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Exploring Mathematics Learners' Conceptual Understanding of Coordinates and Transformation Geometry through Concept Mapping

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ABSTRACT

In this article, we explored learners' conceptual understanding of coordinates and transformation geometry through concept mapping. A qualitative case study was employed in this study, and data was collected using an investigative task, observation and reflective interviews on a sample of 34 Grade 12 mathematics learners. The study was guided by the following two research questions: What are the learners' understanding of coordinate and transformation geometry? and How do concept maps improve learners' understanding of coordinate and transformation geometry? The study revealed that, although some leaners struggled with linking words and omitted some concepts in their concept maps, there were some indications of conceptual understanding of coordinate and transformation geometry when the learners were probed during the interview. Thus, we argue that the learners' conceptual understanding of coordinate and transformation geometry could be improved when they are taught, using concept mapping. The learners actively situated their knowledge of small pieces of coordinates and transformation geometry; and their understanding of coordinate and transformation geometry was expanded to show the conceptual structure of representing given information and a complete picture of the situation. It is therefore recommended that the teachers allow the learners to use the concept mapping strategy to construct their knowledge and that concept maps assist the learners in focusing on main ideas of identified concepts and explain the relationship.

Keywords: concept map, conceptual understanding, coordinate geometry, transformation geometry

INTRODUCTION AND BACKGROUND

New education transformation in South Africa concerns learners learning in a meaningful way. Many learners are struggling to understand mathematics. Pillay and Bansilal (2010) indicate that the learners find it difficult to solve mathematical problems. Naidoo and Bansilal (2010) are in agreement that many learners want to quickly resort to the procedures without understanding the concepts. The learners do not draw pictures or diagrams as solution strategies to help them with mathematical problems. (Machaba, 2018). From our experiences, we noted that Grade 12 learners tend to leave out the sections on coordinate and transformation geometry during the examination at the end of the year or perform poorly in these sections (Limpopo Department of Education (LDoE), 2009). The leaners' lack of conceptional understanding of coordinate and transformation geometry resulted in the researchers conducting this study. We therefore argue in this study that the learners' conceptual understanding of coordinate and transformation geometry could improve when taught by using concept mapping as teaching strategy.

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Contribution of this paper to the literature

- Learners learn better and in a meaningful way when using a concept map. Concept mapping is a representation of a relationship of a group of concepts.
- Concept mapping could be used as an effective teaching strategy in mathematics education.

LITERATURE REVIEW

Definition of a Concept Map

The relationship between mathematical concepts can be shown by means of a concept map. A concept map can be described as a graphical tool that shows the relationship between concepts in a structured manner. The concepts which are interrelated and the lines used to connect and show how the concepts are related are often written in a circle (Asan, 2007; Grevholm, 2008; Novak, 1990; Novak & Cañas, 2008). In their research, Mwakapenda and Adler (2003); Borda, Burgess, Plog, and Luce, (2009); and Chichekian and Shore (2013) show that a concept map could be used as a powerful teaching tool in teaching and learning mathematics.

Conceptual Understanding

Kharatmal (2009) refers to conceptual understanding as connections of mathematics ideas which are not disconnected to one another. This implies that a learner has conceptual understanding when he or she is able to explain, describe and apply the same concept in different ways and in different situations. This further refers to a situation where a learner can see the integration of concepts – that understanding takes place when mathematical concepts are not isolated.

According to Kilpatrick, Swafford and Findell (2002), conceptual understanding refers to the ability to comprehend mathematics concepts to perform operations and relate concepts. They state that there are three components of conceptual understanding: comprehension, operations and relations. These three components form the basis of conceptual understanding of coordinate and transformation geometry. Learning with understanding improves retention, promotes fluency and facilitate learning material (Kilpatrick, Swafford, & Findell 2002).

There are seven layers which are connected to the constructs of conceptual understanding, namely primitive thinking, image making, image having, property noticing, formalising, observing, structure and investigating. These layers show how leaners could develop conceptual understanding of any mathematical concept, using concept mapping (Pirie & Kieran, 1994). These layers are referred to as the Pirie-Kieren Theory. The theory explains the layers as follows: "Primitive knowing, which in constructivism perspective could be referred as prior knowledge, refers to the knowledge that an individual brings to a setting. The process of growth of understanding begins at this level and it contains what a researcher assumes a learner can do at the beginning of instruction" (Pirie & Kieren 1994, p. 170). This is the kind of prior knowledge and, according to Piaget (1964), it gives meaning to the new knowledge. Image making is "the level wherein a learner can make distinctions in his or her previous knowing and can use that knowledge in new ways that involve actions and activities with that knowledge" (p. 170). This suggests that image knowledge is a knowledge which could be put in practice when integrated to the primitive knowledge.

At the image layer the learner is not tied to an activity any longer, he or she can carry out a mental plan for the activities and use it accordingly. Property noticing is a level of understanding that occurs when one can manipulate or combine aspects of his or her images to construct context-specific, relevant properties. At the formalising layer, the learner can think consciously about the generalised properties and work with the concept as a formal object without specific reference to a particular action or image. These seven layers could be connected to the theory of constructivism to elaborate an individual understanding as the continual process of organising and reorganising knowledge structures (Pirie & Kieren, 1994). Furthermore, these layers show that a leaner could develop a conceptual understanding of any mathematical concepts. For example, conceptual understanding, according to Khartmal (2009), is the level where a learner can make distinctions in his or her preceding knowledge and can use the specific knowledge in new ways that engage actions and deeds with the new knowledge. Similarly, image making is when a leaner can make distinctions in his or her previous knowledge and can use the knowledge in new ways that involve actions and activities with the specific knowledge.

The views of conceptual understanding by Kilpatrick and Swafford (2002), Pirie, and Kieren (1994) were adopted in this study. Both views give details of the stages of conceptual understanding. Kilpatrick and Swafford (2002) give components that one could look at when concluding that there is a conceptual understanding while Pirie and Kieren (1994) give the levels that could be used to identify if there is conceptual understanding. The explanations of Kilpatrick and Swafford (2002) and Pirie and Kieren (1994) were used because their descriptions are similar although written differently; and they provide for components that could be looked at when identifying that a learner has a conceptual understanding. Kilpatrick, Swafford and Findell (2001), for example, point out that for an evidence of conceptual understanding, learners will be able to verbalise connections among concepts and representation. For this study, drawing from Pirie and Kieren (1994) and Kilpatrick et al. (2001), a learner conceptually understands coordinate and transformation geometry when he or she could draw a representation of the concepts in a clear structure. The structural layout, which is referred to as a concept map, is the most important first pillar in understanding a concept. The second pillar of concept mapping is referred as linkage (linking words used). This is when the learners connect the concepts with prepositions and verbalise these connections among concepts. We have used the structural layout and linking words of learners in this study as evidence of a conceptual understanding of coordinates and transformation geometry.

