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Examining the least preferred topics among advanced placement calculus students

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Abstract

This mixed-methods study investigates the perceptions of Grade 12 students enrolled in Advanced Placement (AP) Calculus regarding their least preferred topics, to identify instructional and cognitive challenges that hinder their engagement. Data were collected from 53 students across five schools in the United States and Iraq using a survey that included both quantitative and qualitative items. Quantitative analysis revealed that topics related to integration, particularly Applications of Integration and Accumulation of Change, were most frequently identified as difficult. Latent class analysis further distinguished two distinct student groups based on topic preferences. Thematic analysis of open-ended responses highlighted key challenges such as cognitive overload, lack of engagement, struggles early in the semester, and anxiety related to assessments. Students also expressed negative emotional responses, emphasizing the affective dimensions of learning advanced mathematics. Despite these difficulties, participants identified effective instructional methods, including concise teacher explanations, visual aids, guided practice, and the use of multimedia resources. The findings suggest that student-centered, scaffolded instruction can help alleviate conceptual barriers and improve learning outcomes in AP Calculus. This study adds to the growing body of literature advocating for responsive and differentiated teaching practices in high-stakes mathematics courses. Implications for instruction, curriculum design, and teacher training are also discussed.

Keywords: AP calculus, students, least preferred topics, integration, differentiation

INTRODUCTION

Advanced Placement (AP) Calculus is essential for preparing high school students for college-level mathematics, as it promotes analytical thinking, problem-solving skills, and logical reasoning. However, students often show reluctance or disengagement towards specific calculus topics, which can negatively impact their academic performance and long-term confidence in mathematics (Hammoudi & Grira, 2023). Identifying these less preferred topics can offer valuable insights into areas for improving teaching strategies and curriculum design.

Recent studies highlight that cognitive, emotional, and contextual factors influence students' preferences in mathematics. Calculus topics, such as limits, formal definitions of continuity, and applications of integrals, often involve abstractions and require a high cognitive load to process both symbolic and graphical representations simultaneously. This can overwhelm students with weak foundational knowledge who often find these topics difficult (Ruamba et al., 2025). Thompson and Harel (2021) examined the challenges students face in calculus, emphasizing how fragmented early understandings can result in superficial reasoning. They noted that learners often stay in procedural modes of thinking instead of developing a coherent structural comprehension of the topic. This observation is consistent with the findings of Biza et al. (2022), which suggest that students initially approach calculus procedurally, often struggling to transition to a structural understanding of concepts and mathematical objects.

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Contribution to literature

- The study provides empirical evidence on how students' perceptions of specific AP Calculus topics, particularly those related to integration, are influenced by cognitive overload, instructional quality, and emotional engagement. This area has been insufficiently explored in prior research on advanced mathematics education.
- Additionally, the study employs a novel application of latent class analysis to categorize student preferences in calculus, providing a more detailed understanding of the diversity among learners in terms of topic difficulty and engagement.
- By prioritizing student perspectives, this research reinforces the need for learner-centered and differentiated instructional practices in high-stakes mathematics courses. It connects theoretical frameworks, such as self-determination theory and cognitive load theory, with practical classroom strategies.

Affective factors such as mathematical anxiety, a fixed mindset, and perceived self-efficacy significantly influence students' willingness to engage with challenging content (Boaler, 2022; Capuno et al., 2019). Hannula (2019) found that students' emotional responses to specific topics can predict their persistence and performance. When students anticipate failure, they tend to avoid those topics altogether, which increases gaps in their understanding. Instructional strategies play a crucial role in shaping students' views of particular mathematics topics. According to self-determination theory, students are more likely to engage with challenging material when they feel competent, supported, and autonomous in their learning (Chiu, 2021; Ryan & Deci, 2000).

Instructional quality plays a vital role in students' preferences and success in calculus. Traditional lecture-based models often fail to address the diverse learning needs of students, particularly in abstract topics (Lee & Paul, 2023). More recent approaches emphasize active learning, the use of real-world applications, and student-led exploration to increase engagement (Johnson et al., 2025). Technology-enhanced instruction, including graphing tools and dynamic visualization platforms, has shown promise in improving conceptual understanding when implemented thoughtfully (Schoenherr et al., 2024). Additionally, specific scaffolding techniques, such as the Feynman Technique (Adeoye, 2023) and worked example modeling (Barbieri et al., 2023), have proven effective in enhancing comprehension and retention of complex topics. However, studies emphasize that these strategies must be tailored to students' prior knowledge and cultural backgrounds to realize their potential fully.

There is a growing consensus that students' perspectives should play a crucial role in curriculum development and teaching practices (Källberg & Roos, 2025). Understanding which mathematics topics students dislike the most and the reasons behind their preferences can help educators modify their instruction to enhance engagement and comprehension. As Boaler (2022) emphasizes, employing inclusive pedagogies that validate students' experiences and learning identities is crucial for transforming attitudes toward mathematics, particularly in high-stakes courses such as AP Calculus. By identifying and addressing the topics that students find most challenging, educators can create more supportive learning environments and foster a deeper understanding of mathematics.

Identifying students' least preferred topics goes beyond being just a diagnostic tool; it serves as a foundation for developing inclusive and targeted teaching strategies. When students can express their learning experiences, educators can adopt more responsive methods that encourage perseverance, reduce anxiety, and promote meaningful learning outcomes (Boaler, 2022). This study aims to investigate the perceptions and learning experiences of students regarding their least preferred topics in AP Calculus. By analyzing patterns of difficulty and students' perceptions of instruction, this research aims to inform the development of targeted strategies that enhance engagement and improve understanding in the areas that need it most. The specific research questions to be addressed in this study are as follows:

RQ1 Which topics in the AP Calculus curriculum are least preferred by students?

RQ2 What challenges do students experience when learning the least preferred topics?

RQ3 How do students perceive the effectiveness of instructional methods used to teach the least preferred topics?

Previous research has examined students' difficulties in learning calculus concepts, primarily focusing on cognitive and procedural aspects, such as misconceptions about limits and integrals, as well as the challenges of transitioning from procedural to structural understanding (for example, Biza et al., 2022; Thompson & Harel, 2021). However, there have been few systematic studies exploring students' subjective experiences with their least preferred topics in AP Calculus. This includes an examination of cognitive, emotional, and instructional dimensions, leaving a crucial gap in understanding how the least liked AP Calculus topics relate to instructional practices, mindset, and self-efficacy—factors that are especially relevant in a high-stakes course like AP Calculus. By specifically focusing on students' perceptions of the least preferred AP Calculus topics and the underlying reasons for their preferences, this study addresses a significant yet underexplored area in mathematics education.

This study takes a unique approach by focusing on student feedback to identify and analyze the topics they find least appealing or most challenging in AP Calculus. Unlike previous research that often generalizes conceptual difficulties across broad populations, this study targets the AP Calculus context explicitly, which is characterized by accelerated pacing, high academic pressure, and college-level expectations. Additionally, by examining students' topic preferences alongside their reported challenges and perceptions of teaching methods, the study offers a more comprehensive understanding of the barriers to learning in calculus education. This multidimensional approach makes the research particularly relevant, considering the growing demand for student-centered and inclusive mathematics instruction (Boaler, 2022; Källberg & Roos, 2025).

