowledge, thereby enhancing the delayed effects of instruction. Direct presentation instruction provides explanations, largely disregarding the process of reflection. Therefore, even students with high prior knowledge and more developed mathematical knowledge links; they did not have the chance to receive teachers' questioning prompts.

Students receiving Polya questioning instruction had more opportunities to solve problems on their own and therefore demonstrated a stronger willingness to participate in the lessons. This also helped those with high prior knowledge to develop their understanding more effectively than did those with low prior knowledge. Another reason may be that students with high prior knowledge are more confident in mathematic problem solving.

The results of this study have two important implications with respect to instructional design: (1) Polya questioning instruction improved posttest performance and enhanced participation willingness beyond that of direct presentation instruction. This supports the contention that the questioning prompt approach should be integrated into teaching design to guide the independent thinking of students as well as the active processing of information from understanding the problem, developing and executing a strategy, to examination and review. (2) The similarity in the performance of students with low prior knowledge, regardless of the instruction method employed, may simply be due to the small sample size in this experiment. Students with low prior knowledge should be provided other teaching strategies, such as scaffolding, dynamic software, and cooperative learning to prepare students for the challenges of integrating new information and developing reasoning skills.

This study presents a framework of Polya questioning instruction based on four stages of problem solving and the theory of questioning prompts, tailored specifically for the instruction of geometry reasoning. In future research Polya questioning instruction could be applied to other topics of mathematics education (such as algebra and probability) or other fields (such as physics and chemistry) to verify its effectiveness. In addition, the impacts of the individual difference of students (such as learning style) or other teaching strategies (such as peer assessment and concept mapping) on the learning performance of Polya questioning instruction could also be taken into consideration.

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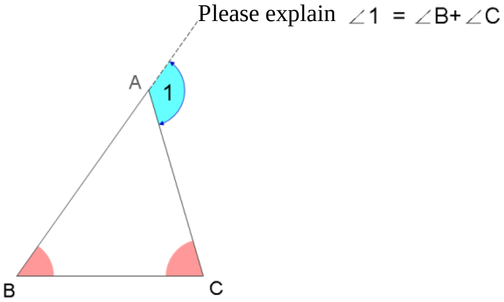
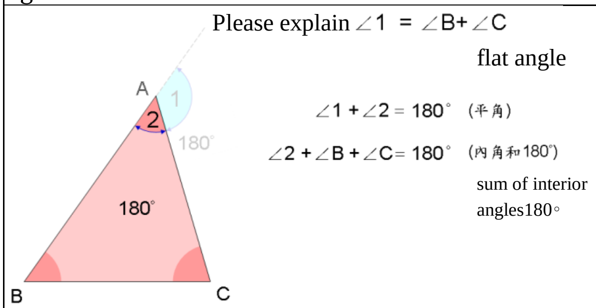
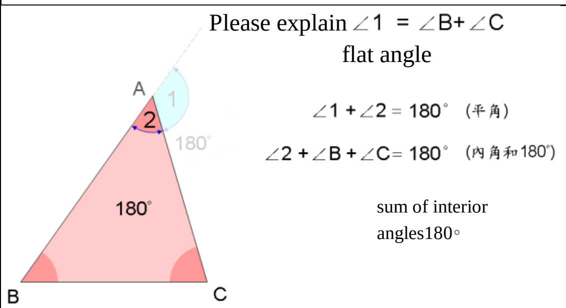
Appendix 1

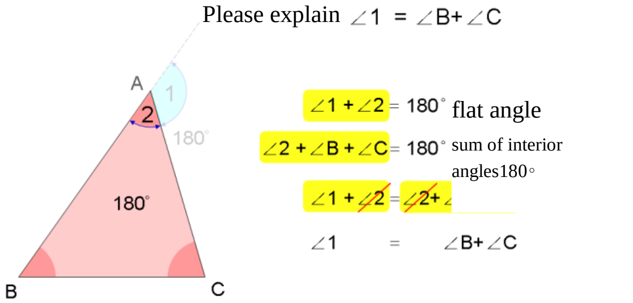
Framework of Poyla Questioning Instruction

