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Exploring preservice teachers' PCK on fractions: Insights from the fraction-as-operator sub-construct

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Abstract

Teachers' understanding of fractions—especially the fraction-as-operator interpretation—is essential for students' learning of rational numbers, yet research shows persistent weaknesses in this area. This study presents an empirical analysis of Spanish preservice primary teachers' pedagogical content knowledge (PCK) regarding the fraction-as-operator, using a content representation aligned with the mathematical knowledge for teaching framework. A total of 263 third- and fourth-year students of different specialization completed six fraction-focused items. Through directed content analysis, 53 indicators linked to PCK subdomains were generated. The results show low PCK, with limited identification of teaching and learning difficulties. The operator interpretation appeared in fewer than 15% of responses, despite being explicitly included in the task. Differences between academic years and specializations were minimal, indicating limited PCK development. These findings point to program-level challenges in initial teacher education within a European generalist-teacher model and highlight the need to strengthen opportunities for developing deeper, more diagnostically oriented PCK.

Keywords: pedagogical content knowledge, fractions, fraction-as-operator, preservice teachers, teacher education, mathematical knowledge for teaching

INTRODUCTION

Fractions constitute a vital yet consistently difficult area in elementary mathematics. In many countries and curricula, students struggle to build conceptual understanding, often confusing fractions with whole number operations and overlooking their multiple interpretations and uses (Kieren, 2020; Olanoff et al., 2014). Although these difficulties have been documented for decades, recent empirical research confirms that they persist in contemporary educational contexts, both among primary school students and preservice teachers (Pramudiani & Dolk, 2025; Llinares et al., 2025). International assessments show students worldwide perform far worse on fraction than on whole number items, indicating fractions remain a conceptual boundary where teaching often fails (Ismail et al., 2024).

However, this difficulty should not be interpreted as inherent in the sense of being less learnable than other mathematical concepts. Rather, unlike integers, fractions involve multiple interconnected meanings (part-whole, measure, quotient, ratio, and operator) that pose substantial epistemological and didactical challenges when not explicitly addressed in instruction. Teachers' knowledge and practices determine whether students build deep, connected understanding or only meaning-less procedures. Meta-analyses and longitudinal studies show that teachers with stronger pedagogical content knowledge (PCK)—especially as measured by the mathematical knowledge for teaching (MKT) framework—produce greater student gains in fractions and rational-number learning (Hill et al., 2005; Kelcey et al., 2019). A one-standard deviation rise in teachers' MKT scores corresponds to student gains equal to two to three extra weeks of instruction—an effect comparable to socioeconomic status (Hill & Ball, 2009; Hill et al., 2005). This highlights a key point: improving fraction

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Contribution to the literature

- This study provides the first empirical analysis of preservice primary teachers' PCK on the fraction-as-operator sub-construct, a rarely examined yet crucial area in rational-number learning.
- Using a content representation (CoRe) aligned with the MKT framework, it identifies key conceptual and pedagogical gaps behind teachers' limited attention to operator meanings.
- The findings reveal structural weaknesses in initial teacher education and help explain the minimal progression observed across academic years and specializations.

learning requires strengthening teachers' knowledge and preparation.

Many teacher-education programs fail to prepare future teachers adequately for teaching fractions. In generalist primary-teacher programs—common in Europe and North America—preservice teachers often receive limited training in mathematics and math pedagogy. They may complete their programs with procedural fluency in fraction algorithms yet lack the deep knowledge needed to detect student errors, address misconceptions, or link fractional ideas to related areas such as proportional reasoning or algebra (Copur-Gençturk, 2015; Vallespín, 2024). This gap between procedural skill and conceptual-pedagogical knowledge (PK) is well documented and constitutes a major weakness in initial teacher education (Li & Kulm, 2008; Olanoff et al., 2014; Tirosh, 2000).

This study addresses this gap by examining preservice teachers' PCK on fractions, with explicit focus on the fraction-as-operator sub-construct. The context is a Spanish public university offering a four-year bachelor's degree in primary education. Spain operates within the European higher education area, where degree structures are nationally regulated (by the Ministry of Education and ANECA) and comparable to those of other countries in the region. Analyzing initial teacher education in this context provides insights relevant to systems with similar structures. Moreover, the generalist primary-teacher model—requiring future teachers to teach all subjects, including mathematics, without prior specialization—is common in many European countries (Blömeke & Delaney, 2012; Vergara & Cofré, 2014), making the findings applicable to similar programs elsewhere.

This study addresses three research questions (RQs):

- **RQ1.** Overall level: What level of PCK on the fraction-as- operator sub-construct do preservice teachers display?
- **RQ2.** Group differences: How does this PCK vary by academic year (3rd vs. 4th) and by degree specialization?
- **RQ3.** Component profile: Which CoRe components (learning objectives, educational relevance, learning and teaching difficulties, teaching strategies, and assessment) show the greatest strengths and weaknesses?

By documenting preservice teachers' strengths and weaknesses in PCK on this critical yet understudied sub-construct, the study provides an empirical basis for understanding the state of preparation in this area. The findings highlight the need for programs to explicitly address the fraction-as-operator sub-construct and the pedagogical dimensions of its teaching. Thus, the study expands knowledge on preservice mathematics-teacher education and offers guidance for future research and program development.

THEORETICAL FRAMEWORK

PCK in Mathematics Teacher Education

Shulman (1986), a pioneer in theorizing teachers' professional knowledge, defined PCK as a distinct category arising at the intersection of content knowledge (CK) and general PK. PCK enables teachers to transform disciplinary ideas into forms learners can understand, distinguishing educators from subject-matter specialists. Since its introduction, PCK has become a major research focus across subjects—including physics, science, and mathematics (Sakaria et al., 2023; Star, 2023)—yet conceptual ambiguity remains regarding its boundaries with CK and the relationships among PCK subdomains (Eraut, 1994; Mientus et al., 2022). Recent bibliometric and systematic reviews confirm that PCK remains a central and evolving construct in contemporary mathematics teacher education research, with sustained international attention to its conceptualization and development in preservice contexts (Asvat, 2024; Fukaya, 2024).

To operationalize PCK for research and practice, Ball et al. (2008) developed the MKT framework. Rather than resolving theoretical ambiguities, the framework offers a structured model distinguishing subject matter knowledge (SMK) from PCK and specifying practice-based subdomains (see **Figure 1**). This operational structure has been highly productive for research and professional development despite ongoing debates about domain boundaries. Although alternative frameworks exist (e.g., COACTIV and TEDS-M), MKT terminology remains widely used in mathematics-education research and is adopted in this study.

Recent research shows that PCK develops through multiple, interacting learning opportunities. Conceptual models describe it as a trajectory shaped by university

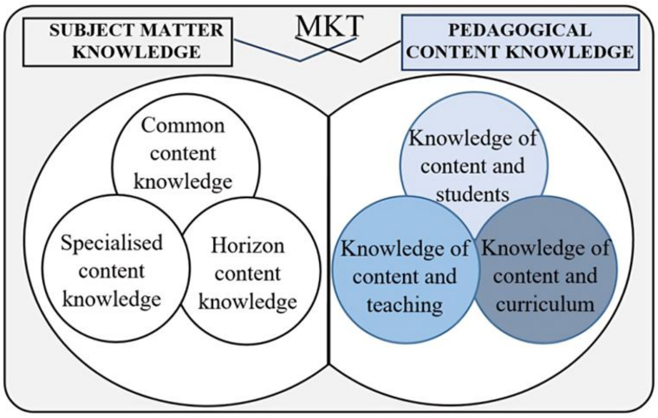

Figure 1. Representation of the MKT framework based on Ball et al. (2008, p. 403)

coursework, supervised practicums, collaborative reflection, and repeated classroom enactment (Gess-Newsome, 2015). Longitudinal interventions show that tools such as the CoRe instrument, peer-coaching cycles, and structured observations can accelerate preservice teachers' progress, especially in knowledge of students and instructional strategies (Ekiz-Kiran et al., 2021). Qualitative studies with novice teachers indicate that such progress depends on continuous cycles of planning, teaching, and reflection linking university learning with real classroom challenges (Loughran et al., 2012). Together, prior research suggests that variations in learning opportunities—such as practicum or advanced methods coursework—are associated with differential development of teachers' PCK (Blömeke & Delaney, 2012; Dragnić-Cindrić & Anderson, 2025; Kleickmann et al., 2013). However, whether such differences systematically emerge in generalist primary teacher-education programs remains an open empirical question that this study examines.

Despite ongoing refinements regarding interconnections among PCK categories (Eraut, 1994; Mientus et al., 2022), studies agree on a key point: higher teacher PCK consistently correlates with stronger student learning outcomes (Copur-Gencturk, 2015). This robust finding has driven the development of multiple frameworks defining the knowledge needed for effective teaching, examined next with specific focus on mathematics (Depaepe et al., 2015).

MKT and Alternative Frameworks

Three widely used and representative international programs currently guide efforts to conceptualize and measure mathematics teachers' professional knowledge, including the MKT framework (Ball et al., 2008), the COACTIV project (Kunter et al., 2013), and TEDS-M (Blömeke & Delaney, 2012). Although all have significantly advanced the field, they differ notably in scope, theoretical foundations, and applicability to preservice contexts.

The MKT framework, developed in the United States, refined Shulman's CK-PCK distinction through practice-oriented subdomains capturing the complexity of mathematics teaching. Ball and colleagues built the first large-scale item bank—the learning mathematics for teaching (LMT) assessments—to evaluate the specialized knowledge needed for effective instruction. Developed and validated with item response theory, these instruments show strong reliability and predictive validity. Although initially created for in-service teachers, MKT has been effectively adapted to preservice contexts, proving useful for fine-grained analyses of developing knowledge in domains such as fractions and proportional reasoning (Sin, 2021).

