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Enhancing grade 9 teachers' geometric thinking through Van Hiele theory-based geoboard workshops: A reflective study

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Abstract
Mathematics teachers in Namibia struggle to use geoboards when teaching geometry using the levels of the Van Hiele theory of geometric thinking. This research investigated how grade 9 mathematics teachers interacted, participated, and learnt or didn't learn during intervention workshops on the Van Hiele theory-based geometric thinking using geoboards, through a reflective practice. Mathematics teachers teach geometry theoretically, without using interactive tools that could help learners to be active participants in the classroom. Mathematics teachers involved in this research were unaware of how the Van Hiele theory could be used to teach geometry using geoboards. Teaching geometry using interactive tools, such as geoboards, helps learners to grasp concepts. The study followed a participatory action research design to understand how mathematics teachers use geoboards to teach geometry within the various levels of the Van Hiele theory. Four mathematics teachers were selected purposively, as they shared the same context and had similar knowledge gained from professional development (PD) workshops. An intervention with the lead author and four mathematics teachers took place over a period of three days, divided into three phases: orientation, theory, and practical application of the levels of the Van Hiele theory. Data were collected from group discussions and reflective journals. Findings revealed that some teachers effectively used the Van Hiele theory in teaching geometry without using geoboards. However, other mathematics teachers were at first unaware of how to teach geometry using geoboards and mastered it during the intervention workshops. The study recommends PD initiatives and curriculum revisions to use geoboards as an interactive tool in teaching geometry. Mathematics teachers from higher institutions need to be involved in practical initiatives for how geoboards could be used in teaching geometry.

Keywords: Van Hiele theory, geometry, geoboard, mathematics, pedagogical strategies, reflective practice

INTRODUCTION

Mathematics teachers in Namibian secondary schools are not provided with professional development (PD) support by the Ministry of Education in Namibia. In professional learning communities (PLCs), less attention is paid to developing mathematics skills through reflective practices, such as using geoboards when teaching geometry at the various levels of the Van Hiele theory of geometric thinking (Hamukwaya, 2019). A PLC allows teachers to engage in reflective practice, where they analyze and refine their teaching methods through collaboration, discussions, and shared experiences (Alzayed & Alabdulkareem, 2020). This reflection is particularly vital in mathematics education, as it enables teachers to identify effective strategies, address common challenges, and integrate innovative teaching tools to enhance learning (Uugwanga & Aipinge, 2022). However, this research used intervention workshops to work with teachers on how to teach geometry using geoboards. The aim was for teachers to interact with each other and learn from each other's experiences.

Mathematics teachers in Namibia do not use interactive tools such as a geoboard in their lessons, limiting the effectiveness of their teaching (Hamukwaya,

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Contribution to the literature

- This article contribute on enhancing mathematics teachers using Geoboard to teach geometry through van Heile theory.
- The authors took grade 9 mathematics through intervention and reflected on their activities and tasks.
- There is a need to develop a modeling approach to help grade 9 teachers to understand the fundamental aspects of geometry in practice.

2019; Uugwanga & Aipinge, 2022). The absence of PD support means that teachers are less exposed to modern pedagogical techniques, leading them to rely on traditional textbook-based teaching instead of interactive, hands-on learning approaches. Some teachers may also be unaware of the benefits of geoboards or may feel they do not align well with curriculum requirements (Sibiya, 2020). According to Chauraya and Brodie (2018), PD fosters collaborative learning in mathematics teachers, allowing them to reflect on their teaching methods and refine their instructional strategies. Mathematics teachers involved in the intervention workshops shared their experience and knowledge of teaching geometry using geoboards. Reflective practice is crucial for addressing teaching challenges and integrating innovative techniques (Lerman, 2000). However, the lack of institutional support and structured professional learning networks leaves many mathematics teachers relying on traditional teaching methods, thus limiting the effectiveness of their instruction.

In Namibia, PD for teachers is primarily conducted through initiatives such as the National Institute for Educational Development and the Regional Directorate of Education continuous PD (CPD) programs (Bestman & Chiwhetu, 2024). However, many mathematics teachers still face challenges in reflecting on their practice, particularly in geometry teaching. Research indicates that teachers often rely on chalk-and-board methods, which limit learners' understanding of geometric concepts. The Van Hiele theory of geometric thinking presents additional challenges, as many teachers are unfamiliar with its five levels (Van Hiele, 1986). While mathematics teachers acquire content knowledge during their four years of study at higher education institutions, interactive tools such as geoboards are not commonly included in their training (Sibiya, 2020). Intervention workshops have shown that teachers find the Van Hiele theory helpful in improving their instructional strategies (Forman, 2020). Chauraya and Brodie (2018) illustrated that mathematics teachers can collaborate, share best practices, and enhance their teaching methods, particularly when using geoboards to teach geometry. This was evident during the intervention workshops with the four mathematics teachers. Through collaborative efforts between researchers and mathematics teachers, this study aims to foster a culture of continuous improvement in mathematics education, ultimately enriching teachers' experiences of teaching geometry. To address this, the study was guided by the following research questions: *How to enhance grade 9 teachers' geometric thinking through Van Hiele theory-based geoboard workshops: a reflective study?*

LITERATURE REVIEW

Mathematics

Mathematics is not only a discipline of logical reasoning and problem solving, but also a cognitive and social process that evolves within the PLC. Learning mathematics in collaborative settings allows teachers to refine their understanding through interactions, shared problem solving, and reflective thinking. This fosters deeper engagement with mathematical concepts, particularly in geometry, where visual reasoning and logical deductions benefit from discussions and peer explanations (Yadav, 2019). Geometry, as a significant branch of mathematics, is fundamental to spatial reasoning and logical proof construction (Battista, 2007; Candiotes, 2023). Mathematics teachers involved in the intervention workshops developed geometric thinking that aligns with the Van Hiele theory, which outlines five stages of cognitive growth: visualization, analysis, informal deduction, formal deduction, and rigor (Van Hiele, 1986). Collaborative learning environments reinforce this cognitive progression by enabling teachers to articulate reasoning, challenge assumptions, and construct proofs in a structured and supportive setting.