This study contributes to the field of mathematics education by providing practical insights into how students' perceptions of specific calculus topics can guide the creation of more inclusive, supportive, and differentiated instructional strategies. By highlighting the intersection of emotional and pedagogical factors in shaping students' topic preferences, the study helps refine instructional practices that consider students' lived experiences and limitations related to cognitive load. This work enriches both theoretical and practical discussions on learner-centered curriculum design and assessment in advanced mathematics courses. Additionally, the findings may inform teacher training programs, curriculum development, and classroom interventions aimed at enhancing engagement, retention, and equity in AP Calculus and related mathematics pathways.

This research is grounded in a social constructivist theoretical framework, which posits that students create knowledge through their interactions with their surroundings, classmates, and teaching methods (Vygotsky, 1978). It is also informed by self-determination theory, which posits that students engage more deeply when their psychological needs for autonomy, competence, and motivation are met (Ryan & Deci, 2000), as well as cognitive load theory, which illuminates how instructional design affects the comprehension of complex mathematical concepts (Sweller, 1988). Collectively, these frameworks provide a comprehensive perspective to analyze students' views on difficulty, emotional disengagement, and the effectiveness of instruction in AP Calculus.

MATERIALS AND METHODS

Research Design

This study employs a concurrent mixed-methods design, in which quantitative and qualitative data were collected simultaneously through a single survey instrument. The design allows for the integration of statistical trends and personal insights, providing a more comprehensive understanding of students' learning experiences with their least preferred AP Calculus topics. Quantitative data were used to identify patterns in topic preferences, while qualitative responses provided a deeper insight into students' perceptions, challenges, and attitudes. This combination enabled both broad generalizations and nuanced interpretations.

Participants and Context

The study sample consisted of 53 grade 12 students (25 males and 28 females) drawn from three public schools in the United States (US) and two private schools in Iraq that follow the American curriculum. Data collection took place during the 2024–2025 academic year. A purposive sampling technique was employed to select participants based on their relevance to the study phenomenon. **Table 1** presents the distribution of participants across the research sites. Participation was entirely voluntary, and informed consent was obtained from all students, as well as from their parents and the respective school administrators.

In the US, AP Calculus is typically offered to high-achieving students in grades 11 or 12 who have demonstrated strong performance in foundational mathematics courses, such as Algebra I, Geometry, Algebra II, and Pre-Calculus. Enrolment is often restricted to students on a college preparatory track, particularly those planning to pursue degrees in science, technology, engineering, or mathematics. In some school districts, academically advanced students may begin AP Calculus as early as grade 10 if they follow an accelerated mathematics pathway. Placement is typically

Table 1. Composition of participants per research site

Research site code	Number of students enrolled in AP calculus: 2024/2025 academic year	Number of students who participated	School status
A	20	13	Private
B	21	11	Private
C	22	10	Public
D	22	11	Public
E	20	8	Public

determined by a combination of academic performance, teacher recommendations, and, in some cases, diagnostic assessments (College Board, 2023; Education Commission of the States, 2021).

Stirling Schools in Iraq offer the American curriculum and provide AP Calculus primarily to students in grades 11 and 12. This course is designed for individuals preparing for international university admissions or seeking to advance their academic standing. Admission to AP Calculus typically depends on a student's performance in prerequisite subjects, such as Algebra II and Pre-Calculus, as well as their proficiency in English, as both instruction and assessment adhere to the College Board's AP standards. The inclusion of AP Calculus is part of a broader initiative to provide students with globally recognized academic credentials and to prepare them for standardized tests, such as the Scholastic Aptitude Test (SAT) (Stirling Schools, 2025).

Research Instruments

Quantitative and qualitative data were collected using an online survey designed by the researchers, which comprised three sections: Demographics, Section 1, and Section 2. The demographic data gathered included gender, grade level, school status (private or public), and school region (Iraq or the US). In Section 1, students were asked to select all AP Calculus topics they found challenging and unenjoyable from a provided list. The topics included:

1) Limits and continuity,
2) Differentiation,
3) Applications of differentiation,
4) Differential equations, integration, and
5) Applications of integration.

Section 2 included open-ended questions that prompted students to explain the specific challenges they experienced when learning their least preferred AP Calculus topics, as well as their perceptions of the instructional approaches used by teachers to present those topics. The content validity of the survey instrument was evaluated by asking ten mathematics education experts from various universities to assess the relevance and clarity of the questionnaire items using a criterion adapted from Zamanzadeh et al. (2015). Items were evaluated for relevance and clarity using a 4-point ordinal scale (see **Table 2**).

Table 2. Content validity scoring criteria

Relevance	Clarity
1 = Not relevant	1 = Not clear
2 = Item requires major revisions	2 = Item needs major revision
3 = Relevant but requires minor revisions	3 = Clear but needs minor revision
4 = Highly relevant	4 = Very clear

Adapted from Zamanzadeh et al. (2015, p. 168)

The content validity index (CVI) for each survey item was determined by dividing the number of raters who scored the item as 3 or 4 (for both relevance and clarity) by the total number of raters. Items with a CVI below 0.7 were removed from the survey (Zamanzadeh et al., 2015). Items that received a rating of 3 for either relevance or clarity were revised. The scale-level CVI (S-CVI) was calculated by averaging the item-level CVIs of the remaining items after excluding those that did not meet the minimum threshold of 0.7 (Zamanzadeh et al., 2015). The S-CVI (Scale-Level Content Validity Index) of the survey was 0.96, exceeding the minimum threshold of 0.9 for averaging CVIs across items (Naye et al., 2022).

Data Collection Procedure

Data collection for this study began only after receiving the necessary ethical approval from the Human Research Ethics Committee at Tishk International University in Iraq. This approval was granted on April 13, 2025, under Protocol number 16/2025. To ensure the protection and rights of the participants, explicit permission was also obtained from the school authorities of the selected schools, allowing their students to take part in the research.

Once the school principals approved the study, comprehensive information regarding the research was provided to both the AP Calculus students and their teachers. This information outlined the purpose and scope of the study, as well as the research methods to be employed. Before distributing the survey, it was essential to inform the participants that their participation was entirely voluntary. They were guaranteed that any data collected would be kept confidential and used exclusively for academic purposes, thus ensuring the integrity of their responses was maintained. It was made clear that no names or any identifiable information would be used in the data collection and data reporting. Participants in the study were assigned identity codes for confidentiality. Students from School A were designated codes A1 through A13. Students from School B received codes B1 through B11. Students from School C were coded from C1 to C10. Those from School D were assigned codes D1 through D11, while students from School E were coded E1 through E8. The students received clear instructions regarding their right to refuse participation without any consequences or penalties. This emphasis on informed consent was crucial for fostering an environment of trust and transparency.

Data collection took place during the second semester of the AP Calculus course in the 2024-2025 academic year, specifically after the students had completed most of the major topics included in the curriculum. Surveys were distributed in a digital format to facilitate easy access and completion. The distribution process was carried out by the teachers responsible for instructing the

AP Calculus course in the selected schools, ensuring that students felt supported and guided throughout the process. A total of 53 complete surveys were received.

Data Analysis

RQ1 was investigated from two perspectives. Firstly, for each topic, the relative number of students who identified it as their least preferred was analyzed. Secondly, structural equation modelling was employed to identify different cohorts of students based on their interests in AP Calculus topics. Given the dichotomous nature of the collected data (participants could either select a least preferred topic or not), latent class analysis (LCA) was applied, which is appropriate for categorical data (Weller et al., 2020). LCA uses maximum likelihood estimation to fit a hypothesized model to the data, based on pre-selected indicators, and groups participants into latent classes. Consequently, each latent class can be interpreted as a subpopulation with homogeneous profiles on the various observed measures included in the analysis. In contrast, the differences between the latent classes indicate heterogeneity in the studied population (Bu et al., 2024).