Conceptual Understanding of Coordinates and Transformation Geometry

Learners who have a conceptual understanding of coordinate and transformation geometry would be able to apply this knowledge in another knowledge domain, such as measurements. New Jersey (Assessment of Skills and Knowledge [NJASK], 2009). These skills give learners more options to answer questions related to coordinate and transformations geometry. NJASK (2009) states that when a learner has come to understand a concept in mathematics, he or she would be able to clearly and consistently demonstrate procedural fluency when calculating, transforming and analysing the problem. This would further assist the learners with abstract thinking. They would able to provide explanations that are consistent and connected and not in isolation to one another. Conceptual understanding would help the leaners to think and reason inductively and deductively when solving mathematical problems. Conceptual understanding encourages learners to use various forms of representation when solving complex problems and to abstract relevant information, such as multiple strategies and reasoning methods. (NJASK, 2009). These abilities are firmly situated in the components of conceptual understanding. To demonstrate these abilities, the teaching strategy of concept mapping should be used. Conceptual understanding emanates from the theory of constructivism.

THORETICAL FRAMEWORK

A Constructivist Perspective

Since this study is about exploring learners' conceptual understanding of coordinates and transformation geometry through concept mapping, the theory of constructivism was found to be helpful and useful to explore and describe the solution strategies of the learners. The theoretical framework that informs this study is derived from Piaget's (1964, 1968) theory of constructivism, which is also informed by the interpretations of Hatano (1996) and Van de Walle (2016). Van de Walle (2016) noted that learners should be active participants in the process of their own learning according to a widely accepted theory, known as constructivism (Hatano, 1996; Van de Walle, 2016). This also suggests that each learner constructs his or her own knowledge. When learners build on their prior knowledge (the knowledge that they already possess) they can understand the new information. This implies that what learners learn is constrained and enhanced by what they know; and what they think, say and do make sense to them in relation to what they know. The current knowledge is not merely built on as behaviourist theories suggest, but it is restructured and re-organised into richer, more connected and more powerful knowledge (Hatano, 1996). Therefore, in the context of this study it was very interesting to see the solution strategies of learners using structural layout and linking words in the coordinate and transformation geometry mathematical domain.

The theory of constructivism is related to concept mapping in the sense that concept mapping has frequently been used as an instructive tool to help learners learn more meaningfully and form a "conceptual understanding of the subject (Novak, 1990). In other words, the learners interact with the content by picking up the concepts that are related and how they are related. Concept mapping has the potential to make a knowledge discipline more "conceptually transparent" (Novak, 1998:162) and to "convey ideas that are not easily put into words" (Raymond, 1997, p. 1). Concept meanings are constructed by determining relationships between concepts (Linkage). According to Roth and Roychoudhury (1992), the network of propositions interlinking a group of concepts tells us much about the meaning of the concept from the perspectives of the map makers.

In concept mapping, interrelationships between concepts are an "essential property of knowledge" (Ruiz-Primo & Shavelson, 1996, p. 592). The material to be learned must be conceptually clear and presented with language and examples relevant to the learner's prior knowledge which is related to the theory of constructivism. Concept maps can be helpful to meet this condition by identifying large general concepts held by the learner prior to instruction on more specific concepts. They can further be helpful in that they assist in sequencing learning tasks through gradually more precise knowledge that can be fastened into developing conceptual frameworks. In order to do this, a learner must possess relevant prior knowledge. This condition could be met after a long time for any field of subject matter, but it is necessary to be careful and clear when building concept frameworks if one hopes to present detailed specific knowledge in any field in succeeding lessons. We therefore see that two conditions are interrelated and both are important. The learner must choose to learn significantly. The one condition over which the teacher or mentor only has indirect control is the motivation of learners to choose to learn. Learners attempt to incorporate new meanings into their prior knowledge, rather than simply memorising concept definitions, propositional statements or computational procedures.

As supported in Machaba (2014, 2016), constructivism discards the idea that learners are "blank slates"; they do not come to class with no knowledge and they are not empty vessels to build on. They come up with an existing amount of knowledge which they use to build on to access new knowledge which they would make sense of. This implies that, in their teaching, teachers have to recognise learners' existing knowledge so that they could build on it in their teaching. This suggests that a learner's existing knowledge will give meaning to the incoming knowledge, after it has been organised, structured and restructured. Concept mapping could be used in the process of constructing knowledge. Interrelated concepts could be used to give meaning to other concepts.

How Ideas are Constructed

A concept map is a tool that shows how ideas are constructed. Van de Walle (2016) noted that mathematical ideas cannot be "poured into" a passive learner with an inactive mind (p. 50). Learners must wrestle with new ideas to work at fitting them into an existing network of ideas and to challenge their own ideas and those of others. Van de Walle (2016, p. 50) uses the term "reflective thought "to explain this endeavour on the part of the learner to actively think about or mentally work on an idea. It means sifting through existing ideas to find those that seem to be most useful in giving meaning to the new idea.

A concept map could be referred to as the integrated network of ideas. It is a cognitive schema, which is the product of constructing knowledge and the existing knowledge that an additional knowledge could be build on Van de Walle (2016). Thus Van de Walle (2016, p. 45) defines understanding as "the quality of and quantity of connections that an idea has with existing ideas". This implies that an understanding solely depends on the learners' prior knowledge. An understanding will only take place in a learner's cognitive structure, if there is connections to a network of ideas.

"Understanding at this rich and strong interconnected end of a continuum will be referred to as relational understanding, while at the other end of the continuum, where the ideas are completely isolated is referred as Instrumental understanding" Van de Walle (2016, p. 45). This means that instrumental understanding is a knowledge learned thought rote learning. It is a knowledge which has an isolated and disconnected idea; while relational understanding is a knowledge learned through self-discovery. In the context of this study, relational understanding will be observed through the structural layout and the linkage of words in learners' concept mapping; while instrumental understanding will be reflected by the lack of structural layout and the omission of connected concepts in the learners' concept mapping responses. When a learner has learned through self-discovery, he or she could make sense of the information and it would be easier for him or her to recall the information (Machaba, 2014). Learning through concept mapping can produce relational understanding because learners would be able to produce a link or network of connected ideas which makes sense to them. Learners who learn trough concept mapping can be independent learners who are able to think, express ideas and solve problems.

