MODESTUM EURASIA Journal of Mathematics, Science and Technology Education, 2024, 20(7), em2473
ISSN:1305-8223 (online)
OPEN ACCESS Research Paper https://doi.org/10.29333/ejmste/14750

Enhancing the acquisition of basic algebraic principles using algebra tiles

José Antonio Núñez-López [1], David Molina-García [1*], José Luis González-Fernández [1], Iván Fernández-Suárez [1]

[1] Department of Mathematics, Faculty of Education, University of Castilla-La Mancha, Ciudad Real, SPAIN

Received 05 March 2024 ▪ Accepted 04 June 2024

Abstract

Introducing initial algebraic principles poses a significant challenge, often compounded by the inherent abstract nature of algebra. This article introduces an innovative pedagogical approach that promotes the use of algebra tiles, a didactic manipulative material formed by a collection of geometric pieces symbolizing distinct algebraic monomials. Additionally, this article includes the findings of a quasi-experimental study that applied this inventive teaching method. This research was carried out across two separate classes of 15 students (10 boys and five girls) of first year of compulsory secondary education. One group of students adhered to the conventional teaching approach (the control group), while the other class embraced the proposed methodology using algebra tiles (the experimental group). The disparities in algebraic proficiency observed between these two student groups, as assessed through various examinations conducted during the intervention were statistically significant, with the experimental group consistently achieving superior results. Moreover, mathematical and algebraic errors of students were assessed using eight distinct indicators. In all cases, the experimental group demonstrated lower error percentages, and these errors showed a marked decrease as the intervention progressed. In summary, this innovative methodology markedly enhanced students' comprehension of algebra, their knowledge, and their motivation while significantly reducing mathematical errors.

Keywords: algebra tiles, algebraic language, compulsory secondary teaching, mathematics

INTRODUCTION

In the realm of mathematics education, particularly when working with children and young adolescents, challenges often arise owing to its formal and inherently abstract nature (Arcavi, 1995; Blanton et al., 2015; Ndemo & Ndemo, 2018; Muchoko et al., 2019). Consequently, the incorporation of hands-on, manipulative materials has emerged as a cornerstone strategy relied upon by educators and advocates to enhance comprehension over time (McNeil & Jarvin, 2007). In fact, it has been demonstrated that the use of this type of materials enhances both motivation and the students' overall understanding of mathematics (Larby & Mavis, 2016; Laski et al., 2016; Swan & Marshall, 2010).

Among the various branches of mathematics, algebra stands out as one of the most challenging for students due to the complexities associated with transitioning from arithmetic and grasping the multiple interpretations and applications of variables (Caylan & Haser, 2021; Guner, 2020; Jupri et al., 2015). Its learning is carried out throughout children's development: in early childhood education (three-six years) the so-called early algebra, which has been proposed in recent years (Blanton et al., 2015; Radford, 2022), use mainly games, songs or situations of everyday life are used to allow the child to develop logical-mathematical reasoning and, thus, to be initiated in algebra. During primary education (six-12 years), this early algebra tends to be taught through arithmetic and its axioms (Carraher & Schliemann, 2007), the relationships between quantities and the use of the first mathematical symbols to represent operations (Radford, 2022).

In the first years of compulsory secondary education (12-16 years) algebraic contents taught experience a significant qualitative leap in terms of complexity

✉ joseantonio.nunez@uclm.es ✉ david.molina@uclm.es **(*Correspondence)** ✉ jluis.gonzalez@uclm.es
✉ orientacion@marianistas-cr.org

Contribution to the literature

- The study presents a detailed innovative methodology, along with recommendations, key findings and limitations, for teaching basic algebraic concepts using the didactic manipulative material algebra tiles.
- The evidence provided reveal how the use of algebra tiles in the classroom of first year of compulsory secondary education can improve students' comprehension of basic algebraic concepts.
- The study provides an analysis of the main primary algebraic errors of the students using the classical and proposed methodology.

(Andini & Prabawanto, 2021). At these levels, a significant focal point is the initial shift from arithmetic to formal algebra, marked by the introduction of variables in calculations. Indeed, numerous prior studies have highlighted the challenges that secondary school students face during this transition (Filloy et al., 2008; Kieran, 1989, 2007).

Algebra instruction plays a pivotal role in the realm of mathematics education, serving as the cornerstone upon which more advanced mathematical concepts are built. When it comes to teaching algebra, which must be assessed over the years during relevant and irrelevant mathematical practices (Tsamir & Tirosh, 2022), the prevailing approach often relies on memorization-based instructional methods, leading to suboptimal learning outcomes and restricted knowledge retention (Garzón & Bautista, 2018). Conversely, multiple studies have examined various facets of algebra education, such as the effects of technology, the influence of student characteristics, and teacher preparation (Veith et al., 2023).