The COACTIV project, conducted in Germany, broadened MKT by integrating video-based classroom analyses and teacher-belief inventories with knowledge assessments. Its findings showed that strong SMK does not guarantee high-quality teaching without solid PCK and productive instructional beliefs (Kleickmann et al., 2013; Kunter et al., 2013). This framework effectively documented the interplay among knowledge, beliefs, and practice, though its complexity has limited its wider adoption.

The TEDS-M study, an international comparative project involving over 22,000 preservice teachers in 17 countries, assessed both mathematics content knowledge and mathematics pedagogical content knowledge using psychometrically validated instruments. It revealed substantial international variation in knowledge outcomes and showed that program-level factors shape teacher-knowledge development during initial preparation (Blömeke & Delaney, 2012). Although it did not directly measure student learning, TEDS-M provided crucial systemic insight into the institutional conditions influencing teacher learning across diverse contexts.

For this study, MKT provides the strongest analytical leverage. Unlike COACTIV, which highlights belief-practice interactions, or TEDS-M, which maps program-level influences without linking knowledge to student outcomes, MKT unites well-validated item banks, practice-oriented subdomains, and value-added evidence. Its adaptability to preservice contexts has made it the preferred framework for topic-specific research. A bibliometric review of 725 fraction-learning papers (Ismail et al., 2024) identified a "teacher knowledge and its impact on mathematics teaching" cluster whose most frequently co-cited works are MKT landmarks along with empirical studies applying MKT to fraction multiplication (Izsák, 2008), fraction division (Lo & Luo, 2012), and cross-cultural analyses of teacher knowledge (Hill et al., 2005; Ma, 1999). This expanding literature establishes MKT as the de facto framework for international research on teachers' fraction knowledge and instruction.

Assessing Topic-Specific PCK: CoRe Instrument

Although scholars disagree on PCK's exact boundaries, most agree that curricular knowledge, knowledge of learners, and knowledge of teaching interact dynamically (Zakaryan et al., 2018). This diversity appears in the instruments used to measure PCK. Large-scale multiple-choice surveys based on the MKT/LMT item bank offer international comparability (Chick, 2012; Vergara & Cofré, 2014) but are criticized for masking the pedagogical reasoning behind teachers' choices (Depaepe et al., 2015). In contrast, narrative or performance-based tools—such as pedagogical and professional experience repertoires (PaP-eRs), lesson-plan analyses, and video-stimulated interviews—provide rich qualitative data but demand substantial resources, limiting sample sizes (Loughran et al., 2004).

The CoRe instrument, developed by Loughran et al. (2004) to capture PCK systematically, occupies a useful middle ground. Whereas PaP-eRs provide narrative accounts of pedagogical experience, CoRe structures essential content ideas, anticipated misconceptions, teaching strategies, and assessment approaches. This systematic format makes CoRe especially valuable for identifying recurring patterns in teacher preparation (Loughran et al., 2012). Kind (2009) adds that CoRe is adaptable for both research and instructional planning because it clarifies key concepts, establishes conceptual connections, and supports the design of learning activities, which makes it even more valuable in retrospective analyses.

CoRe prompts respondents to articulate learning objectives, curricular links, anticipated student errors, teaching challenges, instructional strategies, and assessment approaches for one topic. A validation study by Herreros-Torres et al. (2025) confirmed via confirmatory factor analysis that objectives load onto knowledge of content and curriculum (KCC), student difficulties onto knowledge of content and students (KCS), and the remaining prompts onto knowledge of content and teaching (KCT), reinforcing CoRe's alignment with the MKT framework used here. Its feasibility with preservice mathematics teachers is well established (Maryono et al., 2017; Suripah et al., 2021), and recent studies show that CoRe reveals nuances in fraction teaching missed by multiple-choice measures (Rafiepour et al., 2019).

Despite its potential, no published study has used CoRe to examine preservice teachers' knowledge of fractions as operators—a sub-construct requiring coordination of ratio, scaling, and transformation. CoRe was thus chosen because it

(1) elicits written reflections that reveal the operator perspective,

(2) aligns with the MKT categories guiding our analysis,

(3) serves as a formative tool suitable for large lecture groups without video equipment, and

(4) complements—rather than duplicates—the information provided by widely used LMT surveys.

The Fraction-as-Operator Sub-Construct: Didactical Challenges

Fractions present three interconnected didactic challenges that align with MKT subdomains. First, the persistent procedural-conceptual disconnect—seen in lessons emphasizing algorithms like "invert and multiply" without integrating part-whole, measure, and operator meanings—reflects weaknesses in KCT. Teachers trained to prioritize procedures over conceptual understanding (Butto, 2013; Tsai & Li, 2016) struggle to design instructional sequences that link algorithms with underlying meanings (Copur-Gençturk & Li, 2023; Stelzer et al., 2016).

Second, routinely correcting student errors—such as numerator-denominator reversals—without diagnosing their conceptual sources reflects gaps in KCS. Teachers often fail to anticipate misconceptions or use mistakes as opportunities for collective reflection and conceptual restructuring (Candray, 2021; Fernández & Roa, 2022; Parra-Sandoval et al., 2023).

Third, relying on a narrow set of representations—typically area diagrams—limits what Buforn et al. (2018) and Murniasih et al. (2020) call fraction-sense flexibility: coordinating part-whole, ratio, quotient, and operator interpretations across area, set, number line, and symbolic registers. This limitation signals weaknesses in KCC—linking concepts across developmental progressions—and constrains KCT by reducing available instructional strategies.

The fraction-as-operator sub-construct adds further complexity because interpreting expressions like $\frac{2}{3} de\ 9$ as dynamic quantity transformations requires coordinated use of all three PCK strands. Observational and assessment studies (Behr et al., 1997; López-Martín et al., 2022; Rafiepour et al., 2019) show that teachers and students default to part-whole imagery, producing systematic errors whenever tasks demand multiplicative scaling or whole reconstruction. Within Kieren's (1988) five fraction interpretations—part-whole, measure, quotient, ratio, and operator—the operator view is uniquely demanding because it treats fractions not as static quantities but as transformations. A whole split into three parts of which two are taken (part-whole) is conceptually different from the operation "multiply by two-thirds" applied to a whole (operator).

Mastery of this sub-construct requires strong KCS to anticipate operator-specific misconceptions, well-developed KCT to design tasks integrating multiplication, division, and ratio reasoning, and solid

KCC to situate operator work within coherent curricular progressions (Contreras, 2013; Ríos-Cuesta, 2021). By mapping each didactic challenge onto its corresponding PCK subdomain and using CoRe to elicit preservice teachers' topic-specific reflections, this study captures how they plan to enact their CK in teaching.

METHOD

Context and Participants

The exploratory study was conducted at a Spanish public university offering a four-year bachelor's degree in primary education. In Spain, primary teachers are not required to hold a mathematics degree; a generalist program suffices, which can lead to uneven mathematical preparation among future teachers (Copur-Gencturk, 2015; Vallespín, 2024). Therefore, the design and implementation of initial teacher education are crucial for preservice teachers to develop adequate PCK.

Program structure and fraction-related coursework

Regarding mathematics and didactics, all students complete two common foundation modules in year 1-year 2. "General didactics" provides general PK and principles of effective teaching without addressing specific mathematical content, and "mathematics for teachers" introduces the natural, integer, and rational number systems—including fractions and proportional reasoning—providing the mathematical foundations underlying the primary curriculum.

From year 3 onward, students may remain on a general track or choose a specialization. Those opting for science and mathematics (S&M) complete 24 additional credits in mathematics-education electives that other specializations do not take (e.g., history of ideas and of the S&M curriculum or didactic approaches in mathematics). Regardless of track, all students must pass two compulsory didactics modules:

The third-year module, "didactics of arithmetic and problem solving" (6 ECTS), is the only point in the program where the teaching and learning of fractions is addressed explicitly. The official syllabus requires coverage of the didactic analysis of whole numbers, fractions, decimals, ratio, proportion, and proportionality, spanning both conceptual structures and algorithmic aspects. Importantly, the module explicitly incorporates:

(1) common difficulties and errors in rational-number understanding,

(2) cognitive processes and representational decisions in rational-number learning, and

(3) the selection and use of mathematical representations, including ICT.

Students analyze teaching sequences and textbook approaches and design classroom activities, thereby working directly with PCK subdomains.

Critically, the operator meaning of fractions is not explicitly named or isolated as a distinct pedagogical focus. Although the syllabus addresses proportional reasoning, multiplicative structures, and real-world problem contexts—conceptual foundations of the operator sub-construct—these are not framed as "fraction as operator" or distinguished from other interpretations (part-whole, measure, quotient, ratio). This silence reflects a pedagogical gap: if the operator is not explicitly recognized as a distinct, conceptually demanding sub-construct, teacher educators may not systematically highlight its teaching challenges or design activities specifically targeting its development. Consequently, preservice teachers may acquire procedural fluency with proportional reasoning without recognizing the pedagogical significance of interpreting fractions as multiplicative transformations.

The fourth-year module, didactics of geometry, measurement, probability and statistics (6 ECTS), further develops PCK through analysis of student errors, representational decisions, and problem-solving approaches. However, its content is limited to these four domains; arithmetic and fractions are not revisited. Thus, students receive no formal instruction on rational numbers or on the operator meaning of fractions after year 3, leaving any conceptual or PCK gaps unaddressed during the final year.

The curriculum includes no standalone course dedicated exclusively to PCK development; instead, PCK elements are embedded within the two didactics modules. Although different lecturers may teach different groups and staffing may vary across specializations, all lecturers follow the same faculty-approved syllabus, ensuring coherent content coverage. This structure reflects an implicit model of PCK development: the assumption that PCK will emerge through situated engagement in didactic analysis and classroom-activity design rather than through explicit, scaffolder instruction in PCK frameworks or their application to specific sub-constructs.