Reflection is central to learning mathematics. The ability of teachers to critically analyze problem-solving approaches and refine thinking strategies strengthens mathematical proficiency (Schoenfeld, 2017). Within the intervention workshops, mathematics teachers become reflexive practitioners by engaging in reflection through a shared process of dialogue, justifying solutions, and developing metacognitive awareness of mathematical reasoning (Tall, 2006). In Namibia, where geometry is integrated into the curriculum from grade 1 to grade 12, structured reflection and collaboration help learners transition through different Van Hiele levels, ensuring a gradual and meaningful progression in their understanding. Mathematics is enriched through community-based learning, where geometric thinking and reflective practices converge to facilitate knowledge construction (Battista, 2007). The integration of the Van

Hiele theory and reflective thinking creates a powerful framework for teaching mathematics in a way that supports collaborative learning. These insights are particularly relevant in the Namibian educational context, where structured approaches to geometry can enhance learners' ability to reason, reflect, and apply mathematical knowledge in broader fields. Within mathematics teaching, teachers must learn from each other through a PLC to improve their pedagogy.

Geometry Teaching and Learning in Namibia

Geometry in Namibian schools is taught by using chalk and chalkboard, as these are the only teaching materials available to the teachers, apart from textbooks. A study conducted in the Oshikoto Region of Namibia highlights that many mathematics teachers face significant challenges in teaching geometry due to limited resources. These include a lack of teaching aids beyond chalk, chalkboard, and textbooks, which restricts the use of more interactive or visual tools that could enhance learners' understanding (Dongwi, 2014). This teaching style presents several challenges for secondary school mathematics teachers. As an integral component of the mathematics curriculum from grade 1 to grade 12, geometry has historical significance in shaping learners' spatial reasoning, logical deduction, and problem-solving skills (Armah et al., 2018). However, teachers often encounter obstacles such as limited resources, insufficient training, and learners' difficulties in understanding geometric concepts. Despite its enduring presence in educational curricula, learners often find geometry challenging due to its abstract nature and their weak foundational understanding (Ugulu, 2008). Similarly, Hassan et al. (2020) assert that geometry is commonly perceived as a difficult topic within the mathematics curriculum. Research indicates persistent struggles among learners, even in grade 12, suggesting a pervasive weakness in geometry knowledge (Ugulu, 2008). Moreover, studies such as Dongwi's (2014) highlight the challenges faced by mathematics teachers, particularly in grade 8 to grade 12, elucidating the difficulties encountered in teaching geometry, which possibly contribute to learners' struggles as identified by (Ugulu, 2008).

Efforts to address these challenges are under way, with a focus on employing the Van Hiele theory of geometric thinking to enhance geometry instruction (Ugulu, 2008). PLC workshops involving grade 9 mathematics teachers aim to refine teaching methodologies in geometry (Ugulu, 2008). However, gaps persist; notably, the curriculum's failure to integrate the Van Hiele levels of geometric thinking (Ministry of Education, 2018) hinders teachers' ability to effectively guide learners through the conceptual stages. This disjointed approach to teaching geometry, as observed in Namibia's educational landscape, perpetuates challenges in learners' mastery of geometric concepts (Dongwi, 2014; Muhongo, 2008; Muyeghu, 2008). Despite pockets of success, such as learners demonstrating proficiency in certain geometric thinking levels (Muyeghu, 2008), the overarching lack of alignment between curriculum and pedagogy impedes optimal learning outcomes (Machisi & Feza, 2021, Muhongo, 2008). Thus, addressing this misalignment is paramount to ensuring learners' holistic understanding and mastery of geometry.

Figure 1. Geoboard (Source: Authors' own elaboration)

Teaching Geometry Using Geoboards

Teaching geometry using geoboards in the Namibian context can be highly effective, as it aligns with the national curriculum's emphasis on creativity and innovation in teaching methods (Chikiwa & Schäfer, 2019). Geoboards, which are manipulative tools, allow learners to visualize and explore geometric concepts through hands-on activities (Bestman & Chiwhetu, 2024). According to a study by Owusu and Sallah (2023), geoboards are rarely used in Namibian classrooms, despite their potential to enhance learners' understanding of geometric properties. By integrating geoboards (**Figure 1**), teachers can create a more engaging and interactive learning environment, helping learners to better grasp abstract concepts through concrete experiences (Chikiwa & Schäfer, 2019). Various research studies support the use of geoboards in teaching geometry. Schäfer (2021) found that geoboards facilitate a visual approach to teaching, making mathematical ideas more accessible and understandable for learners. In addition, Owusu and Sallah (2023) emphasized the importance of manipulative tools like geoboards in promoting problem solving and exploratory learning. In the Namibian context, integrating geoboards into the geometry curriculum can address the current challenges in mathematics education by providing learners with the opportunity to actively engage with geometric concepts, thereby enhancing their conceptual understanding and problem-solving skills (Schäfer, 2021).

Geoboards are versatile educational tools that have increasingly been recognized for their effectiveness in enhancing learners' understanding of geometric concepts through hands-on learning experiences (Bestman & Chiwhetu, 2024). By allowing learners to physically manipulate shapes, geoboards help them visualize and explore geometric properties and relationships. This interactive approach aligns with constructivist learning theories (McLeod, 2024), which emphasize active engagement and discovery-based learning. Sibiya (2020) found that using geoboards in teaching Euclidean geometry significantly improved learners' conceptual understanding and engagement. The study demonstrated that when learners use geoboards, they could better grasp abstract geometric concepts, which in turn fostered a deeper understanding and retention of the material. Similarly, Ajere (2023) researched the impact of geoboards on learners' achievement in geometric shapes within primary education. The findings revealed that geoboards not only enhanced learners' performance in geometry but also increased their motivation and interest in the subject. This aligns with the principles of active learning, where learners are more likely to be engaged and motivated when they can tangibly interact with learning materials.

Workshop-Based Professional Development

The workshops were structured to enhance teachers' geometric thinking, thus improving their instructional abilities and mathematics teaching methods. Research demonstrates a strong link between teachers' geometric proficiency and their effectiveness in teaching geometry, directly influencing learners' understanding and performance in the subject (Pavlovičová & Bočková, 2021). Using the Van Hiele theory, which outlines progression from basic visual recognition to formal deductive reasoning, the workshops aimed to develop teachers' higher-order geometric thinking. Engaging activities, including solving complex problems and exploring spatial relationships, were employed to facilitate this growth (Driscoll et al., 2007). In addition, integrating hands-on tools and technologies such as GeoGebra software and manipulatives further supported a deep understanding of geometric concepts (Mwiikeni, 2017; Ndungo et al., 2025).