The use of manipulative materials has proven to improve students' algebraic abilities, such as representing and interpreting algebraic expressions, making connections between algebraic concepts, and to establishing meaningful connections in algebraic thinking. However, surprisingly, there is a limited availability of manipulative materials designed for algebra, despite the well-documented advantages of this approach in enhancing learning and comprehension in various other mathematical disciplines (Carbonneau et al., 2013; Moyer, 2001). Among them, algebra tiles stand out as it is specifically designed to enhance comprehension of algebraic concepts using geometric figures (Kablan, 2016). The possibility of creating and examining mathematical terms and breaking them down, then reconstructing them in various structural configurations stand out as the main benefits of using algebra tiles. Previous research on the application of algebra tiles in algebraic teaching and learning has mostly focused on teaching how to solve linear equations in one variable (Agrawal & Morin, 2016; Saraswati et al., 2016), a system of two linear equations (Akpalu et al., 2018), factor algebraic expressions (Larbi & Mavis, 2016), polynomial multiplication or solve quadratic equations by completing a square (Vinogradova, 2007). However, there is still a lack of research into its potential to enhance algebraic understanding.

The aim of this article is to show the benefits of using algebra tiles with 12-13-year-old students in their first contacts with algebra. For this purpose, this study showcases the outcomes of an intervention conducted with two groups of first-year compulsory secondary education students using algebra tiles. Furthermore, this document includes details on the particular methodology used, the sequencing of topics taught in the experimental group, and a description of the algebra tiles materials developed.

MATERIALS & METHODS

Participants

This study involved 30 students from two different groups of first year of compulsory secondary education (students aged 12-13 years). Both groups were taught by the same instructor, with the control group taking traditional lessons, whereas the experimental group received a specialized approach using algebra tiles. Initially, the control group consisted of 17 students, with two of them being frequently absent, resulting in a total of 15 students (10 boys and five girls). In contrast, the experimental group comprised 19 students, with two of them being regular absentees and an additional two receiving therapeutic pedagogy support. As a result, 15 students (consisting of 10 boys and five girls) from this group were included in the study.

Algebra Tiles

Algebra tiles are manipulative materials designed to enhance the comprehension of algebraic principles. They serve diverse educational purposes, including visualizing additive and multiplicative algebraic expressions, factoring polynomials, and solving linear equation systems. These tiles are readily obtainable and can even be crafted in the classroom using templates, as was the case in the current study. In essence, they comprise a collection of squares and rectangles, with their respective areas symbolizing various algebraic monomials (Garzón & Bautista, 2018). **Figure 1** displays different components from a set of algebra tiles.

algebra tiles comprise geometric elements that symbolize terms in algebraic expressions of various

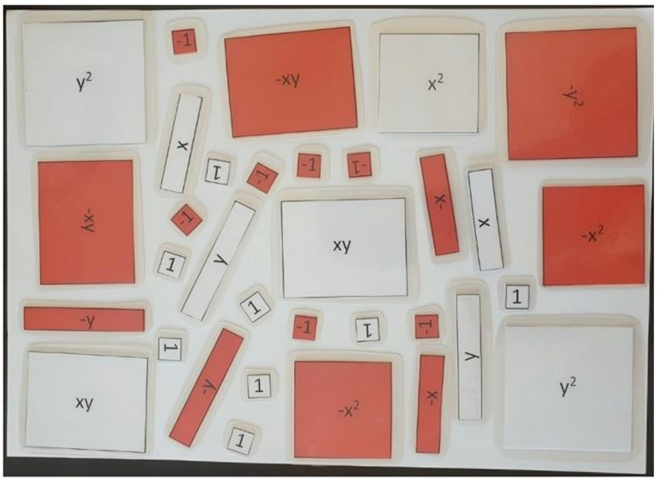

Figure 1. Algebra tiles pieces built & used for this study (Source: Authors' own elaboration)

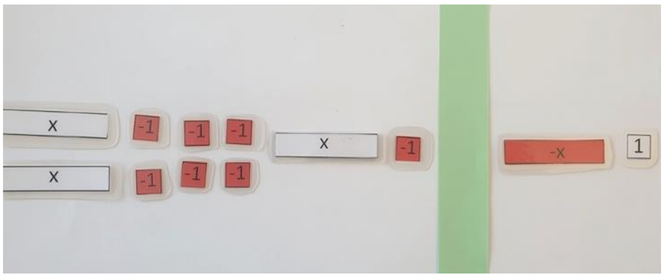

Figure 2. Representation of equation $2(x-3)+x-1=-x+1$ using algebra tiles (green vertical separation between two expressions symbolizes "=" symbol in equation) (Source: Authors' own elaboration)

degrees: degree zero (1x1 squares), degree one (rectangles with varying areas based on the unknown value), and degree two (squares and rectangles with areas influenced by the unknowns) (Reinschlüssel et al., 2018). In the material developed for this study, the white pieces represent positive terms, whereas the red ones denote their negative counterparts.

Representing an algebraic expression using algebra tiles is straightforward; you merely group the appropriate pieces until you achieve the desired representation. Conversely, to depict an equation, the workspace is divided into two sections, one for each side of the equation, and the necessary pieces are arranged to match the expression on both sides. **Figure 2** shows the representation of the equation $2(x-3)+x-1=-x+1$ with algebra tiles.

Particularly for this research, 16 sets of algebra tiles were manufactured: 15 sets for the experimental group and 1 set for the teacher. Each set comprised 56 pieces, resulting in a total of 896 pieces utilized. **Table 1** displays the allocation of pieces per set and the overall number of pieces created.

