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Exploring teachers' perceptions of mathematical connections in the mathematics curriculum and teaching

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Abstract

Learners' poor mathematical knowledge is often associated with teachers' ineffective teaching. This problem affects the development of society. Meanwhile, teachers' perceptions of mathematics influence their teaching of the subject. By implication, teachers' perceptions of mathematics are crucial to learners' success in mathematics. This study investigated South African junior secondary mathematics teachers' (JSMTs) perceptions of mathematical connections. This study purposively sampled six teachers. Data were collected through semi-structured interviews and analysed using an inductive thematic data analysis approach. The findings suggest that teachers' views of mathematical connections are limited and lack the inclusion of using different representations of mathematics. Their perceptions that mathematical connections involve scaffolding and improve learners' mathematical knowledge are consistent with existing research. This study recommends targeted professional development, focusing on teachers' knowledge, perceptions, and access to curriculum resources for effective mathematical connections in practice. This study contributes to effective mathematics teaching and teachers' professional development.

Keywords: teachers' perceptions of mathematical connections, mathematical connections in the curriculum, quality mathematics teaching, mathematics education

INTRODUCTION

The low quality of mathematics teaching is a concern in many societies, including South Africa. This problem significantly contributes to learners' poor mathematical understanding (Mabena et al., 2021; Mosvold, 2022; Pournara et al., 2015). Students' poor mathematical understanding often arises from insufficient prior knowledge and difficulties in abstracting information and formalising mathematical concepts (Arenas-Peñaloza et al., 2024). Addressing these issues necessitates quality teaching for enhanced students' ability to establish connections. The low quality of mathematics teaching requires an urgent solution, as it reduces citizens' access to employment opportunities (Mhakure, 2019). Otherwise, the unemployment rate will continue to rise, leading to a poor standard of living (Priambodo, 2010). This study focuses on the low quality of mathematics teaching resulting from teachers' limited knowledge of mathematical connections.

Many studies highlight the significance of mathematical connections for high-quality mathematics teaching (Bingölbali & Coskun, 2016; Hatisaru et al., 2024; Rodríguez-Nieto et al., 2021). These connections refer to the ability to associate mathematical concepts with one another, other disciplines, and real-life situations (Bingölbali & Coskun, 2016; Diana et al., 2020). For instance, Bingölbali and Coskun (2016) propose a conceptual framework for enhancing the skill of making connections in mathematics teaching, demonstrating its importance for learner understanding and learning. Similarly, Rodríguez-Nieto et al. (2021, 2024) analyse mathematical connections in learners' understanding of derivatives by networking the extended theory of mathematical connections (ETC) and the onto-semiotic approach. The authors provide a detailed analysis that

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Contribution to the literature

- This study provides teachers' perceptions regarding the role of mathematical connections in mathematics curriculum and teaching.
- This study contributes practical insights into how teachers perceive mathematical connections to enhance students' conceptual understanding.
- This study sheds light on how mathematical connections within the curriculum shape teaching practices.

can help teachers improve their practices. Rodríguez-Nieto and Alsina (2022) further explore mathematical connections in daily practices by integrating ethnomathematics, STEAM education, and a globalised approach, emphasising the relevance of mathematics in diverse socio-cultural contexts and how these contexts shape the connections learners make. De Gamboa et al. (2022) explore the relationship between mathematical connections and the mathematics teacher's specialised knowledge (MTSK) model and suggest that mathematical connections are a key component of pedagogical content knowledge (PCK). These studies collectively support the critical role of mathematical connections in enhancing the quality of mathematics teaching and learning.

It is essential to highlight that mathematical connections are becoming more and more powerful every day because in the broad research agenda, mathematical connections made by people with a mathematical focus, such as mathematics teachers, engineers, public accountants, statisticians, etc., are recognised (Campo-Meneses & García-García, 2023; Font Moll & Rodríguez-Nieto, 2024; Garcia-Garcia, 2024; Rodríguez-Nieto et al., 2023, 2024). In addition, there are ethnomathematical connections, which consist of relating the mathematics used by people in everyday life with the institutional or public mathematics found in curricular materials such as textbooks (Rodríguez-Nieto, 2021; Rodríguez-Nieto & Escobar-Ramírez, 2022). These types of connections are used to explore everyday practices associated with gastronomy (Olivero-Acuña et al., 2025), brick making (Rodríguez-Nieto et al., 2025), fishing and food marketing methods (Sudirman et al., 2024), everyday games for children, young people and adults (Manchego Palacio et al., 2024), among others. Other types of connections are those identified in real-world situations, such as detailed modeling connections with the modeling cycle (Ledezma et al., 2024) and the new vision of connection theory referring to neuro-mathematical connections associated with mathematical activity in the brain (Cantillo-Rudas et al., 2024).