These workshops also emphasized collaboration among mathematics teachers, creating a space to exchange strategies and best practices for teaching geometry. PD focusing on collaborative learning and reflective practices leads to lasting improvements in teaching methodologies (Pavlovičová & Bočková, 2021). Participating in such initiatives not only enhanced teachers' understanding of geometry but also provided them with valuable insight into designing engaging lessons tailored to diverse learners (Ndungo et al., 2025). This comprehensive approach equips teachers with the tools necessary to foster a deeper understanding of geometry among learners, thereby boosting overall mathematical literacy and problem-solving abilities across the curriculum (Driscoll et al., 2007).

The workshops integrated experiential techniques, such as creating geoboards or engaging in simulations, which allowed participants to practice applying new concepts in practical scenarios, enhancing both understanding and long-term retention (Mayombe, 2023). By integrating collaborative and reflective practices, as well as ongoing support mechanisms like mentoring and peer networks, this workshop provided a robust framework for CPD, fostering sustained improvements in teaching practices and professional growth (Antúnez-Montes et al., 2021; Darling-Hammond et al., 2017).

Reflective Practice

Reflective practice is a crucial process in PLCs and in personal development, as it allows individuals to critically analyze their experiences to improve future actions (Alzayed & Alabdulkareem, 2021). It involves self-examination and evaluation of one's thoughts, teaching strategies, and decisions to enhance learning and effectiveness (Nahmias & Teicher, 2021). Mathematics teachers enhance their understanding of geoboards when teaching geometry. The focus was for the co-researchers to reflect after each intervention workshop without the influence of the researchers. This gave the co-researchers enough time to critically reflect on what happened during the workshop. According to Finlay (2008), reflective practice fosters deeper understanding and helps professionals navigate complex situations by making informed decisions. In this research, for instance, mathematics teachers used reflective practice to assess their understanding of the Van Hiele theory, identify areas for improvement, and adapt teaching strategies accordingly. This continuous cycle of reflection over three different intervention workshops helped the teachers learn and improve their professional growth (Sellars, 2021).

Sellars (2021) highlighted that reflective practice enables individuals to develop critical thinking skills and improve problem-solving abilities. In addition, Harvey et al. (2022) suggested that integrating reflective practice enhances self-awareness and leads to more effective decision making. By engaging in reflective practice, mathematics teachers can refine their skills, foster innovation, and maintain ethical standards in their respective fields (Finlay, 2008; Harvey et al., 2022).

RESEARCH METHODOLOGY

Following the participatory action research design, data were generated from four co-researchers who were part of the group discussion during intervention workshops and reflective journals. The PAR cycle involves four steps: identifying the problem, collaboration, actions, and reflection (Walter, 2009). Intervention workshops allowed mathematics teachers to collaborate, and reflections formed the cornerstone of this research, which helped them to learn from each other (Huijboom et al., 2021). Mathematics teachers were able to collaborate and reflect, giving and receiving feedback and experimenting with hands-on activities in line with PAR and intervention workshops (Huijboom et al., 2021). The researchers identified a need for change in teaching grade 9 geometry. Collaboration between co-researchers and researchers facilitated the planning and implementation of geoboard usage in grade 9 mathematics. To ensure structured engagement, intervention workshops were conducted for the co-researchers to understand the Van Hiele theory's levels and the use of geoboards in teaching geometry. Actions were carried out collaboratively, with all co-researchers working together by sharing their knowledge and experiences. Reflection was emphasized through journal entries, allowing co-researchers to document insights after workshops, ensuring continuous learning and improvement.

In this study, PAR was structured to foster a collaboration among mathematics teachers, ensuring active participation through three stages: orientation and theory presentation (levels 1-3), advanced theory presentation (levels 4-5), and practical work (Chevalier, 2019). Mathematics teachers fostered a learning community to continue after the intervention workshops, to enable them to continue working together (Brydon-Miller & Maguire, 2009). By engaging teachers in intervention workshops, qualitative data such as transcripts, reflective journals, and triangulated findings were collected to enhance understanding of geoboard usage in geometry teaching. These reflective practices align with Wenger's (1998) concept of legitimate peripheral participation, where teachers collaboratively construct and refine knowledge, reinforcing the shared learning process inherent in both PLC and PAR. However, the number of mathematics teachers was limited to only four, restricting the data that was collected and presented in this research.

Sampling Method

Purposive sampling was used to select four grade 9 mathematics teachers based on accessibility and relevance to the study (Gay et al., 2011). Four mathematics teachers explored different strategies for teaching geometry using the Van Hiele theory, with data collected through group discussions and reflective journals. Mathematics teachers participated in an intervention workshop during planning, actin and reflection sessions. All co-researchers completed the reflective journals and were part of a group discussions (reflection session). The co-researchers were all part of the intervention workshops to learn from each other's experiences. The aim was for them and the researchers to continue working and share their knowledge of using geoboards to teach geometry within the levels of the Van Hiele theory.

Interventionist Workshops

The intervention unfolded over three workshops; each strategically focused on distinct phases of geometric thinking. Workshop 1 addressed the recognition (visual) and analysis (descriptive) levels, laying the foundation for understanding basic geometric concepts. Teachers used simple activities of visualization and analysis. The researcher used geoboards to explain to the co-researchers the two levels of geometric thinking and how they could be used to teach geometry in mathematics classrooms. Mathematics teachers were able to link geometric shapes to the levels of understanding of geometry. During this phase, the researchers and co-researchers shared their experiences that allowed them to work as a team and learn from each other's experiences. In workshop 2, the researcher engaged the co-researchers on levels 3-5 of Van Hiele's theory (informal deduction, deduction and rigor). The researcher presented how geoboards can be used to teach geometry through the levels of Van Hiele's theory. Finally, workshop 3 emphasized hands-on geoboard applications, where teachers had the opportunity to create and present geometric figures. During these phases, teachers were able to share their experiences gained by working with geoboards. This created space for them to learn from and help each other and also allowed for active engagement and practical application of concepts, strengthening teaching methodologies and understanding of geometry. Co-researchers had to reflect on the activities that were conducted over three days and reflective journals contained guiding questions for participants to answer when reflecting. These allowed them to focus on what was requested by the researchers.