It's important to highlight that algebra tiles were distributed to the participants at the outset of the intervention, granting them access both inside and

Table 1. Algebra tiles pieces constructed for each set & in total

Algebra tiles piece	Number of pieces included in each set	Total number of pieces produced
1	15	240
-1	15	240
x	5	80
$-x$	4	64
y	2	32
$-y$	5	80
x^2	2	32
$-x^2$	2	32
y^2	1	16
$-y^2$	1	16
$x \cdot y$	2	32
$-x \cdot y$	2	32
Total	56	896

outside the classroom. This approach enabled the students to utilize the algebra tiles at their convenience.

Methodology of Intervention

The intervention detailed in this article was conducted at the beginning of the third term. This intervention, which is the focus of this article, marked the students' initial exposure to algebra. Thus, the first two evaluation periods exclusively covered arithmetic topics, including integers, rational numbers, exponentiation, proportions, and operations. The students' performance in these initial two evaluations served as a basis for assessing the prior disparities in mathematical proficiency between the two groups.

The intervention was developed in a total of 16 sessions. However, the use of algebra tiles was taught uninterruptedly during the first seven sessions, allowing students to use this material freely from this point forward. The concrete methodology followed in this first seven sessions was the following:

- *Session 1*: A preliminary test aimed at gauging students' foundational understanding of algebra was administered. This assessment entailed solving seven problems that required determining the value of an unknown variable, and it was filled in by the participants before any specific algebraic instruction. Students were informed that they were free to employ any strategy of their choice to solve the assessment.
- *Session 2*: During this session, each student received a set of algebra tiles. The instructor provided a comprehensive explanation of the significance of each tile, emphasizing the unique geometric attributes of each piece and highlighting that different pieces could not be combined through addition or subtraction. The teacher used this opportunity to introduce the concept of an unknown variable, as several pieces within the set symbolized these unknowns.

Subsequently, both the teacher and the students utilized algebra tiles to depict their initial algebraic expressions. As homework, three exercises were assigned, all focused on the representation of algebraic expressions using algebra tiles.

- *Session 3*: The initial segment of this session involved addressing previous homework and resolving uncertainties. For instance, some students had misconceptions about the algebraic expressions "8+x" and "8x", erroneously considering them identical. Consequently, the concept of an unknown variable was revisited. Then, the instruction shifted to teaching the numerical value of algebraic expressions using algebra tiles. This entailed instructing students to match a piece representing an unknown variable with a given number and subsequently perform mathematical operations among these numerical values using the visual aid of the tiles. Subsequently, the class delved into the initial practices of addition and subtraction with monomials, always emphasizing the importance of using pieces with matching geometric forms when performing mathematical operations. To reinforce these concepts, two fresh exercises were assigned as homework: one focusing on determining numerical values and the other focused on executing mathematical operations with monomials, both to be tackled with the aid of algebra tiles.
- *Session 4*: After resolving homework assignments, the session comprised similar exercises that were introduced and solved in class, and two of them designated as homework. In retrospect, the teacher observed that dedicating this session to reviewing previously covered contents significantly contributed to the enhancement of students' grasp of algebraic concepts and their overall enthusiasm for learning. For instance, the instructor pinpointed a prevalent misconception regarding the relative sizes of variables "x" and "y": due to "y" tiles being larger than "x" tiles, students erroneously concluded that the variable "y" must always be greater than "x".
- *Session 5*: After resolving homework assignments, the teacher introduced the concept of first-grade equations. Initially, the teacher had the students concurrently represent two distinct algebraic expressions using algebra tiles. Subsequently, the students were asked to discern how these expressions could be made equivalent. Initially, the students faced some confusion, prompting the teacher to reiterate the notion of an unknown variable and the flexibility in its value, emphasizing the need to find the appropriate value. Since none of the students could arrive at an answer, the teacher explained that they could achieve equivalence by adding or subtracting equivalent terms from both expressions. As the students began to work on this concept, the teacher provided individual guidance and support. Two additional examples were presented using the same approach, and another pair of examples were assigned as homework.
- *Session 6*: This session, following the resolution of prior homework, was dedicated to review contents of previous session and to introduce equations with parentheses, and its representation with algebra tiles. They started presenting difficulties in the use of parentheses, and so the teacher realized that using the classical methodology, this procedure could be easier and faster, keeping algebra tiles as a resource for these situations.
- *Session 7*: In this session, an intermediate exam was administered to assess the acquisition of the material taught at the halfway point of the intervention. This intermediate examination encompassed six problems that required students to articulate quantities in algebraic terms, compute numerical values, simplify algebraic expressions, and solve elementary equations.

After this seventh session, the teaching methodologies in both groups were unified and no use of algebra tiles was imposed in the experimental group. This decision was made for two main reasons: in one hand, to observe the influences of the initial methodologies and whether the algebra tiles continued to be used by the students in the experimental group. On the other hand, algebra tiles could lose effectiveness representing algebraic fractional equations in a natural and agile way, something not previously reported in the literature.

The subsequent nine sessions, leading up to session 16, which marked the final exam, were dedicated to instructing students on fractional equations, problem-solving techniques using equations, and reviewing all previously covered concepts.

After the last session, a simple satisfaction survey was administered to the 15 students in the experimental group to qualitatively assess the benefits and drawbacks of their use in class, as well as each student's individual impression regarding their usefulness for learning algebra.

Primary Algebraic Errors

The mathematical errors found in the two different exams across the intervention developed in this study, the intermediate and final exams, were classified in different categories to analyze the quantity and distribution of different error sources in both the control and experimental group.