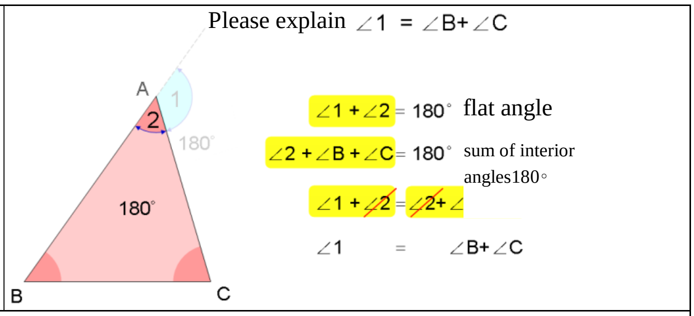
Problem solving stage	Questioning prompt
Understanding the problem	Recognizing the problem from a description and then isolating key elements. 1. What is the unknown quantity? What are the known data? 2. What are the conditions? Is it possible to meet all conditions? Are there adequate unknown quantities and conditions? 3. Draw a diagram to introduce the proper unknown quantity. 4. Write down all parts of conditions.
Developing the plan	Motivating students requires that problem solving ideas, hints, questioning prompts and suggestions be kept simple, unobtrusive, and generalizable. In this manner, students will feel that they have discovered the solutions by themselves. 1. Have you seen it before? Have you seen this question in a slightly different form? 2. Do you know any problems related to this? Do you know any theory which could be useful in solving this? 3. Try to think of a familiar situation related to the problem you want to solve. 4. You have solved another problem related to this one; can you make use of it? Can you make use of the result? Should you introduce auxiliary elements in order to make use of it? 5. Can you re-describe this problem? Can you re-describe this problem using different methods? 6. Can you think of a related question which is easier to process? Can it be a more general question, a more specific question, or an analog question?
Executing the plan	Implementing the problem-solving plan derived in the previous step, writing down every step of the plan, and carefully examining the correctness of these steps. 1. Are you sure that this step is correct? 2. Can you prove that this step is correct?
Examining and reviewing	Examining the argumentation of final solution, and considering the possibility for other arguments. 1. Can you verify this result? 2. Can you verify this argument? 3. Could you derive this result by different methods? 4. Could you figure it out immediately? 5. Could you apply this result or method to other questions?

Appendix 2

Overview of comparison between the teaching materials and teaching strategies for the experimental and control groups – taking the explanation of sum of exterior angles theory as an example.

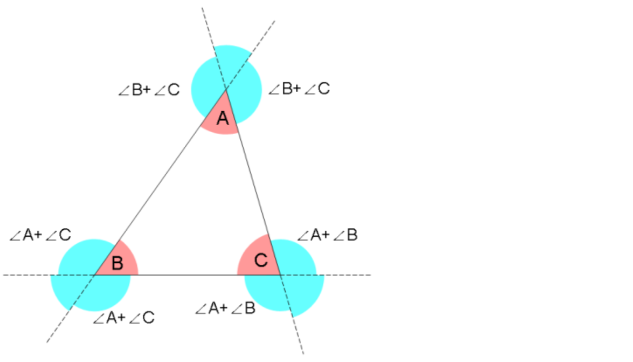
<table>
<tr><th>Experimental group (Polya questioning instruction)</th><th>Control group (direct presentation instruction)</th></tr>
<tr><td>Please explain</td><td>
</td></tr>
<tr><td colspan="2">Stages of understanding the problem</td></tr>
<tr><td>Given this triangle ABC, the angle between the extended side AB and the other side is called∠1. What is the relationship between ∠1 and the two interior angles ∠B∠C?
We are asked to explain ∠1=∠B+∠C using this problem. The blue angle is on the outside, while the red angles are on the inside. These two sets of angles are very different; however, the sum of red angles is actually equal to that of the blue angles. Is there any bridge for establishing the correlation between them? What is this bridge?
What kind of usefule relationship can be found in this figure?</td><td>We are asked to explain ∠1=∠B+∠C using this problem. This means that the exterior blue angle is equal to the sum of two interior red angles. How can we solve this problem?</td></tr>
<tr><td>
</td><td>
</td></tr>
<tr><td colspan="2">Stages of plan development</td></tr>
<tr><td>We just said that the bridge is∠2, and then…?
We write down this relationship on the right side as “flat angle”. What’s next?</td><td>To explain this problem, we will first look at the top half of this figure. We need help from this angle (the blue arc of ∠2 as shown in the figure). We name this angle ∠2.</td></tr>
<tr><td>We write down this relationship on the right side with the reason that the sum of the interior angles of a triangle equals 180°.</td><td>We just learned that, with placing ∠1 and ∠2 next to each other to form a straight line will add up to 180° (as shown in the figure), which means ∠1 plus ∠2 equals 180°. We write this down on the right side with the reason “flat angle”.
Next, look at the red triangle. We just learned that the sum of these three interior angles∠2, ∠B, ∠C equals 180° (as shown in the figure). We also write this down on the right side such that ∠2 plus ∠B plus ∠C equals to 180°. The reason is that the sum of the interior angles of a triangle equals 180°.</td></tr>
</table>