Group discussions were organized to allow the co-researchers and researchers to work together. Participatory action research allowed participants to discuss how geoboards could be used to teach geometry, using the levels of the Van Hiele theory. The discussions were arranged in four steps: The first discussion focused on looking at the problems that teachers face when teaching geometry and how geoboards could be used to teach geometry. This led to the second step, where mathematics teachers collaborated on best practices for teaching geometry. This collaboration helped the mathematics teachers act by developing the lessons that were taught by two mathematics teachers. The last step was reflection, where teachers reflected on their reflective journals.

Data Analysis

A thematic analysis approach was used, as recommended by Braun and Clarke (2019), which involved identifying and analyzing patterns within the data. Reflective journals were used to collect data from the co-researchers. This allowed the co-researchers to reflect after the intervention workshops. Data from group discussions were recorded and transcribed with the permission of the co-researchers. Group discussions were used to collect data during and after the intervention workshops. Data from each instrument that had similar patterns were identified and grouped into categories. Data on the understanding of the Van Hiele theory of geometric thinking were mostly generated from reflective journals from co-researchers. Data on the use of geoboards were generated from group discussions during the intervention workshops. Group discussions were recorded and transcribed to obtain data and data from both group discussions during the intervention workshops and reflective journals were analyzed thematically. This method allowed the researchers to organize data into meaningful themes based on recurring patterns and insights. After transcribing the data, coding involves organising raw data into meaningful units by assigning labels to recurring ideas, which helps researchers to identify patterns and structure their analysis (Williams & Moser, 2021). These codes were then synthesized into broader themes which were used to discuss and analyze the data (Naeem et al., 2023). Theme development was a critical interpretive step that transformed fragmented data from reflective journals and focus group discussions into coherent insights and ideas. To enhance the validity of these findings, the researchers employed triangulation, which involves using multiple data sources, methods, or analysis to cross-check and confirm interpretations (Jonsen & Jehn, 2009). This multi-angle approach reduces bias and strengthens the trustworthiness of the research outcomes. Thematic analysis is particularly suitable for qualitative research as it provides a flexible yet rigorous approach to data analysis. Initially, data from the presentation and group discussion were transcribed and then coded to identify the participants with codes. Data from reflective journals and group discussions were grouped into categories, which were then refined into overarching themes. This method ensured that the analysis was data-driven. By employing thematic analysis, the study effectively highlighted key themes that emerged from the data.

Ethical Considrations

In qualitative research, ethical rigor is anchored in the principles of informed consent, anonymity, confidentiality, and researcher positionality. Informed consent was given by the co-researchers, and an explanation was made by the researcher. This allowed the co-researchers to understand their role as participants in the research. The researcher visited the participants at their school to build rapport and a one-time agreement with co-researchers required the researcher to revisit consent throughout the study to respect participants' autonomy and evolving perspectives (Xue et al., 2025). Confidentiality was achieved by keeping the data in secure storage on a password-protected laptop. Anonymity was achieved by coding the teachers according to the instrument by which the data was generated (Vacek et al., 2017). Meanwhile, researcher positionality demands reflexive awareness of how one's social location, values, and assumptions shape the research encounter and interpretation (Corlett & Mavin, 2018). The researcher positioned himself as a co-learner during the intervention and was involved in the study to work with the teachers. These principles form the ethical backbone of a qualitative case study, guiding researchers to act with integrity, transparency, and care.

RESULTS AND DISCUSSION

Findings from the group discussions during intervention workshops and reflective journals are discussed under one theme and three categories as posed below. Theme: Enhancing grade 9 teachers' geometric thinking through Van Hiele theory-based geoboard workshops: a reflective study. The five categories are understanding the Van Hiele theory of geometric thinking, geoboard as a useful tool in geometry, and a workshop based on the Van Hiele theory for geometry teaching, enablers of geometric teaching, and constraints of geometric teaching.

Understanding the Van Hiele Theory of Geometric Thinking

The Van Hiele theory of geometric thought is a framework that describes how teachers hierarchically understand geometry, progressing through five distinct levels of thinking. The levels range from visualization, where teachers recognize shapes by their appearance, to rigor, where teachers can understand and construct formal proofs (Arnal-Bailera & Manero, 2024). This theory emphasizes that teachers must achieve mastery at one level before moving to the next, highlighting the sequential nature of geometric understanding. For teachers, understanding these levels is crucial for effectively guiding learners through their geometric learning journey. Mathematics teachers were not familiar with the levels of the Van Hiele theory before the intervention workshops. This was evident when all mathematics teachers indicated this during a group discussion before the intervention. For example, T2 noted, "*I am not aware of Van Hiele theory*", and T3 indicated that they were "*not aware*". T4, argued that "*I never came across this during my time at an institution of* *higher learning*". This shows that they were not exposed to the Van Hiele theory at institutions of higher learning, suggesting that some teacher education programs are not integrating the Van Hiele theory into their curricula. This concern is echoed in research by Mbatha and Bansilal (2023), who found that many pre-service teachers struggled with geometric reasoning because they had not developed the appropriate Van Hiele levels of thinking. Their study emphasized the need for explicit instruction in the Van Hiele theory to help teachers recognize and use geoboards to address learners' misconceptions in geometry.

However, after the intervention workshops on using geoboards to teach geometry and the Van Hiele theory levels, mathematics teachers showed an understanding. Data in this theme were collected from reflective journals from mathematics teachers, and thematic data analysis was used to analyze the data. Some teachers noted the following:

> The intervention workshops helped me to understand Van Hiele theories and how important it is to the learners and how learners must learn through the levels (T1RJ).

> Through the workshops, now I understand all the levels and how to use them to teach geometry to the learners (T3RJ).

> The intervention workshop helped me to understand the theory of Van Hiele through the five level of geometric thinking (T2RJ).

These three excerpts revealed that the intervention workshops helped the mathematics teachers to understand the Van Hiele theory and helped them in teaching geometry and understanding the levels of geometry. Workshops provide a collaborative environment where mathematics teachers share best practices and strategies for teaching geometry, fostering a community of CPD. The intervention workshops focused on the application of the Van Hiele theory, coupled with hands-on activities using tools like the geoboard, empowering teachers to enhance their pedagogical content knowledge (Chauraya & Brodie, 2018). The application of Van Hiele's theory in classroom settings necessitates that teachers be well-versed in identifying the geometric thinking levels of their learners. This understanding enables teachers to design lessons that meet learners at their current level and help them advance (Fitriyani et al., 2018). For instance, at the visualization level, teachers might focus on activities that help learners recognize and classify shapes, while at higher levels, the focus might shift to understanding properties and relationships between shapes. Effective implementation of this theory requires teachers to be adaptable and responsive to their learners' needs.