While mathematical connections studies focus on teachers' mathematical connections in practice, research on mathematics teachers' perceptions of mathematical connections is scarce. Teachers' perceptions and practices have a positive relationship (Benken & Brown, 2008; Eley, 2006). Hence, this study bridges this gap by investigating South African junior secondary mathematics teachers' (JSMTs) perceptions of mathematical connections. The aim is to understand the teachers' perceptions of mathematical connections in the mathematics curriculum and teaching to support them in improving the quality of their teaching. In this study just as in Chand (2022) we are inclined to use teachers' perceptions as their beliefs, we are first to admit that while the two constructs have some similarity (Millar, 2014; Smith, 2001), their definitions are different. For instance, Smith (2001) posts that perceptions are individual external experiences of events that occur at a specific time, while beliefs are internal dispositional states of the mind that endure over time. Studies about teachers (Chand, 2022; Meeran & Van Wyk, 2022) that have used interviews as main data collecting methods have swayed towards the use of their perceptions and beliefs interchangeably. This study conducted semi-structured interviews with six JSMTs in the Tshwane region of South Africa and analysed data through inductive thematic analysis. The theoretical basis of this research is shown below.

BACKGROUND OF STUDY

Researchers categorise mathematical connections broadly into intra-mathematical and extra-mathematical connections, also known as internal and external connections, respectively (García-García & Dolores-Flores, 2021; Rodríguez-Nieto et al., 2022; Siregar & Surya, 2017). Intra-mathematical connections refer to making connections within and among the concepts of mathematics. Bingölbali and Coskun (2016) propose a framework consisting of four components for making connections in teaching and learning mathematics; the authors identify intra-mathematical connections as connections between concepts and further categorise them as connections between a mathematics concept and its sub-concepts or among the sub-concepts or connection between a concept and others within mathematics, similar to Yang et al.'s (2021) closest superordinate and convertible concepts. Extra-mathematical connections involve connections between mathematics and other subjects and between mathematics and real-life contexts. Connecting with other subjects includes connecting mathematics with geography, economics, science, and technology. This category encompasses teaching a mathematical concept within the context of a different discipline and referencing other disciplines through verbal examples

(Bingölbali & Coskun, 2016). The connections with real-life contexts involve applying mathematical concepts and skills to solve problems and analyse real-world phenomena, either teaching mathematics within a real-life context or stating real-life connections through verbal examples (Bingölbali & Coskun, 2016). Businskas (2008) describes mathematical connections as relationships in the forms of procedural, part-whole relationships, different representations, instruction-oriented connections, and implications. These five categories have been extended to include meaning, reversibility, and metaphoric (García-García & Dolores-Flores, 2021; Hatisaru, 2022; Rodríguez-Nieto et al., 2022).

Connection in the Mathematics Curriculum and Teaching

Mathematical connections allow learners to see mathematics as interconnected concepts, crucial for profound understanding. Jauchen (2019) and Ormond (2016) emphasise connections is reflected in curriculum materials designed for teacher education, highlighting its foundational importance. The National Council of Teachers of Mathematics (NCTM) further reinforces this view, advocating for learners to recognise and utilise connections across mathematical concepts, fostering a holistic perspective and a more cohesive understanding of the discipline. This ability to connect concepts is essential for learners to develop a conceptual understanding of mathematics and apply it effectively. However, fostering these connections in the classroom poses challenges. While research highlights the importance of connecting concepts with their visual or symbolic representations, including linking angles to shapes and measurement, teachers often struggle to implement these strategies effectively. Clark and Linn (2003) and Mhlolo et al. (2012) highlight that insufficient PCK can hinder teachers' ability to weave concepts together meaningfully. This lack of PCK may manifest in difficulties in selecting appropriate examples, designing activities, or even recognising the connections themselves. Furthermore, as Clark and Linn (2003) note, simply allocating instructional time is insufficient; teachers need targeted support and training to develop the skills necessary to facilitate connections.

Several studies have explored strategies for fostering mathematical connections. Ormond (2016) suggests an innovative approach to preparing teachers for algebra instruction, emphasising scaffolding to promote the development of mathematical connections. Similarly, Remillard and Kim (2017) stress the importance of teachers' understanding of curriculum-embedded mathematics, including the interrelationships between mathematical concepts. Ormond (2016) also provides evidence for the effectiveness of integrating mathematical connections into teacher preparation programs, demonstrating the benefits of explicitly linking number concepts and algebraic principles within the Australian curriculum context. This approach calls for teacher education that explicitly develops the pedagogical skills needed to teach for making connections. Research also emphasises the critical role of mathematical representations in facilitating connections. In the South African context, Mwakapenda (2008) advocates for an inquiry-based approach that leverages learners' prior knowledge and emphasises the interconnected nature of mathematical concepts and procedures. This approach allows learners to build on their existing knowledge and develop a relational understanding of mathematics to tackle real-world problems. Barmby et al. (2007) argue that connecting concepts with representations is crucial for learners' mathematical understanding. While significant progress has been made in understanding the importance of mathematical connections, further research is needed on mathematics teachers' perceptions of mathematical connections. For example, JSMTs' perceptions of mathematical connections regarding fractions teaching and in the curriculum. This study aims to understand the teachers' perceptions of mathematical connections in the curriculum and teaching to support them in improving the quality of their teaching.

