MODESTUM EURASIA Journal of Mathematics, Science and Technology Education, 2025, 21(7), em2665
ISSN:1305-8223 (online)
OPEN ACCESS Research Paper https://doi.org/10.29333/ejmste/16603

Examining the impact of Galilean and Euclidean methods on students' performance in calculating triangle areas based on given vertices

Abdullah Kurudirek [1], Eric Machisi [2*], Bekzod Sultanov [3], Sherzodbek Ismoilov [4], Botirjon Mamadaliyev [5]

[1] Department of Mathematics Education, Tishk International University, Erbil, IRAQ
[2] Department of Mathematics, Fujairah Boys High School, Fujairah, UAE
[3] Department of Algebra and Mathematical Engineering, Urgench State University, Urgench, UZBEKISTAN
[4] Department of Mathematics, Tashkent State Transport University, Tashkent, UZBEKISTAN
[5] Department of Mathematics, Fergana State University, Fergana, UZBEKISTAN

Received 27 April 2025 ▪ Accepted 22 June 2025

Abstract
The research compares Euclidean and Galilean methods in calculating triangle areas. It assesses how these different geometric frameworks influence problem-solving strategies and computational efficiency amongst thirty grade 10 male students at a selected school in Iraq. A mixed-methods approach was employed, using quantitative data from student assessments and qualitative data from interviews to evaluate the impact of these methods. The findings indicate that curriculum modifications, such as integrating Galilean methods alongside traditional Euclidean approaches, can enhance students' understanding of geometric principles and improve their problem-solving skills. Additionally, research indicates that students find Galilean methods enjoyable, motivating, and novels which clarifies concepts and positively influences their perceptions of mathematics. The study highlights the importance of curriculum development incorporating diverse methodologies to cater to varied learning styles and promote a more inclusive and effective learning environment. The implications extend to educational practices, emphasizing the need for educators to adopt multifaceted approaches that foster enhanced mathematical literacy and prepare students for real-world applications.

Keywords: Euclidean methods, Galilean methods, student performance, triangle area

INTRODUCTION

A comparison of Euclidean and Galilean methods for calculating the area of a triangle highlights fundamental differences in their geometric principles and computational techniques. Euclidean methods, grounded in rigid transformations, provide precise formulas based on distances, angles, and coordinate geometry (Tomkins, 1911). These methods are particularly effective when accurate measurements and absolute geometric properties are essential. In contrast, Galilean methods utilize affine transformations and invariance principles to simplify area calculations, focusing on properties that remain unchanged under specific transformations. This approach is beneficial in scenarios where the absolute shape or orientation of the triangle is less important than its area, or when transformations can be applied to reduce computational complexity. Ultimately, the choice between Euclidean and Galilean methods depends on the specific problem at hand, the information available, and the desired level of accuracy.

Euclidean methods are widely used in practical applications, such as surveying, computer graphics, and engineering, where precise measurements and accurate representations of geometric shapes are essential. In contrast, Galilean methods are applied in fields like image processing, pattern recognition, and physics simulations, where affine transformations help analyze and manipulate shapes while preserving area and other

✉ abdullah.kurudirek@tiu.edu.iq ✉ e.machisi@yahoo.com **(*Correspondence)** ✉ sultanov.b@urdu.uz ✉ sh.ismoilov@nuu.uz ✉ bm.mamadaliyev@pf.fdu.uz

Contribution to the literature

- This research addresses the lack of empirical studies comparing the effectiveness of Euclidean and Galilean methods in geometry education. By offering evidence on how these methods affect student performance, the paper provides valuable insights that can help improve instructional practices.
- The findings provide practical guidance for curriculum developers. By emphasizing the potential advantages of integrating Galilean methods with Euclidean approaches, the research suggests ways to improve students' understanding of geometry and their problem-solving skills, leading to a more versatile and effective mathematics education.
- The research explores how Galilean methods affect students' perceptions and engagement with geometry, offering valuable insights for creating a more inclusive and effective learning environment. This study contributes to a broader understanding of how various teaching methodologies influence student motivation and learning outcomes, supporting the goal of improving mathematical literacy in contemporary mathematics education.

important properties. In cartography, which relies heavily on geometric computations, both Euclidean and Galilean geometries contribute to map projections and distance calculations (Papadopoulos, 2020). Additionally, geometry plays a significant role in trigonometry, integrating algebraic, geometric, and graphical reasoning. Moreover, the application of geometric principles extends to various fields, including computer-aided design and virtual reality, where geometric algorithms are essential for tasks such as collision detection and object modelling (Jiménez et al., 2001). Data-driven approaches in geometry processing also utilize geometric concepts for shape analysis and processing (Xu et al., 2015).

The mathematical concept of determining the area of a triangle based on its vertices is a fundamental problem in geometry that can be addressed using various methods from different geometric systems. In Euclidean geometry, which emphasizes constructions and congruence, traditional approaches are typically used. In contrast, Galilean geometry focuses on affine transformations and their invariance under shear transformations, providing alternative perspectives (Błaszczyk & Petiurenko, 2020). A comparative analysis of these two approaches reveals how different geometric frameworks can influence problem-solving strategies and affect computational efficiency. The Euclidean method usually involves calculating side lengths, applying Heron's formula, or using coordinate geometry with determinants. On the other hand, the Galilean approach may leverage the invariance of area under specific transformations to simplify calculations or provide geometric insights (Hızarcı & İpek, 2003). A comprehensive understanding of both methods enhances the problem-solving toolkit and highlights the relationship between geometric axioms and analytical techniques.

Although literature provides numerous proofs and examples of Euclidean and Galilean methods for calculating the area of a triangle, empirical research is scarce on how high school students perform when exposed to these methods. This study aims to fill this research gap. The present study will address the following research questions:

1. Do students who use Galilean methods to find the area of a triangle perform better than those using Euclidean approaches?
2. What are students' perceptions of using Galilean methods to calculate the area of a triangle given its vertices?

The importance of comparing Euclidean and Galilean methods for calculating the area of a triangle in education lies in their ability to enhance students' understanding of geometric principles and improve problem-solving strategies. This study highlights the fundamental differences between these two geometric frameworks and their respective approaches to addressing different computational needs. By examining the rigid transformations in Euclidean methods alongside the transformative invariance principles found in Galilean methods, educators can provide students with a diverse set of tools for solving geometric problems. This comparative analysis not only deepens students' conceptual understanding but also fosters an appreciation for the versatility inherent in mathematical methodologies.

This study addresses a significant gap in the existing literature regarding the performance of high school students when exposed to different geometrical approaches. It raises important questions about whether students who use Galilean methods perform better than their peers who rely on traditional Euclidean techniques. Understanding these dynamics can inform instructional practices, encouraging educators to incorporate diverse geometric perspectives into their curricula. Additionally, by examining how Galilean methods influence students' perceptions of geometry, this research may provide valuable insights into student engagement and motivation, both crucial elements for effective learning outcomes.