Adding to this, T1 and T2 wrote in their reflective journal,

> It was well presented whereby Van Hiele theory should be used and how it is helpful to learners to understand the connections between the shapes' appearance and their properties (T1RJ).

> It was presented very well, which was a model of most of the teachers to apply during the teaching of geometry chapter (T2RJ).

T3 in her reflective journal just wrote *"excellent"*; this indicating that she understood the topic that was presented to her on how the Van Hiele theory could be used when teaching geometry.

It is evident that mathematics teachers learned how Van Hiele theory can be used to teach geometry in grade 9, following the five hierarchical levels of geometric thinking proposed by Van Hiele. The theory emphasizes progression from visual recognition to formal deduction, aiding teachers in structuring lessons that align with learners' cognitive development (Van Hiele, 1986). As teachers deepen their understanding of the Van Hiele theory framework, they become more equipped to scaffold geometric concepts effectively and promote higher-order thinking (Usiskin, 1982). Moreover, the Van Hiele theory underscores the importance of language and communication in teaching geometry. Teachers must use precise and clear language that aligns with learners' current levels of understanding. For example, at lower levels, teachers might use everyday language and simple terms, while at higher levels, they introduce formal geometric vocabulary (Naufal et al., 2021). PD workshops that focus on the Van Hiele theory can help teachers develop the necessary skills and strategies to communicate effectively and foster a deeper understanding of geometry among their learners.

Mathematics teachers showed that they had learned about geometry by illustrating the following in the reflective journals:

> Using the Van Hiele levels into my teaching can completely transform my learners' understanding of geometry. By teaching learners through the levels of Van Hiele theory can help them to have a greater sense of direction and purpose in learning geometry in school (T2RJ).

> As learners progress through each level, my confidence as a teacher will grow and that will make me confident and grow significantly. Teaching learners from visualization level to complex deductive reasoning is incredibly rewarding for me as a teacher and my learners (T1RJ).

> Transitioning from traditional methods to Van Hiele's theory has given me an entirely new meaning to geometric teaching. The structured progression through the levels is a game-changer for learners' understanding (T3RJ).

The three extracts reveal the understanding of mathematics teachers on the levels of the Van Hiele theory. Through their interactions, they were able to reflect on how they could teach geometry to their learners (as illustrated above). Their reflective practices helped them engage in a deep understanding of the concepts after the intervention workshops. During PD courses, teachers found that reflecting on their learning experiences helped them align their teaching practices with recommendations for mathematical understanding (Chamberlin, 2009). The Van Hiele theory of geometric thinking has had a transformative impact on learners' understanding of geometry over the past five years. Integration of the Van Hiele theory into teaching practice provides learners with a structured framework, enhancing their comprehension and giving them a sense of direction and purpose in learning geometry (Lwanga, 2022). Additionally, as learners advance through the Van Hiele levels, teachers experience reciprocal growth in their confidence and effectiveness. Armah et al. (2018) noted that teachers' confidence grew as they witnessed their learners advancing through these levels. Transitioning from traditional methods to the Van Hiele theory revolutionizes geometric teaching, offering a more systematic and game-changing approach. Armah and Kissi (2019) demonstrated that this structured progression yields marked improvements in learners' understanding compared to traditional methods. Armah et al. (2018) highlighted the significant enhancement in learners' comprehension and engagement when adopting the Van Hiele structured approach.

The Geoboard as a Useful Tool in Geometry

Data was collected on whether mathematics teachers felt that geoboards can be a useful teaching tool in geometry, and answered research question 1: *How did grade 9 mathematics teachers interact, participate and learn (or not) during the intervention workshops on the use of geoboards to teach geometry?* Geometry, as a fundamental aspect of mathematics, often poses challenges for learners due to its abstract nature. Traditional methods of teaching geometry, which rely heavily on theoretical explanations and static visual representations, can sometimes fail to engage learners and promote deep understanding. To address this challenge, mathematics teachers need to turn to hands-on learning tools such as geoboards, as indicated in **Figure 1**. The intervention workshops helped teachers participate in hands-on activities and learn from each other's experiences. A geoboard is a mathematical manipulative that consists of a square board with a grid of pegs onto which rubber bands are stretched to create various geometric shapes (Poloamina et al., 2024). This interactive tool offers a tangible way for learners to explore geometric concepts, visualize relationships between shapes and develop spatial reasoning skills. Geoboards can significantly enhance teachers' and learners' engagement, motivation, and understanding in geometry lessons. Geoboards are a valuable resource in teaching geometry, particularly within the context of constructivist learning theories (Vygotsky, 1978), which emphasize active, experiential learning (Bada & Olusegun, 2015). The geoboard was developed by researchers to help mathematics teachers understand the levels of Van Hiele theory of geometric teaching and how they can teach learners using the geoboard. The geoboard was used during the interventions to explain how the Van Hiele theory of geometric thinking could be taught in the classroom.

Figure 1 illustrates the geoboard used during the intervention workshops to help mathematics teachers understand different levels of the Van Hiele theory of geometric thinking. The geoboard helps teachers visualize different figures that could be constructed during the lessons to allow learners to understand the levels. Four mathematics teachers discussed how a geoboard could be useful in mathematics during the group discussion. The data were from group discussions during the intervention workshops. T2 illustrated:

> Using a geoboard to teach geometry could help learners to grasp more knowledge about geometry and how the level could be connected from level 1 to 5. Geoboards are fantastic for visual learning. They help learners understand geometric concepts by allowing them to physically manipulate shapes and see how they fit together (T2GD).

As T2 indicated, geoboard tools are useful for teaching geometry, especially for learners. By allowing learners to physically manipulate shapes, geoboards help them better understand geometric concepts and visualize how shapes fit together. Research supports the effectiveness of such interactive tools in boosting engagement, motivation, and retention of mathematical concepts (Bicer & Lee, 2019; Chen et al., 2018).

T4 and T1 indicated the following during group discussions:

> Using geoboards in my classroom might make geometry lessons more interactive and engaging as it helped us to engage during this intervention workshop. Learners are more motivated to learn when they can use hands-on tools (T4GD).