In the data analysis, contemporary recommendations were followed by incorporating several model fit statistics. The overall model fit was first examined using χ^2 (Chi-square) and G^2 (likelihood ratio) statistics. Subsequently, additional fit indices were assessed, including the Akaike Information Criterion (AIC), the Bayesian Information Criterion (BIC), the consistent Akaike Information Criterion (CAIC), which imposes an additional penalty for model complexity, and the sample-size-adjusted BIC (ABIC). These criteria were utilized to compare models with varying numbers of classes (ranging from 1 to 5), favoring models that displayed lower values alongside satisfactory G^2 and χ^2 statistics, which indicate an optimal balance between model fit and parsimony (Van Lissa et al., 2023). Additionally, we compared the various models using the Bayes Factor (BF), which is calculated as the exponential of half the difference between the BIC statistics of the two models (Zhao et al., 2025). This statistic enables direct comparisons between models, where values of 3 or higher are considered acceptable. Lastly, we ensured that the lowest average latent class posterior probability did not fall below the established threshold of 0.80 (Băjenaru et al., 2022).

A post hoc analysis of the supplementary covariates, namely gender and type of school, was conducted to enhance the interpretation of the identified latent classes. Nonetheless, these additional variables were excluded from the model specification as predictors of class membership because of the small sample size. The latent class analysis was conducted using the poLCA package in R version 4.4.2.

To address **RQ2** and **RQ3**, students' written responses to open-ended questions were analyzed.. The qualitative data were analyzed using thematic analysis with inductive coding. Inductive coding is the process of analyzing raw data to identify emerging patterns, ideas, or themes without predefined categories (Braun & Clarke, 2024). It begins with reading the textual data multiple times to gain a deep understanding of the content, followed by initial coding, category formation, theme development, review, and refinement. The process is data-driven, adaptable, and allows for exploration without a fixed endpoint.

The following section presents the study's results.

RESULTS

This section provides a detailed overview of the results, organized according to the three research questions outlined in the introduction. To enhance the analysis, selected quotes illustrating key themes emerging from participants' responses have been included. These quotes serve as powerful examples of the insights gained from the research. Additionally, a thematic analysis was conducted to explore the data more deeply, revealing nuanced patterns and relationships among the themes. To enhance clarity and understanding, we have also included visualizations that provide a graphical representation of the data trends, reinforcing the overall narrative of our findings.

The Least Preferred AP Calculus Topics

A quantitative analysis of 53 student responses to a multiple-response item revealed that the top three least preferred topics in AP Calculus were applications of integration (45.3%), analytical applications of differentiation (35.8%), and integration and accumulation of change (30.2%) (see **Figure 1**). These topics are broad and encompass several subtopics. The specific content falling under each of these broad categories will be highlighted.

The *Applications of Integration* involve using integral calculus in geometric and physical contexts. This topic begins with finding the average value of a function over a specified interval and then explores concepts such as position, velocity, and acceleration using integrals. Students engage with accumulation functions and definite integrals in various real-life situations. This unit includes determining the areas between curves in relation to both the x- and y-axes, as well as addressing regions that feature more than two intersection points. It also examines the volumes of solids by utilizing cross-sections in the shapes of squares, rectangles, triangles, and semicircles. Finally, the unit covers the Disc and Washer Methods for calculating the volumes of solids of revolution around the x-, y-, or other axes. This reinforces a comprehensive understanding of volume calculation through integration.

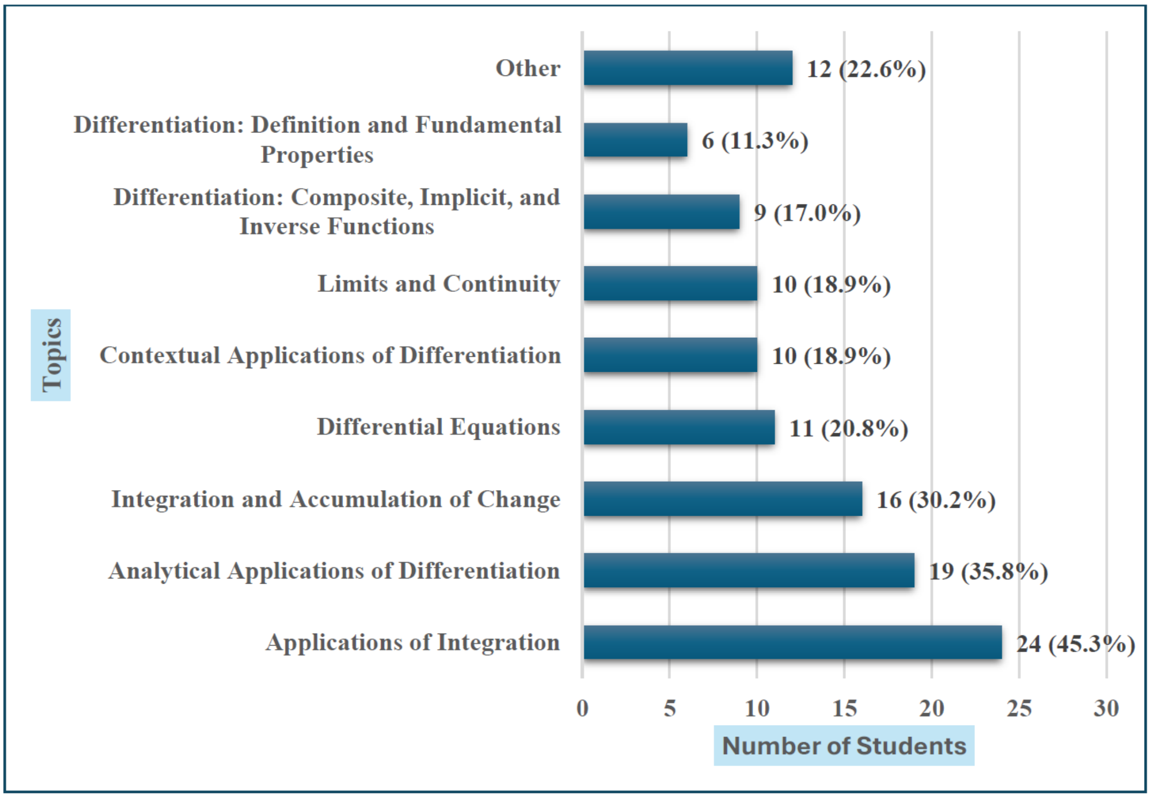

Figure 1. Student-reported least preferred topics (Source: Authors' own elaboration)

Table 3. Overview of all models and their corresponding fit statistics

Number of classes	AIC	CAIC	BIC	ABIC	G^2	χ^2	BF
1	468.72	492.49	484.49	459.36	131.65	9816.78	--
2	446.03	496.53	479.52	426.13	90.96	344.66	12.00
3	452.53	529.75	503.75	422.09	79.45	270.82	0.00
4	456.72	560.68	525.68	415.75	65.65	106.24	0.00
5	467.98	598.66	554.66	416.46	58.90	138.33	0.00

Note: CAIC = Consistent Akaike Information Criterion, ABIC = Adjusted Bayesian Information Criterion. The Bayes Factor (BF) of each model was calculated against the baseline model, which consisted of only one class

Analytical Applications of Differentiation explores how derivatives affect the behavior of functions. This section starts with an examination of the Mean Value Theorem and the Extreme Value Theorem, which helps identify critical points and distinguish between global and local extrema. It then analyzes the intervals where functions increase or decrease, employing the First and Second Derivative Tests and the Closed Interval Method (also known as the Candidates Test) to find absolute extrema. Additionally, students learn to assess concavity, sketch graphs using derivatives, and interpret the relationships between a function, its first derivative, and its second derivative. The unit concludes with an introduction to optimization problems and a discussion of implicit relationships.