Sample

A convenience sample of 263 preservice teachers—146 in year 3 and 117 in year 4—volunteered from eight of the twelve lecture groups running in 2021-2022. We used convenience sampling because access was limited to the groups whose lecturers agreed to collaborate. **Table 1** shows that the sample includes 34.8 % of all year 3 enrolments (N = 419) and 23.2 % of year 4 enrolments (N = 504) across four specializations. Physical education (PE) specialization appears only in the fourth-year cohort because no year 3 groups in that specialization were available during data collection. Participants' ages

Table 1. Participants by year and degree specialization

Specialization	Participants (%)	Third year	Fourth year
S&M	114	96	18
A&H	54	12	42
PE	45	-	45
SEN	50	38	12
Total	263	146	117

ranged from 20 to 25 years (mean [M] = 22.4), and 23.9 % were male. None had professional teaching experience; therefore, their responses reflect solely knowledge gained during initial training.

At the moment of testing, fourth-year students differed from third-year students in two respects:

(a) they had already completed "didactics of arithmetic and problem solving", whereas third-year students were enrolled in but had not finished the course and

(b) they had undertaken an additional 1.5-month practicum during the preceding term.

Participation was voluntary, informed consent was obtained from all participants, and the study was conducted in accordance with relevant ethical guidelines, with approval obtained from the appropriate institutional ethics committee.

Instrument

This study employed the CoRe instrument developed by Loughran et al. (2004), which invites teachers to analyze a specific disciplinary situation through open-ended questions that elicit the core PCK elements. Although the original CoRe contains eight questions, this study adopted the six-question version validated for Spanish preservice teachers by Herreros-Torres et al. (2025) (**Figure 2**), itself derived from Verdugo-Perona et al.'s (2018) earlier science-education adaptation. In that version, two questions—"What else do you know about this idea?" and "What other factors influence your teaching of this idea?"—were removed because pilot testing showed they duplicated information already captured through learning objectives and teaching difficulties, adding little to PCK analysis.

To align the instrument with this study, the focal content was fractions as taught in year 4-year 6 of Spanish primary education. This grade band was selected because Spanish national curriculum guidelines (Royal Decree 126/2014) introduce the fraction-as-operator sub-construct during these years. The curriculum specifies seven key content areas:

(1) the concept of a fraction,

(2) the interpretation of a fraction as the division of two natural numbers,

ACADEMIC YEAR: ______

SEX: MALE___ **FEMALE**_____

AGE: ______

DEGREE SPECIALISATION: ______________

STARTING SITUATION

Fractions are studied in 4th, 5th and 6th grade of Primary School. Some of the contents of this topic included in the relevant curriculum are:

- Concept of fraction as a division of natural numbers. Relationship between fractions and decimals.
- Graphic representation of proper fractions (for example, 1/2) and improper fractions (for example, 5/3).
- Meaning and utility of fractional and decimal numbers in personal and social contexts (commercial invoices, sales, taxes, etc.). Solving everyday problems involving fractions.
- Calculation of the product of a fraction by another number, either natural or fractional.

QUESTIONS:

Q1. What would you try to get students to learn about this particular situation (objectives)?

Q2. Why do you think it is important for students to learn what has been stated above (relevance of the topic or situation)?

Q3. Do you know the possible learning difficulties of children or their alternative ideas about this situation? Justify your answer.

Q4. Do you know the difficulties or limitations in the teaching about the mentioned aspects? Justify your answer.

Q5. What teaching methodology would you use to obtain greater learning from the students in the case presented? What specific activities would you propose?

Q6. How would you evaluate if the students have really achieved the objectives set at the beginning?

Figure 2. CoRe instrument for analyzing preservice teachers' PCK on fractions (Source: Authors' own elaboration)

(3) decimal representation of fractions,

(4) graphical representation of proper and improper fractions,

(5) the meaning and use of fractions in social contexts,

(6) solving everyday problems involving fractions, and

(7) calculating the product of a fraction by another number, whether natural or fractional—a content area explicitly linked to the operator interpretation.

Rather than restricting CoRe solely to the operator sub-construct, we selected content areas related to it and commonly linked to learning difficulties, allowing respondents flexibility while maintaining focus on the target domain.

As validated through confirmatory factor analysis by Author (2025), each CoRe question maps to specific PCK subdomains within the MKT framework:

- **Q1** (didactic objectives) loads on a "curriculum" factor, probing KCC, the translation of syllabus goals into lesson aims.
- **Q3** (expected learning difficulties) loads on a "student thinking" factor, probing KCS, the anticipation of misconceptions and prior conceptions.
- **Q2** and **Q4-Q6** cluster on an "instructional strategies" factor, probing KCT, the planning, enactment, and evaluation of teaching.

Variables, Indicators, and Measures

Each of the six CoRe questions defined one study variable: Q1 (instructional objectives), Q2 (educational relevance), Q3 (learning difficulties), Q4 (teaching difficulties), Q5 (teaching methodology and activities), and Q6 (assessment).

For each variable, multiple indicators were created to capture specific ideas, concepts, or pedagogical dimensions in participants' answers. In total, 53 indicators were defined (listed in **Appendix A**). These indicators were generated through expert judgment: a three-member panel iteratively reviewed participants' responses and refined the indicator set to ensure comprehensive coverage while avoiding the privileging of a single instructional approach, a common limitation of open-ended tools (Chick, 2012).

Once the indicator system was established, all responses were scored using a three-point ordinal scale: 0 for incorrect or missing conceptual content; 0.5 for correct but incomplete responses; and 1 for correct and complete responses. All participants provided substantive answers to every question, ensuring a complete dataset. **Appendix A** include examples illustrating each scoring level.

Inter-rater reliability was then assessed. The three researchers independently scored a subsample of 20 randomly selected responses (≈ 7.6% of the dataset) for each CoRe question. After three rounds of independent scoring and consensus refinement, Cohen's kappa exceeded .80 for all six variables (κ > .80, Q1-Q6), indicating substantial agreement (Landis & Koch, 1977).

Data Collection and Analysis Procedure

After obtaining permission from course lecturers, one researcher introduced the study, explained the pencil-and-paper CoRe task, and allotted 60 minutes for completion. Participation was voluntary and anonymous; only academic year, sex, age and degree specialization were recorded.

A directed content-analysis approach (Hsieh & Shannon, 2005) was used, combining deductive and inductive steps. Deductively, the six CoRe questions served as a priori categories aligned with the PCK subdomains in the MKT framework. Inductively, the three-member coding team independently reviewed responses and generated 53 indicators, later refined through iterative comparison, extending Verdugo-Perona et al.'s (2018) adaptation. A color-coding system classified ideas and assigned them to indicators within each variable. Links between questions were also examined to find whether particular ideas contributed to multiple indicators (see one example on **Figure 3**).

Following qualitative analysis, each of the six variables (Q_i) was quantified as the sum of the values of its corresponding indicators (Q_{ij}), and the resulting values were rescaled to a 0-10 scale:

$$Q_i = \frac{\sum_{j=1}^{k} Q_{ij}}{1 \cdot k} \times 10, \tag{1}$$

where k represents the number of indicators for variable Q_i, with i ranging from 1 to 6. The overall result of the CoRe instrument was obtained by averaging the scores of the six variables. This global score was also rescaled to a 0-10 range. As an example, **Table 2** illustrates the specific case of Q5, which comprises seven indicators.

Accordingly, Eq. 1 applied to Q5 is expressed as follows:

$$Q_5 = \frac{\sum_{j=1}^{7} Q_{5j}}{1 \cdot 7} \times 10. \tag{2}$$

To facilitate qualitative description, within each indicator we analyzed the frequency of participants who provided substantive information for that indicator, expressing results as percentages. For example, if 25 out of 263 participants provided relevant content for indicator Q5.1, the frequency was calculated as $\frac{25}{263} \times 100 = 9.5\%$. This approach allowed us to characterize both the quantitative depth of responses (via scores) and the breadth of participant engagement (via frequency percentages) across all indicators within each question.

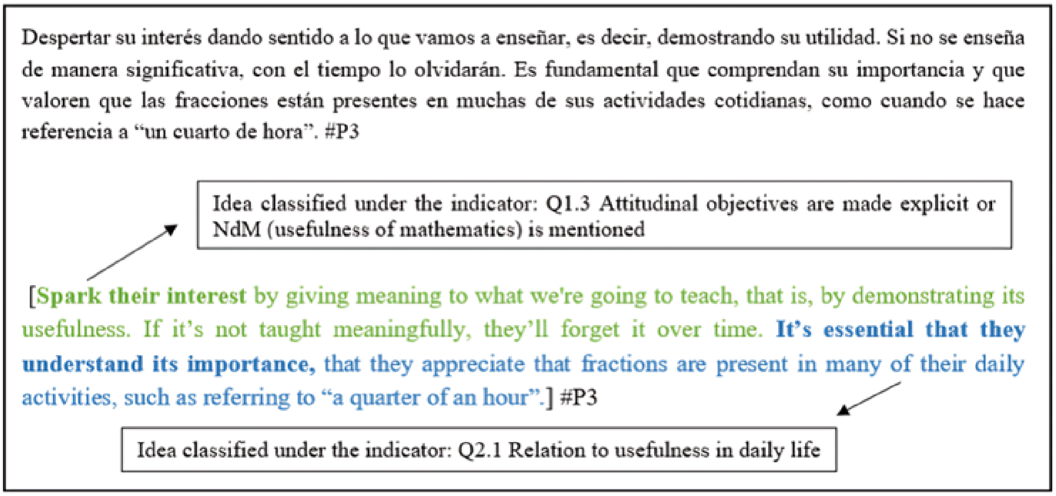

Figure 3. Example of analysis of the response produced in Q1. Didactic objectives (Source: Authors' own elaboration)

Table 2. Indicators for Q5 (teaching methodology and activities)

Variable	Indicators
Q5. Teaching methodology and activities	Q5.1 General instructional approach (active, constructivist, etc.)
	Q5.2 Arguments for or against specific methodologies
	Q5.3 Organization: roles of the teacher and students, learning environment, etc.
	Q5.4 Task types (observing, experimenting, debating)
	Q5.5 Specific activities linked to the stated learning objectives
	Q5.6 General and vague description of methodology
	Q5.7 General and vague description of activities

Following quantification, data were analyzed using the statistical software jamovi (version 2.3.16, Jamovi Project, 2022), following these procedures:

- **Inferential analysis** (Kruskal-Wallis test, 95% confidence level) to identify significant differences by academic year or degree specialization, followed by Mann-Whitney U tests with Holm correction to compare third- and fourth-year students within each specialization.
- **Quantitative analysis of variables (Q1-Q6).** Since the assumptions of normality were not met ($p < 0.05$, Shapiro-Wilk test), the median and interquartile range (IQR) were used.
- **Descriptive analysis** of response frequency by variable and indicator, expressed as percentages (noting that a single participant could provide responses for multiple indicators).
- **Descriptive analysis of response types** (complete with a score of 1 or incomplete with 0.5), presented as percentages and illustrated with examples to assess the quality of preservice teachers' PCK.