> Geoboards can be a great tool to introduce learners to geometry lessons. They can experiment with different shapes and patterns,

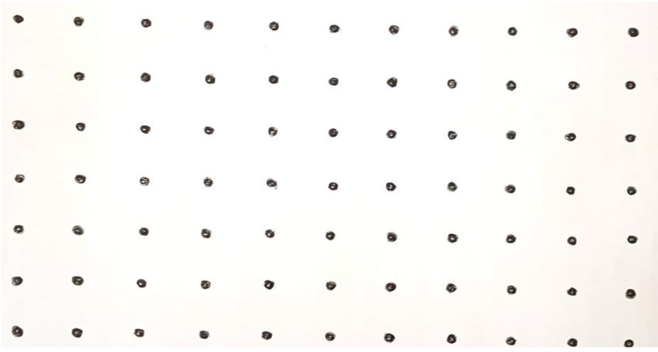

Figure 2. Geoboard developed by the teachers (Source: Authors' own elaboration)

> which help build a strong foundation for more complex concepts later on the topic (T1GD).

T3 said that

> Geoboards could be useful for teaching concepts like symmetry, area, and perimeter. It can make abstract ideas more concrete and easier for us to grasp as we are learning during this workshop (T3GD).

The quotes indicate the experiences of mathematics teachers when teaching geometry using geoboards to enhance learners' understanding. Mathematics teachers revealed how a geoboard could be useful in their classroom when teaching geometry. Integrating technology and hands-on learning tools, such as dynamic geometry software, GeoGebra software, and manipulatives, can further support teachers in developing a deeper understanding of geometric concepts (Mwiikeni, 2017). Geoboards are effective educational tools for making geometry lessons more interactive and engaging. They allow learners to physically manipulate shapes, enhancing their understanding of geometric concepts (Ajere, 2023). This hands-on approach is supported by Bicer et al. (2019), who found that using practical tools can significantly boost learners' motivation and interest in STEM subjects. Sibiya (2020) highlights that integrating interactive tools into the curriculum improves learners' engagement and learning outcomes in mathematics.

As the teachers were interactively engaged during the group discussions in the intervention phase, they shared their teaching experiences on the use of geoboards when teaching geometry. The engagement helped one mathematics teacher develop the easiest geoboard that could be used by learners in the class. T4 used paper, a ruler, and a pencil to develop the geoboard in **Figure 2**.

During group discussions, mathematics teachers developed the geoboard without using nails, a board and a hammer. It was interesting to see how innovative teachers could be when teaching geometry. It was easy to make copies for the learners to draw different shapes

Activity done during the intervention workshop

Name and draw the 2 – dimensional shapes describe below

1. A quadrilateral with opposite sides equal, all angles are equal, sides meet at right angles, and it possesses two lines of symmetry.
2. A figure with three edges, two edges identical. Two angles are equal, and it possesses one line of symmetry.
3. A quadrilateral featuring two parallel sides of varying lengths, where the upper line is shorter than the lower line. It features two equal opposing sides that are not parallel, along with two pairs of equal angles (the top angles are obtuse and equal, while the bottom angles are acute and equal), and there is one line of symmetry.
4. A quadrilateral where all sides are equal, all angles are equal, and it possesses four lines of symmetry.
5. A quadrilateral featuring two pairs of equal opposite sides, two pairs of equal opposite angles (two acute and two obtuse angles, with no lines of symmetry)

Figure 3. Activities given to the teachers (Source: Field study)

instead of using the wooden geoboard in **Figure 2**. Mathematics teachers were able to demonstrate how learners could use the geoboard developed by T4 to draw different geometrical shapes. Mathematics teachers were given questions on the properties of different geometric shapes from the levels of Van Hiele theory to answer during phase 3 of the intervention workshop. The hands-on activities helped mathematics teachers engage and participate during the workshop by drawing different shapes. The following instructions were given to the mathematics teachers to draw the shapes on the developed geoboard.

All four teachers worked on the activities of by drawing the different quadrilaterals using the properties given. The data were generated during the group discussion in phase 3 of the intervention workshop, shown in **Figure 3**. The four teachers followed the properties, and the results of two of the mathematics teachers' drawings are seen in **Figure 4**.

Figure 4 showcases the drawings created by T2 and T4 using a geoboard. In **Figure 4**, various geometric shapes are represented through the careful manipulation of pencil and ruler. This visualization aids in understanding the geometric properties and relationships between shapes.

For T2 and T4, the focus may have been on basic geometric shapes such as triangles, rectangles, and squares. The simplicity of these shapes helps to grasp the foundational concepts of geometry, including angles, sides, and symmetry. By creating these shapes on the geoboard, T2 and T4 demonstrate an understanding of how to construct and differentiate between basic polygons (**Figure 4**). According to Sibiya (2020), hands-on tools like geoboards can significantly enhance learners' understanding of geometric concepts through active learning. In the case of this study, it enhanced mathematics teachers' understanding of the use of a geoboard when teaching geometry.