The topic of *Integration and Accumulation of Change* focuses on the principles of integral calculus. It begins with understanding the concept of accumulated change and approximating area through Riemann sums. The unit then progresses to summation notation and the definite integral. A key aspect of this unit is the Fundamental Theorem of Calculus, which connects accumulation functions to definite integrals and includes an interpretation of area-related behaviors of these functions. Students learn important characteristics of definite integrals and methods for finding antiderivatives. They explore various integration techniques, such as substitution, polynomial long division, and completing the square. Students develop the skills to select appropriate techniques for antidifferentiation.

It is essential to note that not all participants reported a disinterest in the AP Calculus topics discussed in the foregoing analysis. As shown in **Figure 1**, fewer than half of the participants identified each of the AP Calculus topics as their least preferred. To better understand the different interests among students regarding these topics, we conducted a latent class analysis. Model fit statistics for all the models under consideration are presented in **Table 3** and visualized in **Figure 2**.

Model comparisons based on fit indices indicated that the two-class solution provided the best balance between model fit and parsimony. The two-class model yielded the lowest AIC value (446.03) and a lower G^2

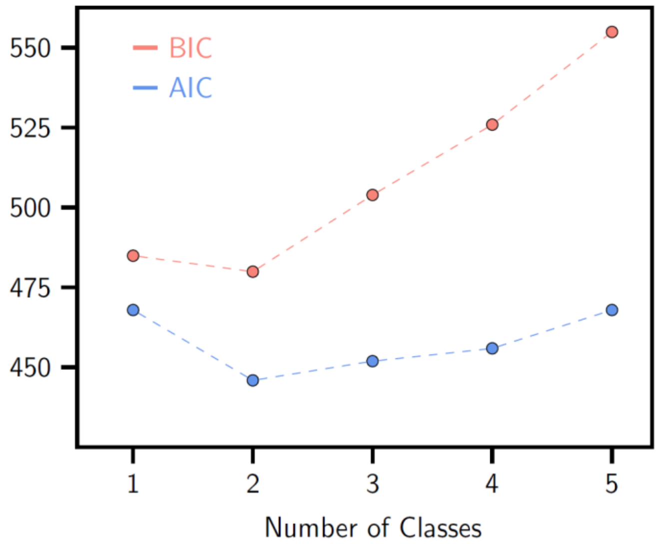

Figure 2. AIC and BIC values for all models as a function of the number of latent classes (Source: Authors' own elaboration)

(90.96) and χ^2 (344.66) compared to the one-class solution, indicating improved fit. The BIC for the two-class model (479.52) was lowest and notably better than those for models with more classes, which showed increasing BIC values and only marginal improvements in fit statistics. This is consistent with the adjusted values. While the ABIC values drop off significantly after the one-class solution and do not differ among the multi-class solutions, the CAIC values for the one- and two-class solutions are the lowest. In conjunction with the BIC being the most reliable fit statistic in LCA (Lezhnina & Kismihók, 2022) and the Bayes Factor only identifying the two-class model as superior to the baseline model, the two-class model was selected as the optimal representation of the latent class structure. Thus, a more detailed analysis of the two-class model will be presented.

On local model fit, the two-class model was investigated using posterior class membership probabilities, with all but one participant exhibiting a classification probability of at least 80% for their assigned class. Additionally, this participant has a 75% classification probability, which is only slightly below the commonly used threshold. Furthermore, we examined the agreement between the estimated class population shares and the predicted class memberships based on the modal posterior probabilities. The estimated shares were 84.19% (for Class 1) and 15.81% (for Class 2), while the predicted shares were similar at 84.91% and 15.09%, respectively, indicating a close correspondence between the model estimates and individual classifications. The conditional item response probabilities by topic (that is, the probability of observing each category within the two classes) are presented in **Table 4** and illustrated in **Figure 3**.

The conditional item response probabilities paint a clear picture: the vast majority of students (84.2%) can be assigned to Class 1, where very few students selected the AP Calculus topics as their least preferred choice. Marginal spikes can be observed for T5 (29% probability of selecting Analytical Applications of Differentiation)

Table 4. Probabilities to observe 0 (not selected as least preferred) or 1 (selected as least preferred) in each class, for each of the eight topics T1 to T8

T1: Limits and continuity	Pr (0)	Pr (1)
Class 1	0.84	0.16
Class 2	0.52	0.48
T2: Differentiation - definition and fundamental properties	Pr (0)	Pr (1)
Class 1	1.00	0.00
Class 2	0.28	0.72
T3: Differentiation - compose, implicit, and inverse functions	Pr (0)	Pr (1)
Class 1	0.91	0.09
Class 2	0.40	0.60
T4: Contextual applications of differentiation	Pr (0)	Pr (1)
Class 1	0.91	0.09
Class 2	0.29	0.71
T5: Analytical applications of differentiation	Pr (0)	Pr (1)
Class 1	0.71	0.29
Class 2	0.26	0.74
T6: Integration and accumulation of change	Pr (0)	Pr (1)
Class 1	0.78	0.22
Class 2	0.26	0.74
T7: Differential equations	Pr (0)	Pr (1)
Class 1	0.86	0.14
Class 2	0.39	0.61
T8: Applications of integration	Pr (0)	Pr (1)
Class 1	0.58	0.42
Class 2	0.24	0.76

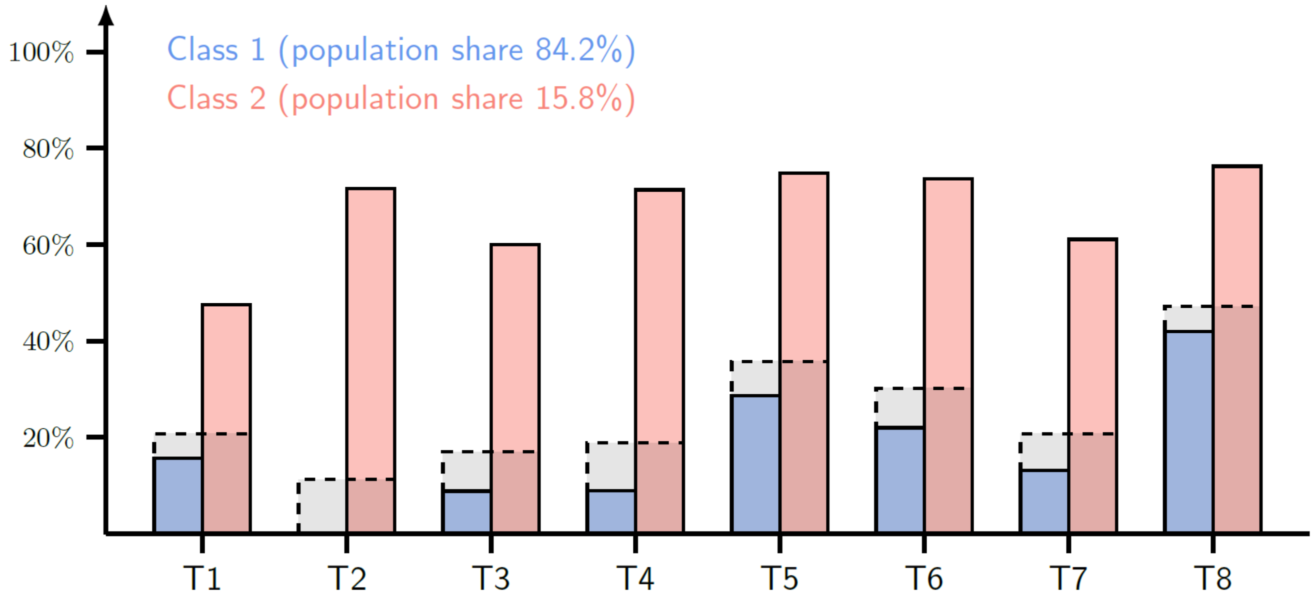

Figure 3. Probability of selecting a topic as least preferred, split by classes. Blue bars belong to Class 1, and the pinkish-red bars belong to Class 2. For reference, absolute values for the total sample are indicated by dashed bars (Source: Authors' own elaboration)