Mathematics Teachers' Perceptions of Mathematical Connections

Teachers' beliefs about teaching and learning are crucial in shaping their pedagogical approaches. These beliefs, encompassing ideas about the roles of teachers and learners, have been shown to impact teaching practices and their effectiveness (Masduki et al., 2019). Teachers' understanding of mathematical connections directly influences how they design and deliver mathematics lessons (Siregar & Siagian, 2019). Effective curriculum implementation hinges on teachers' ability to enact their roles (Masduki et al., 2019; Ring-Whalen et al., 2018). Therefore, understanding teachers' perceptions is critical for quality mathematics teaching and learning. For instance, examining how teachers conceptualise mathematical definitions can provide valuable insights into their use of examples during instruction (Johnson et al., 2014). While Yang et al. (2021) suggest a pedagogical approach where teachers initially explain a mathematical definition and then guide learners in constructing specific examples for deeper understanding, research indicates that teachers' ability to provide such examples varies. A study of middle school mathematics teachers found that while teachers generally understood the importance of mathematical connections, some struggled to generate appropriate examples to illustrate these connections (Siregar & Siagian, 2019). This struggle may stem from insufficient PCK to bridge the gap between abstract definitions and concrete examples. Teachers' inability to provide

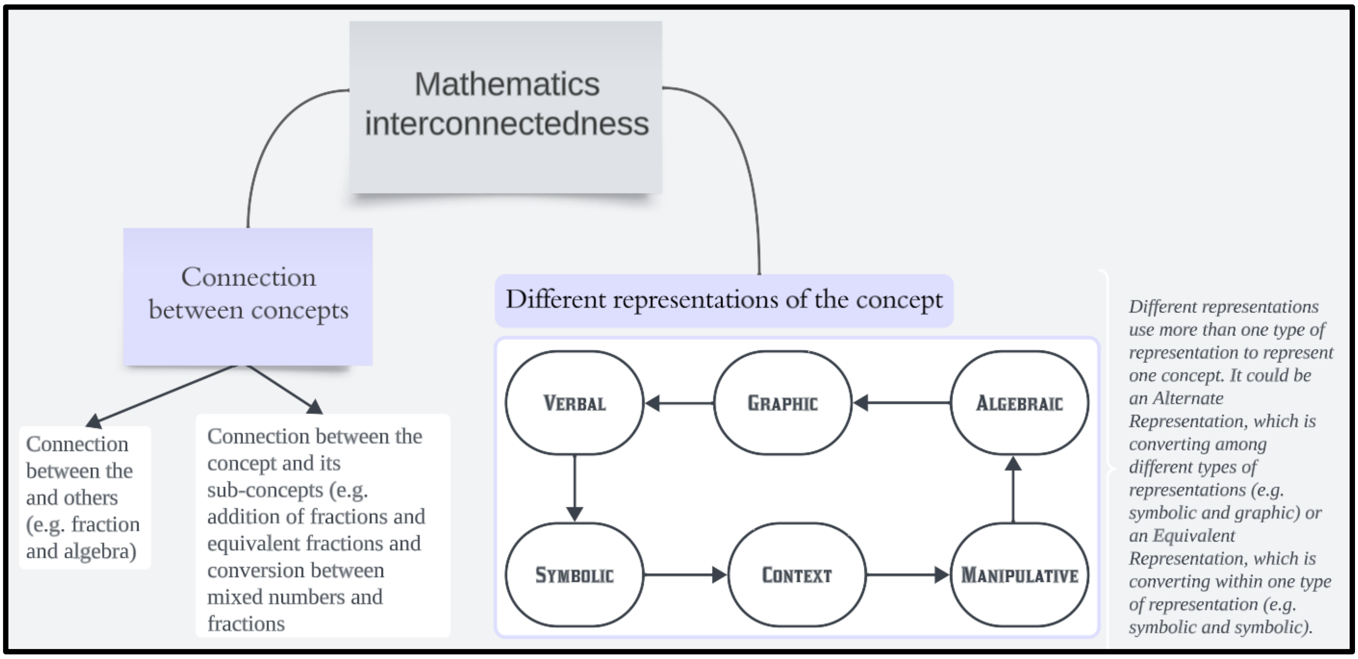

Figure 1. Mathematical connections between concepts and different representations (Source: Authors' own elaboration)

examples can hinder learners' mathematical connection ability.

These insights from the literature, particularly regarding the importance of teacher perceptions, inform the conceptualisation of mathematical connections in this study consisting of the connections between concepts and among different representations. **Figure 1** illustrates these dimensions, highlighting the interplay between conceptual understanding and representational fluency-connections between concepts include the ability to link a given concept to its related superordinate and subordinate concepts, and connections to other related mathematical ideas while connections between different representations recognise that mathematical concepts can be expressed in various ways, including verbal, graphic, algebraic, symbolic, contextual, and manipulative representations. This study considers representations as equivalent or alternate forms of expressing the same mathematical idea (Mhlolo et al., 2012). Translating between multiple representations is crucial for developing a profound and conceptual understanding of mathematical concepts. For example, the connection between the algebraic representation of a function and its graphical representation allows learners to visualise the relationship between variables and gain a conceptual understanding of functions. Similarly, connecting a mathematical concept to a real-world context enables learners to see the practical relevance of mathematics and apply their knowledge to solve authentic problems.