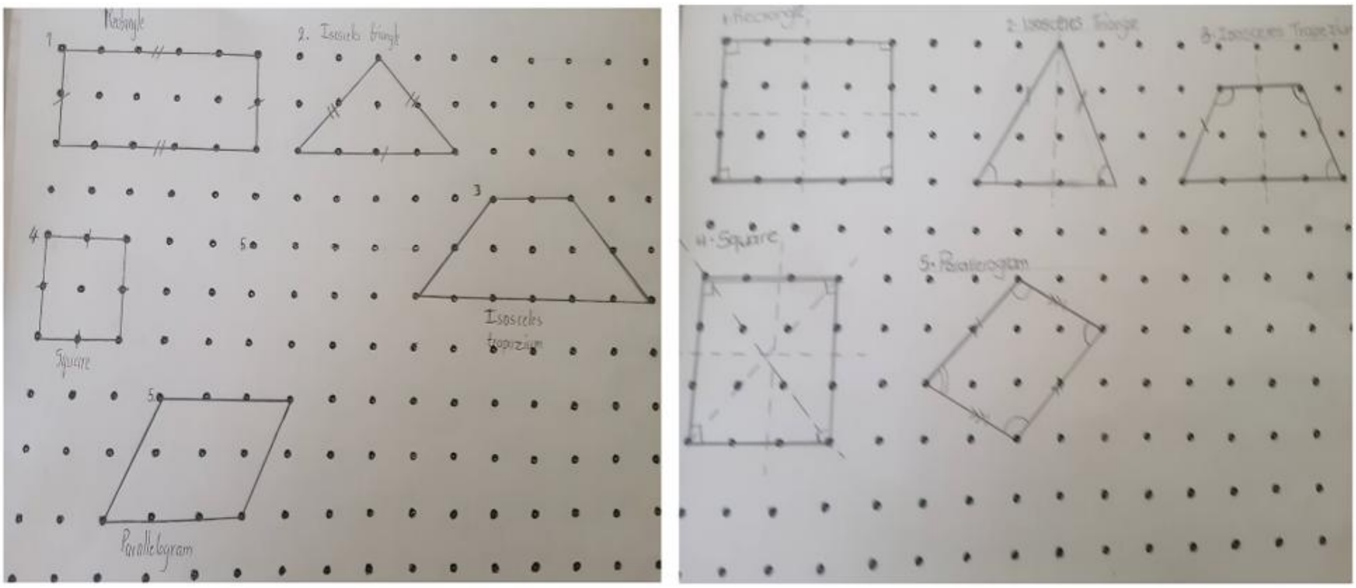

Figure 4. T2 and T4's drawings of different geometrical shapes on the geoboard (Source: Field study)

The use of geoboards in the context of the intervention workshop provided a hands-on learning experience that enhanced teachers' spatial reasoning and visualization skills. It allowed teachers to actively engage with the material, making abstract concepts more tangible and easier to comprehend. This interactive approach supports the constructivist theory of learning, which emphasizes the importance of active participation and experiential learning in building knowledge (Sibiya, 2020).

Workshop Based on the Van Hiele Theory for Geometry Teaching

The workshop that was based on the Van Hiele theory for geometry teaching was a dynamic approach that provided teachers with the opportunity to collaborate, reflect, give and receive feedback, as well as hands-on, practical experience (Dongwi, 2014). The data from the focus group and reflective journal allowed the researcher to answer research question 2: *How does the Van Hiele theory enable and/or constrain grade 9 mathematics teachers in their teaching of geometry in their mathematics lessons?* The result of the intervention workshops was that the teachers were able to build rapport with the researchers and among themselves. The sharing of their knowledge and experiences on teaching geometry was the cornerstone of the intervention workshops, and mathematics teachers indicated that they would continue sharing their experiences and knowledge. Regarding the use of the Van Hiele theory in geometry teaching, teachers shared their knowledge on how they could integrate the levels of the Van Hiele theory. These workshops offered an opportunity for teachers to explore and practice new instructional strategies, receive feedback, and reflect on their teaching practices (Armah & Kissi, 2019). By engaging in different activities, mathematics teachers could better understand how to facilitate geometric learning that aligns with the Van Hiele levels of thinking (Van Hiele, 1986).

Two mathematics teachers had this to say.

> This workshop helped me to understand the level of geometry that I was not aware of before this workshop. For me to teach geometry I must follow the levels and connect them before moving to the next level. Yaaa, I saw how the presenter explains how to teach geometry using that level, it's really fascinating to learn new things that I was supposed to learn at the university before I became a teacher (T4GD)

> The workshop was excellent, and it helped me to understand how Van Hiele theory could be used when teaching geometry in grade 9. The knowledge I gained will be useful for me when teaching my learners in future (T1GD).

> Teaching geometry was challenging before the workshop, now I understand the levels that learners must follow when learning geometry. The workshop helped me to be confident and understand the levels of Van Hiele theory that I will be using when teaching geometry to the learners. It was interesting to learn that geometry topics have a theory that can be used to teach it, which I was not aware of (T3GD).

The excerpts from three mathematics teachers revealed that they were not aware of the theory of Van Hiele, and the intervention workshops helped them to learn how to use the theory to teach geometry through the levels. The reflection of mathematics teachers illustrates that the four constructs of PD were integrated during the intervention workshops to enhance teachers' understanding of the Van Hiele theory. Workshops that use experiential learning techniques provide participants with opportunities to engage actively with the material, fostering a deeper understanding of the subject matter (Bestman & Chiwhetu, 2024). The workshops often included simulations and role-playing exercises that mimicked real-world challenges, allowing teachers to practice and refine their skills in a safe environment. This was illustrated by T3 and T4 during the workshop discussions.

> The workshops allowed us to work together as a team and learn from each other both our challenges and strengths when teaching geometry using Van Hiele theory. This was not common to most of us as teachers (T3GD).

> As a mathematics teacher, this workshop helped me to engage with other teachers on how to teach geometry using geoboards that could make the lesson more interesting to the learners. Teachers need to collaborate with each other when they face challenges on different topics in mathematics. This kind of collaboration is needed to improve our teaching as teachers (T4GD).

During the reflection, two mathematics teachers illustrated the need for collaborative work with other teachers for them to improve their teaching. This highlights the importance of teachers sharing their best practices on different mathematics topics. Vygotsky (1978) underscores the importance of collaborative learning, which not only helps in building a sense of community but also provides a support system that can lead to more sustained and impactful changes in practice. Vygotsky (1978) supports this, highlighting that social interaction plays a critical role in cognitive development. Providing ongoing support and follow-up activities helps to reinforce learning and sustain changes in practice. Research emphasizes that effective PD includes mechanisms for continuous support, such as coaching, mentoring, and peer networks (Antúnez-Montes et al., 2021; De Grave et al., 2024). This ongoing engagement helps participants to implement new strategies, overcome challenges, and continue their professional growth long after the workshop has ended. Mathematics teachers and researchers agreed to continue supporting and collaborating after this research was concluded.

A critical component of successful workshops was the integration of interactive and collaborative elements. Teachers benefited from working together to design lesson plans, solve geometric problems, and discuss best practices (Smith et al., 2023). This collaborative approach allows teachers to share their experiences, challenges, and solutions, fostering A sense of ownership (Lahann & Lambdin, 2020). Workshops can also integrate role-playing and peer-teaching exercises, where teachers practice new strategies and receive constructive feedback from their colleagues. This hands-on practice is invaluable for building confidence and competence in applying the Van Hiele theory in the classroom.

Furthermore, effective intervention workshops should be ongoing rather than one-time events. T1GD indicated that "we need to continue supporting each other after this workshop for us to keep learning and updating each other on the new teaching strategies". CPD allows teachers to revisit and refine their understanding and implementation of the Van Hiele theory over time. Follow-up sessions can address emerging challenges, introduce new instructional tools, and provide opportunities for further collaboration and reflection (Mann & Webb, 2022). This sustained support helps ensure that the insights and skills gained during workshops translate into lasting improvements in teaching practice, ultimately enhancing learners' geometric learning experiences. The participants indicated that they had enjoyed the intervention workshop by outlining the enablers and constraints of the Van Hiele theory in teaching geometry.