Table 5. Demographic statistics for both classes identified in LCA, including those of the total sample for comparison

	Male	Female	Public school	Private school
Class 1	48.9	51.1	48.9	51.1
Class 2	37.5	62.5	87.5	12.5
Total Sample	47.2	42.8	54.7	45.3

and T8 (42% probability of selecting Applications of Integration). Overall, students of Class 1 exhibit a broad interest in almost all topics. The opposite is true for Class 2, which constitutes 15.8% of the sample. The probability that students of Class 2 will select a topic as least preferred ranges from 0.48 in the case of T1 (Limits and Continuity) to 0.76 in the case of T8 (Applications of Integration). In this cohort, which constitutes only a small portion of the sample, disinterest is stable across all topics, with an average probability of over 50% for each topic to be selected as the least preferred. An overview of demographic data within both cohorts is presented in **Table 5**. While the total sample and Class 1 are well-balanced in terms of gender and school type, Class 2 is skewed towards both female students and students from public schools.

Challenges Students Experience When Learning Least Preferred Topics

Analysis of the participants' responses revealed several recurring themes that highlight the specific challenges students encounter when dealing with their least preferred topics in AP Calculus. These themes include *cognitive overload, topic complexity, cross-disciplinary confusion, lack of engagement or attendance, early semester challenges, assessment pressure or memorization difficulties, negative emotional responses, and general struggles*. Each theme is discussed below and accompanied by direct quotations from student participants.

Theme 1: Cognitive overload, topic complexity, and cross-disciplinary confusion

A notable theme among students was the inherent difficulty of the material, particularly when topics involved abstract mathematical reasoning or intersected with concepts from physics. These interdisciplinary connections often led to cognitive overload and confusion.

> Participant C1: Delving back into physics topics and interpreting graphs made it difficult to understand what was going on.

This indicates that while integrating mathematical concepts with real-world applications is pedagogically valuable, it can overwhelm students if they do not receive sufficient support. Students seemed to struggle when topics required not only procedural fluency but also a deeper understanding and interpretation of concepts.

A typical example that illustrates the cognitive demands placed on students is the analysis of motion under gravity, such as determining a ball's maximum height, velocity, and acceleration using the function:

$$s(t) = -4t^2 + 64 \quad (1)$$

This example requires students to apply calculus concepts, such as differentiation, to real-world physics contexts — a process that many find confusing due to the abstract nature of rate-of-change graphs and the need for interdisciplinary thinking.

Several participants reported challenges with specific subtopics that required the application of multiple concepts or methods.

Participant C2: Applications of Integration... had the most confusing problems.

Participant E7: I didn't understand implicit differentiation,

highlighting how some calculus units posed conceptual barriers even when students had prior exposure to related content.

These difficulties extended beyond conceptual understanding to the structural features of the curriculum.

Participant A2: It was Unit 8, and I had difficulties in it because it was too long.

A sentiment echoed by Participant A10, who also found Unit 8 (Applications of Integration) to be overwhelming due to its length and complexity. The cited unit comprised 12 sub-concepts (8.1 - 8.12) and covered challenging topics, including the area between curves, the volume of solids of revolution, and the volume of cylindrical shells. Similarly,

Participant A13: I struggled with Units 5, 6, and 8 because they consist of problems that require applying multiple topics.

underscoring the cognitive demands of integrative problem-solving. A typical integrative calculus problem that mirrors Participant A13's experience is given below.

$$Given\ F(x) = \int_x^{2x} t^3 dt,\ find\ F'(x) \quad (2)$$

(Gilbert et al., 2016, p. 553)

To tackle this problem, students need to utilize the Fundamental Theorem of Calculus for an integral that has variable upper and lower bounds. Successfully solving this problem demands an understanding of the chain rule, properties of integration, and proficiency in variable substitution. This reflects the multi-topic challenge that some participants found overwhelming.

In some cases, students encountered meta-cognitive challenges related to strategy selection.

Participant E5: I had to solve a variety of complex integrals involving multiple methods such as substitution, logarithmic, and trigonometric identities. What made it especially difficult was not the techniques themselves, but knowing which method to choose for a given integral.

This highlights the advanced reasoning required for success in calculus and the challenges students face when making procedural decisions under pressure.

Participant B4: The topic itself was hard and challenging; it was difficult for me to apply what I learned in the lesson when solving questions.

pointing to a gap between instruction and practical application.

Other students experienced difficulty with spatial reasoning and geometric visualization, particularly in topics involving the solids of revolution.

Participant B5: I had difficulty calculating the volume of solids using the shell approach. We were asked to calculate the volume of a region bordered by two curves that revolved around a vertical line other than the y-axis. What made it tough was that I couldn't clearly imagine how the form appeared in 3D, and I kept getting confused about which portion of the expression indicated the radius, and which represented the height.

Similarly,

Participant D6: I struggled with calculating the volume of the solid of revolution. It's a complex topic which needs thorough understanding of application and the use of proper formulas.

A typical example of a calculus problem that captures the concerns of participants B5 and D6 is presented below. Let $\boldsymbol{R}$ be the region bounded by:

$$f(x) = 2x - x^2 \quad (3)$$

and the x-axis on [0, 2]. Find the volume of the solid obtained by revolving $\boldsymbol{R}$ about the vertical line x = 3 using the method of cylindrical shells (Adapted from Gilbert et al., 2016, p. 659).

To solve this problem, students need to visualize the 3D solid and accurately identify the radius, which is the horizontal distance from the slice at x to the line x = 3, and the height, which is the function value above the x-axis. The shell method requires them to clearly define both the radius and height—an area where confusion often arises.

The challenges were not limited to procedural and visual aspects but extended into highly abstract reasoning.

> Participant B11: I remember struggling with limits at infinity in calculus. What made it particularly difficult was the abstract nature of the concept; trying to understand what happens to a function as x gets infinitely large or small felt disconnected from anything concrete.

This reflection highlights how abstraction in mathematical thinking can hinder conceptual understanding, particularly when students lack concrete representations or intuitive anchors.

A representative example of the abstract nature of limits at infinity is when students are asked to evaluate $\lim_{x\to\infty} \frac{sinx}{x}$. This problem exemplifies the conceptual difficulties described by Participant B11, as it requires students to reason about infinite processes and horizontal asymptotic behavior without concrete numerical references. The absence of visual intuition, combined with the necessity of applying the squeeze theorem or limit laws, increases the cognitive load.

Overall, the responses from the students highlight a multifaceted form of cognitive overload, encompassing confusion about concepts, uncertainty in procedures, and difficulties with spatial reasoning. This overload is especially noticeable in content-dense topics such as integration and limits. These findings underscore the importance of providing clear instructional support, targeted strategy instruction, and employing multiple teaching methods to help students master the most challenging aspects of the AP Calculus curriculum.