RESULTS

This section has two parts: The first part reports differences by academic year and degree specialization. The second part analyses each variable and its indicators to better understanding how preservice teachers justify their ideas about teaching and learning fractions.

Analysis by Academic Year or Degree Specialization

The Kruskal-Wallis test reveal significant differences by academic year in Q5 (KW = 12.41; $p < 0.001$), Q6 (KW = 4.07; $p = 0.044$), and in the overall score (KW = 7.84; $p = 0.003$), supporting a year-by-year analysis. Overall scores in both years indicate very low PCK on fractions: on a 0-10 scale, medians did not exceed 2.50 and IQR values ranged from 0.00 to 1.50. Looking at the median scores by year, third-year students scored highest in Q1 (instructional objectives), with a median of 2.08 (IQR = 1.25), and in Q2 (educational relevance), with a median of 2.00 (IQR = 1.00). Meanwhile, fourth-year students also performed best in Q1 (median = 2.50; IQR = 1.25) and in Q5 (teaching methodology and activities), with a median of 2.14 (IQR = 1.43). Taken together, the results show that academic year is the main source of variation, after which we examine differences by degree specialization. Results by specialization appear in **Figure 4**.

We highlight two points: First, the fourth-year group (part b in **Figure 4**) shows slightly higher median scores than the third-year group (part a in **Figure 4**) because these are independent cohorts, this reflects a cross-sectional contrast rather than actual progression. Second, the group with the highest PCK is not S&M but special educational needs (SEN), which shows a slightly

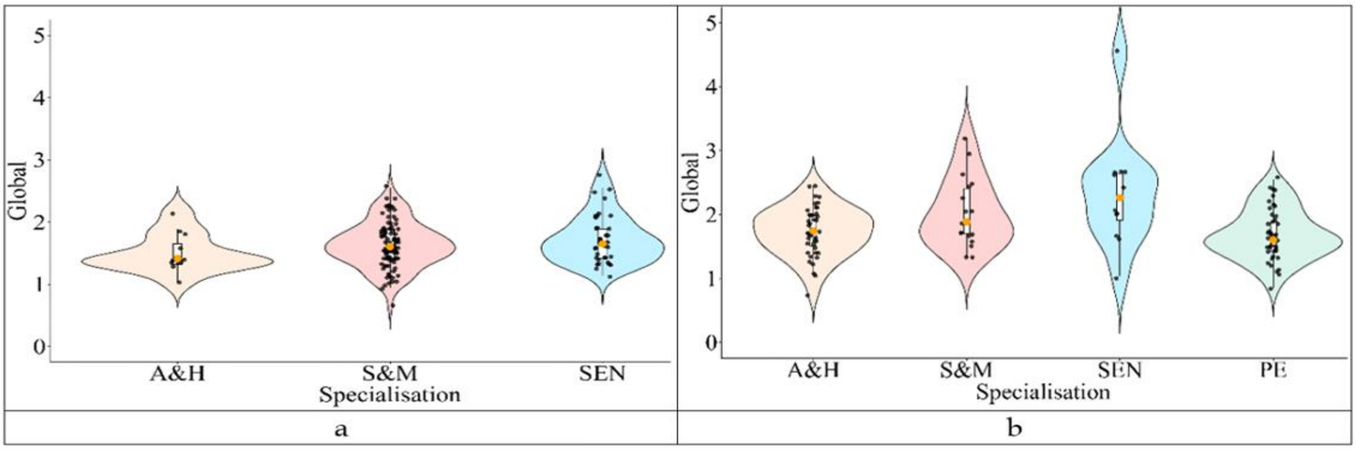

Figure 4. Differences by specialization in (a) third year & (b) fourth year (Source: Authors' own elaboration)

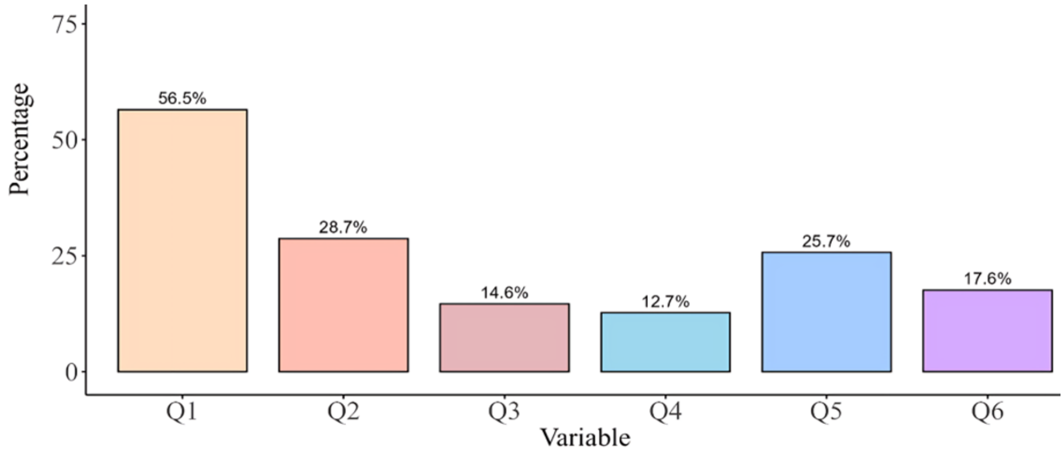

Figure 5. Percentage of responses for each variable (Source: Authors' own elaboration)

higher median than the other specializations and a wider score spread, indicating greater response variability. Examining each specialization across academic years reveals two notable patterns. In S&M, fourth-year students outperform third-year students both in the overall index (Med4th-3rd = 1.89 vs. 1.60; U = 490; pHolm = 0.021) and especially in Q5 (methodology and activities: Med4th-3rd = 3.57 vs. 1.43; U = 157; pHolm < 0.001). SEN shows a similar pattern (overall pHolm = 0.032; Q5 pHolm = 0.014), although the fourth-year subsample is small (n = 12), so results must be interpreted cautiously. No significant differences appear between years in arts and humanities (A&H) and PE cannot be tested because it occurs only in the fourth-year cohort.

Detailed Analysis of Variables and Indicators

Each CoRe question is a composite variable whose meaning depends on the indicators that make it up; thus, all Q1-Q6 results reflect the combined scores of their respective indicators. This section examines the knowledge preservice teachers display about fractions—particularly the fraction-as-operator concept—by analyzing their ideas and justifications related to this content. It also evaluates the quality of their responses, considering the clarity, strength, and coherence of their justifications.

What do they know about fractions as operators?

As noted before, the sample shows an overall lack of PCK on fractions. But the higher proportion of responses appears in instructional objectives (Q1 in **Figure 5**), with 56.5%, while no other variable exceeds 30%.

An analysis of response frequencies by indicator shows clear variation across variables, with specific indicators standing out within each CoRe component. As observed in **Table 3**, in instructional objectives (Q1), conceptual (Q1.1), procedural (Q1.2), and attitudinal goals (Q1.3) are the most frequently mentioned indicators, all exceeding 70% in both third- and fourth-year students (Q1.1: 71.9% and 76.1%; Q1.2: 76.7% and 76.1%; Q1.3: 73.3% and 76.1%). In educational relevance (Q2), references to the everyday usefulness of fractions (Q2.1) clearly dominate this variable (83.3% overall), appearing with high frequency in both courses (89.7% in third year and 71.8% in fourth year), whereas references to relationships and sequencing among contents (Q2.3) are much less frequent, and explicit lack of awareness of educational relevance (Q2.4) is marginal in both groups (below 3%).