Ultimately, this study is significant not only for its theoretical implications but also for its practical

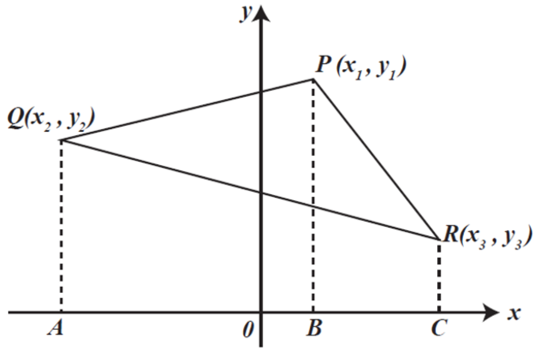

Figure 1. The triangle configuration into trapezoids (Source: Authors' own elaboration)

applications in educational settings. By highlighting the relationship between geometric axioms and analytical techniques through empirical research, educators can create a more inclusive and effective learning environment that caters to diverse cognitive styles. This approach fosters enhanced mathematical literacy among students, preparing them to tackle complex problems with confidence and creativity.

Euclidean Methods for Calculating the Area of a Triangle Given the Vertices

Euclidean geometry, renowned for its rigid transformations, offers several methods for calculating the area of a triangle given its vertices (Beeson, 2022). One common method employs coordinate geometry, where the vertices are represented as points in a Cartesian plane, and the area is computed using a determinant formula (Aliyev et al., 2021). For example, let us consider a triangle PQR with its vertices represented in the coordinate plane as points $Q(x_1, y_1)$, $P(x_2, y_2)$, and $R(x_3, y_3)$. By drawing vertical lines from points P, Q, and R perpendicular to the x-axis, we create three trapezoids within the coordinate plane: QABP, PBCR, and QACR. **Figure 1** shows the triangle configuration into trapezoids.

The area of triangle PQR equals the sum of the areas of trapezoids QABP and PBCR minus the area of trapezoid QACR. This leads to the **determinant formula** given in Eq. (1).

$$Area\ of \Delta PQR = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1 + x_3(y_1 - y_2)] = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}. \quad (1)$$

Alternatively, the Euclidean distance formula can be used to compute the lengths of the triangle's sides, followed by **Heron's formula**, which connects the area to the semi-perimeter and side lengths. Heron's formula is phrased as Eq. (2).

$$Area\ of \Delta PQR = \sqrt{s(s-p)(s-q)(s-r)}, \quad (2)$$

where s is the semi-perimeter ($s = \frac{p+q+r}{2}$) and p, q, and r are the side lengths of the triangle.

Another Euclidean method for finding the area of a triangle given the coordinates of the vertices is to use the cosine rule to determine the magnitude of one of its interior angles and then apply the **area formula** as Eq. (3), Eq. (4), or Eq (5).

$$Area\ of \Delta PQR = \frac{1}{2}p.q.\sin R, \quad (3)$$

or

$$Area\ of \Delta PQR = \frac{1}{2}q.r.\sin P, \quad (4)$$

or

$$Area\ of \Delta PQR = \frac{1}{2}p.r.\sin Q. \quad (5)$$

The Galilean approach is an alternative method for calculating the area of a triangle, given that the vertices are known. The following section provides a more in-depth description of the Galilean methods.

Galilean Methods for Triangle Area Calculation When the Vertices are Known

In contrast to Euclidean geometry, Galilean geometry emphasizes affine transformations that preserve parallelism and the ratios of distances along a line, but not necessarily angles or absolute distances. This geometric framework offers alternative strategies for calculating areas, utilizing invariance under shear transformations. In this context, calculating the area of a triangle often involves transforming the triangle into a simpler shape, such as one with a base aligned along an axis, while preserving its area. For example, a shear transformation can be applied to make one side horizontal without changing the area, thereby simplifying the calculation of the altitude and the area itself. Additionally, the concept of "equi-affine geometry," a subfield of affine geometry, focuses on transformations that preserve area. This provides a natural framework for Galilean methods. Here, the area of a triangle can be viewed as an invariant under specific types of transformations, leading to the development of area formulas that are independent of the triangle's orientation (Fjelstad,1996). These methods may involve expressing the area in terms of determinants or cross products of vectors, similar to the Euclidean approach, but with a focus on properties that remain unchanged under Galilean transformations. Galilean transformations maintain the parallelism of lines but do not preserve perpendicularity, which affects the approach to geometric measurements.

The lack of a well-defined notion of angles, as found in Euclidean geometry, affects the applicability of trigonometric formulas for calculating area. Thus, Galilean geometry offers a unique perspective on area, emphasizing its affine invariance and presenting alternative computational strategies that may be more suitable in certain contexts (Rovira-Más & Sáiz-Rubio, 2013).

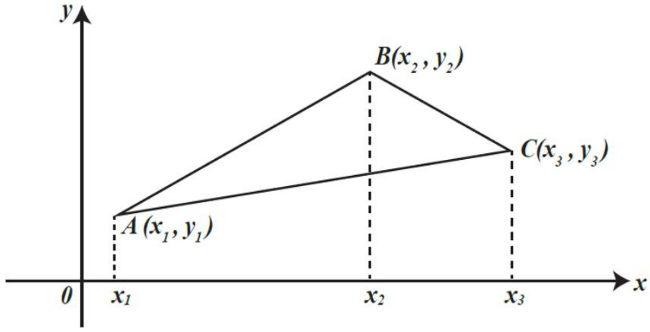

Figure 2. Triangle in the Galilean plane (Source: Authors' own elaboration)

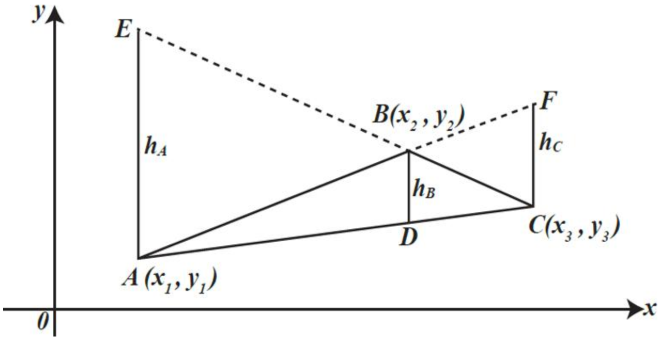

Figure 3. Heights of a triangle in the Galilean plane (Source: Authors' own elaboration)

In Galilean geometry, a triangle's area is defined the same way as it is in Euclidean geometry. The fundamental formula for calculating the area of a triangle is half the product of its base and height. The base and height of a triangle in the Galilean plane are fundamentally different from those in the Euclidean plane. To understand the geometric meaning of a triangle's side length and height in the Galilean plane, we can examine a representation of the triangle in coordinates, as shown in **Figure 2**. Here, $AB = |x_2 - x_1|, BC = |x_3 - x_2|,$ and $AC = |x_3 - x_1|$. So, the length of the segment in Galilean geometry is equal to the projection of the length of this segment in Euclidean geometry in the direction of the Ox axis.

If points lie on a straight line parallel to the Oy axis, their abscissas will be equal, and the distance between them corresponds to the projection of these points onto the Oy axis. This means that the distance is determined solely for points located on a straight line parallel to the Oy axis. In modern geometry, straight lines parallel to the Oy axis are referred to as special lines. In Galilean geometry, the height of the vertex of ΔABC is denoted as h, and the length of a special line passing through this vertex is the length of the segment contained within the triangle (see **Figure 3**). In **Figure 3**, the heights of the triangle ABC are h_A, h_B, and h_C.