The categories considered were

(1) algebraic situation description,

(2) numerical values,

(3) monomial operations,

(4) solving for the unknown variable,

(5) term transposition,

(6) proper use of parentheses,

(7) equations involving fractions, and

(8) algebraic problems.

Statistical Methods

All the collected data has been analyzed with statistical package for social sciences, SPSS (v.29). In all statistical tests, a two-tailed significance level of p-value lower than 0.05 was applied.

The Wilcoxon signed-rank test (Wilcoxon, 1945) was used for the intragroup statistical analyses. This is a nonparametric test used to compare two related samples and therefore does not rely on a particular distribution of the data. In contrast, for intergroup comparisons, the Mann-Whitney U test (Mann & Whitney, 1947), which is the generalization of the Wilcoxon test for unrelated samples, has been employed.

To assess the relationships between students' scores on various evaluations, a nonparametric Spearman's correlation analysis (Spearman, 1904) was conducted. Consistent with established criteria, a Spearman's correlation coefficient (S) greater than 0.7 (or less than -0.7) was deemed to indicate a robust positive (or negative) correlation.

Along with the aforementioned, standard statistics such as mean, standard deviation, maximum and minimum value, median and coefficient of variation were computed over the different exams.

Beyond a global perspective, the results achieved by both student groups on the intermediate and final exams for each specific algebraic task were examined. This approach allowed to explore the primary challenges encountered by both groups and assess the potential impacts of both algebra tiles and the instructional methods utilized during the intervention. To achieve this, the error percentage for each activity was measured, as follows: if a task was answered perfectly, it was assigned a 0.00% error rate. On the other hand, for instance, if one-third of the maximum score was achieved, it was recorded as a 66.00% error rate.

RESULTS

Assessment of Same Initial Mathematical Level Between Groups

Initially, it was confirmed that both groups of students exhibited comparable arithmetic proficiency at the outset of the intervention. To verify this, the marks attained by the students in the first two assessments, which focused on arithmetic content, were compared using the Mann-Whitney test. When scores were rated on a scale from zero to 10, with 10 representing the highest possible score, the control group achieved a mean mark ± standard deviation of 5.92±1.99 and 6.75±1.11 in the first and second evaluation, respectively. In contrast, the experimental group obtained scores of 5.69±1.76 and 7.19±1.53 in the first and second evaluations, respectively. The results revealed no significant differences between both groups, with p-values of 0.783 while comparing the first evaluation and 0.546 for the second evaluation, indicating an equivalent level of arithmetic proficiency.

To assess the baseline algebraic understanding prior the intervention, Mann-Whitney test was also applied to the preliminary test scores, where average marks 6.09±1.83 and 6.47±1.78 were obtained in the control and experimental group, respectively. The non-significant outcome of this test (p-value=0.882) showed an equivalent initial level of algebraic knowledge in both groups.

Intergroup Comparisons During Intervention

Regarding the intermediate exam, after seven teaching sessions employing each methodology, significant disparities in algebra acquisition emerged between both groups of students (p-value=0.007). The most substantial contrast in achievement was observed during this exam, with scores of 4.22±1.80 for the control group and 6.04±1.86 for the experimental group.

These differences in algebraic knowledge persisted until the final exam, although they did not show statistical significance (p-value=0.083). Nevertheless, they remained substantial, as evidenced by scores of 5.90±2.31 in the control group and 7.21±1.80 in the experimental group.

Table 2 and **Table 3** show the statistical measures calculated for each of the evaluations and assessments for the control group and the experimental group, respectively.

No substantial correlations were observed among the scores of any evaluation or exam, indicating that the results of one test did not impact those of another. However, it is noteworthy that a correlation of 0.68 was observed between the results of the second evaluation and the preliminary test, which can be attributed to their close temporal proximity. **Figure 3** presents the Spearman correlation results for each pair of evaluations/assessments examined in this study.

Intragroup Comparisons During Intervention

In the control group, the Wilcoxon test revealed significant differences between the scores of the

Table 2. Statistics computed (using 10 as highest mark) over control group

Statistic	First evaluation	Second evaluation	Preliminary test	Intermediate exam	Final exam
Mean	5.92	6.75	6.09	4.22	5.90
Standard deviation	1.99	1.11	1.83	1.80	2.31
Maximum	9.75	9.13	8.57	8.75	9.00
Minimum	2.83	5.33	1.43	1.50	2.40
Median	5.49	6.75	5.71	4.00	5.55
Coefficient variation	33.67%	16.48%	29.98%	42.63%	39.08%

Table 3. Statistics computed (using 10 as highest mark) over experimental group

Statistic	First evaluation	Second evaluation	Preliminary test	Intermediate exam	Final exam
Mean	5.69	7.19	6.47	6.04	7.21
Standard deviation	1.76	1.53	1.78	1.86	1.80
Maximum	9.79	9.40	10.00	9.00	9.85
Minimum	3.09	4.65	4.29	3.50	4.50
Median	5.98	7.50	5.71	5.90	6.90
Coefficient variation	30.86%	21.35%	27.48%	30.83%	24.92%