Table 3. Response percentage for some of the most prominent indicators of each variable

Variables	Indicators	Percentage (%)		
		3rd	4th	M
Q1.Instructional objectives (56.5%)	Q1.1 Conceptual objectives	71.9	76.1	73.4
	Q1.2 Procedural objectives	76.7	76.1	76.4
	Q1.3 Attitudinal objectives/ usefulness of mathematics	73.3	76.1	74.5
Q2. Educational relevance (28.7%)	Q2.1 Relation to usefulness in daily life	89.7	71.8	83.3
	Q2.3 Relationship, precedence, and sequence among contents	25.3	34.2	29.3
	Q2.4 Lack of awareness of its educational importance	2.70	0.90	1.90
Q3. Learning difficulties (14.6%)	Q3.1 Insufficient or incorrect prior knowledge	17.8	26.5	21.7
	Q3.2 Level of abstraction of certain concepts	15.1	36.0	24.3
	Q3.3 Inability to develop procedural skills or a specific content	41.8	41.0	41.4
	Q3.6 Ignorance of the usefulness of the concepts presented	11.0	18.8	14.4
	Q3.9 Lack of knowledge about learning difficulties	13.0	11.1	12.2
Q4. Teaching difficulties (12.7%)	Q4.1 Making abstract or difficult concepts understandable	24.7	16.2	20.2
	Q4.2 Overcoming low prior knowledge or misconceptions	13.0	6.80	10.3
	Q4.6 Using the history of mathematics effectively	9.0	22.2	14.8
	Q4.7 Lack of resources, places and/or materials	15.1	23.1	18.6
	Q4.9 Lack of teacher preparation/skills/type of methodology	37.7	31.6	35.0
Q5. Teaching methodology and activities (25.7%)	Q5.2 Arguments for/against certain methodologies	48.6	70.1	58.2
	Q5.5 Task types (observe, experiment, discuss, etc.)	50.7	57.3	53.6
	Q5.6 Specific activities linked to objectives for this topic	32.2	23.1	28.1
Q6. Assessment (17.6%)	Q6.1 General assessment approach	29.5	29.1	29.3
	Q6.3 Ways of assessing, techniques and tools	48.6	58.1	52.9
	Q6.4 Justification and reasoning about assessment	11.6	13.7	12.6
	Q6.5 Specific indicators or aspects to assess prior knowledge, etc.	6.20	19.7	12.2

In learning difficulties (Q3), difficulties related to procedural skills or to specific content (Q3.3) constitute the most salient indicator within this variable (41.4% overall), with similar frequencies in third- and fourth-year students (41.8% and 41.0%, respectively). Other indicators, such as insufficient or incorrect prior knowledge (Q3.1) and the level of abstraction of certain concepts (Q3.2), appear less frequently, although they are more visible among fourth-year students (26.5% and 36.0%) than among third-year students (17.8% and 15.1%). Mentions of ignorance of the usefulness of the concepts presented (Q3.6) and lack of awareness of learning difficulties (Q3.9) remain comparatively low in both courses.

Regarding teaching difficulties (Q4), lack of teacher preparation or methodological skills (Q4.9) stands out as the most frequently mentioned indicator (35.0% overall), followed by difficulties related to making abstract content understandable (Q4.1) and lack of resources or materials (Q4.7), which appear with moderate frequencies in both academic years. Indicators such as the use of the history of mathematics (Q4.6) or overcoming low prior knowledge or misconceptions (Q4.2) are mentioned less frequently and show greater variability between courses.

Finally, in teaching methodology and activities (Q5), arguments for or against specific teaching methodologies (Q5.2) and references to task types (Q5.5) are the most prominent indicators, together accounting for over half of the responses in this variable. While both indicators appear in the two courses, references to methodological arguments (Q5.2) are more frequent among fourth-year students (70.1%) than among third-year students (48.6%), whereas references to specific activities linked to instructional objectives (Q5.6) are less frequent overall and appear more often in third year (32.2%) than in fourth year (23.1%). In assessment (Q6), ways of assessing techniques and tools (Q6.3) clearly dominate the variable (52.9% overall), while references to specific assessment indicators (Q6.5) and to justification or reasoning about assessment (Q6.4) remain comparatively scarce in both academic years.

These findings show that preservice teachers give very limited attention to the fraction-as-operator meaning when developing their PCK on fractions. Although the initial prompt explicitly included the content "calculating the product of a fraction by a number, whole or fractional," students mentioned this idea only in Q1 (instructional objectives) and at very low rates. Specifically, references appeared in the conceptual-objectives indicator (Q1.1B) for 1.7% of responses (all from fourth-year students) and in the procedural-objectives indicator (Q1.2B) for 8.2% of third-year and 14.5% of fourth-year students. This detail can be observed in the **Appendix A**.

Quality of preservice teachers' responses regarding PCK on the fraction-as-operator

The qualitative analysis shows marked differences in how preservice teachers justify their ideas.

In **instructional objectives (Q1)**, most responses rated 1 present detailed learning goals and precise descriptions of what pupils are expected to understand, for example:

> "Que tengan claro el concepto de fracción, su significado más allá de su expresión matemática y que puedan pensar en ejemplos las fracciones para darle su utilidad en la resolución de problemas." ["That they clearly understand the concept of a fraction, its meaning beyond its mathematical expression, and that they can think of examples of fractions to make them useful in problem-solving."] (participant 16, score 1)

In contrast, responses rated 0.5 typically state general aims without addressing specific aspects of fractions, for example:

> "Que aprendan lo básico de la fracción y conozcan y entiendan su utilidad en situaciones cotidianas." ["That they learn the basics of fractions and know and understand their usefulness in everyday situations."] (participant 5, score 0.5)

Only in **Q1** do participants explicitly mention the specific contents provided in the initial situation of the CoRe instrument, albeit to varying degrees of depth. Among these, the nature of fractions and their relationship with other numbers stand out. As the examples show, complete responses explain why understanding the meaning of a fraction and its links to other number systems is important, while incomplete responses simply mention these ideas without justification:

> "Que sepa transformar fracciones en decimales y porcentajes, y que comprenda en qué situaciones es más útil una representación u otra. Por ejemplo, pedir ¼ de carne en lugar de un 25% de carne." ["That they know how to convert fractions into decimals and percentages and understand in which situations one representation is more useful than another. For example, asking for ¼ of meat instead of 25% of meat."] (participant 195, score 1)

> "Saber identificar y clasificar fracciones propias, impropias y mixtas." ["To know how to identify and classify proper, improper and mixed fractions."] (participant 15, score 0.5)

Conversely, the fraction-as-operator was the least mentioned content, with incomplete responses such as:

> "Resolver productos fraccionarios y fracción por número natural." ["To solve fractional products and fraction by natural number."] (participant 151, score 0.5)

Notably, complete and accurate responses include examples and explanations demonstrating understanding of the operator meaning, for instance:

> "El alumnado debe entender la fracción como un multiplicador de otra cantidad. Por ejemplo, al calcular $\frac{2}{3}$ de 9, deben ver que se efectúa la operación $\frac{2}{3} \times 9 = 6$, comprendiendo así, cómo la fracción actúa sobre el número base." ["Students must understand the fraction as a multiplier of another quantity. For example, when calculating $\frac{2}{3}$ of 9, they should see that the operation $\frac{2}{3} \times 9 = 6$ is carried out, thereby understanding how the fraction acts upon the base number."] (participant 196, score 1)

For **educational relevance (Q2),** score 1 responses dominate, especially those connecting relevance to everyday usefulness and explaining why such usefulness is educationally meaningful, often with concrete examples, such as:

> "Porque les ayudarán a resolver situaciones en su día a día de una manera sencilla y eficaz, como, por ejemplo, medir ingredientes o partir una pizza." ["Because it will help them solve everyday situations simply and effectively, such as measuring ingredients or cutting a pizza."] (participant 8, score 1)

Score 0.5 responses mention usefulness only briefly and without linking it to teaching practice, for example:

> "Las fracciones son útiles para el día a día … aunque en mi opinión hay varios apartados matemáticos más concretos que no ayudan en su desarrollo." ["Fractions are useful in daily life … although in my opinion there are other, more specific mathematical topics that do not support their development."] (participant 98, score 0.5)

In **learning difficulties (Q3)**, difficulty developing procedural skills is one of the most frequent indicators, with a balance of score 1 and score 0.5 responses. Score 1 answers clearly identify problematic procedures and explain their causes or misconceptions, for example:

> "Muchos confunden numerador y denominador cuando la fracción es mayor que la unidad porque creen que las fracciones siempre representan parte de un entero." ["Many confuse the numerator and the denominator when the fraction is greater than one because they believe that fractions always represent part of a whole."] (participant 249, score 1)

By contrast, responses rated 0.5 only mention a procedural difficulty without explaining its origin:

> "A veces se confunde la suma con la multiplicación, pero no sé muy bien por qué ocurre." ["Sometimes addition is confused with multiplication, but I don't really know why it happens."] (participant 78, score 0.5)

Additionally, many responses also show lack of knowledge about learning difficulties. The scoring difference depends on whether this lack is acknowledged with or without explanation:

> "No sé qué problemas concretos pueden surgir con las fracciones, ya que es muy difícil conocer las dificultades de los alumnos porque cada uno tiene un ritmo de aprendizaje diferente." ["I don't know what specific problems may arise with fractions, since it's very difficult to know the students' difficulties because each one has a different learning pace."] (participant 26, score 0.5)

> "Admito no conocer en detalle las dificultades, pero me he dado cuenta de que hay que ahondar en la forma de enseñar porque muchos desconocen la relación entre estas y los números decimales y presentan problemas al realizar cálculos." ["I admit I don't know the difficulties in detail, but I've realized that we need to delve into how to teach, because many are unaware of the connection between fractions and decimal numbers and struggle when performing calculations."] (participant 5, score 1)

For **teaching difficulties (Q4)**, the most frequent indicator is insufficient teacher preparation and methodology. Score 1 responses describe concrete examples of training gaps or propose methodological solutions; score 0.5 responses point out problems without strategies. For example:

> "El docente solo enseña con la teoría, lo que dificulta el entendimiento de los conceptos, debería combinar con tareas prácticas." ["The teacher only teaches theory, which makes it hard to understand the concepts. They should combine it with practical tasks."] (participant 29, score 1)

> "Las fracciones se enseñan de manera mecánica, con mucha memorización y a la carga, eso es algo que el alumnado acaba olvidando …" ["Fractions are taught mechanically, with a lot of memorization and pressure, which students eventually forget …"] (participant 9, score 0.5)

In **Methodology and Teaching Activities (Q5)**, most responses focus on arguments for or against specific methods or task types. Score 1 responses provide detailed implementation strategies and justify their didactic value, for example:

> "Propongo una metodología activa y experimental que incluya material concreto (regletas) con ejemplos y contraejemplos para que el alumnado manipule y valide distintas representaciones de la fracción." ["I propose an active and experimental methodology that includes concrete materials (Cuisenaire rods) with examples and counterexamples, so that students can manipulate and validate different representations of fractions."] (participant 22, score 1)

In contrast, score 0.5 responses mention methods or activities in general terms without describing how they would be applied or why they are effective:

> "Propongo un enfoque experimental donde utilizar materiales manipulativos." ["I propose an experimental approach using manipulatives."] (participant 10, score 0.5)