Given triangle △ABC, the three possible ways to compute the area using heights perpendicular to the x-axis are:

Using base BC and height from point A (Eq. [6]):

$$Area\ of\ \Delta ABC = \frac{1}{2}|x_3 - x_2| \cdot h_A = \frac{1}{2}|x_3 - x_2| \cdot |y_E - y_1|. \tag{6}$$

Using base AC and height from point B (Eq. [7]):

$$Area\ of\ \Delta ABC = \frac{1}{2}|x_3 - x_1| \cdot h_B = \frac{1}{2}|x_3 - x_1| \cdot |y_2 - y_D|. \tag{7}$$

Using base AB and height from point C (Eq. [8]):

$$Area\ of\ \Delta ABC = \frac{1}{2}|x_2 - x_1| \cdot h_C = \frac{1}{2}|x_2 - x_1| \cdot |y_F - y_3|. \tag{8}$$

The values of y_E, y_D and y_F can be obtained by substituting x_1, x_2 and x_3 into the equations of lines BC, AC, and AB, respectively. This leads to the following results: $y_E = y_2 + \frac{y_3-y_2}{x_3-x_2} \cdot (x_1 - x_2)$, $y_D = y_1 + \frac{y_3-y_1}{x_3-x_1} \cdot (x_2 - x_1)$, and $y_F = y_2 + \frac{y_2-y_1}{x_2-x_1} \cdot (x_3 - x_2)$.

Another Galilean method to find the area of a triangle given its vertices involves using the vector cross product. For triangle ABC with vertices $A(x_1, y_1), B(x_2, y_2)$, and $C(x_3, y_3)$, area of triangle ABC can be found as follows:

$$A = \frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right| = \frac{1}{2}|(x_2 - x_1)(y_3 - y_1) - (y_2 - y_1)(x_3 - x_1)|. \tag{9}$$

While the concepts of Euclidean and Galilean geometry differ, the numerical values of the areas obtained using the different formulae will be the same.

Comparison of Euclidean and Galilean Methods for Calculating Triangle Area

Table 1 illustrates the key differences between the Euclidean and Galilean approaches to calculating the area of a triangle. Euclidean geometry operates within a metric space, where both distance and angle measures are preserved. This allows for standard geometric interpretations that involve the use of distance formulas and angular measurements. In contrast, Galilean geometry exists in a degenerate affine space, where only spatial components are considered and angles are not defined. As a result, the area in Galilean geometry is interpreted through affine relationships that maintain parallelism and ratios, rather than metric distances.

Table 1 also shows that while orientation sensitivity is present in both geometries, it has a more algebraic role in Galilean space. Angle-based formulas commonly used in Euclidean contexts, such as those involving trigonometric functions, are not applicable in Galilean settings due to the absence of angles. Consequently, Euclidean geometry is well-suited for measuring physical space such as in construction and mapping, while Galilean geometry is primarily used in non-relativistic kinematic models, focusing on spatial relationships without reference to angular or temporal components.

Table 1. Euclidean versus Galilean methods for calculating triangle areas

Aspect	Euclidean geometry	Galilean geometry
Space type	Metric (distance and angles are preserved)	Degenerate affine (no angular measure; time-space split)
Distance formula	Uses full distance: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Only spatial components are used: $\lvert x_2 - x_1 \rvert$
Interpretation of area	Actual area in the Euclidean plane (based on distances and angles)	Affine area: preserves parallelism and ratios, but is not based on distance
Orientation sensitivity	Orientation affects the sign (positive or negative area)	Same; orientation is considered, but the sign is only algebraic
Angle relevance	Angles are used in alternative area formulas such as: $\mathrm{A} = \frac{1}{2}\mathrm{ab}\sin\mathrm{C}$	Angles do not exist in Galilean geometry
Applications	Real-world measurements involving physical space (e.g., construction, maps)	Physics-based models, especially in non-relativistic kinematics

Current Research on Geometry Methods in Education

Research in geometry education is increasingly recognizing the critical role that spatial reasoning and deep conceptual understanding play in improving students' mathematical performance. According to Novita et al. (2018), students' spatial abilities play a pivotal role in determining their success in solving geometric problems. This is particularly evident in tasks requiring visualization of shapes and accurate measurement, where students with well-developed spatial skills tend to perform better than their peers (Atit & Rocha, 2021).

Recent studies have highlighted the transformative impact of dynamic geometry software and various visual representations on students' comprehension of geometric properties and relationships (Jones & Tzekaki, 2016; Ye et al., 2023). These educational tools create interactive learning environments that empower learners to engage actively with geometric concepts. By allowing students to manipulate shapes and explore geometric invariants (properties that remain unchanged under certain transformations), these technologies facilitate the development of robust conceptual frameworks, enhancing students' engagement and understanding.

Furthermore, educational scholars have emphasized the importance of integrating culturally responsive pedagogy into geometry instruction. Ethnomathematics serves as a prime example, as it connects mathematical concepts to the real-world experiences of students. This connection not only increases engagement but also fosters a deeper contextual understanding of geometry (Fouze & Amit, 2023; Sunzuma & Maharaj, 2020). For instance, in teaching area calculation, linking abstract geometric concepts to familiar spatial contexts can help students better grasp the relevance and application of these ideas in their everyday lives.

Despite these advances, research in geometry education has revealed persistent misconceptions among students regarding geometric measurement and the effective application of formulas, particularly in contexts involving coordinates (Clements & Sarama, 2021; Wilson & Sztajn, 2020). Many students tend to misapply area formulas or struggle to accurately interpret coordinates within a geometric framework. This highlights the urgent need for instructional strategies specifically designed to enhance conceptual clarity and address these misunderstandings.

Another important aspect in geometry education is the distinction between Euclidean and Galilean geometries, a topic that remains underexplored at the secondary education level but holds significant instructional potential. While traditional classroom instruction predominantly leans towards Euclidean methods, introducing alternative systems, such as Galilean geometry, which maintains properties like parallelism and area without relying on angle measures, can offer students new insights into geometric structure and reasoning (Dreyfus et al., 2018). Exposure to these diverse geometric perspectives may help to alleviate rigid thinking patterns among students, promoting greater adaptability in their problem-solving approaches.

Finally, research has demonstrated that interventions incorporating visual-spatial strategies, such as drawing geometric figures or mentally rotating shapes, can significantly enhance students' abilities in geometric thinking (Lowrie et al., 2022). These strategies encourage learners to view the concept of area not merely as a formulaic computation, but as a spatial concept deeply rooted in the intrinsic properties of shapes and the coordinate systems that describe them. By fostering this understanding, educators can help students develop a more nuanced and flexible approach to geometric reasoning.

Gap in Literature

Despite growing interest in diverse geometry pedagogies, few empirical studies have directly compared Galilean and Euclidean approaches in terms of their impact on student learning. Most existing research focuses on technological or cultural enhancements within Euclidean frameworks, leaving a gap in understanding how fundamentally different geometric systems affect comprehension and problem-solving strategies. This study addresses this gap by evaluating how exposure to both Galilean and Euclidean methods influences students' ability to compute triangle areas based on vertex coordinates.

The following section will provide a detailed outline of the research methods employed in this study, including the design, participant selection, and data collection techniques.

MATERIALS AND METHODS

Design of the Study

A sequential explanatory mixed-methods design was used to thoroughly evaluate the effects of Euclidean and Galilean approaches on students' proficiency in calculating the area of triangles. This design combined both quantitative and qualitative methodologies to rigorously analyze data and enhance the credibility of the findings (Flores, 2019). It is grounded in pragmatist philosophical principles, which advocate for researchers to choose any method deemed appropriate to achieve the study's objectives (Mosese & Ogbonnaya, 2021). The use of both quantitative and qualitative methods to assess the impact of Euclidean and Galilean techniques on students' performance in triangle area calculations aimed to provide a comprehensive understanding of the topic and gain deeper insight into its effects.