METHODOLOGY

Research Design

In order to gather data significant to teacher knowledge and to how learners understand coordinate and transformation geometry, the research followed a qualitative case study design. The data was purposed to explore how learners think, come to decision-making and for teaching and learning coordinate and transformation geometry.

Sample

The study involved 34 Grade 12 mathematics learners in the 2010 Grade 12 mathematics classes at a school from the Malokela circuit; Sekhukhune district in Limpopo. The Grade 12 mathematics class was chosen because it was accessible and it was found to be the group that had problems with coordinates and transformation geometry.

Teaching Coordinates and Transformation Geometry using Concept Mapping

During the exploration, learners were taught concept mapping techniques in the three guided practice sessions. During session one the 34 learners were walked through the process and how to construct a concept map. This formed one of the three sessions planned.

During session two the 34 learners were given a content area (number and number relationship) to generate their own terms and use those terms to draw and practice concept mapping. After that, they drew a concept map to show the relationship (connections) of the terms. During session three the learners generated and constructed a concept map on the topic compound angles. The two topics for sessions two and three were chosen because, according to the DoE (2010) work scheduled, they should be taught prior to teaching them the topics of coordinate geometry and transformation geometry. Thus, the data collected from session three was not captured in this study. Furthermore, it formed part of the prior knowledge of the topics coordinate and transformation geometry in the study. During session four, the learners were given a list of 29 concepts to represent the concept map and an investigative task (See **Appendix**) based on a coordinates and transformation geometry task to answer.

DATA COLLECTION AND ANALYSIS

Sources of information were literature, recorded video tapes and learners (through investigative task responses and reflective interview answers).

An unstructured open-ended reflective interview was used to find out the meaning the 34 learners had in drawing their concept maps and the explanation of their concept maps, as interpreted by the learners, to convey their ideas. The other reason for the interviews was to gather extra information from the 'primary source', not that they were experts in the subject, but simply that they provided information. As indicated in Malatjie (2012)'s work all 34 Grade 12 learners took part in the investigative task and were also interviewed since they were the only learners sampled to form part of the study. Furthermore, participant observation was used because the process occurred at the same time as the teaching and observation took place. The video tape was used throughout the sessions to make field notes as part of the observations and was viewed after every session. The video footage was used as field notes. In this study, investigative tasks were also given to the 34 learners to explore their conceptual understanding of coordinate and transformation geometry.

For data analysis in this study, the analysis style of Mwakapenda and Adler (2003) was adopted. The maps were examined to determine whether the learners regarded certain concepts as central in developing links among concepts. The central concepts that learners used were identified; and the meanings learners associated with these concepts were examined (Mwakapenda, 2003; Mwakapenda & Adler, 2003). The learners' descriptions of their concept maps were then analysed to examine the adequacy of the connections made by means of interviews. The data was analysed inductively. The term "inductive analysis" was defined as 'a process of making sense of field data' by sorting data into categories to establish patterns and themes (Creswell, 2007). All the interview conversations were transcribed one by one and read, studied and analysed; and the text derived from the transcripts were re-read to search for relationships, themes and categories. The first author went through the responses by writing them down and by highlighting significant statements or quotes: sentences that provided an understanding of how the participant experienced the phenomena. This is called horizontalisation (Creswell, 2007).

When viewing and assessing the concept maps, the following themes emerged: Structural layout, Omission and Linkage. Sense of the data was made by grouping information into themes emerged (structural layout, omission, and linkage), categories and codes. According to McMillan and Schumacher (1993), qualitative analysis is a systematic process of selecting, categorising, comparing, synthesising and interpreting data to provide explanations of the single phenomenon of interest. The learners' responses were selected based on emerging themes. The selected data segments were then separated into categories.

There were nine categories in this instance: Under the theme, Structural layout there were those with a non-hierarchical structure and those with a linear structure; under the theme Omission there were those who omitted linking words and concepts, while under the theme Linkage, there were those whose concepts were not well-linked and who used irrelevant linking words. Then the data were grouped into these categories. Each of the themes had two categories, with the exception of Miscellaneous. Structural layout consisted of hierarchy and linear, Omission included the concept "omission and linkage omissions"; finally, Linkage included relevant and irrelevant link categories.

The themes were used to write the description of what the participant experienced – textural description. This was refined progressively as further data became re-analysed. The data, collected through an interview of the teacher and the learners were presented, using pseudonyms to substantiate the findings from the concept map data. Pseudonyms were used in the place of the learners' L1 for learner one, L2 for learner two and so forth while the pseudonyms for the teacher (interviewer) were JF (First Author). During analysis codes were also used for the categories. As in Malatjie (2012) the following codes were used: OC (omission of concepts), OLW (omission of linking words), H (hierarchical structure), L (linear structure), OCL (omission of cross links), ILW (irrelevant linking words) and CNWL (concepts not well linked).

Although this was the case, some learners' concept maps could not be categorised or coded and the data were reanalysed. During the re-analysing of the data, it was found that the learner's concept map findings were crossing among the themes that were identified initially. During the re-analysing process, three themes were identified and it was found that Miscellaneous emerged. Miscellaneous contains a further three categories as follows: correct linking words with one hierarchy structure; omission of concepts and other linking words; irrelevant linking words with one hierarchy structure; all concepts used and no omission of linking words; correct linking words with two hierarchy structures; concepts used and no omission.

This is because some findings of each learner's map fell in all the other themes and categories. The fourth theme has emerged in order to avoid repetition of using one concept map twice; the fourth theme emerged to discuss the concepts maps thoroughly.

In this study a variety of data collection and analysis techniques were employed to ensure the credibility of data and data analysis. Through prolonged engagement with the participants, with interviews and in their classroom, an effective vehicle for obtaining and processing reliable information was established and maintained. Triangulation was another means of strengthening data collection and analysis, to seek multiple and comparative opinions about the same issue. The type of triangulation used is called methodological triangulation; confirm-ability was maintained by providing raw data that could be traced to original sources and by describing how the data were to be interpreted and placed into themes and categories (Lincoln & Guba, 1985).