In **assessment (Q6)**, the most common indicator concerns assessment methods, techniques, and instruments. Score 1 responses specify tools and link them clearly to learning objectives, such as:

> "Aplicaría una rúbrica que evalúe la comprensión conceptual, la capacidad de resolver problemas y la justificación de los pasos …" ["I would use a rubric that evaluates conceptual understanding, problem-solving ability, and justification of steps …"] (participant 28, score 1)

In contrast, score 0.5 responses refer only generally to tests or assessment methods without examples or justification:

> "Haría un examen final para ver si aprendieron las fracciones." ["I would give a final exam to see if they learned fractions."] (participant 117, score 0.5)

Finally, although no significant differences were found across specializations, students in S&M tended to use more technical terminology (e.g., "Cuisenaire rods," "validation," and "representations"), suggesting stronger conceptual appropriation—particularly in responses related to the general instructional approach, as illustrated by the following examples:

> "Propongo una metodología activa y experimental que incluya material concreto (regletas) con ejemplos y contraejemplos para que el alumnado manipule y valide distintas representaciones de la fracción." ["I propose an active and experimental methodology that includes concrete materials (Cuisenaire rods) with examples and counterexamples, so that students can manipulate and validate different

representations of fractions."] (S&M participant, score 1)

"La metodología tendría relación directa con lo real, con situaciones que vayan donde vayan las puedan encontrar. De esta forma aumentan las experiencias y el aprendizaje se hace más adecuado, relacionando, complementando y mejorando sus conocimientos." ["The methodology would have a direct connection with real life, with situations they can encounter wherever they go. This increases their experiences and makes learning more appropriate, by linking, complementing, and enhancing their knowledge."] (SEN participant, score 1)

DISCUSSION

Overall PCK and Key Findings

Preservice teachers showed generally low PCK across all CoRe components, a finding consistent with previous research (Depaepe et al., 2015; Zolfaghari et al., 2021). However, this study provides more fine-grained evidence that identifies where these gaps lie. As in earlier work, participants expressed learning objectives (Q1) and content relevance (Q2) more confidently (Rodríguez Rojas & Navarrete Rojas, 2020) but struggled to diagnose and address students' misconceptions and teaching challenges (Q3-Q4). This supports Li and Kulm's (2008) and Tirosh's (2000) argument that teachers often know "what" and "why," but have difficulty anticipating and addressing learners' errors.

The most significant contribution of this study is the documented invisibility of the operator interpretation. Although it is part of the curriculum, preservice teachers almost never mentioned it, consistent with Rafiepour et al.'s (2019) observation that teachers tend to avoid seeing fractions as multiplicative transformations. Our results deepen this insight by showing that this absence persists even when the operator is explicitly presented as focal content in the instrument. This suggests the presence of broader program-level challenges in teacher preparation, rather than merely isolated conceptual oversights, although the available data do not allow us to distinguish between curricular structure and instructional approaches.

Patterns by Academic Year and Specialization

Differences between third- and fourth-year students were small and cross-sectional. This mirrors TEDS-M findings showing that additional coursework does not automatically improve PCK (Blömeke & Delaney, 2012). However, the indicator-level analysis reveals a more nuanced pattern: while instructional objectives and procedural difficulties appear with similar frequencies in both courses, fourth-year students more frequently refer to conceptual aspects of learning difficulties and to methodological considerations, particularly in relation to teaching approaches and assessment. Although these differences are modest, they may be related to fourth-year students' greater exposure to didactic analysis and classroom practice, through the completion of the arithmetic didactics module and an additional practicum period, which could increase their sensitivity to instructional and learning-related challenges without necessarily leading to substantial gains in overall PCK.

The fact that fourth-year students—after an extra didactics course and an additional practicum—did not obtain substantially higher scores suggests that current programs may not systematically develop PCK.

The unexpected pattern across specializations also reinforces the idea that PCK is not automatically linked to mathematical background. Although S&M students used more precise technical language—indicating stronger CK—this did not translate into higher PCK. This aligns with Kleickmann et al. (2013), who showed that PCK requires explicit pedagogical reflection in addition to disciplinary knowledge.

PCK Subdomains: Strengths and Weaknesses

Strengths in KCC (Q1), along with partial strengths in KCT (Q2, Q5, Q6), suggest that preservice teachers can articulate goals and propose general strategies. However, persistent weaknesses in KCS (Q3-Q4) indicate limited opportunities within their preparation to analyze actual student thinking. This echoes international evidence showing that KCS is the most difficult domain to develop without explicit scaffolding (Tirosh, 2000; Zolfaghari et al., 2021). Recent research further suggests that PCK development tends to be uneven across subdomains, with some components remaining underdeveloped despite participation in methods courses (Dragnić-Cindrić & Anderson, 2025).

Crucially, the near absence of operator-related reasoning suggests that teacher-education programs may not be treating this interpretation as a distinct conceptual entity interconnected with other fraction meanings. When operator ideas do appear, they are framed procedurally rather than conceptually, confirming Copur-Gençturk and Li's (2023) argument that teachers often conflate multiplicative operators with algorithms instead of viewing them as transformations.

Limitations

This study has several limitations. First, it focuses on preservice teachers at two late stages of preparation (third and fourth year), which may limit the detection of developmental changes in PCK that could be more visible when including earlier stages or using longitudinal designs.

Second, the study does not directly analyze the content of teacher-education modules or the instructional approaches used by university instructors; therefore, interpretations of program-level challenges should be considered tentative.

Finally, the study is situated in a specific context—a Spanish public university within a European generalist primary-teacher model—so caution is needed when transferring the findings to programs with different structures or specialization models.

CONCLUSION

This study provides the first empirical examination of preservice primary teachers' PCK on the fraction-as-operator sub-construct using a qualitative, MKT-aligned approach. It confirms the generally low PCK reported in prior research but extends existing work by offering a fine-grained analysis of how these limitations manifest across different PCK components, and, crucially, by documenting the near-absence of the operator interpretation even when it is explicitly presented as focal content.

This finding indicates that preservice teachers do not yet conceptualize fractions as multiplicative transformations—one of the most demanding fraction interpretations—and suggests, rather than reflecting isolated gaps or individual shortcomings, preservice preparation in the context examined still lacks the conceptual and pedagogical grounding required to support this understanding. The minimal differences observed between third- and fourth-year students further point to persistent challenges in the systematic development of topic-specific PCK during initial teacher education, while acknowledging that the present study does not allow for a direct distinction between curricular structure and instructional approaches.

Taken together, these findings provide an empirical basis for characterizing the current state of preservice teacher preparation within a European generalist-teacher model, contributing to ongoing debates about how PCK develops—or fails to develop—during initial training. They also highlight several directions for strengthening teacher preparation in similar contexts. Programs should explicitly name and foreground the operator sub-construct, offer systematic opportunities to develop diagnostic KCS through engagement with authentic student work, and distribute fraction-related instruction across multiple years rather than concentrating it in a single module. They should also model how mathematical CK informs instructional decisions. Beyond the specific context studied, these results offer actionable guidance for redesigning preservice mathematics education and contribute to international discussions on how to prepare teachers to teach one of the most conceptually challenging areas of elementary mathematics.

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APPENDIX A

Table A1. Indicators

Variables	Indicators	S	Example of response
Q1. Instructional objectives (56.5%)	**Q1.1 Conceptual objectives are made explicit (3rd: 71.9%; 4th: 76.1%; M: 73.4%)**		
	Q1.1A Nature of fractions and their relationship with other numbers (3rd: 71.9%; 4th: 71.8%; M: 71.9%)	0	That they acquire a foundation on the topic of fractions. #P1
		0.5	That they learn the basics of fractions and become familiar with their usefulness in everyday situations. #P5
		1	That they have a clear understanding of the concept of a fraction, its meaning beyond the mathematical expression, and that they are able to think of examples involving fractions. #P16
	Q1.1B Fraction as operator (3rd: 0%; 4th: 1.7%; M: 0.8%)	0	They know that a fraction is a number with a line in the middle, and that it is used to modify a number. #P32
		0.5	That they understand that a fraction can be applied to a quantity, for example, when we talk about half of something #P45
		1	That they see that a fraction acts as an operator on a base number #P34
	Q1.1C Arithmetical treatment (3rd: 4.1%; 4th: 1.7%; M: 3%)	0	That pupils understand that fractions are only used to divide things and that they cannot be added like normal numbers. #P2
		0.5	After understanding what a fraction is, they practice basic operations to become familiar with its use #P160
		1	That they understand the meaning of basic operations with fractions, and how these relate to operations with whole numbers and decimals, identifying similarities and differences. #P202
	Q1.1D Graphical representation (3rd: 2.7%; 4th: 3%; M: 4.6%)	0	I would use drawings in class so that they understand it, because children like them more than numbers. #P15
		0.5	To know the concept and representation of proper and improper fractions through everyday elements. #P183
		1	I believe that graphical representation is very helpful for visually understanding the meaning of a fraction. #P142
	Q1.1E Problem solving (3rd: 2.1%; 4th: 5.1%; M: 3.4%)	0	That they learn to solve the problems they are given, as always. #P89
		0.5	That pupils know how to tackle problems involving fractions and solve them without difficulty, since it is an important and new topic for them. #P3
		1	That they understand the concept of a fraction and apply it when solving real problems, such as sharing quantities or interpreting measurements, as understanding its meaning is key to using it correctly in different contexts. #P166
	Q1.2 Procedural objectives are made explicit (3rd: 76.7%; 4th: 76.1%; M: 76.4%)		
	Q1.2A Nature of fractions and their relationship with other numbers (3rd: 58.2%; 4th: 47.9%; M: 53.6%)	0	That they do the exercises with fractions correctly without getting confused. #P90
		0.5	To be able to identify and classify proper, improper, and mixed fractions. #P15
		1	To know how to convert fractions into decimals and percentages and understand in which situations one representation is more useful than another. For example, ordering ¼ of meat instead of 25%. #P195
	Q1.2B Fraction as operator (3rd: 8.2%; 4th: 14.5%; M: 11%)	0	That they learn to operate with fractions as if they were regular numbers, without worrying about why it works that way. #P234
		0.5	To solve fractional products and fraction multiplied by whole number. #P151
		1	Pupils should understand the fraction as a multiplier of another quantity. For example, when calculating 2/3 of 9, they should see that the operation $2/3 \times 9 = 6$ is performed and thus understand how the fraction acts on the base number. #P196
	Q1.2C Arithmetical treatment (3rd: 32.9%; 4th: 22.2%; M: 28.1%)	0	That they practice fractions so that their mistakes disappear. #P111
		0.5	That they know how to do basic operations with fractions, such as addition or subtraction, even if at first only with the same denominators. #P13
		1	Achieving assimilation and mastery of operations with fractions and decimals. #P18
	Q1.2D Graphical representation (3rd: 29.5%; 4th: 31.6%; M: 30.4%)	0	To use colors so that fractions appear clearer in class. #P116
		0.5	That they learn to represent fractions in drawings, such as dividing a shape into parts, even without addressing the different types of representation. #P25
		1	To represent graphically (and know what type of graph to use) for fractions and decimals. #P9
	Q1.2E Problem solving (3rd: 28.1%; 4th: 31.6%; M: 29.7%)	0	That they memorize the steps to solve any fraction problem as they are usually all the same. #P189
		0.5	That they can apply the concept of fractions to their daily life, in other words, that they find the knowledge useful. #P19