While quantitative assessments provide a straightforward way to measure student progress, they often do not capture the psychological aspects of the learning process. To address this gap, qualitative methods, particularly student interviews, were incorporated to monitor and evaluate these subtle changes. This mixed-methods approach enhances the study's methodological rigor, allowing for the identification of trends and differences that might otherwise be missed (Poth, 2018). Combining qualitative and quantitative methods provides a more holistic understanding of educational phenomena, capturing both the numerical changes in student performance and the underlying psychological processes.

Ethical Considerations

This study prioritized ethical research conduct by strictly following established guidelines to protect the well-being of all participants. Before any data collection began, informed consent was carefully obtained from all student participants and their parents or legal guardians. This process confirmed their voluntary agreement to take part in the research. Participants were provided with comprehensive information about the study's objectives, methods, potential risks and benefits, and their right to withdraw without facing any negative consequences. This transparency alleviates potential anxieties and encourages honest participation in research activities. To ensure privacy and confidentiality, we took stringent measures to anonymize the data collected. Participants were guaranteed that their responses would remain confidential and not be shared with third parties.

This study's ethical framework was further reinforced by obtaining formal ethical approval from the Research Ethics Committee at Tishk International University (approval date: 03/03/2024, reference number: 021), which demonstrated a commitment to upholding the highest ethical standards in research. This approval was an independent validation of the study's adherence to ethical principles and guidelines, ensuring the research was conducted responsibly and ethically.

Sample Selection

The present study carefully selected its participants to focus on mathematical understanding within a specific educational setting. It was conducted in Erbil, Iraq, among students at Stirling Schools, renowned for their commitment to academic excellence (Aguhayon et al., 2023). The sample consisted of 30 tenth-grade students from a single branch of Stirling Schools, which ensured a degree of homogeneity regarding curriculum exposure and the educational resources available to the participants. This intentional selection strategy aimed to minimize extraneous variables that could interfere with interpreting the research findings.

To maintain comparability within the sample, all participating students had a relatively uniform mathematical background. This indicated that they had received similar instruction in previous grades and had achieved a comparable level of mathematical proficiency before the study commenced. Students were randomly selected from five different classes within the same school, ensuring that each student had an equal opportunity to participate, making the sample representative of the broader tenth-grade population at the institution (Connor et al., 2017).

Furthermore, the study was conducted exclusively with male students, as the research site was a boys' school. This is an important contextual factor when interpreting and generalizing the results to other populations or settings (Aguhayon et al., 2023).

Instruments

To evaluate the impact of the Euclidean and Galilean methods on students' performance quantitatively, we used a 5-item pre-assessment to establish baseline prior knowledge and a 5-item post-test to measure learning outcomes related to calculating the area of a triangle given its vertices (See **Appendix A**). The total mark allocation for the pretest was 17, while that for the posttest was 16. Both assessments included free-response questions, allowing for a comprehensive evaluation of students' understanding and problem-solving skills beyond simple multiple-choice formats (López-Caudana et al., 2020). The validity of the assessment items was determined through expert judgment, using a sample of ten teachers who had a minimum of five years of experience teaching the grade 10 mathematics curriculum in Iraq. These teachers were asked to evaluate the items based on their relevance and clarity. A content validity index of 1.00 was achieved, indicating that there was perfect agreement among the experts that the items in the assessment instruments were valid and unambiguous (Sullivan, 2011).

A test-retest strategy was employed to assess the consistency of the test items. The same test was administered to the same group of students at two different times, and the correlation between their scores was measured. The post-test instrument achieved a test-retest reliability coefficient of 0.86, based on the Pearson product-moment correlation, indicating strong consistency and temporal stability (Jariyah, 2017). This coefficient falls within an acceptable range, suggesting that the instrument provides reliable measurements of student learning outcomes over time (Ajan et al., 2021).

The researchers developed a semi-structured interview guide based on the findings from the quantitative phase of their study. This guide aimed to delve deeper into students' perspectives on the Euclidean and Galilean methods for calculating the area of a triangle. Only one main question was used (See **Appendix A**). The interview question was reviewed by two mathematics specialists with over ten years of teaching experience. The question was revised in response to the experts' feedback.

Data Collection Procedures

We randomly assigned the students to experimental and control groups, each comprising 15 members. Students in the experimental group were assigned the codes E1, E2, E3, …, E15, while those in the control group were coded C1, C2, C3, …, C15. We administered the prior knowledge assessment to both groups before teaching the students the concept of the area of a triangle. Assessment scripts of the participating students were graded by an experienced teacher who is not part of the research team. The students' scores were converted to percentages and recorded for analysis. For four weeks, students in the control group received instruction on area calculations using Euclidean geometry methods. This involved six lessons, each lasting 40 minutes. At the same time, experimental group students were taught area calculations using Galilean geometry, also through six lessons of 40 minutes each, during the same four-week period. Both groups were instructed by the same teacher, who was one of the researchers in the study, to prevent the teacher factor from becoming a confounding variable in the results. Lessons took place after regular school hours, and arrangements were made for parents to pick up their children after class. The school authorities organized refreshments for the participating students to ensure they had enough to eat during the lessons.

Lessons were presented using PowerPoint slides and worksheets. We also utilized dynamic geometry software to help students comprehend the properties of geometric figures and demonstrate that certain transformations of triangles do not affect the area of the triangle. The teaching strategies used to deliver the lessons included whole-class discussions, exploration, guided discovery, questioning, think-pair-share, and group work. Both classes were placed under the same conditions, with the only difference being the formulas and methods used to calculate the area of triangles based on given vertices. Sample questions were solved with students, ensuring they learned concepts of calculating triangle areas using vertex coordinates. Students were given practice questions to complete at home to reinforce the formulas for calculating the area of triangles using Euclidean and Galilean geometry methods. The following week, the students graded these assignments in class, and solutions were explained.

After four weeks of treatment, both groups completed a post-test. The test lasted 45 minutes, with two invigilators in each group. The students were informed about the post-test in advance to ensure they were fully prepared and understood that their scores would accurately reflect their grasp of the solution methods used. The same teacher who evaluated the prior knowledge assessment graded and recorded the post-test scripts. Students' post-test scores were converted to percentages and rounded to the nearest whole number.

After analyzing the students' pre- and post-test scores, we conducted interviews with a small sample of students to gather their opinions on the methods used to find the area of a triangle when given the coordinates of its vertices. The interviews were conducted by a professional moderator and recorded using a digital recorder.

Data Analysis Procedures

The data analysis for this study used a mixed-methods approach to thoroughly evaluate the impact of Galilean and Euclidean methods on students' performance in calculating the areas of triangles. Quantitative data, collected from pre- and post-test assessments, were analyzed using both descriptive and inferential statistics. Additionally, qualitative data obtained from student interviews were analyzed through content analysis.