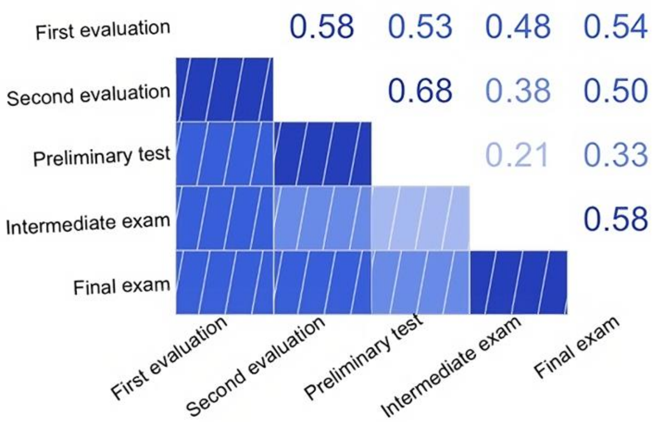

Figure 3. Spearman correlations between different evaluations & exams (upper right triangle shows exact correlation coefficients, while lower left triangle shows corresponding color map) (Source: Authors' own elaboration)

intermediate exam and both the preliminary test and the final exam (p-values of 0.007 and 0.005, respectively).

Table 2 illustrates that the average scores attained in the intermediate exam (4.22±1.80) were considerably lower compared to those of the preliminary test (6.09±1.83) and the final exam (5.90±2.31). These findings underscore a limited comprehension of foundational algebraic concepts among students instructed through the traditional teaching method.

Conversely, focusing on the experimental group, the Wilcoxon test did not reveal significant differences between the preliminary test and the intermediate exam (p-value=0.389). This suggests a more proficient grasp of fundamental algebraic concepts in this group when compared to the control group. Additionally, significant disparities emerged between the intermediate and final exams (p-value=0.005), with the latter exhibiting superior scores (6.04±1.86 versus 7.21±1.80, respectively), as shown in **Table 3**.

Primary Algebraic Errors

Error percentages were assessed by the instructor for the intermediate and final exam and for both groups. Concerning the intermediate exam, it became evident that the average error rates in each of the tasks were lower in the experimental group compared to the control group: algebraic situation description (28.44% vs. 38.89%, respectively), numerical values (40.00% vs. 42.22%), monomial operations (52.00% vs. 79.17%), solving for the unknown variable (18.00% vs. 18.33%), term transposition (31.66% vs. 59.17%) and proper use of parentheses (54.17% vs. 70.83%). Consequently, the overall average error rates were 37.39% for the experimental group and 51.44% for the control group.

However, regarding the final exam, while the experimental group continued to exhibit superior performance, different results could be found: algebraic situation description (49.00% vs. 45.00% for the experimental group and control group, respectively), numerical values (31.13% vs. 18.33%), monomial operations (20.66% vs. 63.66%), solving for the unknown variable (14.33% vs. 12.66%), term transposition (9.00% vs. 31.00%) and proper use of parentheses (28.44% vs. 41.11%), equations involving fractions (33.33% vs. 40.44%) and algebraic problems (26.00% vs. 58.33%). Consequently, the overall average error rates were 26.49% for the experimental group and 38.82% for the control group.

Interestingly, there was a noticeable reduction in the average error rate across the exams, evident in both the experimental group (decreasing from 37.39% to 26.49%) and the control group (decreasing from 51.44% to 38.82%).

Table 4 provides the precise percentages of primary algebraic errors for each activity and both exams.

Table 4. Primary algebraic error rates for both groups for each activity & both intermediate & final exams

Algebraic activity	Intermediate exam		Final exam	
	Experimental group	Control group	Experimental group	Control group
Algebraic situation description	28.44%	38.89%	49.00%	45.00%
Numerical values	40.00%	42.22%	31.13%	18.33%
Monomial operations	52.00%	79.17%	20.66%	63.66%
Solving for the unknown	18.00%	18.33%	14.33%	12.66%
Term transposition	31.66%	59.17%	9.00%	31.00%
Proper use of parentheses	54.17%	70.83%	28.44%	41.11%
Equations involving fractions	-	-	33.33%	40.44%
Algebraic problems	-	-	26.00%	58.33%
Means	37.39%	51.44%	26.49%	38.82%

Satisfaction Survey

The results of the satisfaction survey conducted in the experimental group at the end of the intervention showed a high acceptance of algebra tiles for teaching initial algebraic concepts. Specifically, 73.30% of the students (11 out of 15) expressed that the material greatly helped them understand the concepts and found algebra tiles interesting and easy to use. They also considered this didactic material a straightforward way to learn algebra, highlighting its playful aspect. On the other hand, 26.70% (4 out of 15) expressed that algebra tiles lost its utility and even became confusing when dealing with complex algebraic content. For this reason, they decided to stop using it in the later sessions of the intervention.

DISCUSSION

The algebraic concepts covered in the final years of primary education and the initial stages of compulsory secondary education introduce a higher level of mathematical abstraction. The challenges that students in these levels face with algebra have been extensively examined (Jupri & Drijvers, 2016; McCrory et al., 2012; Muchoko et al., 2019) and are a significant factor contributing to the widespread apprehension and aversion to mathematics. This article introduces an innovative methodological approach to teach introductory algebra, utilizing the manipulative didactic material algebra tiles. Additionally, it presents the findings of a quasi-experimental exploratory study conducted on two distinct groups of students of first-year compulsory secondary education using this methodology.