FINDINGS AND RECOMMENDATIONS

This section reports on the findings and discussions. To organise our analysis, we use the emerged themes drawn from literature (structural layout and linkage) and from the data (omission of either concepts or linking words). Within each theme, we further use categories that emerged from the data; and within these categories we explore the learners' conceptual understanding guided by its three components: comprehension, operation and relations.

Each of the themes had two categories. Structural layout comprised hierarchy and linear; omission comprised the concept "omission and linkage omissions"; finally, linkage comprised relevant and irrelevant link categories.

From the themes, it is observed that the concept maps drawn give a visual relationship of the coordinate and transformation geometry. Themes such as "use of linking words", "hierarchical structure" and "omission" give clarity of how the learner conceptually understands coordinates and transformation geometry to a certain extent. Therefore, "omission" and "lack of linking words" meant conceptually missing the interrelatedness of the concept. For this paper we generally provide a description of the findings of the 34 participants in terms of how they had structurally laid out and linked concepts in their concept mapping because of the lack of space and requirement of the length of the paper. We then focus on the comprehensive findings of two learners L4, who, correctly linked words with one hierarchy structure, omission of concepts and other linking words, and L6, who correctly linked words with two-hierarchy structure, concepts and used no omission.

Structural Layout

Category 1: Hierarchy layout

Each of the concept maps was directly viewed to see the layout. The study found that most learners were not familiar with the drawing of concept maps. They were able to understand that the map should follow a hierarchy and they had drawn their maps hierarchically. It was found that only two learners separated the concept maps from the two main concepts: analytical and transformation geometry.

Category 2: Linear layout

Under this category, it was found that out of the 34 concept maps, there was no linear concept map. It is therefore evident that, although the learners were not able to determine the main central concept through which they could centralise and draw the hierarchical concept map around, they were able to draw and remember it when drawing the concept map the main important concept should be drawn on the highest rank of the concept map. This therefore, encouraged them to draw a hierarchy map and not a linear one.

Table 1. Table on Concept used and Unused

Group	Number of concepts used	Number of concept omitted	Number of learners
1	30-26	0-5	17
2	25-21	6-10	12
3	20-16	11-15	03
4	15-10	16-20	02
5	09-05	21-25	00
6	04-00	26-30	00

Omission

Some basic counting was used only to show the degree of interlinking displayed in the maps. The counting involved: counting the number of linked concept pairs; the number of linking phrases used; the number of concepts used or omitted from the 28 concepts given in the task; and the extra concepts or terms learners included apart from those given in the task.

Category 1: Omission of concepts

It was found that out of 34 learners, 27 learners omitted some of the concepts given. Only seven learners from the samples managed to use all the concepts given. Out of this category, it was found that some sub-categories also existed as groups where the omission of concepts was quantitatively measured and grouped into the following six groups:

Group 1: Those with 0-5 omitted concepts

Group 2: Those with 6 -10 omitted concepts

Group 3: Those with 11-15 omitted concepts

Group 4: Those with 16 – 20 omitted concepts

Group 5: Those with 21 – 25 omitted concepts

Group 6: Those with 26 – 30 omitted concepts

The learners in each group are classified in **Table 1**.

From the table, it can be seen that 17 learners omitted at least between zero and five concepts. It indicated that out of the 34 learners, 17 were able to use almost all the concepts given by working. It can then be said that half the sample used almost all the concepts. The table further points out that only few learners were unable to use more concepts, five learners. They used between 10 -20 concepts. It can, therefore, be assumed that they might have not understood how to relate the remaining concepts; or they could not locate the relationships on their maps.

Category 2: Omission of linking words (OLW)

Out of the 34 learners, it was found that 15 omitted the linking words.

As indicated above, during the reanalysis of the data, it was found that some learners' concept maps did not have a common threat that could be classified under a specific theme identified initially. Out of the three themes identified initially, it was found that the fourth theme, which we called Miscellaneous emerged. Miscellaneous in this study means the data that do not have a common threat to other themes and categories. This was because some data findings of each learner's map fell in some of the themes and categories while others did not. The other reason was to avoid repetition of using one concept map twice in each theme. Therefore, much of the findings in this article would come from this theme.

MISCELLANEOUS

The theme Miscellaneous contains three more categories: correct linking words with one hierarchy structure; omission of concepts and other linking words; irrelevant linking words with one hierarchy structure; all concepts used and no omission of linking words**;** correct linking words with two hierarchy structures; concepts used and no omissions. In this section, we report the data of two learners, L4, who correctly linked the words with one hierarchy structure, omission of concepts and other linking words; and L6 who correctly linked the words with two hierarchy structures, concepts and used no omission. Due to the length of the paper, we decided to exclude the third category: irrelevant linking words with one hierarchy structure, all concepts used and no omission of linking words.