Theme 2: Lack of engagement or attendance

Some students identified inconsistent engagement and absences as reasons for their struggles. For example,

> Participant C2: It made it difficult for me when I didn't engage in the class sessions.

while

> Participant C4: I skipped some sessions, and skipping made it difficult for me to catch up.

Similarly,

> Participant A2: I had difficulties with Unit 8 because I was sick.

indicating that even involuntary absences due to illness can disrupt continuity and understanding. Additionally, Participant D3 concisely identified *"lack of focus"* as a contributing factor, suggesting that disengagement can occur even in the classroom when attention or cognitive presence is compromised.

> Participant A12: I really struggled… because I hadn't paid enough attention during classes.

highlighting the cumulative effect of inattentiveness on later understanding. This theme emphasizes the importance of consistent attendance and active participation, particularly when studying cumulative subjects like calculus. Absences, lapses in focus, and low engagement levels can accumulate difficulties over time, particularly if foundational knowledge is missed or misunderstood.

Theme 3: Early semester and beginning of a new topic challenges

Several students shared their experiences regarding the difficulties they encountered at the beginning of the academic term and when they were first introduced to new topics within their AP calculus course. These challenges can often be linked to the transitional phase that students undergo as they adjust to the demanding pace and heightened expectations that come with a new course structure. For instance, Participant A1 referred to *"the beginning of Unit 6"* as particularly challenging, indicating that this shift to more complex material posed significant hurdles in their learning journey. In a similar vein, Participant A4 recalled, *"First month of school,"* as a difficult period, emphasizing that the initial adjustment stage can reveal the challenges of grasping new concepts and maintaining motivation.

Moreover,

> Participant E3: I struggled with limits at the beginning of the school year.

which highlights how early exposure to foundational yet abstract mathematical concepts, such as limits, can be especially daunting for students. The confusion and frustration experienced during these first few weeks can lead to a diminishing sense of motivation and, if not promptly addressed, may adversely impact students' overall performance throughout the course. It is essential to tackle these initial challenges effectively to help students build a solid foundation and foster their confidence in handling more advanced material as the term progresses.

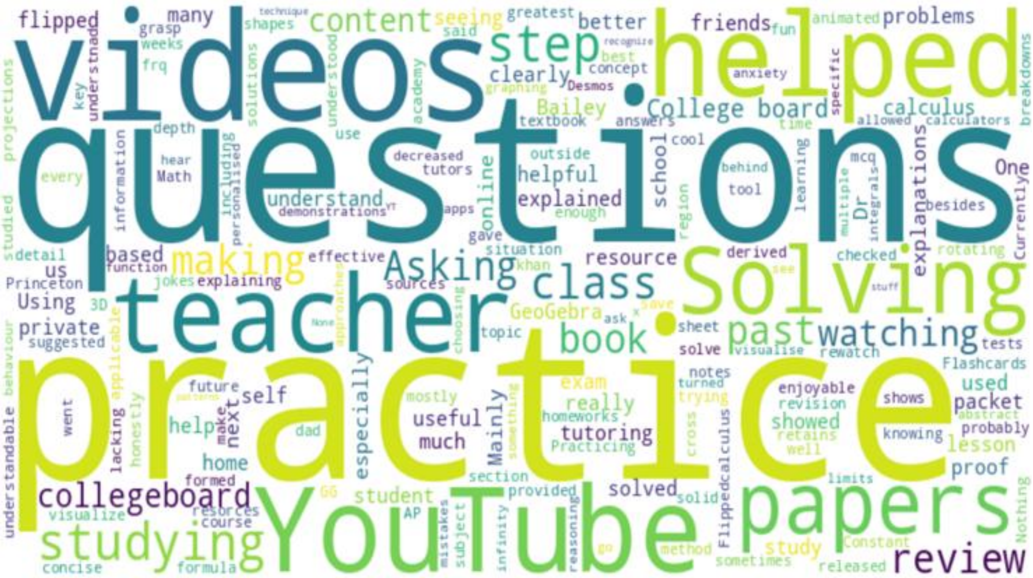

Figure 4. Word cloud illustrating frequently used terms in open-ended responses regarding teaching approaches (Source: Authors' own elaboration)

Theme 4: Assessment pressure and memorization difficulties

Several participants reported challenges with assessments and the perceived need for rote memorization.

> Participant A5: In the Chapter test, I didn't know what to do with the questions,

and

> Participant A6: The key points that students should memorize made it difficult.

These comments highlight an overemphasis on performance and memorization rather than understanding.

> Participant A9: At the end of Semester 1, I couldn't remember the topics because I had memorized them and not understood them.

which shows the limits of surface learning in a subject that requires deep conceptual understanding.

Additionally,

> Participant C8: I had a challenge with tests and timing.

pointing to assessment-related pressure and time constraints as compounding factors. Such perceptions may increase test anxiety and hinder meaningful engagement with the subject matter, impacting performance and long-term retention.

Theme 5: Negative emotional responses

The emotional impact of these academic challenges was painfully evident among participants. For instance,

> Participant C3: The whole year was unenjoyable.

which highlights a profound sense of dissatisfaction and struggle throughout the academic period. Another participant succinctly expressed their feelings with the powerful statement, *"I hate it,"* indicating a deep-seated aversion that may stem from repeated difficulties with the subject matter.

These honest responses highlight that ongoing struggles with specific mathematics topics can create a lasting dislike of the subject overall. Such emotional reactions are important to recognize because they significantly influence students' motivation, confidence, and their long-term educational and career choices. The emotional experiences associated with learning, particularly in subjects like mathematics, can significantly influence students' attitudes and decisions.

Theme 6: General struggles and non-specific challenges

Several participants reported experiencing general difficulties in their studies without pinpointing a specific cause. For example,

> Participant A2: I had difficulties with Unit 8 because I did not understand anything.

Although these responses may lack depth, they highlight a common reality: some students struggle with challenges that are vague or stem from an overall sense of inadequacy.

The analysis suggests that students' difficulties with their least favorite calculus topics are multifaceted, stemming from cognitive, behavioral, and emotional factors. Addressing these challenges may necessitate a combination of instructional redesign, emotional support, and proactive engagement strategies. These findings suggest that both the way content is delivered, and the classroom culture play a significant role in shaping students' experiences with AP calculus topics.

Students' Perceptions of Instructional Methods Used to Teach the Least Preferred Topics

Based on the analysis of participant responses regarding the instructional methods used to teach the least preferred topics, several key themes emerged from the data. These themes reflect students' evaluations of which instructional strategies either facilitated or hindered their understanding of challenging calculus concepts. **Figure 4** illustrates some of the dominant themes.

Theme 1: Value of concise teacher explanations and notes

Students consistently noted that resources prepared by teachers, especially concise notes and simplified explanations, were effective. These supports were seen as essential in making complex topics more understandable.

> Participant C1: My teacher gave us concise notes and suggested YouTube videos.

indicating that organized materials coupled with outside resources aided in reinforcing understanding.

> Participant C3: Easier step-by-step explanation as well as examples.

highlighting the importance of clarity and scaffolding in instruction. This finding indicates that when encountering cognitively demanding subjects, students benefit from streamlined instructional materials that prioritize clarity and progressive learning. Instructional methods that simplify abstract calculus concepts into manageable steps were regarded as particularly effective.