Algebra tiles stand out as one of the limited manipulative resources designed specifically for the early stages of algebra, despite the well-documented advantages of manipulative materials in mathematics learning (Carbonneau et al., 2013). Furthermore, the transition from arithmetic to algebra represents a substantial shift for most students (Caglayan et al., 2013). In fact, owing to the inherent challenges, suggestions have previously been made to incorporate algebraic concepts into earlier coursework, encompassing patterns, relationships, situational representations, and algebraic structures, among other strategies (Blanton et al., 2015; Kusumaningsih et al., 2018; NCTM, 2000).

Although research involving algebra tiles remains somewhat limited, prior studies have already highlighted the advantages of incorporating this instructional tool for teaching algebra across various student populations. Specifically, Garzón and Bautista (2018) observed a significant enhancement in the algebra performance of 40 undergraduate engineering students who utilized virtual algebra tiles. Long et al. (2020) further demonstrated its effectiveness among students with intellectual and developmental disorders. Additionally, Reinschlüssel et al. (2018) developed a tailored user interface for working with algebra tiles and achieved promising outcomes. Finally, Leitze and Kitt (2000) also noted improvements in algebra learning when employing algebra tiles with elementary and high school students, as well as incarcerated students pursuing their high school diplomas. However, it is worth noting that, to date, no study has examined the impact of this material during students' initial exposure to algebra.

This quasi-experimental exploratory study developed in this paper involved two groups of first year compulsory secondary education, each comprising 15 students. Both groups were instructed by the same instructor. In one group, referred to as the experimental group, the teacher employed the outlined methodology based on algebra tiles, while in the other group, the control group, the teacher used the traditional/classical teaching approach. Prior to the intervention, the study ensured that the students in both groups exhibited similar levels of proficiency in arithmetic and algebra. This similarity was confirmed by the results of the Mann-Whitney test during the first and second evaluations, as well as in a preliminary test developed at the beginning of the intervention.

The examination results from the students indicate a superior performance of the experimental group in their comprehension and execution of algebraic operations. Notably, significant differences emerged at the midpoint of the intervention when contrasting the outcomes of the two teaching methodologies. Moreover, these findings underscore the challenges faced by students who followed the traditional teaching approach, particularly in areas such as monomial operations and proper usage of parentheses. Conversely, the experimental group consistently outperformed the control group in all the activities presented. It is important to point out that previous studies have shown that interaction with algebra tiles in group work has some positive effect on students' algebraic thinking and motivation (Chappel & Strutchens, 2001).

Upon reaching the midpoint of the intervention, the teacher faced the challenge of reconciling the remaining curriculum with the potential incompatibilities of algebra tiles. However, it became evident that a significant number of students in the experimental group persisted in using algebra tiles as a supportive tool, gradually transitioning to independence. As the intervention concluded, the results of the final examination revealed that over half of the students in the control group continued to encounter challenges in areas such as monomial operations and algebraic problem-solving. In contrast, the students who had adhered to the methodology focused on algebra tiles consistently demonstrated superior performance.

Results obtained emphasize that the use of the manipulative material algebra tiles significantly enhanced the comprehension of basic algebraic contents. It effectively improved the challenge of visualizing algebraic concepts, a primary hurdle in early algebra education (Muchoko et al., 2019). Furthermore, it is crucial to highlight the substantial reduction observed in nearly all categories of primary algebraic errors among the group that utilized algebra tiles, particularly in "monomial operations" and "term transposition". The students in the experimental group demonstrated a remarkable proficiency, particularly in these domains, underscoring the significant impact of learning foundational algebraic concepts using algebra tiles, which proved to be an extremely beneficial initial tool. Nonetheless, it's worth noting that once fundamental algebraic concepts were firmly grasped, many students in the experimental group opted to stop using algebra tiles. This is a common occurrence, leading to the conclusion that while this manipulative material is valuable, its utility in the classroom diminishes over time.

Regarding the limitations of the study, it is worth noting that the study was conducted with a relatively small sample, consisting of just two groups of 15 students. Despite this limitation, the results obtained were robust, and it is hoped that the research can be expanded in the coming years. On the positive side, the small group sizes could have contributed to more focused interactions. Also, as a limitation, it is important to note that this intervention was developed for the first time, and the instructor has collected significant information on its development to enhance it in upcoming courses. Furthermore, it is crucial to underscore the constraints of algebra tiles when instructing equations involving fractions. Consequently, the authors recommend their utilization primarily for grasping fundamental algebraic concepts and equations, which is certainly valuable. Based on these limitations and prompted by the promising results from this study, the researchers are actively engaged in crafting an expanded iteration of this intervention for six groups of first-year compulsory secondary education students. Additionally, the authors are working to improve algebra tiles to more effectively represent equations involving fractions, addressing one of the current limitations of this resource. Finally, the authors plan to compare the effectiveness of virtual algebra tiles, which have been previously utilized in the literature and may increase student engagement through the use of computers and learning applications specifically designed for learning algebra, as opposed to manipulative algebra tiles.