Table A1 (Continued). Indicators

Variables	Indicators	S	Example of response
		1	That they are able to solve problems involving fractions both in school contexts and in everyday situations, such as sharing quantities or interpreting recipes. #P205
	Q1.3 Attitudinal objectives are made explicit or NdM (usefulness of mathematics) is mentioned (3rd: 73.3%; 4th: 76.1%; M: 74.5%)	0	That they learn to solve fractions the way I teach them, so that they do well in the exam. #P97
		0.5	That pupils can solve mathematical situations they encounter in life, such as applying discounts or calculating taxes. #P1
		1	To spark their interest by giving meaning to what we are going to teach, that is, showing its usefulness. If it is not taught meaningfully, they will forget it over time. #P3
	Q1.4 General or vague response (3rd: 1.4%; 4th: 2.6%; M: 1.9%)	0	The important thing is that they learn fractions because it's part of the curriculum at that stage and it's in the textbook. #P37
		0.5	That they understand what they are working on. It's a concept that is too abstract for them not to understand it and just be guessing blindly. #P4
		1	From my point of view, I believe all the aforementioned contents can be adjusted to Year 5 or 6 level. Maybe topics like sales, taxes, and invoices should be left for later years, like Year 7. #P27
Q2. Educational relevance (28.7%)	Q2.1 Relation to usefulness in daily life (3rd: 89.72%; 4th: 71.79%; M: 83.27%)	0	Fractions are not used in everyday life because everything is done with calculators or whole numbers. #P18
		0.5	Fractions are useful for daily life … although in my opinion there are more specific mathematical topics that are less helpful for their development. #P98
		1	Because they will help them solve daily situations in a simple and effective way, such as measuring ingredients in the kitchen or slicing a pizza. #P8
	Q2.2 Importance in personal development (3rd: 29.45%; 4th: 22.22%; M: 26.24%)	0	Fractions are part of personal development because they must be memorised properly, just like multiplication tables. #P235
		0.5	Fractions provide useful knowledge that can be applied in daily life, although it's not always clear how they help personally. #P103
		1	Fractions are important because they will be useful in their future academic, professional, and personal life, by fostering reasoning and problem-solving. #P13
	Q2.3 Relationship, precedence, and sequence among these and other contents (3rd: 25.34%; 4th: 34.19%; M: 29.28%)	0	Fractions don't have much connection with other topics; they're just learnt and that's it. #P66
		0.5	Fractions are useful for progressing and expanding knowledge in the long term. #P100
		1	(Fractions) are the foundation for acquiring future knowledge such as proportions or percentages. #P40
	Q2.4 Lack of awareness of its educational importance (3rd: 2.7%; 4th: 0.9%; M: 1.9%)	0	With a calculator available, fractions don't matter because they can be converted into decimals. #P76
		0.5	It doesn't really matter much if we hardly use them when we grow up. #P38
		1	I think it's a somewhat abstract topic for primary children, as it's a rather ambiguous topic with little relevance for them. #P1
	Q2.5 General or vague response (3rd: 0.68%; 4th: 5.13%; M: 2.66%)	0	-
		0.5	It is useless for a pupil to have a lot of knowledge if they do not have the tools to apply it in their personal life. #P187
		1	-
Q3. Learning difficulties (14.6%)	Q3.1 Insufficient or incorrect prior knowledge (3rd: 17.81%; 4th: 26.5%; M: 21.67%)	0	No prior knowledge is needed to learn fractions, because it almost always starts from scratch. #P32
		0.5	Mixing up concepts and not being able to differentiate their function makes learning difficult, as it is not clear to them what they are learning. #P28
		1	Not having acquired the basic knowledge of fractions hinders progress in acquiring new knowledge. #P11
	Q3.2 Level of abstraction of certain concepts (3rd: 15.07%; 4th: 35.9%; M: 24.33%)	0	Abstract concepts do not affect learning much because, with clear formulas and steps, they can be learnt anyway. #P154
		0.5	Fractions are abstract and difficult, like something far removed from their reality. #P4
		1	Fractions require abstract reasoning that is not always well developed in previous years. #P56
	Q3.3 Inability to develop procedural skills or a specific content (3rd: 41.78%; 4th: 41.03%; M: 41.44%)	0	I don't think they have problems with operations because, if they are given the steps, they always do them correctly. #P165
		0.5	Sometimes addition is confused with multiplication, but I don't really know why it happens. #P78
		1	Many confuse numerator and denominator when the fraction is greater than one because they believe fractions always represent a part of a whole. #P249

Table A1 (Continued). Indicators

Variables	Indicators	S	Example of response
	P3.4 Overcoming negative attitude/motivation towards mathematics (3rd: 7.53%; 4th: 1.7%; M: 4.94%)	0	Sometimes they don't like it, but that doesn't matter if they study what they are taught. #P162
		0.5	It's not usually a topic they enjoy, since it has always been presented as boring rather than fun and useful for everyday life. #P217
		1	Lack of interest in mathematics makes it difficult to learn fractions, especially when pupils do not understand their usefulness or the teacher's explanations. Lack of motivation prevents them from engaging in class. #P27
	Q3.5 Pupil diversity / Low development of cognitive abilities (3rd: 7.53%; 4th: 5.98%; M: 6.84%)	0	If they are all in the same year group, they should all learn the same. #P4
		0.5	Some children need to learn in a more visual way or through familiar everyday examples. #P115
		1	Not all children are the same or learn at the same pace, so for those who struggle, the topic of fractions will feel like an uphill battle. #P38
	Q3.6 Ignorance of the usefulness of the concepts presented (3rd: 10.96%; 4th: 18.80%; M: 14.44%)	0	Even if they don't know what fractions are for, what matters to them is doing the exercises correctly. #P57
		0.5	I understand that pupils struggle with mathematics… that's why it should be explained in an engaging way, regardless of its usefulness. #P107
		1	They ignore the meaning of fractions and are unaware of how or where they can use them in daily life, which limits their ability to learn. #P1
	Q3.7 Recently introduced curriculum content (3rd: 2.74%; 4th: 5.13%; M: 3.8%)	0	Even if it's a new concept, they usually learn whatever they are taught. #P89
		0.5	They may have difficulties because they are combinations of numbers they hadn't encountered before. #P25
		1	When this topic is introduced, children don't understand it because they have never seen a division expressed as a fraction or a decimal number presented like that. #P226
	Q3.8 Inappropriate teaching methodology (3rd: 15.75%; 4th: 11.11%; M: 13.69%)	0	If the teacher explains it well on the board, the pupils won't have learning problems. #P12
		0.5	In most cases, traditional teaching is followed, without methodological renewal, taking theoretical instruction as the foundation of education. #P15
		1	Fractions are usually explained on the board and they don't understand them because they can't see their form, so they should be taught using manipulatives (like ice cream sticks). #P6
	P3.9 Lack of knowledge about learning difficulties (3rd: 13.01%; 4th: 11.1%; M: 12.17%)	0	-
		0.5	I don't know what specific problems may arise with fractions, as it is very difficult to know pupils' difficulties because each one has a different learning pace and diverse alternative ideas. #P26
		1	I admit I don't know the difficulties in detail, but I've realized that we need to delve deeper into how we teach, because many don't understand the relationship between fractions and decimal numbers and have problems when doing calculations. #P5
	Q3.10 General or vague response (3rd: 1.37%; 4th: 0%; M: 0.76%)	0	-
		0.5	I think it is more interesting to ask whether children should develop these mathematical competences or follow their preferences. #P98
		1	-
P4. Teaching difficulties (12.7%)	Q4.1 Making abstract or difficult concepts understandable (3rd: 24.66%; 4th: 16.24%; M: 20.15%)	0	Fractions are easy to understand if they are explained in the usual way; there's no need to complicate them with other things. #P238
		0.5	Fractions are often treated as an isolated topic and seem like something completely new and unfamiliar. #P67
		1	Many find it hard to understand that a fraction represents a part of a whole and not just a number with two digits. To avoid it being seen as something abstract, manipulatives such as fraction blocks or drawings in real contexts can be used. #P22
	Q4.2 Overcoming low prior knowledge and/ or misconceptions (3rd: 13.01%; 4th: 6.84%; M: 10.27%)	0	If pupils make mistakes when learning fractions, it is because they are not paying attention, not because they lack correct prior ideas. #P129
		0.5	It may be that the incorrect ideas some pupils have are due to poor teaching. #P95
		1	Pupils should have a solid foundation so that difficulty can gradually be added and knowledge expanded in line with learning objectives. #P37
	Q4.3 Adapting to learner diversity (3rd: 9.59%; 4th: 6.83%; M: 8.37%)	0	All pupils should follow the same explanation, so no one gets confused, ensuring the same pace for everyone. #P37
		0.5	I use the textbook for convenience, even though it means not all pupils can follow the same pace, as the books are not very flexible. #P92