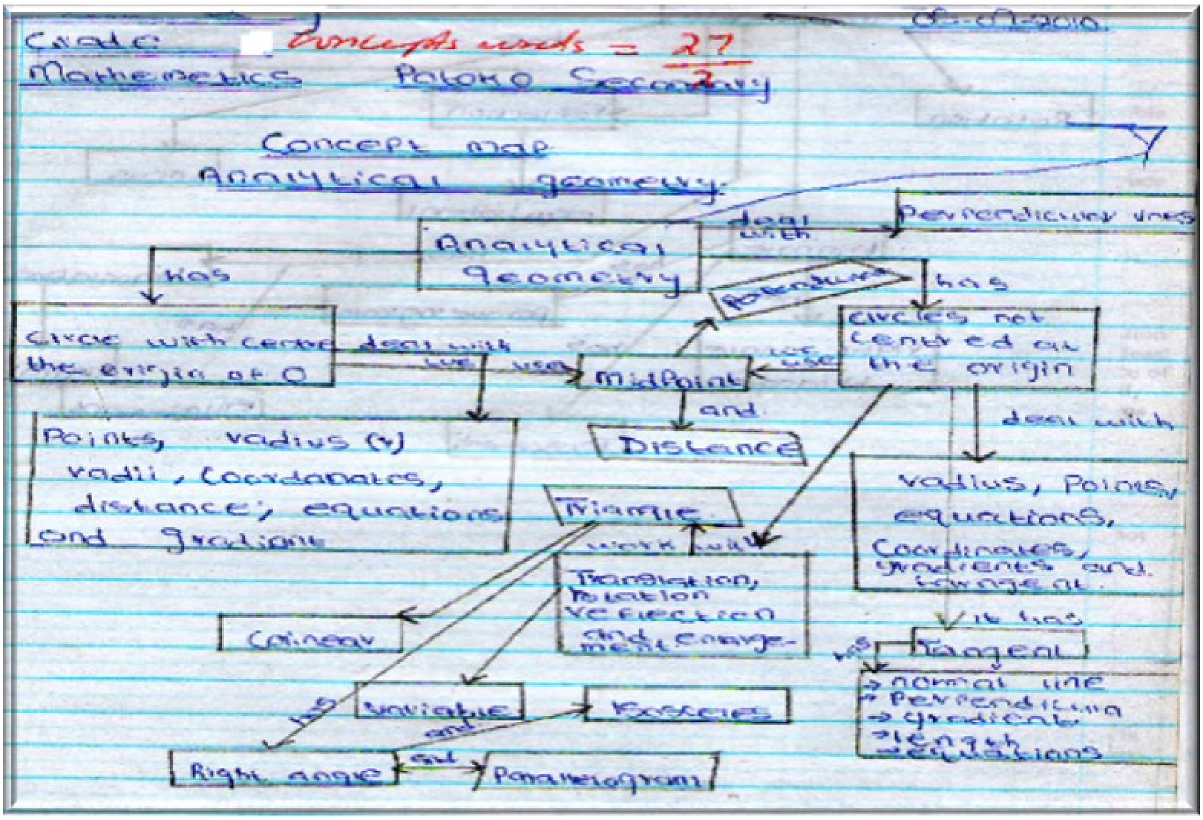


Figure 1. Correct linking words with two hierarchy structure, omission of concepts and other linking words

Correct Linking Words with One Hierarchy Structure, Omission of Concepts and other Linking Words

In this review section each of the learner's concept maps had been directly reviewed to see the layout. It was found that even though most learners were not familiar with the drawing of the concept maps, they could understand that the map should follow a hierarchical structure and they drew their maps hierarchically. **Figures 1** and **2** are the excerpts that are taken from L4 who correctly linked the words with one hierarchy structure, omitted concepts and other linking words.

Although the learner used two different maps, as depicted in **Figures 1** and **2**, the structure layout of the map followed the hierarchy. The most important items were placed on the top of the concept map with the less important ones below. The problem identified with the two maps, as indicated here, is that there was some missing linking words. The maps have some omissions.

It is seen in **Figure 1** from the concept triangle to collinear, there is just a line without linking word(s). Again, from the concept: circle not centred at the origin it was linked to the concept: translation, rotation, reflection and enlargement without any description words or linking words. At the same time, the learner used irrelevant linking words. Circle not centred at origin deals with radius, points, equations, coordinate gradients and tangents. When one looks directly at the two concepts, as depicted by the learner, the phrase 'deal with' was just used to try to mention that when determining the equation of a circle which is not centred at the origin, we still use the points, the radius to get that equation. The learner was not able to link the given concept and used a relevant word or preposition to explain that.

On the second concept map, attached as **Figure 2**, the learner managed to draw a hierarchical structure in terms of layout. However, when looking at the map thoroughly, it can be seen that, there is still a problem of omission. From the concept to "rotation" and to "triangle", nothing was said about the line connecting them; and from the concept "translation" to "variable", there is just a line without any explanation or a descriptor word. Further, it was found that on the same concept map the concept of variable was joined with the concept, "enlargement" without a line or any linking words, even though the learners had been told about the importance of the linking words and lines during the introduction session of concept map.