CONCLUSIONS

Algebra tiles proved to be a valuable resource for teaching basic algebraic concepts. The intervention implemented in this study using algebra tiles clearly outperformed the conventional methodology, demonstrating the effective resolution of the primary challenges related to the visualization and comprehension of algebra teaching through the utilization of an algebra tiles-based approach. Furthermore, the use of this manipulative material reduced the primary algebraic errors of the students, especially in "monomial operations" and "term transposition". It is worth highlighting that the results acquired are consistent, robust, and align with prior research, underscoring that employing algebra tiles for algebra instruction offers significant benefits to students, facilitating a more intuitive grasp of mathematical abstraction.

Author contributions: JAN-L, DM-G, JLG-F, & IF-S: conceptualization, methodology, & writing-review & editing; **JAN-L:** data curation; & **DM-G:** formal analysis. All authors have agreed with the results and conclusions.

Funding: No funding source is reported for this study.

Acknowledgements: The authors would like to thank Cristina Solares Martínez & Rocío Blanco Somolinos for their valuable assistance during initial phases of study design & for their support in comprehending algebra tiles material.

Ethical statement: The authors stated that the ethical approval was obtained for this work from the institutional ethics committee of IES Pedro Álvarez de Sotomayor here on 23 November 2021 (Approval code: PAS-2021-17). Written informed consents were obtained from the participants.

Declaration of interest: No conflict of interest is declared by the authors.

Data sharing statement: Data supporting the findings and conclusions are available upon request from the corresponding author.

REFERENCES

Agrawal, J., & Morin, L. L. (2016). Evidence-based practices: Applications of concrete representational abstract framework across math concepts for students with mathematics disabilities. *Learning Disabilities Research and Practice, 31*(1), 34-44. https://doi.org/10.1111/ldrp.12093

Akpalu, R., Adaboh, S., & Boateng, S. S. (2018). Towards improving senior high school students' conceptual understanding of system of two linear equations. *International Journal of Educational Research Review, 3*(1), 28-40. https://doi.org/10.24331/ijere.373336

Andini, M., & Prabawanto, S. (2021). Relational thinking in early algebra learning: A systematic literature review. *Journal of Physics: Conference Series, 1806*, 012086. https://doi.org/10.1088/1742-6596/1806/1/012086

Arcavi, A. (1995). Teaching and learning algebra: Past, present, and future. *The Journal of Mathematical Behavior, 14*(1), 145-162. https://doi.org/10.1016/0732-3123(95)90033-0

Blanton, M., Stephens, A., Knuth, E., Gardiner, A. M., Isler, I., & Kim, J. S. (2015). The development of children's algebraic thinking: The impact of a comprehensive early algebra intervention in third grade. *Journal for Research in Mathematics Education, 46*(1), 39-87. https://doi.org/10.5951/jresematheduc.46.1.0039

Caglayan, G., Olive, J., & Izsák, A. (2013). How middle school students understand polynomial sums and products using algebra tiles model in a "course dialogue". *Recherches en Didactique des Mathématiques* [*Research in Mathematics Didactics*], *33*(3), 267–306.

Carbonneau, K. J., Marley, S. C., & Selig, J. P. (2013). A meta-analysis of the efficacy of teaching mathematics with concrete manipulatives. *Journal of Educational Psychology, 105*(2), 380-400. https://doi.org/10.1037/a0031084

Carraher, D. W., & Schliemann, A. D. (2007). Early algebra and algebraic reasoning. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning*. NCTM. https://doi.org/10.1007/978-94-007-4978-8_54

Caylan, B., & Haser, C. (2021). Students' algebra achievement, algebraic thinking and views in the case of using algebra tiles in groups. *Necatibey Eğitim Fakültesi Elektronik Fen ve Matematik Eğitimi Dergisi* [*Necatibey Faculty of Education Electronic Journal of Science and Mathematics Education*], *15*(2), 254-281. https://doi.org/10.17522/balikesirnef.1019292

Chappell, M. F., & Strutchens, M. E. (2001). Creating connections: Promoting algebraic thinking with concrete models. *Mathematics Teaching in the Middle School, 7*(1), 20-25. https://doi.org/10.5951/MTMS.7.1.0020

Filloy, E., Puig, L., & Rojano, T. (2008). *Educational algebra. A theoretical and empirical approach*. Springer. https://doi.org/10.1007/978-0-387-71254-3

Garzón, J., & Bautista, J. (2018). Virtual algebra tiles: A pedagogical tool to teach and learn algebra through geometry. *Journal of Computer Assisted Learning, 34*, 876-883. https://doi.org/10.1111/jcal.12296

Guner, N. (2020). Difficulties encountered by high school students in mathematics. *International Journal of Educational Methodology, 6*(4), 703-713. https://doi.org/10.12973/ijem.6.4.703

Jupri, A., & Drijvers, P. (2016). Student difficulties in mathematizing word problems in algebra. *EURASIA Journal of Mathematics, Science & Technology Education, 12*(9), 2481-2502. https://doi.org/10.12973/eurasia.2016.1299a

Jupri, A., Drijvers, P., & van den Heuvel-Panhuizen, M. (2015). Improving grade 7 students' achievement in initial algebra through a technology-based intervention. *Digital Experiences in Mathematics Education, 1*(1), 28-58. https://doi.org/10.1007/s40751-015-0004-2

Kablan, Z. (2016). The effect of manipulatives on mathematics achievement across different learning styles. *Educational Psychology, 36*(2), 277-296. https://doi.org/10.1080/01443410.2014.946889

Kieran, C. (1992). The learning and teaching of school algebra. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 390-419). Macmillan.