The literature reviewed suggests that teachers' beliefs about mathematics and teaching significantly influence their ability to foster mathematical connections in their learners. The research also highlights the challenges of making these connections explicit and using multiple representations effectively. These findings have led to this study's conceptualisation of mathematical connections as the connections between concepts and connections between different representations. **Figure 1** provides a lens through which to examine teachers' perceptions of mathematical connections in the curriculum and teaching to support them in improving their quality of teaching.

Figure 1 provides a conceptual framework for mathematical connections in the curriculum and teaching, illustrating connections between concepts and how teachers can represent those concepts. The connection between concepts is divided into connections between the concept and other concepts and between a concept and its sub-concepts. Different representations of a concept can be either alternate or equivalent.

METHODOLOGY

This study employed a qualitative approach to identify JSMTs' perceptions of mathematical connections in the South African junior secondary mathematics curriculum and teaching. A qualitative approach emphasises the social construction of knowledge and individuals' experiences (Blaxter et al., 2006; Scotland, 2012; Walsh & Downe, 2005). Furthermore, this study followed an exploratory design (Creswell & Creswell, 2018; Korstjens & Moser, 2017). The exploration involves collecting direct insights from participants, providing a nuanced understanding of their views on mathematical connections.

Participants and Context

This study purposively selected six JSMTs from Tshwane based on specific criteria, including being qualified JSMTs, teaching Grade eight or Grade nine, using English as the medium of instruction, working within the Tshwane region, and expressing willingness to participate. The participants were pseudonymously labelled as T1, T2, T3, T4, T5, and T6. Among them were three males and three females, and their age distribution was even, with two participants each falling within the age groups of 40-49, 30-39, and less than 30. Regarding their highest level of qualification, four participants held a Bachelor of Education degree, one had a BSc with PGCE, and another possessed a master's degree. Their teaching experience spanned a range, with two participants having over 20 years of experience; one had 12 years, and another had nine years. At the same time, the remaining two were younger teachers with two years and nine months of experience, respectively.

Data Collection

This study collected data through document analysis and semi-structured interviews. The document analysis examined the South African mathematics curriculum (curriculum assessment policy statement, CAPS), and semi-structured interviews explored the teachers' perceptions of mathematical connections. While the initial plan was to conduct face-to-face interviews, the COVID-19 pandemic necessitated the use of WhatsApp for two teachers. Interview questions one and eight aimed to gain teachers' overview of mathematical connections, specifically in teaching (**Figure 2**). Question two aimed to see how teachers would use mathematical connections to teach fraction division. In question three, this study is intended to determine if teachers' use of mathematical connections aligns with the curriculum's learning objectives. In contrast, this study aimed to use question five to establish teachers' views about the learning outcomes-question four documents teachers' perspectives regarding mathematical connections in the junior secondary mathematics CAPS. Lastly, questions six and seven assess teachers' knowledge of representations, particularly about word problems and diagrams in the context of fraction calculations.

1. After our conversation about mathematics interconnectedness, how would you define it?
2. Do you use mathematics interconnectedness to teach mathematics? Consider the division problem $5\frac{3}{4} \div \frac{3}{2}$. How will you use mathematics interconnectedness to explain the solution to your learners?
3. Why do you use mathematics interconnectedness in teaching mathematics?
4. Does the CAPS (or ATP) document make mathematics interconnectedness easy and interesting to teach common fractions in grade 8? How?
5. Are your learners aware of mathematics interconnectedness? How do/ did they know?
6. Which of the following story problem could be used to illustrate $1\frac{1}{4} \div \frac{1}{2}$? Why do you think so?
   A. You want to split $1\frac{1}{4}$ pies evenly between two families. How much should each family get?
   B. You have $R1,25$ and may soon double your money. How much money would you end up with?
   C. You are making some homemade taffy, and the recipe calls for $1\frac{1}{4}$ cups of butter. How many sticks of butter will you need (if each stick = $\frac{1}{2}$ cups)?
7. Given that: $\frac{6}{7} \div \frac{1}{3}$ What diagram would you draw to assist learners in making sense of the task? Please draw the diagram.
8. What more can you tell me about mathematics interconnectedness for teaching and learning mathematics?

Figure 2. Interview protocol comprising eight primary guiding questions (Banjo, 2023)

Data Analysis

The CAPS analysis involved four steps: document selection, open coding, clustering of codes into themes, and integration of emergent themes from data and literature (Wood et al., 2020). This study examined the fraction content covered in the curriculum to determine the inclusion of mathematical connections and suggested teaching strategies and resources supporting connections. This study examined terms and phrases associated with mathematical connections, such as relate, connect, integrate, apply, transfer, and generalise. This helped determine how the curriculum presents mathematical connections for teachers to use. For example, CAPS recommends that grade 9 learners work with fractions as coefficients in algebraic expressions and equations. The curriculum expects learners to be proficient in performing multiple operations using fractions and mixed numbers and appropriately applying rational numbers' properties. This suggests making connections between various concepts.