Descriptive statistics, including the mean, median, standard deviation, and range, were calculated to provide an overview of student performance in both the experimental and control groups. To ensure that any observed differences in post-test scores were not due to

Table 2. Descriptive statistics

	Groups	Mean	Number	Standard deviation	Minimum	Maximum	Range	Median
Pre-test	Control	53.27	15	2.434	50	58	8	53
	Experimental	53.60	15	2.772	50	59	9	53
Post-test	Control	69.80	15	6.889	61	85	24	68
	Experimental	96.20	15	2.396	91	99	8	96

Table 3. Normality test on pre-test scores

	Groups	Kolmogorov-Smirnov[a]			Shapiro-Wilk		
		Statistics	df	Significance	Statistics	df	Significance
Pre-test	Experimental	.159	15	.200*	.937	15	.345
	Control	.144	15	.200*	.951	15	.537

Note. *This is a lower bound of true significance & [a]Lilliefors significance correction

pre-existing disparities, the Shapiro-Wilk test was performed to assess the normality of the pre-test scores for both groups. If the data were normally distributed, an independent samples t-test was performed in statistical package for the social sciences (SPSS) to determine if there was a statistically significant difference in mean scores between the two groups before the treatment. The post-test data were analyzed using a similar approach to evaluating the impact of each method on student performance. Furthermore, effect sizes (Cohen's *d*) and 95% confidence intervals are included to quantify the magnitude of the observed learning gains and assist in the interpretation of findings beyond p-values.

After conducting a quantitative analysis, semi-structured interviews were conducted with a small group of students to gather in-depth insights into their experiences and perceptions of the geometry methods they used. The interview questions were developed based on the findings from the quantitative phase and were reviewed by mathematics specialists to ensure their validity and relevance. The interviews were recorded and transcribed verbatim. Thematic analysis was conducted with inductive coding. Inductive coding is a qualitative data analysis method that involves creating codes and categories directly from the data itself, rather than relying on pre-existing theories or frameworks. This approach is bottom-up, highly flexible, and exploratory, making it ideal for understanding participants' perspectives without imposing a predetermined structure. Two researchers independently coded the data. Discrepancies were resolved through discussion. Codes were reviewed and grouped into themes, and credibility was enhanced through peer debriefing and member checking. Themes included enjoyment, conceptual clarity, motivation, and perceived novelty.

Finally, the quantitative and qualitative findings were integrated to provide a comprehensive understanding of the impact of Galilean and Euclidean methods on student performance and perceptions. This mixed-methods approach enabled the researchers to view the research phenomenon from multiple angles, thereby enhancing the credibility and robustness of the study's conclusions.

RESULTS

The findings are organized into two sections. The first section analyzes the students' performance on pre- and post-test assessments, examining whether significant differences existed in their test scores before and after the treatment. This is followed by feedback from students that supports the experimental results.

Quantitative Data Analysis

Table 2 provides a general overview of the quantitative results. In the pretest, the control and experimental groups had similar average, median, standard deviation, and range values. However, huge differences are evident in the posttest results. The average score for the experimental group (mean [M] = 96.2) is much higher than that of the control group (M = 69.8). Additionally, the experimental group's standard deviation and range values (2.396, 8) are considerably lower than those for the control group (6.889, 24), indicating greater consistency in students' performance in the experimental group. Notably, the experimental group's minimum score exceeds the control group's maximum score, demonstrating a complete lack of overlap in posttest results between the two groups.

To ensure that the differences observed in the post-test were not influenced by any existing disparities before the treatment was administered, we conducted statistical tests on the pre-test data. The Shapiro-Wilk test for normality indicated that the pre-test scores for both the experimental and control groups were normally distributed (see **Table 3**). Therefore, we were able to analyze the data using a parametric test.

We performed a t-test to assess whether the difference in average scores between the experimental and control groups before treatment was statistically significant. The results of the t-test indicated that there was no significant difference in mean scores between the two groups ($t[28] = .350, p = .729$), confirming baseline equivalence (**Table 4**).

Table 4. Independent samples t-test on pre-test scores

Variable(s)	Groups	N	Mean	Standard deviation	t	df	Significance
Pre-test	Experimental	15	53.60	2.772	.350	28	.729
	Control	15	53.27	2.434			

Table 5. Normality test on post-test scores

	Groups	Kolmogorov-Smirnov[a]			Shapiro-Wilk		
		Statistics	df	Significance	Statistics	df	Significance
Post-test	Experimental	.200	15	.109	.896	15	.082
	Control	.155	15	.200*	.931	15	.285

Note. *This is a lower bound of true significance & [a]Lilliefors significance correction

Table 6. Independent samples t-test on post-test scores

Variable(s)	Groups	N	Mean	Standard deviation	t	df	Significance	95% CI	
								Lower	Upper
Post-test	Experimental	15	96.20	2.396	14.018	28	< .001	22.542	30.258
	Control	15	69.80	6.889					

Table 7. Independent samples effect sizes

		Standardizer[a]	Point estimate	95% CI	
				Lower	Upper
Post-test	Cohen's d	5.158	5.119	3.594	6.621
	Hedges' correction	5.301	4.980	3.497	6.442
	Glass's delta	6.889	3.832	2.243	5.394

Note. [a]The denominator used in estimating the effect sizes; Cohen's d uses the pooled standard deviation; Hedges' correction uses the pooled standard deviation, plus a correction factor; & Glass's delta uses the sample standard deviation of the control (i.e., the second) group

Analysis of the t-test results indicates that the experimental and control group students had comparable levels of prior knowledge. Consequently, any differences observed in the students' performance on the posttest can be attributed to the effects of the treatment administered to each group.

The normality test on the posttest scores also showed a nonsignificant Shapiro-Wilk's test result ($p > .05$) (**Table 5**). To assess whether the posttest scores differed significantly between the experimental and control groups, we conducted a two-sample t-test using SPSS.

The results of the t-test on posttest scores showed a statistically significant difference in mean scores in favor of the Galilean group ($t[28] = 14.350, p < .001$, 95% confidence interval [CI] [22.5, 30.3]) (**Table 6**). The 95% confidence interval does not include zero, confirming a statistically significant difference between the means. Students who calculated the area of a triangle using Galilean methods performed better than those who used Euclidean geometry methods.

The effect size calculated using Cohen's d was extremely large ($d = 5.119$, 95% CI [3.59, 6.62]), showing a substantial treatment effect (**Table 7**). This suggests that the mean score of the experimental group is over five standard deviations higher than that of the control group. In practical terms, there is almost a complete separation between the two groups, indicating that experimental group's performance is significantly better than that of the control group. The 95% confidence interval for Cohen's d does not include zero, confirming that the effect size is statistically significant.

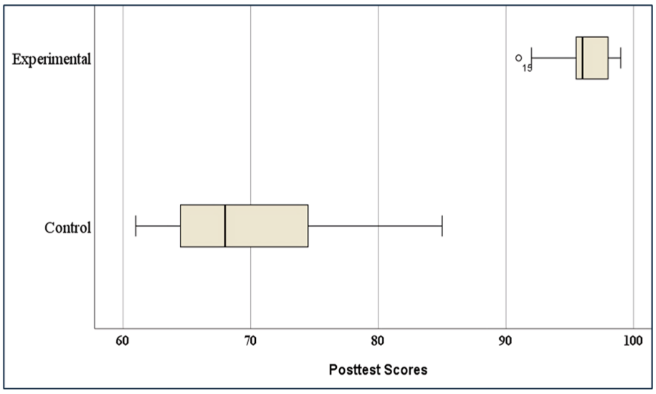

Figure 4. Comparison of post-test score distributions between experimental and control groups (Source: Authors' own elaboration)

Boxplots revealed greater consistency within the experimental group. Additionally, the minimum score of the experimental group exceeded the maximum score of the control group, further reinforcing the effectiveness of the Galilean methods (see **Figure 4**).