Enablers of geometric teaching

Teaching geometry using Van Hiele's theory can be significantly enhanced by leveraging various enablers. Technology integration, such as Geometer's Sketchpad, plays a crucial role in improving learners' understanding of geometric concepts and their spatial visualization abilities (Elbehary, 2022). Additionally, teacher education and PD are essential to equip teachers with the necessary skills and knowledge to effectively implement the Van Hiele theory in their teaching practices (Abdullah & Zakaria, 2013). Learner-centered instructional strategies, which focus on active learning and hands-on activities, also contribute to better engagement and understanding of geometric principles (Yunus et al., 2019). By combining these enablers, teachers can create a more effective and enriching learning environment for geometry learners.

Mathematics teachers in their reflective journals illustrated:

> Learners to get the names and sides of different regular polygons according to the level (T3RJ).

> It enables teachers to go through the five levels of Van Hiele theory and make the learners understand them very well one by one (T2RJ).

> It enables teachers to teach and to have the understanding that through these levels it will help learners to learn better in geometry (T1RJ).

The three journal reflections of the teachers show how they could teach geometry using the Van Hiele theory to help learners understand the topic of geometry. The Van Hiele levels provide a structured way to learn geometric concepts. At the visualization level, learners can recognize and name shapes based on their appearance. As they advance to analysis level, they begin to understand the properties of shapes, such as the number of sides and angles. This method aligns with the Van Hiele model, ensuring that learners build a solid foundation before progressing to more complex concepts (Vojkuvkova, 2012). The five levels of the Van Hiele model, visualization, analysis, abstraction, deduction, and rigor, provide a clear curriculum path for geometry education. Teachers can guide learners through these levels sequentially, ensuring thorough comprehension of geometric concepts (Fitriyani et al., 2018). Each level requires mastery before moving to the next, helping learners to develop their geometric thinking in a step-by-step manner (Fitriyani et al., 2018).

Constraints of geometric teaching

When teaching geometry using the Van Hiele theory, several constraints may arise. Firstly, the availability of technological resources can be a significant hurdle, as not all schools may have access to tools like Geometer's Sketchpad (Elbehary, 2022). Additionally, teacher preparedness is crucial, and there may be a lack of sufficient PD opportunities to adequately train teachers in Van Hiele theory-based methodologies (Abdullah & Zakaria, 2013). Furthermore, learners' diversity in learning styles and paces can pose a challenge, as the Van Hiele theory requires a sequential understanding of geometric concepts, which may not align with all learners' learning trajectories (Yunus et al., 2019). Addressing these constraints is essential for the successful implementation of Van Hiele theory in geometry education.

Two mathematics teachers wrote the following in the reflective journals:

> To me it was a bit tricky when it comes to geoboard, since I did not have an idea of what geoboard is? At the end I came to know that geoboard is a teaching material which is effective when teaching geometry (T2RJ).

> The lack of knowledge on how to teach geometry is one of the constraints that have been affecting most of us teachers. We are used to teaching geometry using traditional methods without using geoboards (T1RJ).

This is consistent with the findings of Sibiya (2020), who noted that many learners initially find geoboards challenging due to their unfamiliarity with this tool. Sibiya's (2020) study emphasizes that this initial confusion is a common hurdle that can be overcome with proper introduction and practice (Sibiya, 2020). Sibiya and Mudaly (2018) highlight that hands-on tools like geoboards help learners to gain a clearer understanding of geometric concepts and their study shows that learners who use geoboards tend to perform better academically in geometry due to the tactile learning experience (Sibiya & Mudaly, 2018).

CONCLUSION AND RECOMMENDATIONS

The exploration of grade 9 mathematics teachers' use of the geoboard when teaching geometry using the levels of the Van Hiele theory through workshops revealed that geometric understanding among mathematics teachers was enhanced through the use of geoboard. Intervention workshops that focused on this theory and hands-on activities equipped the mathematics teachers with the knowledge and skills needed to identify and address the diverse geometric thinking levels within their classrooms. These workshops fostered a collaborative learning environment where teachers shared experiences, refined their instructional strategies, and received ongoing support. This study provides a significant contribution to the field of mathematics education by demonstrating how teaching the Van Hiele theory using a geoboard, integrated into workshop-based PD, can enhance grade 9 mathematics teachers' understanding and instructional practices in geometry.

The effective use of geoboards in geometry teaching requires mathematics teachers to go through effective PD to employ a variety of teaching strategies. the study is limited to four mathematics teachers in Namibian secondary schools. Furthermore, future studies may scale or diversify the sample. The use of geoboards as a hands-on tool, coupled with targeted PD, would enable teachers to better identify and respond to the diverse levels of geometric thinking in their classrooms. Addressing the challenges associated with using the Van Hiele theory, such as varying levels of prior knowledge and limited instructional time, is crucial for success. This research extends the application of the Van Hiele theory beyond the Namibian context, offering insights that are relevant to global efforts in improving geometry instruction. Continuing PD and sustained support are essential for teachers to effectively translate workshop insights into classroom practice, ultimately leading to improved learner outcomes in geometry, as was evident in this study. Working together as a group helped teachers support each other and share their knowledge and experience on the use of geoboards when teaching geometry. The study provides a replicable model for PD that can be adapted to diverse educational contexts. By showcasing how structured, practical-driven PD empowers teachers to refine their instructional strategies, foster peer learning, and improve learner outcomes, this work extends existing knowledge on teacher PD in geometry. We recommend that mathematics teachers at higher institutions should be provided with ongoing PD opportunities focused on the

Van Hiele theory and its application in geometry teaching. Regular workshops, follow-up sessions, and access to expert mentors can help teachers continuously refine their instructional practices. PD should include hands-on, interactive activities that allow teachers to practice new strategies, give and receive feedback and collaborate with peers. This approach fosters PLCs and will support the effective implementation of the Van Hiele theory.

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AI statement: The authors stated tht Artificial intelligence (AI) tools, including ChatGPT (OpenAI, 2025), were used solely for paraphrasing and improving the clarity of the text. All ideas, analyses, and conclusions belong to the authors.

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