Additionally, the curriculum expects learners to recognise and use equivalent forms for fractions in calculations and when simplifying algebraic fractions (DBE, 2011). The junior secondary CAPS focuses on crucial concepts and skills and connecting related concepts. Therefore, it is recommended that teachers teach fractions in a way that highlights the interconnectedness of these concepts with other mathematical concepts and skills. Thematic analysis followed the steps outlined by Braun and Clarke (2012), allowing flexibility in interpreting participants' perspectives. The interview data were transcribed and analysed inductively. The themes that emerged from the analysis helped answer the research question:

RQ1 What are the perceptions of JSMTs in Tshwane regarding mathematical connections for quality mathematics teaching?

Figure 3 presents how this study generated the set of codes from a sample of an interview response, representing data reduction from the transcribed information to what helped answer the questions. In this case, the codes led to seven categories, and the categories resulted in two themes. Mala is a pseudonym and represents T5 in this study. **Figure 3** identifies the themes by identifying patterns among the codes generated. The sample generated two themes: linking mathematical concepts for teaching and enhancing learners' understanding of mathematical concepts through connections and relationships. The study synthesised all the themes from the six participants' data, resulting in five key themes.

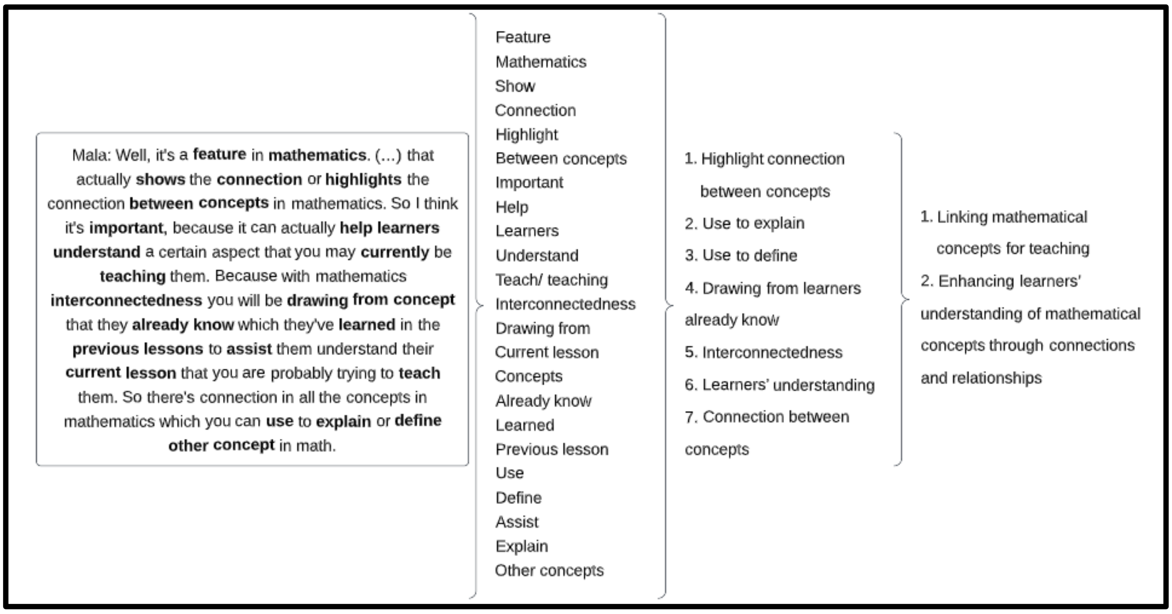

Figure 3. An example of the interview transcripts' coding process and transforming codes into themes (Adapted from Braun & Clarke, 2012)

RESULTS

Junior secondary mathematics CAPS emphasises knowledge applications for solving real-world problems (DBE, 2011, pp. 4-5, 8-9), the connection between concepts (DBE, 2011, p. 122), and the use of different representations to teach learners, especially graphical representations (DBE, 2011, p. 157). The curriculum encourages teachers to use local contexts and real-life examples to help learners understand mathematical concepts better. Five themes emerged from the analysis of JSMTs' responses to the interview questions and are synthesised into three-connecting mathematical concepts for effective mathematics teaching, establishing a solid foundation for mathematical concepts through interconnectedness, and mathematical connections in the curriculum and its influence on mathematics teaching. These findings provide insights towards understanding the JSMTs' mathematical connections in practice, particularly in teaching fractions at the junior secondary level.

Connecting Mathematical Concepts for Effective Teaching

This found that JSMTs are knowledgeable about mathematical connections. The participants defined mathematical connections as the:

> T1: Linking of related topics to teach mathematics.