After analyzing the quantitative results, we asked students to provide feedback on using Galilean methods to solve geometry problems, particularly for calculating the area of a triangle with given vertices. The findings from the follow-up interviews are presented in the next section.

Qualitative Results: Thematic Analysis of Student Reflections on Galilean Methods

This section provides a thematic analysis of students' qualitative reflections regarding their experiences with Galilean geometry methods. The analysis is organized into four main themes: enjoyment, concept clarity, motivation, and perceived novelty. These themes reflect students' emotional, cognitive, and motivational responses, offering valuable insights into the educational potential of transformation-based geometric instruction.

Theme 1: Enjoyment

Many participants described their experiences with Galilean methods as enjoyable and engaging. This emotional connection appears to have had a positive influence on their attitudes toward mathematics. **E2** found the lessons "easy and enjoyable" and even shared them with his parents, suggesting a high level of enthusiasm beyond the classroom:

> "The topic … is truly interesting … easy and enjoyable … The lessons have changed my perspective on geometry … My father eagerly waited to learn new things from me in the evenings."

E5, who initially felt intimidated, later expressed deep enjoyment:

> "I enjoyed the experience so much that I cannot describe it … the sense of overcoming my fears had been instilled."

E9 referred to the lessons as "fascinating" and "enjoyable," particularly when engaging in group activities:

> "When we started using geometry in the classroom activities, we found it enjoyable …"

Similarly, **E13** described the process as a "beautiful experience" and expressed joy in contributing and helping peers:

> "The process was enjoyable and a beautiful experience."

Theme 2: Concept clarity

Students reported a better understanding of geometric concepts, crediting this to clear teacher demonstrations and the use of technology. **E5** emphasized that teacher modeling helped him achieve clarity.

> "Our teacher's motivating words and demonstrating how to solve problems gradually with examples has changed my perspective … now I believe that I can do these things myself."

E9 credited technological tools for enhancing conceptual understanding:

> "We did activities with our friends and used technology to help us understand, and I felt that my geometry skills took a step further."

E13 felt confident enough to support classmates, indicating conceptual mastery:

> "I helped friends who did not have much knowledge about geometry activities."

E14 reflected a deep internalization of geometric ideas:

> "I love mathematics, but now I feel like I am living it."

Theme 3: Motivation

The Galilean approach increased both intrinsic and extrinsic motivation among students. **E2** felt honored to be selected and showed deep appreciation:

> "I thank my teacher for choosing me for this course."

E5 described a transformation in his attitude toward mathematics:

> "Now I cannot express how happy I am… my interest in mathematics has increased."

E14 was encouraged by peer feedback, which spurred continued effort:

> "This further encouraged me, and I started following the lessons with even more excitement."

E13 expressed intense eagerness to continue with the methods:

> "Please let us continue with such activities."

Theme 4: Perceived novelty

The Galilean methods were perceived as innovative and refreshing, contributing to increased engagement. **E9** recognized the non-traditional nature of the experience:

> "It was a different experience."

E13 highlighted innovation and described the experience as transformative:

> "I was pleased to see such innovations in math… I realized that I started using geometry differently for the first time."

E14 saw the lessons as a gateway to deeper mathematical thinking:

> "Later, it became a door opening to a vast and interesting world for me." A general observation from the findings reinforces this sense of innovation and engagement: "Students in the experimental group found that solving problems using Galilean geometry methods was interesting, enjoyable, easy, motivating, and innovative."

The analysis reveals that the use of Galilean geometry methods enhanced students' enjoyment, understanding of concepts, motivation, and perception of novelty. These findings highlight the educational benefits of incorporating transformation-based approaches in geometry instruction and suggest opportunities for future curriculum development that aim to enhance student engagement and conceptual learning.

DISCUSSION AND IMPLICATIONS OF FINDINGS

The study investigates the impact of two distinct geometric methodologies on the performance of grade 10 male students in calculating the areas of triangles. The findings indicate a noteworthy trend: grade 10 male students who utilized Galilean methods performed significantly better than their peers who relied on Euclidean approaches. This discovery underscores the pivotal role that the selection of a geometric framework plays in shaping problem-solving strategies and enhancing computational efficiency in mathematics education.

Traditionally, the Euclidean method necessitates several steps, often requiring students to calculate side lengths through various means. This can involve employing Heron's formula or the area formula. Additionally, students may use determinants to calculate the area. Such methods, while established, can be complex and time-consuming.

In contrast, the Galilean approach introduces a more intuitive perspective by harnessing the concept of area invariance under specific transformations. This simplification allows students to grasp the calculation of triangle areas more quickly and often with greater enjoyment. The study not only reveals enhanced performance levels but also highlights that students found the Galilean methods engaging and less anxiety-inducing. This aligns with contemporary educational trends that advocate for "active" teaching methods, as posited by researchers such as Ambrose et al. (2010). These researchers advocate for instructional strategies that foster student engagement and motivation.

This study's findings indicate that the Galilean approach, although not yet part of the traditional high school mathematics curriculum, was relatively easy for the grade 10 male students to understand. This accessibility enhanced motivation and fostered a more profound comprehension of geometric concepts, aligning with the research conducted by Mosese and Ogbonnaya (2021). The study shows that employing Galilean methods may reduce anxiety associated with mathematical problem-solving and cultivate a greater appreciation for geometry among students. These findings support earlier reports by Galitskaya and Drigas (2020). Additionally, lessons that incorporate Galilean geometry may revitalize geometry classes by engaging students through contextual transformation activities (Sahara et al., 2024).

This study highlights the potential long-term benefits of incorporating Galilean geometry into the high school curriculum. This integration could significantly impact students' overall mathematical development and enhance their enthusiasm for the subject. The findings align with the growing trend of using augmented reality in education, which has been shown to improve students' geometric thinking in various areas, including spatial structuring and representation.

As noted by Kletenik (2013), although the principles underlying Euclidean and Galilean geometries appear different, their formulas are semantically similar, especially when it comes to calculating areas, where the numerical values are consistent. This study addresses a significant gap in the literature related to geometric education. It proposes a promising avenue for innovative teaching methods to enhance student understanding and proficiency in mathematics.

The findings suggest that combining Galilean and Euclidean methods can improve students' understanding of geometric principles and enhance their problem-solving strategies. This suggests the need to integrate both approaches into educational curricula, providing students with a richer and more comprehensive understanding of geometry. Educators can utilize the strengths of each method to accommodate various learning styles and problem contexts. The results offer valuable insights for educators seeking to enhance mathematics teaching by implementing innovative pedagogical techniques. Such improvements potentially enable students to grasp geometric concepts more thoroughly and enhance their problem-solving skills (Mamiala et al., 2022).

Curriculum developers should recognize the advantages of both Euclidean and Galilean methods to meet diverse computational requirements. This could involve creating instructional materials that emphasize the strengths of each approach, enabling students to develop a versatile toolkit for tackling geometry problems.

The study also reveals that the grade 10 male students found Galilean methods to be more engaging. Incorporating these methods into geometry instruction may positively influence students' perceptions of geometry, making the subject more accessible and enjoyable.