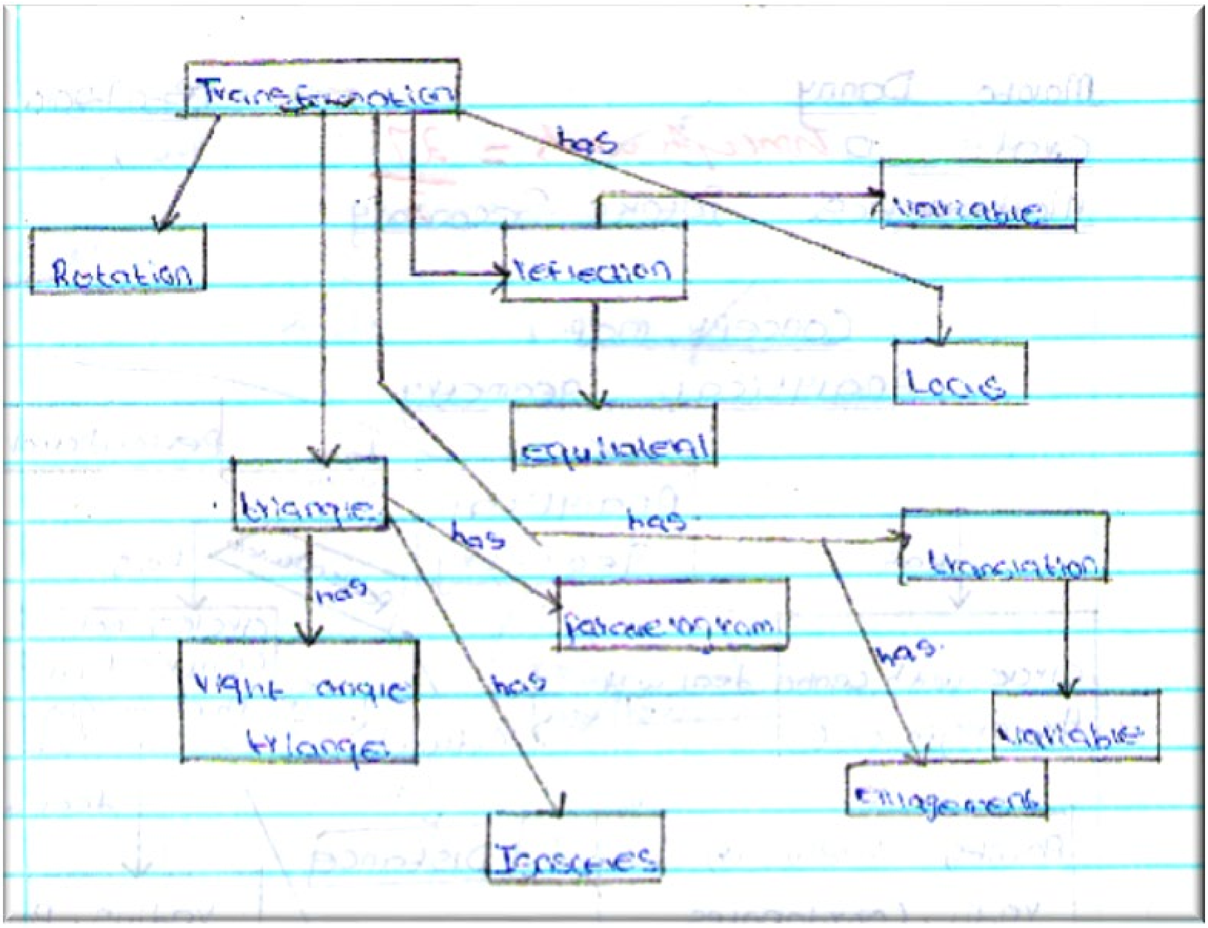


Figure 2. Correct linking words with two hierarchy structure, omission of concepts and other linking words

The direct understanding from the learner's map is that the two topics are not related at all. Nevertheless, when interviewed, L4 seemed to have misunderstood the question (investigative task). He said, he thought that it "needs two separate maps". Here it is how the learner responded when asked:

> JF: Out of twenty-nine concepts, you used only twenty-seven, right, but then I am seeing two Concepts Maps.
>
> L4: Yes Mnr, to have this two Concepts Maps is because as we look there (pointing on the investigative task), there is analytical geometry and on the question paper, I saw as if we are, can separate this two concepts.
>
> JF: Let's go here, your Concept Maps should show and indicate how the Mathematics topics from analytical geometry and transformation geometry are related. This topic must be shown how they are related.
>
> L4: I was not aware of that.
>
> JF: Let us suppose you are now aware, how can you join them, how can you fit transformation into this one?
>
> L4: You can take this, let us separate this (papers). According to my understanding, transformation will join here (pointing on Analytical Geometry) on Analytical.

This conversation shows that L4 did not understand that the two topics are interrelated. L4 thought that every topic in mathematics is treated in isolation. When asked, if he had to show the relationship of transformation geometry with analytical geometry, the learner joined transformation geometry to analytical geometry to show where transformation geometry could fit on the concept map. The explanation or description indicated that the learner had a conceptual understanding of the two concepts. On the other hand, when looking at the concept that the learner used, they seem to be more sensible. The linking words make sense and seem to be correct. During the initial drawing of the concept map, the learner drew two different maps indicating the primitive knowledge of the learner, but the learner was able to connect the two main concepts "coordinate" and "transformation geometry" during the interview. Therefore, the explanation indicates the point where a learner conceptually understands the concepts. Furthermore, this learner was able to manipulate or combine aspects of images to construct context-specific and relevant properties.

Correct Linking Words with Two Hierarchy Structures, Concepts used and no Omission

During the direct basic counting of the concepts used by L6, it was found that 30 concepts were used instead of the given 29 concepts. There was no omission of concepts and L6 managed to give an extra concept. The counting was purposed to identify the number of linking concept pairs used; the number of linking words used; the number of concepts used or omitted from the 29 given concepts; and which concepts and how many extra concepts or terms the learner included apart from those given in the task. As Mwakapenda and Adler (2003) states that in many concept mapping studies, the analysis is mainly based on numbers (quantitative) and then followed by marking on numerous features of the learners' maps. The following features are looked at: the presence of hierarchical levels, propositions; links and cross-links and specific examples provided. This is to illustrate links; and while the marks may indicate the extent to which a learner is able to link given concepts in a subject. In this map, it was checked whether the learner picked up certain concepts to be central in developing links among the concepts among the given concepts; hence, the identification of the two central concepts. The meaning the L6 attached to the concepts was checked.

What could be deduced from L6's concept map? The learner used 30 concepts out of the 29 concepts given. This means that this learner added one extra concept. The concept that was added is "rotation symmetry". From the concept "rotation", we get a linking word joining or rather linking to "rotation symmetry".

Linking Words

Many different linking words were used in the concept map, marked as **Figure 3**. From these different linking words used by L6, it is established that L6's concept map displays a self-construction of new idea, which is supported by the learning theory of Constructivism. The theory of Constructivism emphasises the learner as the influencing subject of learning, not the teacher. Therefore, it is evident that from learning theory perspective, the learning outcomes did not depend on what the teacher presented to the learner, but rather the interactive results of the information encountered and how the learner processed it. This concept map in which the learner has put together thoughts, interpretations and explanations that are personal constructs for him seems to be the best of them all. According to the Constructivist point of view, knowing means being able to do something with information (Yager, 1995), the concept map clearly shows that the learner had done something and hence there is acquired knowledge.