> T2: State of having different parts or things connected or related to each other.

> T5: Feature that shows the connection or highlights the relationship between concepts in mathematics and

> T6: Linkage between one topic or subject to another in mathematics.

They opined that those connections:

> T3: Involve links between topics and concepts, and connections to practical situations and

> T4: Connect two topics that work hand in hand to calculate or thoroughly understand a topic.

These definitions align with existing literature defining mathematical connections as connections between mathematical concepts, mathematics and real-life situations and between mathematics and other disciplines (Sari et al., 2020). Notably, none of the participants described mathematical connections as connections between representations. Moreover, the participants reflected more than a specific view of the nature of mathematics, adding that the interconnected structure of mathematics is instrumental to the teaching and learning of the subject (Amirali & Halai, 2010; Luneta & Giannakopoulos, 2019; Ndlovu et al., 2019).

Participants were asked to explain a fraction division problem $\mathbf{5\frac{3}{4} \div \frac{3}{2}}$ using mathematical connections. Their perceptions of mathematical connections were reflected in their illustrations, suggesting connections between concepts (**Figure 4**). They indicated that learners' understanding of number operations is crucial to solving fraction problems, suggesting links between mathematical concepts. Their explanations included changing a mixed number to an improper fraction, using reciprocals and inverse operations in the division problems, and understanding operations with integers.

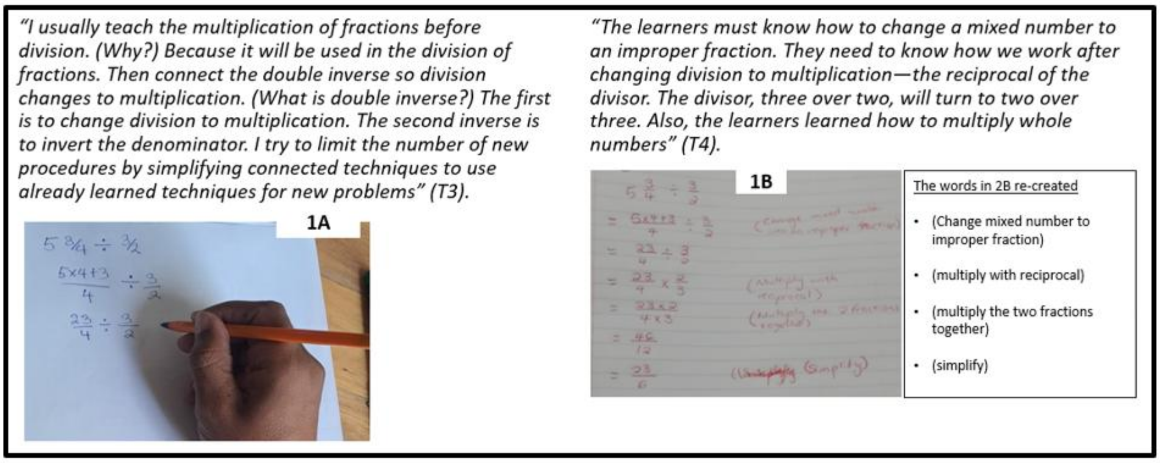

Figure 4. Teachers illustrating and explaining a fraction division problem (Source: Authors' own elaboration)

Figure 4 provides information on teachers' illustrations of dividing fractions. The teachers suggested that learners must know certain concepts related to the division of fractions before learning the concept.

Tchoshanov (2011, pp. 142-143) notes that:

> If a teacher can recall a rule for fraction division or solve simple fraction division problems 1 3/4÷1/2=, then we say that she has procedural knowledge of fraction division. (To) solve 1 3/4÷1/2= in more than one way (e.g., draw a diagram or illustrate it with manipulative) or "Make up a story for the given fraction division problem" requires conceptual rather than only procedural knowledge.

Based on the above suggestion, questions six and seven were included to document teachers' knowledge of using and connecting multiple representations such as stories and diagrams when teaching fraction division in junior secondary. The interviewer requested the teachers to provide an alternative option to a fraction division expression and justify their answers: 1 1/4÷1/2 choosing from multiple choice options:

(A) You want to split 1 1/4 pies evenly between two families. How much should each family get?

(B) You have R1,25 and may soon double your money. How much money would you end up with?

(C) You are making some homemade taffy, and the recipe calls for 1 1/4 cups of butter. How many sticks of butter will you need (if each stick =1/2 cups)?

Excerpt 1

T1: That first one talks about dividing by two, so it is not the answer. Then I think in the second one, when it says double, it should be multiplication, not division. C is the correct one.

T4: No, it is not the first one. Because the first one will be divided into two, B is the one.

T6: [It is] A. The first explains a situation where one and a quarter pie must be split or shared equally among two families; sharing equally by two means they both get half of the pie.