The instructional implications of this study are grounded in fundamental principles of instructional design, specifically constructivist learning theory, cognitive load theory, and visual-spatial reasoning. According to constructivist theory, learning is most effective when students actively build their understanding through meaningful experiences. The Galilean method, which emphasizes transformation-based reasoning over procedural calculations, enables learners to engage more intuitively with geometric structures. This approach encourages students to construct knowledge through visualization and manipulation, rather than relying on rote memorization of formulas. It aligns with the recommendations by Ambrose et al. (2010) for instructional strategies that foster student agency and cognitive engagement.

The Galilean approach helps reduce cognitive load by minimizing the number of steps and abstract symbols that students need to hold in their working memory. This not only improves task efficiency but also reduces anxiety. According to cognitive load theory, effective instruction minimizes extraneous cognitive burdens while still maintaining intrinsic challenges (Ouwehan et al., 2025; Surbakti et al., 2024; Sweller, 2020). This may explain why students found the Galilean method to be more accessible and less frustrating compared to the Euclidean alternatives, which often involve multiple steps, such as deriving lengths and using Heron's formula.

The findings also have important implications for developing visual-spatial reasoning, which is a key skill in learning geometry. Utilizing Galilean transformations enhances spatial thinking by encouraging students to visualize figures in motion and to recognize invariants during translations and shear transformations. These skills are associated with a deeper understanding of geometry and success in STEM fields (Harris, 2023).

The incorporation of Galilean geometry presents both opportunities and challenges in terms of curriculum alignment. Although Galilean methods are not commonly included in most national geometry curricula, they resonate with broader educational goals outlined in various standards frameworks. For instance, they align with the common core state standards in the United States and the curriculum and assessment policy statement in South Africa, both of which emphasize geometric reasoning, multiple representations, and flexible problem-solving strategies. Introducing Galilean methods could enhance conceptual diversity and provide alternative pathways for understanding, particularly for students who struggle with traditional Euclidean approaches.

Limitations of the Study

While the results suggest that Galilean methods were effective in this context, caution should be taken in generalizing these findings beyond triangle area computations or to mixed-gender and culturally diverse educational settings. The study focused on a small, specific group of grade 10 male students, and its findings may not be directly applicable to other age groups, genders, or educational settings. Further research is needed to validate these results across diverse populations. The study's primary focus was calculating the area of a triangle, and the effectiveness of Galilean methods might differ when applied to other geometric problems. It is essential to examine how these methods are applied in various geometric contexts.

Although the study employed a mixed-methods design, additional factors that may have influenced student performance were likely overlooked. Subsequent studies should replicate this design across diverse schools, including female students, and explore additional geometric concepts. Future research could examine additional variables, such as prior knowledge, spatial reasoning skills, and teaching styles, to provide a more comprehensive understanding of the factors that affect student performance. Longitudinal studies may reveal whether Galilean instruction has sustained effects on mathematical achievement and interest.

CONCLUSION

In conclusion, this study provides valuable insights into how geometric frameworks affect students' problem-solving abilities. The findings highlight the importance of integrating both Euclidean and Galilean methods in geometry education to enhance students' understanding and performance. Nevertheless, the limitations regarding sample size, context specificity, and methodological constraints should be considered when interpreting and applying the results in broader educational settings. Future research should address these limitations to offer a more comprehensive understanding of the role different geometric methods play in mathematics education.

Author contributions: AK: conceptualization, writing the original draft, editing, and visualization; **EM:** writing, reviewing, editing, formal analysis, and methodology; & **BS, SI, & BM:** validation and supervision. All authors agreed with the results and conclusions.

Funding: No funding source is reported for this study.

Acknowledgments: The authors would like to thank Prof. Abdullaaziz Artykbaev for his support and guidance throughout the study.

Ethical statement: The authors stated that the study was approved by the Tishk International University's Research Ethics Committee on 3 March 2024 with approval number 021. Written informed consents were obtained from the participants.

Declaration of interest: No conflict of interest is declared by the authors.

Data sharing statement: Data supporting the findings and conclusions are available upon request from the corresponding author.

REFERENCES

Aguhayon, H., Tingson, R., & Pentang, J. (2023). Addressing students learning gaps in mathematics through differentiated instruction. *International Journal of Educational Management and Development Studies, 4*(1), 69-87. https://doi.org/10.53378/352967

Ajan, J. B., Luna, C. A., & Roble, D. B. (2021). Do military students' mathematical self-efficacy and metacognitive awareness matter on their problem solving performance? *American Journal of Educational Research, 9*(6), 330-334. https://doi.org/10.12691/education-9-6-1

Aliyev, S. J., Aghazade, S. M., & Abdullayeva, G. Z. (2021). Using area method in secondary school geometry. *European Journal of Pure and Applied Mathematics, 14*(2), 601-607. https://doi.org/10.29020/nybg.ejpam.v14i2.3942

Ambrose, S. A., Bridges, M. W., DiPietro, M., Lovett, M. C., & Norman, M. K. (2010). *How learning works: Seven research-based principles for smart teaching*. Jossey-Bass.

Atit, K., & Rocha, K. (2021). Examining the relations between spatial skills and mathematical performance: A meta-analysis. *Psychonomic Bulletin & Review, 28*(2), 465-482. https://doi.org/10.3758/s13423-021-02012-w

Beeson, M. (2022). On the notion of equal figures in Euclid. *Beiträge Zur Algebra Und Geometrie/Contributions to Algebra and Geometry, 64*(3), 581-625. https://doi.org/10.1007/s13366-022-00649-9

Błaszczyk, P., & Petiurenko, A. (2020). Euclid's theory of proportion revised. *Annales Universitatis Paedagogicae Cracoviensis|Studia Ad Didacticam Mathematicae Pertinentia, 11*, 37-62. https://doi.org/10.24917/20809751.11.3

Clements, D. H., & Sarama, J. (2021). *Learning and teaching early math: The learning trajectories approach* (3rd ed.). Routledge. https://doi.org/10.4324/9781003083528

Connor, C. M., Mazzocco, M. M. M., Kurz, T. L., Crowe, E. C., Tighe, E. L., Wood, T., & Morrison, F. J. (2017). Using assessment to individualize early mathematics instruction. *Journal of School Psychology, 66*, 97-113. https://doi.org/10.1016/j.jsp.2017.04.005

Dreyfus, T., Hadas, N., Hershkowitz, R., & Schwarz, B. B. (2018). Abstraction in context: Epistemic actions in the construction of mathematical meaning. *Educational Studies in Mathematics, 58*(1), 85-109.