Theme 2: Use of visual and online resources

The integration of multimedia and online platforms was a strategy that received widespread praise. Participants frequently highlighted YouTube, online tutorials, and visual aids as valuable supplements to classroom instruction. For instance, Participant A1 noted the usefulness of *"online resources,"* and others agreed on the importance of video content in improving their understanding of challenging concepts. These insights highlight the importance of multimodal teaching approaches that cater to diverse learning styles and preferences. Visualizations can be particularly helpful in calculus, where students often struggle to develop mental models of dynamic or spatial processes, such as integration or the volume of revolution.

Theme 3: Practice through past papers and repetition

A recurring theme highlighted by several participants was the importance of practice and repeated exposure to diverse types of problems. Many felt that working through past exam papers and solving comparable questions repeatedly was key to mastering challenging topics. For instance,

> Participant C2: Past papers,

while

> Participant A1: Solving questions repeatedly,

as a beneficial strategy. This focus on practice aligns with research in mathematics education, which underscores the value of procedural fluency gained through repeated application. Overall, students appreciated opportunities to reinforce their learning through familiar formats, indicating a preference for active learning strategies over passive ones.

Theme 4: Preference for interactive and engaging lessons

Students also expressed a desire for more interactive lessons, particularly those that incorporate class activities and provide opportunities for active engagement with the content.

> Participant A1: Add more class activities,

while

> Participant C3: Make it easier,

referring to the need for simplified, engaging delivery. These responses suggest that passive lecture-based instruction may not meet the learning needs of all students, particularly when dealing with their least preferred topics. Interactivity, whether through group activities, hands-on problem-solving, or informal discussions, appears to enhance engagement and support comprehension by encouraging learners to process and apply information in real-time.

Theme 5: Self-Regulated Learning and Peer Support

Several responses reflected the importance of student agency and the learning environment. Participant C1 suggested that having *"more hardworking and intelligent classmates would help,"* pointing to the potential influence of peer interactions and classroom culture. Students also advised others to

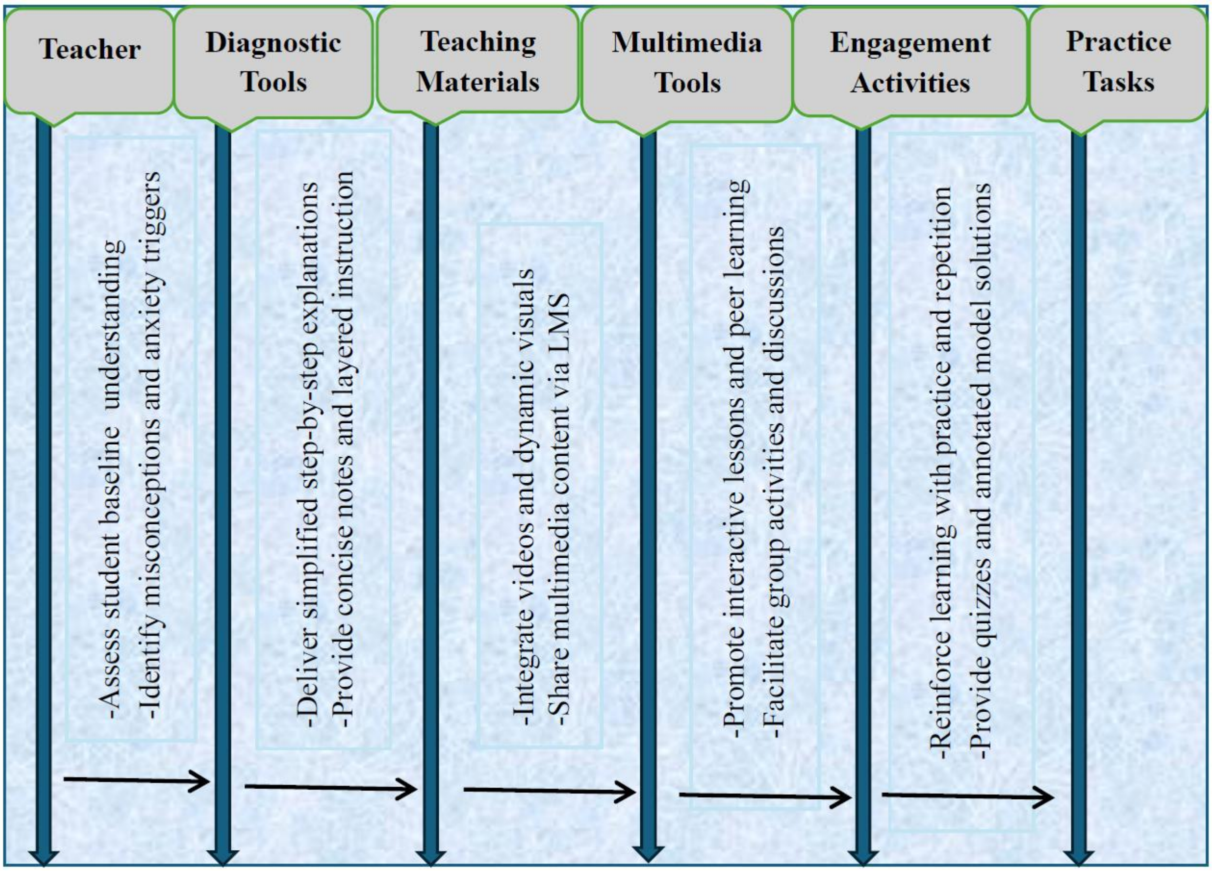

Figure 5. A sequential model to guide teachers in supporting students' understanding of calculus concepts (Source: Authors' own elaboration)

> Participant A1: Seek help from the teacher immediately

and

> Participant C1: Learn each lesson in class,

indicating a recognition of the role of self-regulation and timely support in navigating difficult content.

Proposed Instructional Framework to Tackle Student Difficulties in Calculus

In response to student-reported challenges, teachers can adopt a five-step instructional sequence:

(1) Diagnose learning barriers,

(2) Deliver simplified, step-by-step explanations with notes,

(3) Integrate visual and online multimedia tools,

(4) Promote engagement and peer-supported learning, and

(5) Reinforce through practice and repetition.

This framework responds to students' expressed preferences and aligns with cognitive load theory by reducing extraneous load and enhancing schema construction. When implemented systematically, these strategies may help students build both procedural fluency and conceptual clarity in calculus. **Figure 5** is an elaboration of the possible steps that teachers may take to intervene and address students' concerns.

Diagnosing learning barriers

Before instruction, teachers can utilize brief diagnostic tools, such as online polls, entrance slips, or self-assessment checklists, to assess students' baseline understanding and identify specific misconceptions or sources of anxiety. They may ask students to rate their understanding and confidence regarding upcoming subtopics and gather common points of confusion to provide targeted scaffolding.

Delivering simplified, step-by-step explanations with notes

Calculus teachers should present complex concepts using layered instruction, beginning with a visual or conceptual overview and then breaking the topic into manageable steps. Teachers can also provide downloadable PDF lesson summaries containing color-coded worked examples. Students in this study consistently appreciated concise notes prepared by teachers and simplified verbal explanations. These resources should be provided in class and made available digitally through platforms like the Learning Management System (LMS) for flexible access.

Integrating visual and online multimedia tools

To enhance conceptual understanding, particularly in spatial and procedural calculus topics, teachers should incorporate videos, graphing animations, and dynamic visuals into their lessons. Brief video clips can be created using tools such as GeoGebra, Desmos, or YouTube to illustrate concepts like the volume of revolution or the area under curves. These video clips can be embedded in presentation slides or shared through LMS platforms for revision purposes.