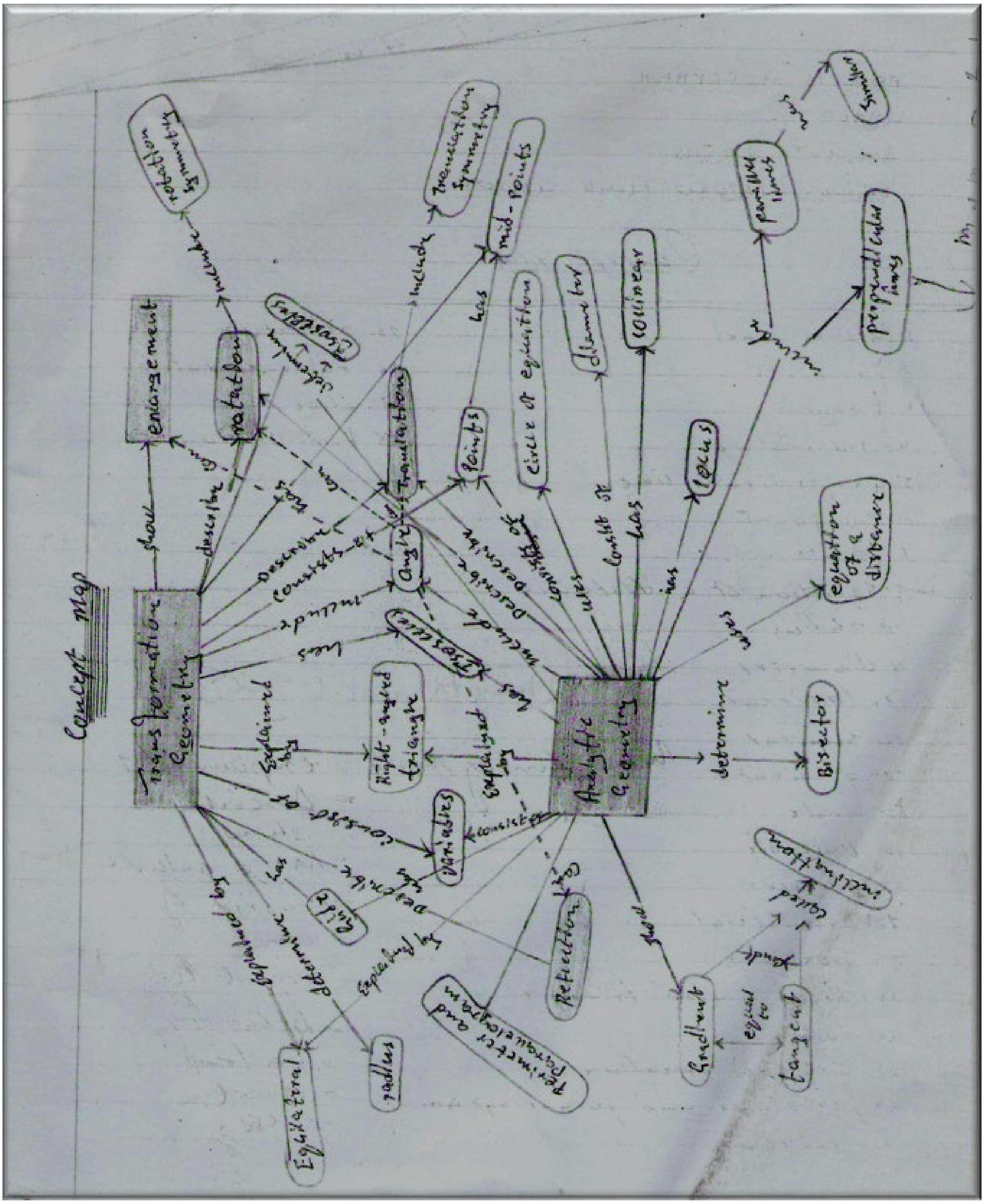


Figure 3. Correct linking words with concepts used and no omission

This concept map is different from those of the other learners who mostly used similar types of linking words. The linking words used in this map are "explained by, determine, has, can, shows, equal to, uses and, called, consist of, includes and describe". For example, "analytical geometry" uses "equation of a distance". "Translation" describes "transformation geometry". Both analytical and transformation geometry consist of variables. When interviewed to find out the meaning and understanding the learner's response was as follows:

JF: Let us look on your map; you used thirty (30) words, one extra concept from the 29 given concepts. Which extra concept did you include in this case?

L6: I have included the concept rotation symmetry.

JF: Why rotation symmetry?

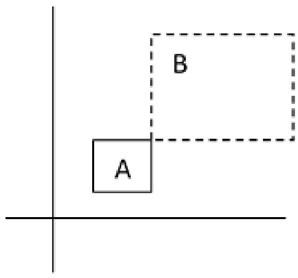
L6: Eh, kera gore di a swana. (meaning: They are the same)

JF: Transformation geometry shows enlargement, what do you mean?

L6: I mean, eh, when we have points, we can make a triangle in analytical geometry. Moreover, in that triangle we can transform it by rotating.

JF: Translation includes translation symmetry explain here.

L6: The object remains the same but it is bigger. With symmetry, it will still look like the smaller object for example.



JF: I am seeing here both analytical geometry and transformation geometry consist of points. Can you explain here?

L6: In Analytical Geometry we use points to draw the figures like ΔPQR where P (2, 4), Q (2, 2) and R (-three,-1). This triangle can be transformed using the transformation rules like enlargement, rotation, reflection and glide

JF: Analytical Geometry determines bisector. What is a bisector?

L6: Eish, Mnr, it means, it divides into two equal parts.

JF: Analytical Geometry uses circle of equation. Can you take me through here?

L6: I mean in analytical geometry we use equation of a circle, like when we are given two points for x and y we can use the formula x2+y2= r2 to get the get the equation of the circle. Moreover, we use the points in analytical geometry.

JF: Should we say Analytical geometry uses circle of equation or uses equation of a circle. What are the differences between these two statements?

L6: Ah, I wanted to say it uses equation or formula to get the equation of a circle.

From the way in which the concept map was drawn, it emerged that the L6 was able to use both transformation geometry and analytical geometry as the central point; and the concept map had been weaved so that someone could see that the concepts are interrelated. This concept map portrays four main intellectual processes: assumption, open-minded variation, integrative understanding and super ordinate learning which Kharatmal (2009) identified. According to Kharatmal (2009), these four cognitive processes help to build a knowledge framework during classroom learning. A concept map is also considered to dwell on issues of meaning-making, understanding conceptual change and knowledge structure and emphasising on quality of knowledge (Kharatmal, 2009).

On the other hand, Novak (1998) says these are the highlights of the prevalent human constructivist era. When looking at the explanations, the learner gave during the interview, it seems that the learner was able to construct meaningful learning; this is because the learner was able to even extend the concepts given, explained and described to him. The learners were trained to draw a hierarchical concept map, but this learner went beyond that in that he indicated much more understanding by weaving the concepts so that no one could classify the concepts under a special main concept. Therefore, the point of weaving the concepts, of course, indicated to us that the learner was creative and constructive. The analysis indicates a bit of fluency on the perceived connection of the learner. This indicates that this learner has developed a specific way of expressing mathematics conceptual understanding that seemed to have developed during their studying of coordinate and transformation geometry. In addition, the way in which the map was drawn and explained shows the characteristics of conceptual understanding, as defined by Kharatmal (2009). This learner further shows another form of knowledge which is called strategic knowledge by Shulman, (1986). This is because the form of knowledge the learner indicated conflicted with known facts of hierarchical structure and that representing given concept only.

From the findings, it emerged that L6's understanding of coordinate and transformation geometry concepts presented here seemed to be very diverse and indicates that the investigative questions helped the learner more to explore the topic. This result suggests that solving difficult and contextual problems through using concept maps to illustrate and exhibit learners' understanding of mathematics could be a powerful tool.