Fjelstad, P. (1996). Extending the Pythagorean theorem to other geometries. *Mathematics Magazine, 69*(3), 222-223. https://doi.org/10.1080/0025570x.1996.11996434

Flores, I. M. (2019). Correlates of mathematics performance of students in public secondary schools in the Division of Batangas, Philippines: Basis for mathematics intervention programs. *World Journal of Educational Research, 6*(2), 248-259. https://doi.org/10.22158/wjer.v6n2p248

Fouze, A. Q., & Amit, M. (2023). The importance of ethnomathematics education. *Creative Education, 14*(4), 729-740. https://doi.org/10.4236/ce.2023.144048

Galitskaya, V., & Drigas, A. (2020). Special education: Teaching geometry with ICTs. *International Journal of Emerging Technologies in Learning, 15*(6), 173-183. https://doi.org/10.3991/ijet.v15i06.11242

Harris, D. (2023). Spatial reasoning in context: Bridging cognitive and educational perspectives of spatial-mathematics relations. *Frontiers in Education, 8*. https://doi.org/10.3389/feduc.2023.1302099

Hızarcı, S., & İpek, A. (2003). Changes in geometry: Invariant areas in Euclid geometry. *Applied Mathematics and Computation, 153*(2), 395-401. https://doi.org/10.1016/s0096-3003(03)00640-4

Jariyah, I. A. (2017). The effect of inquiry combined science-technology-society (STS) learning to enhance critical thinking skills on science. *Jurnal Pendidikan Biologi Indonesia, 3*(1), Article 1. https://doi.org/10.22219/jpbi.v3i1.3888

Jiménez, P., Thomas, F., & Torras, C. (2001). 3D collision detection: A survey. *Computers & Graphics, 25*(2), 269-285. https://doi.org/10.1016/s0097-8493(00)00130-8

Jones, K., & Tzekaki, M. (2016). Research on the teaching and learning of geometry. In Á. Gutiérrez, G. Leder, & P. Boero (Eds.), *The second handbook of research on the psychology of mathematics education* (pp. 109-149). Sense Publishers. https://doi.org/10.1007/978-94-6300-561-6_4

Kletenik, D. V. (2013). *A collection of problems in analytical geometry: Three-dimensional analytical geometry*. Pergamon Press.

López-Caudana, E. O., Soledad, M., Pérez, S. M., & Rodríguez-Abitia, G. (2020). Using robotics to enhance active learning in mathematics: A multi-scenario study. *Mathematics, 8*(12), Article 2163. https://doi.org/10.3390/math8122163

Lowrie, T., Diezmann, C. M., & Logan, T. (2022). Spatial reasoning for geometry learning: An intervention study in upper elementary classrooms. *Mathematics Education Research Journal, 34*(1), 1-18.

Mamiala, D., Simelane-Mnisi, S., & Mji, A. (2022). Framework for articulating relation between teachers' practices and students' self-efficacy in teaching and learning Euclidian geometry in high school. *Universal Journal of Educational Research, 10*(1), 22-35. https://doi.org/10.13189/ujer.2022.100103

Mosese, N. M., & Ogbonnaya, U. I. (2021). GeoGebra and students' learning achievement in trigonometric functions graphs representations and interpretations. *Cypriot Journal of Educational Sciences, 16*(2), 827-846. https://doi.org/10.18844/cjes.v16i2.5685

Novita, R., Putra, M., Rosayanti, E., & Fitriati, F. (2018). Design learning in mathematics education: Engaging early childhood students in geometrical activities to enhance geometry and spatial reasoning. *Journal of Physics: Conference Series, 1088*, Article 012016. https://doi.org/10.1088/1742-6596/1088/1/012016

Ouwehand, K., Lespiau, F., Tricot, A., & Paas, F. (2025). Cognitive load theory: Emerging trends and innovations. *Education Sciences, 15*(4), Article 458. https://doi.org/10.3390/educsci15040458

Papadopoulos, A. (2020). Mathematics and map drawing in the eighteenth century. *Ganita Bharati: Bulletin of the Indian Society for History of Mathematics, 41*(1), 91-126. https://doi.org/10.32381/gb.2019.41.1-2.4

Poth, C. (2018). The contributions of mixed insights to advancing technology-enhanced formative assessments within higher education learning environments: An illustrative example. *International Journal of Educational Technology in Higher Education, 15*(1), Article 9. https://doi.org/10.1186/s41239-018-0090-5

Rovira-Más, F., & Sáiz-Rubio, V. (2013). Crop biometric maps: The key to prediction. *Sensors, 13*(9), Article 12698. https://doi.org/10.3390/s130912698

Sahara, S., Dolk, M., Hendriyanto, A., Kusmayadi, T. A., & Fitriana, L. (2024). Transformation geometry in eleventh grade using digital manipulative batik activities. *Journal on Mathematics Education, 15*(1), 55-78. https://doi.org/10.22342/jme.v15i1.pp55-78

Sullivan, G. M. (2011). A primer on the validity of assessment instruments. *Journal of Graduate Medical Education, 3*(2), 119-120. https://doi.org/10.4300/jgme-d-11-00075.1

Sunzuma, G., & Maharaj, A. (2020). Exploring Zimbabwean mathematics teachers' integration of ethnomathematics approaches into the teaching and learning of geometry. *The Australian Journal of Teacher Education, 45*(7), 77-93. https://doi.org/10.14221/ajte.2020v45n7.5

Surbakti, R., Umboh, S. E., Pong, M., & Dara, S. (2024). Cognitive load theory: Implications for instructional design in digital classrooms. *International Journal of Education and Instructional Innovation, 2*(6), 483-493. https://doi.org/10.70177/ijen.v2i6.1659

Sweller, J. (2020). Cognitive load theory and educational technology. *Educational Technology Research and Development, 68*, 1-16. https://doi.org/10.1007/s11423-019-09701-3

Tomkins, J. A. (1911). The theory of the polar planimeter: An application of the method of barriers. *The Mathematical Gazette, 6*(93), 127-129. https://doi.org/10.2307/3604904

Wilson, P. H., & Sztajn, P. (2020). Understanding students' geometric thinking: Misconceptions and instructional strategies. *Journal for Research in Mathematics Education, 51*(4), 387-415.

Xu, K., Kim, V. G., Huang, Q., & Kalogerakis, E. (2015). Data-driven shape analysis and processing. *arXiv*. https://doi.org/10.48550/arxiv.1502.06686

Ye, H., Liang, B., Ng, O., & Chai, C. S. (2023). Integration of computational thinking in K-12 mathematics education: A systematic review on CT-based mathematics instruction and student learning. *International Journal of STEM Education, 10*, Article 3. https://doi.org/10.1186/s40594-023-00396-w

APPENDIX A

Pre-Test Items

Q1. Visualize sketching a triangle on a sphere's surface. When drawing a triangle on a flat sheet of paper, what distinguishes its angles from those of the other triangle? (3 marks)

Q2. Draw a straight line and a point not on the line on a paper. How many parallel lines to the original line can you draw through that point? (2 marks)

Q3. Find the area of an isosceles triangle with a base $b = 5\ cm$ and height $h = 12\ cm$. (3 marks)

Q4. Find the area of an equilateral triangle whose perimeter is $24\ cm$. (4 marks)

Q5. Find the area of the triangle whose sides measure $4\ cm$, $13\ cm$, and $15\ cm$. (5 marks)

Post-Test Items

Q1. Find the distance between each pair of points $A(1,2)$ and $B(-1,3)$. (2 marks)

Q2. Find the distance between each pair of points $A(1,1)$ and $B(5,1)$. (2 marks)

Q3. Find the area of the triangle whose vertices are $A(1,2)$, $B(-1,3)$, and $C(-3,-2)$. (4 marks)

Q4. Find the area of the triangle whose vertices are $A(1,1)$, $B(-2,-2)$, and $C(2,-5)$. (4 marks)

Q5. Find the area of the triangle whose vertices are $A(3,4)$, $B(-3,5)$, and $C(6,2)$. (4 marks)

Work Sheets

Available at: https://www.kutasoftware.com/.

Interview Main Question

Share your experiences learning geometry during the four weeks of the project.