Table A1 (Continued). Indicators

Variables	Indicators	S	Example of response
		1	It is necessary to address the different learning paces and styles of pupils. Not everyone understands in the same way, so explanations and resources should be varied to reach everyone. #P17
	Q4.4 Carrying out appropriate procedural activities (3rd: 6.85%; 4th: 11.11%; M: 8.75%)	0	Textbook exercises are enough for pupils to learn how to work with fractions. #P253
		0.5	Fractions and their operations can limit learning of content such as decimals and percentages. #P208
		1	There is a lack of experiences that allow pupils to "experiment" with fractions, in order to later build the abstract model (the fraction). #P19
	Q4.5 Capturing and maintaining attention; sparking interest; teacher's attitude (3rd: 6.16%; 4th: 5.98%; M: 6.08%)	0	Pupil interest does not depend on the topic, but on whether the teacher sets clear rules from the start. #P49
		0.5	It might be the stress or uncertainty generated in pupils before the explanation is finished. #P14
		1	Lack of interest from both pupils and teachers. Both need to be motivated—one to teach and the other to learn. #P11
	Q4.6 Using the history of mathematics effectively (demonstrating its usefulness) (3rd: 8.9%; 4th: 22.22%; M: 14.83%)	0	The history of mathematics is not important when teaching fractions; that belongs to the past. #P212
		0.5	It would be useful to show how different mathematical topics are related to each other, so they make more sense. #P202
		1	Mathematics is often taught without personal meaning for the pupils, disconnected from their real-life context. #P30
	Q4.7 Lack of resources, places and/or materials (3rd: 15.07%; 4th: 23.07%; M: 18.63%)	0	No special materials are needed; it's enough to explain the topic well on the board. #P55
		0.5	In many cases, there is no access to materials that allow for that method (graphical). #P14
		1	A common difficulty is the lack of manipulatives to represent fractions. Without them, pupils find it harder to understand the concept, as they cannot visualize parts or make connections with real-life situations. #P165
	Q4.8 Lack of time to develop content (3rd: 5.48%; 4th: 5.98%; M: 5.70%)	0	With good planning, everything can always be covered without any problem. #P273
		0.5	Lack of calm or individual attention—if a child doesn't see the result clearly the first time, they should have opportunities to understand it the tenth time thanks to the teacher's help. #P65
		1	The biggest limitations are time related. There is no time to calmly link everything to real life before having to move on to the next topic. #P23
	Q4.9 Lack of teacher preparation/skills/type of methodology (3rd: 37.67%; 4th: 31.62%; M: 34.98%)	0	If the teacher master's the content, that is enough to teach it well. #P189
		0.5	Fractions are taught mechanically, with lots of memorization and rushing, and pupils end up forgetting them. #P9
		1	The teacher only teaches through theory, which makes it difficult to understand the concepts—practical tasks should be included. #P29
	Q4.10 Lack of awareness of teaching difficulties (3rd: 5.47%; 4th: 12.82%; M: 8.74%)	0	I don't think there are any special difficulties in teaching fractions—you just have to explain it well. #P201
		0.5	I don't know the specific difficulties, but I know that sometimes teachers make mistakes when explaining them. #P198
		1	I don't know them, but I think it should be taught from the very basics to the more complex to avoid difficulties, delays in class, and so on. #P13
	Q4.11 General or vague response (3rd: 1.37%; 4th: 1.7%; M: 1.52%)	0	-
		0.5	When performing operations, they should look for a common denominator. #P225
		1	-
Q5. Teaching methodology and activities (25.7%)	Q5.1 General instructional approach (active, constructivist, etc.) (3rd: 2.05%; 4th: 0.9%; M: 1.52%)	0	I would use the usual methodology because it is already established and works. #P243
		0.5	I suggest an experimental approach using manipulatives. #P10
		1	I suggest an active and experimental methodology that includes concrete materials (Cuisenaire rods), with examples and counterexamples, so that pupils can manipulate and validate different representations of fractions. #P22
	Q5.2 Arguments for/against certain methodologies (3rd: 48.63%; 4th: 70.08%; M: 58.17%)	0	I don't think the methodology matters much as long as the textbook topics are followed. #P123
		0.5	Using the textbook is faster, but it also has its drawbacks. Active methodology is better because pupils learn more by doing through hands-on activities and digital resources. #P67

Table A1 (Continued). Indicators

Variables	Indicators	S	Example of response
		1	Traditional methodology may lead pupils to solve problems mechanically without understanding what a fraction represents. In contrast, constructivist approaches allow for starting from meaningful situations where the concept is built through experience. #P7
	P5.3 Organization: roles of teacher and pupils, environment, etc. (3rd: 15.75%; 4th: 20.50%; M: 17.87%)	0	The teacher should explain clearly and the pupils should follow the lesson—there's no need to complicate things with other dynamics. #P161
		0.5	Practical and everyday situations, as this improves the group's willingness to engage. #P88
		1	I would propose activities that promote peer interaction so that pupils can overcome difficulties collaboratively and take on a more active role. #P2
	Q5.4 General and vague methodology (3rd: 12.30%; 4th: 24.79%; M: 17.87%)	0	-
		0.5	A more active methodology that is meaningful in their lives. #P20
		1	The methodology should focus on pupils understanding what they do and why they do it, not just on repeating steps. It is essential that it encourages reflection and a sense of what is being learnt through examples. #P13
	Q5.5 Task types (observe, experiment, discuss, brainstorming, etc.) (3rd: 50.68%; 4th: 57.3%; M: 53.61%)	0	We would do textbook exercises to practice the procedures, as usual. #P94
		0.5	Where games and exercises are part of the teaching. #P83
		1	I would use examples and counterexamples in tasks where pupils can experiment, observe, and reflect, including elements of embodiment and group discussion. #P22
	Q5.6 Specific activities linked to objectives for this topic (3rd: 32.19%; 4th: 23.08%; M: 28.14%)	0	I would do some activity depending on what comes up at the moment, without planning anything specific. #P38
		0.5	To find some format like wooden pieces and a visual example to enhance and support the prior information we have taught. #P98
		1	To pose problems such as having a cake to share with the children, so they work out how many parts each one gets. #P29
	Q5.7 General or vague activities (3rd: 2.05%; 4th: 0.85%; M: 1.52%)	0	-
		0.5	I don't know what specific activity I would propose, but I would try to make it related to topics they are familiar with or can relate to. #P2
		1	-
Q6. Assessment (17.6%)	Q6.1 General assessment approach (3rd: 29.45%; 4th: 29.06%; M: 29.28%)	0	I would assess what they have learnt in the unit. #P245
		0.5	I would try to assess each pupil's progress from the beginning to the end. #P7
		1	I would use continuous assessment from the start of the unit, gathering information on pupils' progress through observation and varied activities. #P8
	Q6.2 Assessment is related to initial objectives and/or the methodology (3rd: 13.7%; 4th: 11.97%; M: 12.93%)	0	I would assess with exercises different from those used in class to see what they know. #P265
		0.5	I would like to propose situations that reflect what they might encounter in the future and assess the content through pupils' templates where they show their calculations and justifications. #P20
		1	They can correctly separate fractions and carry out operations, solve everyday problems, etc., using a methodology consistent with the content. #P22
	Q6.3 Ways of assessing, techniques and tools (3rd: 48.63%; 4th: 58.11%; M: 52.85%)	0	I would hand out a worksheet with exercises for them to complete, and that would be enough. #P253
		0.5	Through activities closer to real life where they can explain the reasoning behind their final answer. #P60
		1	I would use a rubric that assesses conceptual understanding, problem-solving ability, and justification of steps. #P28
	Q6.4 Justification and reasoning about assessment (3rd: 11.64%; 4th: 13.68%; M: 12.55%)	0	We assess because marks must be given—that's how schools work. #P176
		0.5	I wouldn't limit assessment to just one test; a group project would count for 50% as it helps develop collaborative skills. #P18
		1	I think observation is key in assessment, as it allows us to follow each pupil's process throughout the lesson, beyond what a final test can show. #P16
	Q6.5 Specific indicators or aspects to assess: prior knowledge, etc. (3rd: 6.16%; 4th: 19.66%; M: 12.17%)	0	I would only check if the operations were correct, without worrying about anything else. #P172
		0.5	I would carry out continuous assessment by observing how they distribute game pieces or place decimal numbers. #P206

Table A1 (Continued). Indicators

Variables	Indicators	S	Example of response
		1	Communication between group members, cooperation, creativity, the chosen activity or activities, etc., would also be assessed, as well as procedures and justifications. #P37
	Q6.6 Specific assessment activities (3rd: 10.96%; 4th: 11.11%; M: 11.03%)	0	I would do a couple of typical exercises on the topic to see if they have understood. #P93
		0.5	By presenting situations where they have to solve an operation related to this concept. #P66
		1	By presenting them with real-life situations, asking them to solve a problem where something increases, decreases, or different quantities are simply compared. #P10
	Q6.7 Reject the use of tests (3rd: 7.53%; 4th: 6.84%; M: 7.22%)	0	-
		0.5	I believe assessment is something continuous, not something that should be done through a final test. #P3
		1	I would try not to base assessment on an exam. I would try to carry out a project involving the resolution of real-life practical cases. #P32
	Q6.8 General or vague response (3rd: 2.05%; 4th: 1.7%; M: 1.9%)	0	-
		0.5	Even if you don't try to check it directly and explicitly, you end up realizing whether you've explained it well or not, and whether your pupils have understood you. #P4
		1	-