Promoting engagement and peer-supported learning

Feedback from students indicates a desire for more engaging and interactive lessons. Integrating collaborative learning, discussions, and informal group problem-solving can enhance student engagement and minimize passivity in the classroom. For calculus teachers, beginning lessons with brief assessments using mini whiteboards or interactive platforms like Kahoot can be effective. Additionally, implementing peer pairing for group activities and employing think-pair-share strategies prior to whole-class discussions can further promote active participation and deeper understanding among students.

Reinforcement through practice and repetition

Students highlighted the importance of solving past papers and maintaining a regular practice routine. Teachers should assign formative tasks after each concept is taught. These tasks could include weekly low-stakes quizzes or digital practice activities delivered through LMS. Furthermore, calculus teachers should provide annotated model solutions for past paper questions to aid student understanding.

DISCUSSION OF FINDINGS AND IMPLICATIONS

The findings of this study provide important insights into the perceptions and experiences of AP Calculus students regarding their least preferred topics, particularly those related to integration and the analytical applications of differentiation. A considerable number of students identified "applications of integration" and "integration and accumulation of change" as especially challenging. This suggests that these areas of the curriculum present notable cognitive and conceptual barriers. These findings align with prior research by Ruamba et al. (2025) and Biza et al. (2022), who emphasized that students often struggle with calculus topics that require a shift from procedural knowledge to a structural understanding. Similarly, Thompson and Harel (2021) noted that fragmented conceptual foundations contribute to superficial reasoning in calculus, which aligns with the difficulties reported by students in this study.

The thematic analysis of qualitative responses revealed that students found it challenging to work with topics that required not only procedural fluency but also complex reasoning, spatial visualization, and decision-making under pressure. A considerable number of participants expressed confusion when choosing appropriate integration methods or understanding geometric interpretations, such as those involved in solids of revolution. This aligns with the relevance of cognitive load theory (Sweller, 1988), as students appeared overwhelmed when required to process symbolic procedures, abstract concepts, and visual representations simultaneously. These challenges were exacerbated when students lacked foundational knowledge or were introduced to new content without adequate support, resulting in cognitive demands that exceeded their ability to process the new knowledge.

Affective and behavioral dimensions also significantly shaped students' perceptions of difficulty. Several students cited factors such as lack of attendance, illness, and disengagement as reasons for falling behind. Even brief behavioral lapses had long-term effects on their ability to grasp cumulative content. Additionally, some students described emotional responses ranging from anxiety and confusion to outright dislike of the AP Calculus course. These findings align with self-determination theory, which suggests that students are more likely to engage deeply with content when they feel competent, supported, and autonomous (Chiu, 2021; Ryan & Deci, 2000). The results of this study indicate that students who experience negative emotional reactions or feel overwhelmed are less likely to persist with challenging material, contributing to a cycle of avoidance and deficient performance.

The findings notably highlight how instructional techniques shape students' experiences with challenging AP Calculus topics. Students continually appreciated brief teacher explanations, streamlined notes, and guided examples that simplified complex procedures into manageable parts. The implementation of visual aids, online tutorials, and video content, particularly from YouTube, was found to be particularly effective in helping students visualize abstract mathematical concepts. This aligns with recent studies by Schoenherr et al. (2024) and Adeoye (2023), which emphasize the advantages of multimodal instruction and the Feynman Technique in breaking down abstract material. Additionally, students highlighted the value of practicing with past papers and engaging in repetition, reinforcing existing literature that advocates for deliberate practice as a method to enhance procedural fluency.

Students expressed a wish for more interactive lessons and increased engagement in the classroom. They responded positively to active learning strategies and emphasized the importance of class activities, peer discussions, and real-time problem-solving. This supports the pedagogical changes suggested by Boaler (2022) and Johnson et al. (2025), who argue that conventional lecture-based methods are insufficient for promoting deep interaction with complex content. Interestingly, students also recognized the effect of self-regulation and peer influence on their learning, with several suggesting the need for proactive help-seeking and collaborative learning environments.

The implications of these findings are significant. Instructional strategies in AP Calculus must adapt to meet the cognitive and emotional challenges that students encounter, especially with complex topics such as integration. Clear explanations, along with the incorporation of visual and interactive tools, can help alleviate the perceived difficulty of these abstract subjects. Teacher training should focus on recognizing cognitive overload and implementing scaffolding methods to support diverse learners. Furthermore, school leaders should consider establishing academic counseling and peer mentorship programs to address the emotional and motivational factors that affect student engagement.

However, this study has its limitations. The sample size was small and confined geographically to specific schools in the US and Iraq, which may limit the applicability of the results. Furthermore, relying on self-reported data introduces a degree of subjectivity, as students' perceptions might be shaped by personal biases or their most recent academic experiences. Teacher insights were also not included, which could have shed light on the instructional issues and their responses to student challenges. Finally, the research conducted was cross-sectional and does not reflect changes in student perceptions or performance over time.

Despite these limitations, the results are consistent with existing literature on mathematics education and make valuable contributions to the conversation regarding learner-centered teaching in advanced mathematics. By emphasizing student perspectives and illuminating their cognitive, emotional, and instructional hurdles, this study strengthens the argument for more inclusive, responsive, and strategically distinct approaches to teaching AP Calculus. Future research could benefit from longitudinal studies, larger and more varied samples, and the inclusion of insights from teachers and parents to further validate and elaborate on these significant findings. Examining students' written scripts can provide valuable insights into the challenges they face when learning AP calculus.

The results of this study support earlier research (Biza et al., 2022; Thompson & Harel, 2021), which demonstrates that AP Calculus students often face challenges when transitioning from a procedural to a structural understanding. Specifically, concepts such as integration require students to integrate graphical, numerical, and symbolic representations, which places a significant cognitive burden on them. The emotional and motivational barriers identified in Hannula's (2019) work indicate that students' negative experiences with certain topics can hinder their persistence and exacerbate gaps in understanding. This issue is particularly pressing in high-pressure environments, such as AP Calculus.

Supportive teaching strategies, such as utilizing visuals, providing guided notes, and incorporating humor, have proven effective in reducing obstacles to comprehension. These findings align with the research of Boaler (2022) and Schoenherr et al. (2024), who emphasized the importance of engaging and differentiated instruction for abstract concepts.

The significance of self-efficacy and autonomy, as highlighted in Ryan and Deci's (2000) self-determination theory, was evident in students' suggestions for peer learning, simplified materials, and incremental scaffolding. These preferences highlight the importance of culturally responsive and student-centered teaching approaches in advanced mathematics courses.

CONCLUSION AND RECOMMENDATIONS

This study underscores the importance of aligning instructional strategies in AP Calculus with students' learning needs, emphasizing clarity, engagement, and support. While the findings reflect typical challenges in advanced mathematics, they also highlight practical solutions grounded in responsive teaching. To improve student outcomes, educators should adopt concise, scaffolded explanations and incorporate visual aids and active learning techniques. Schools should invest in professional development that equips teachers to manage cognitive load and foster inclusive classrooms. Additionally, integrating multimodal resources and promoting consistent student engagement from the outset of the course can enhance comprehension and reduce anxiety. Future research should explore the longitudinal impacts and incorporate broader stakeholder perspectives to refine calculus instruction and further support systems.

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AI statement: The authors stated that they utilized Grammarly and QuillBot for proofreading, enhancing coherence, and ensuring grammatical accuracy. These tools aimed to improve readability and uphold academic standards. The authors alone bear

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