There were no common responses (2As, 2Bs, 2Cs). The expected answer is C. This suggests that approximately 70% of the participants needed help to identify the correct story of the fraction problem. Regarding visual representations, the teachers were asked to draw a diagram that could assist learners in making sense of a fraction division problem, 6/7÷1/3. All the participants attempted this problem except T3 (Excerpt 2). T4 suggested using a pie diagram to illustrate a division problem. She divided the pie into seven equal parts and shaded six parts (2C, **Figure 5**). Then, she drew three more pies, each divided into seven parts, with six shaded in each pie. She then determined 18 out of 7 shaded parts in total. Similarly, T5 demonstrated an understanding of representing the multiplication of fractions by the diagram (2D, **Figure 5**). They converted the given problem to an equivalent multiplication and drew a diagram for the multiplication version. Some teachers could not provide correct diagrams (**Figure 6**).

Some teachers could not provide relevant diagrams (**Figure 6**), and T3 expressed that he was not accustomed to teaching the general education and training (GET) phase, which consists of the junior secondary classrooms (Excerpt 2). T3 had more experience teaching higher grades (FET), where they focus on algebraic fractions.

Excerpt 2

T3: I am not sure how to answer this. I do not usually teach GET. I just started about two years back. I am used to the higher Grades. This question is tricky for me, as I highlighted to you

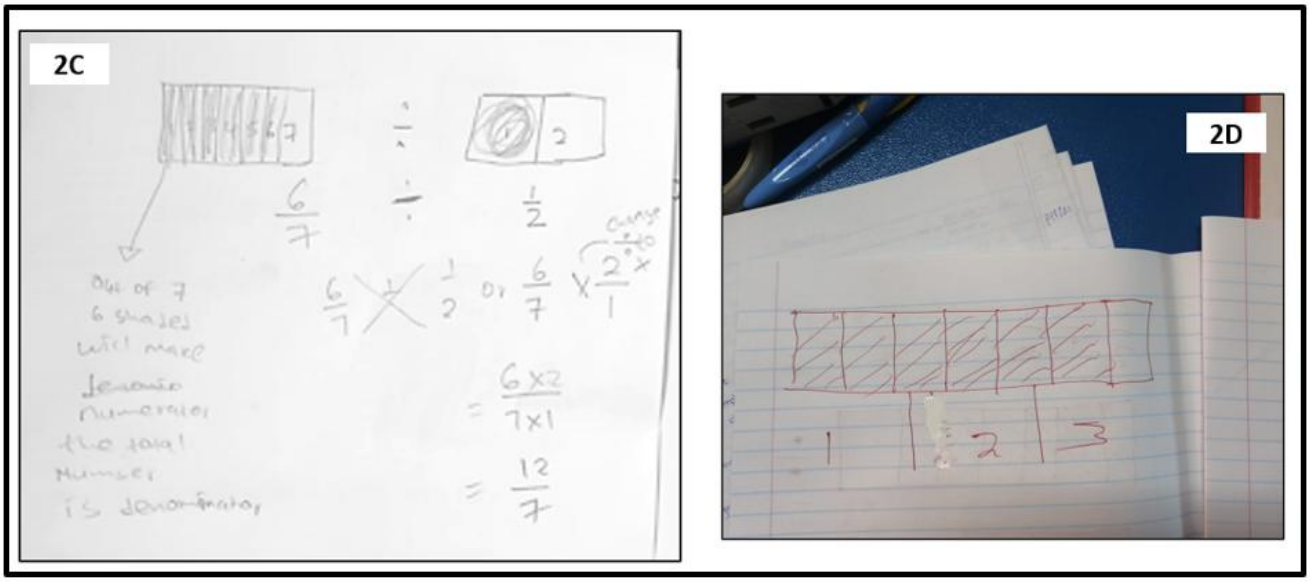

Figure 5. T4 and T5 represent a fraction division problem in diagrams (Source: Authors' own elaboration)