Kieran, K. (2007). Learning and teaching algebra at the middle school through college levels. Building meaning for symbols and their manipulation. In F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 707-762). Information Age Publishing.

Kusumaningsih, W., Darhim, D., Herman, T., & Turmudi, T. (2018). Improvement of algebraic thinking ability using multiple representation strategy on realistic mathematics education. *Journal on Mathematics Education, 9*, 281-290. https://doi.org/10.22342/jme.9.2.5404.281-290

Larby, E., & Mavis, O. (2016). The use of manipulatives in mathematics education. *Journal of Education and Practice, 7*(36), 53-61.

Laski, E. V., Jor'dan, J. R., Daoust, C., & Murray, A. K. (2015). What makes mathematics manipulatives effective? Lessons from cognitive science and Montessori education. *SAGE Open, 5*(2). https://doi.org/10.1177/2158244015589588

Leitze, A., & Kitt, N. (2000). Using homemade algebra tiles to develop algebra and prealgebra concepts.

Mathematics Teacher: Learning and Teaching, PK-12, 93, 462. https://doi.org/10.5951/MT.93.6.0462

Long H., Bouck E., & Domka A. (2020). Manipulating algebra: Comparing concrete and virtual algebra tiles for students with intellectual and developmental disabilities, *Exceptionality, 1*, 1-18. https://doi.org/10.1080/09362835.2020.1850454

Mann, H. B., & Whitney, D. R. (1947). On a test of whether one of two random variables is stochastically larger than the other. *Annals of Mathematical Statistics, 18*, 50-60. https://doi.org/10.1214/aoms/1177730491

McCrory, R., Floden, R., Ferrini-Mundy, J., Reckase, M. D., & Senk, S. L. (2012). Knowledge of algebra for teaching: A framework of knowledge and practices. *Journal for Research in Mathematics Education, 43*(5), 584-615. https://doi.org/10.5951/jresematheduc.43.5.0584

McNeil, N. M., & Jarvin, L. (2007). When theories don't add up: Disentangling the manipulatives debate. *Theory into Practice, 46*(4), 309-316. https://doi.org/10.1080/00405840701593899

Moyer, P. S. (2001). Are we having fun yet? How teachers use manipulatives to teach mathematics. *Educational Studies in Mathematics, 47*(2), 175-197. https://doi.org/10.1023/A:1014596316942

Muchoko, C., Jupri, A., & Prabawanto, S. (2019). Algebraic visualization difficulties of students in junior high school. *Journal of Physics: Conference Series, 1157*, 032108. https://doi.org/10.1088/1742-6596/1157/3/032108

NCTM. (2000). *Principles and standards for school mathematics*. National Council of Teachers of Mathematics.

Ndemo, O., & Ndemo, Z. (2018). Secondary school students' errors and misconceptions in learning algebra. *Journal of Education and Learning, 12*(4), 690-701. https://doi.org/10.11591/edulearn.v12i4.9556

Pilattis, M. (2023). Young students' arithmetic-algebraic structure sense: An empirical model and profiles of students. *International Journal of Science and Mathematics Education, 21*, 1865-1887. https://doi.org/10.1007/s10763-022-10333-y

Radford, L. (2022). Introducing equations in early algebra. *ZDM-Mathematics Education, 54*(6), 1151-1167. https://doi.org/10.1007/s11858-022-01422-x

Reinschlüssel, A., Alexandrovsky, D., Döring, T., Kraft, A., Braukmüller, M., Janßen, T., Reid, D., Vallejo, E., Bikner-Ahsbahs, A., & Malaka, R. (2018). Multimodal algebra learning: From math manipulatives to tangible user interfaces. *i-com, 17*(3), 201-209. https://doi.org/10.1515/icom-2018-0027

Saraswati, S., Putri, R. I. I., & Somakim (2016). Supporting students' understanding of linear equations with one variable using algebra tiles. *Journal on Mathematics Education, 7*(1), 21-32. https://doi.org/10.22342/jme.7.1.2814.19-30

Spearman, C. (1904). The proof and measurement of association between two things. *American Journal of Psychology, 15*, 72-101. https://doi.org/10.2307/1422689

Swan, P., & Marshall, L. (2010). Revisiting mathematics manipulative materials. *Australian Primary Mathematics Classroom, 15*, 13-19.

Tsamir, P., & Tirosh, D. (2023) What is a solution of an algebraic equation? *International Journal of Science and Mathematics Education, 21*, 2303-2323. https://doi.org/10.1007/s10763-022-10342-x

Veith, J. M., Beste, M. L., Kindervater, M., Krause, M., Straulino, M., Greinert, F., & Bitzenbauer, P. (2023). Mathematics education research on algebra over the last two decades: Quo vadis? *Frontiers in Education, 8*. https://doi.org/10.3389/feduc.2023.1211920

Vinogradova, N. (2007). Solving quadratic equations by completing squares. *Mathematics Teaching in the Middle School, 12*(7), 403-405. https://doi.org/10.5951/MTMS.12.7.0403

Wilcoxon, F. (1945) Individual comparisons by ranking methods. *Biometrics, 1*, 80-83. https://doi.org/10.2307/3001968