CONCLUSION

In this study, we answered research questions: What are the learners' conceptual understanding of coordinate and transformation geometry? How do concept maps improve a learner's understanding of coordinates and transformation geometry? An investigative task was given to the learners to guide in constructing a concept map, using concepts generated by the teacher. Thirty-four concept maps were drawn. We analysed and reviewed all the concept maps of 34 learners for similarity and differences in their understanding. For this paper we provided general descriptions of the findings of the 34 participants. We, then focused on learner L4 who correctly linked words with one hierarchy structure, omitted concepts and other linking words and L6 who correctly linked words with two-hierarchy structure, concepts and used no omission. Generally, the following gaps emerged from the study: some learners used double concept maps; some did not include linking words; some used irrelevant linking words and some omitted some given concepts.

When some basic counting of the concepts which had been used (out of the 29 given concepts) was done, it was found that most learners were unable to use all the given concepts. Out of the six representative learners, only two learners were able to use 29 concepts. This indicated that most learners were unable to connect all the concepts given. The direct meaning of this is that the learners could not find the relationship between the used and the omitted concepts. If a relationship existed, they were not able to notice such a relationship. The other thing that emerged from the omission was that the learners did not understand the concepts omitted. By leaving out some of the concepts, it meant they could not join or locate them next to the other concepts. That in itself indicated a missing of a concept in learners' understanding. This meant that the learners lack conceptual understanding of coordinate and transformation geometry.

The identified gaps guided follow-up interviews specifically to search for the meaning of the different linking words they used. Although the data from most learners is not sufficient to support our answer to the second research question: "How do concept maps improve learners' understanding of coordinate and transformation geometry?", the data from L4, specifically when probed more during the interview, could link words with two hierarchy structure correctly , omission of concepts and other linking words. L6 was also impressive in that the leaner could correctly link the words with a two-hierarchy structure, concepts and used no omission. The interviews were also intended to find out extra information that could have been missed in their constructions. In this study, reflective interviews were used to gain more information on the concept map from learners. Interviews helped the researchers to get enough and relevant information on learners' understanding of coordinate and transformation geometry why they connected concepts in the way they did. It also helped to understand how the learners explained and described the relationships of those concepts. We therefore, suggest that, although concept mapping is a very useful strategy, it needs to be used with an intensive probing to give learner responses to their way of doing things and get the explanation or description of their maps.

The mapping strategy facilitated 'self-education' and 'self-discipline'. By requiring of the learners to draw the concept map as a project with an investigative task, they were forced to critically analyse the interrelationships of concepts which resulted in improved conceptual understanding of the concept coordinate and transformation geometry. In this study, concept mapping has been shown as an alternative teaching strategy to teach coordinates and transformation geometry.

The study suggests that the concept map strategy can play a role in developing learning (Mwakapenda & Adler, 2003). It also shows that a learner could develop a conceptual understanding of any topics in mathematics if the learner builds new knowledge from existing knowledge. In other words for learning to take place in mathematics, there must be a connection of previous mathematical ideas with new mathematics concepts.

In answering the focus question, we will say that by giving learners a list of terms and an investigative task with a follow-up interview task helped exploring learners' conceptual understanding of coordinate and transformation geometry. The explanations and views these learners have are easily revealed to be seen. It furthermore helps the learners to summarise what is too large into a short and meaningful way. The concept map strategy further helped the learners to practise and revise to identify the parts they are missing or that they understood concepts they had never understood before.

The study recommends other studies to try to conduct studies into learners from different schools and use a larger sample and to try to use this strategy to explore other mathematical concepts. This study was a case study taken on a smaller scope and on the same positioned learners. For future studies, it is recommended that different learner backgrounds be taken into consideration.

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APPENDIX – INVESTIGATIVE TASK

Mathematics Grade 12

The following questions will guide you to construct a "concept map" to show how the concepts of Analytical (Coordinate) and Transformation Geometry are related.

- Construct a Concept Map about the topic analytical and transformation geometry.
- Your concept mapping should indicate how the mathematics topics from the analytical geometry and transformation geometry are related.
- The following concepts should be used in your concept-mapping: parallel lines, functions, tangent, inclination, perpendicular lines, midpoint, points, equation of a distance, radius, diameter, Collinear, similar, gradient, angle, variable, bisector, Equilateral, Isosceles, Scalene, Right angled triangle, locus, circle equation, perimeter and parallelogram, rotation, reflection, enlargement, glide, translation. Congruency, compound angles, 900 clockwise, 900 anticlockwise, 1800 y-axis, x-axis, the line y=x and the line y= -x.
- Insert linking words or propositions.
- No topic should be left out in your concept map.
- The following set of questions will help you to draw the concept map. This means that your concept map will probably be designed mainly from the questions. You will need to address the questions in your concept map

You will submit the following during the submission date

- The answers to the questions
- The concept map constructed

The following criteria will be used to mark your concept map:

- Memorandum for the questions 40 marks
- A rubric for your concept map 60 Marks

N.B: Some of the questions require of you to explain the process. When explaining a process, you need to give an example so that the one who listens to the process should understand your explanation. This is only done when answering the questions.

Investigative question

1. What is a distance between two points?
2. How do we calculate the distance between two points?
3. What is a midpoint?
4. How to calculate the midpoint of two or more points?
5. What is a perimeter?
6. How does one determine a perimeter of
   6.1. a triangle and when the triangles are collinear?
   6.2. a circle ?
   6.3. a rectangle ?
7. When are lines parallel and what can you say about their gradients?
   7.1 How do we calculate the gradient of a line?
   7.2 How do we calculate the inclination of a straight line?
   7.3 Pretty wants to determine the equation of a straight line. Explain how to do this.
8. When are lines parallel and what can you say about their gradients?
9. What difference is there between parallel lines and perpendicular lines, if any? How do you compare or identify if lines are parallel or perpendicular?
10. How does one calculate the equation of a circle with centre at the origin (0,0)? What are the things needed in this type of problem?
11. How does one calculate the equation of a circle with the centre (a, b) and radius r? How does one compare the process of getting the equations of the circles?
12. What is a locus of a point?
13. If Winnie wants to calculate the equation of a locus under given conditions, how does one explain the process of getting the equation? Explain it.
14. Sometimes a line that touches a circle once may be seen on the circle.
    14.1 What do they call this line?
    14.2 How do we find the equation that defines this line?