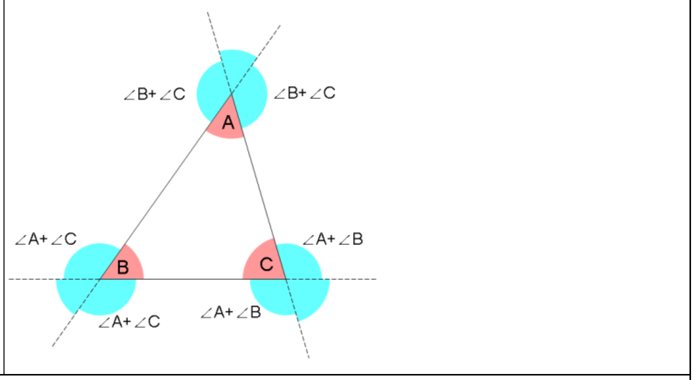




The stage of plan execution

Next, we will attempt to find the relationship between the two equations on the right side. Both of the two yellow bars represent 180, such that they are equal to each other. By looking at the yellow equation and problem, how can we achieve the goal? In this way, we obtain ∠1=∠B+∠C, which we explain using the problem. We also eliminate ∠2 from the figure on the left side.	Next, we see that the yellow bar in the first equation on the right side represents 180, and the yellow bar in the second equation also represents 180; therefore, they are equal. ∠2 is found on left and right sides of the equal sign; therefore, we can eliminate it, in accordance with the axiom of equality in order to obtain∠1=∠B+∠C, which we explain using this problem. We also eliminate ∠2 from the figure on the left side.





The stage of examination and review

This is a triangle. We just learned that the blue angle on the outside equals the sum of the two interior angles farthest away from it, which means that it is equal to∠B+∠C. Is there only one angle outside the triangle? Can you imitate the previous problem and find another blue angle outside this triangle? How would you do this? What would this angle be equal to? If you were asked to explain it, what bridge would you use?	This is a triangle. We just explained that the blue angle on the outside equals the sum of the two interior angles farthest away from it, which means that it is equal to∠B+∠C. However, there is more than one angle outside the triangle, so can imitate the previous problem. For example, we can extend the side BC toward the left to form this blue angle, which equals the sum of two interior angles farthest away from it, meaning that it will be equal to∠A+∠C. To explain how this blue angle is equal to ∠A+∠C, you will need help from ∠B, as in the previous problem. Another example would be extending side BC toward the right to form this blue angle, which will equal to the sum of two interior angles farthest away from it. So what would this be equal to? If you are asked to explain why this blue angle is equal to ∠A+∠B, you will need the help from ∠C, as in the previous problem.