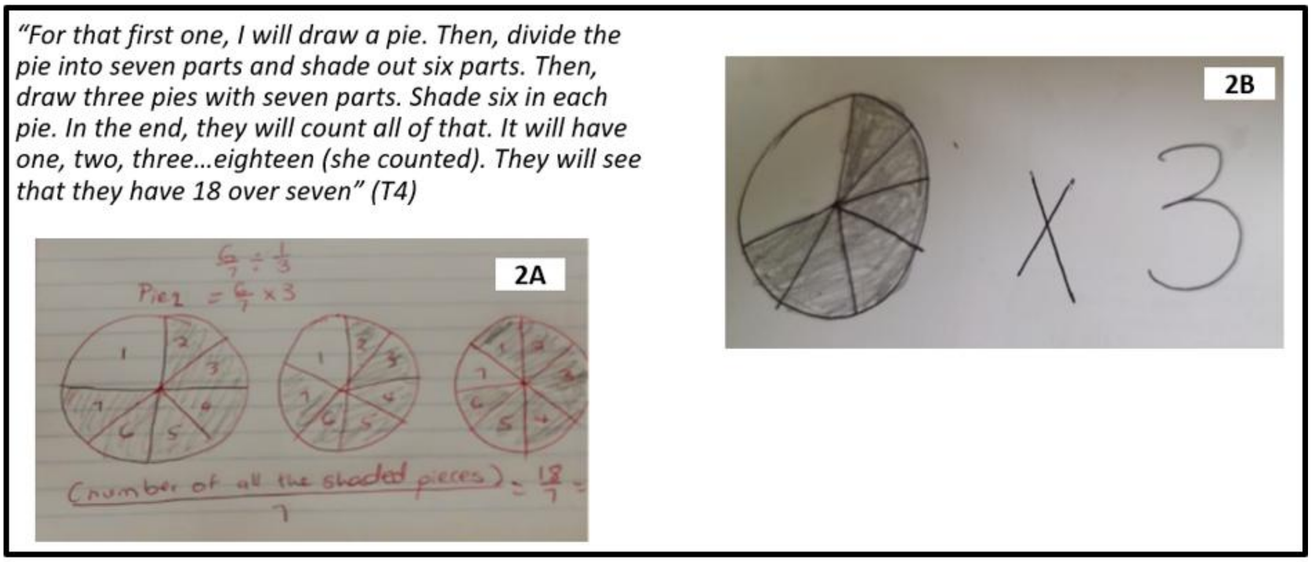

Figure 6. T1 and T6 represent a fraction division problem in diagrams (Source: Authors' own elaboration)

> the other day. The diagrams are a weakness for me. I am used to working with learners who can solve algebraic fractions. So going back to that level is not easy. It is a skill that I also need to learn. To be able to relate numerical fractions in that way.

T3's responses highlight his awareness of his limitations in using visual representations in mathematics teaching. This suggests the importance of teachers' continuous professional development in acquiring the necessary skills for the quality teaching of fractions to enhance learners' mathematical understanding. Despite the participants' limited knowledge of connecting fraction divisions to diagrams, they believe it fosters quality mathematics teaching and learning.

Establishing Strong Foundations for Mathematical Concepts

The participants expressed their views about the role of mathematical connections in teaching and the teacher's responsibilities in facilitating these connections, and they acknowledged their perceptions of learners' awareness of mathematical connections. The teachers emphasised the importance of building a new concept on prior knowledge when introducing new mathematical ideas to learners. They believed that accessing a new concept through learners' prior knowledge facilitates profound mathematical understanding.

Excerpt 3

Interviewer: Why do you use mathematics interconnectedness in teaching mathematics?

T4: Sometimes, certain topics force you to use interconnectedness. Because you can only learn another topic thoroughly after knowing certain topics before it. That is how the curriculum, or the ATP is based. It tries to make you teach a certain topic before the other because you can only teach that topic if you know the topics before it. So, I use interconnectedness to help learners understand another topic. Mathematics works together as a whole.

T5: I use mathematics interconnectedness in teaching because it assists learners in understanding new concepts much better. Because with mathematics interconnectedness, you would be moving them from concepts they are familiar with to concepts they may not be familiar with. So, you would teach a new concept starting with ideas that learners understand or are familiar with. For instance, it is better in mathematics to start with certain topics instead of others because the others that follow would be drawing from the concepts that learners already understand. It is the teacher's responsibility to make it more interesting for learners. So, suppose the teacher keeps reminding learners and showing them that the concepts we are doing are connected to a particular concept they have already learned. Learners will find the concept more interesting. They would also notice how topics interconnect with one another. We sometimes overlook that.

The participants believe mathematical connections enhance learners' mathematical understanding by building upon prior knowledge. They emphasised that it is the teachers' responsibility to help learners become aware of and utilise mathematical connections. However, the participants acknowledged challenges impacting their ability to deliver on this role effectively, which causes their learners to lack adequate understanding. Some challenges they highlighted include CAPS's heavy load and limited instruction time for teachers to teach effectively.

Mathematical Connections in the Curriculum Impact Teaching

The participants used the terms ATP (annual teaching plan) and CAPS interchangeably. The ATP is a micro version of CAPS, which provides a school term's curriculum breakdown. The DBE usually subjects the ATP to an abridged version to meet specific situations that may affect regular schooling, such as during COVID-19. Some participants perceived that CAPS documents are a valuable resource for JSMTs. T2 explained that the ATP teaches the interconnectedness of whole numbers, factors, integers, and fractions, believing it is essential to learn these concepts before going into fractions. The CAPS sequencing does not seem to be a problem for any of the participants but suggests making the use of the mathematical connections easier. The participants' views indicated that CAPS documents reflected the nature of mathematics in terms of mathematical connections.

Excerpt 4

T4: The ATP makes it easier for the learners to understand the ATP puts the topics (ideas) that would help in the topics before. Such a topic has something to do with the first one. And the second one has to do with the third one. Everything is interconnected.

T2: I like how the ATP is. It puts the interconnectedness. …was smart enough to understand that learners must first learn numbers, factors, and integers before going into fractions. I like the way ATP is. If I were to moderate the ATP, I would say kudos to the creator of the ATP.

This study found that teachers noticed mathematical connections in the CAPS documents. This perception aligns with the highlight of existing research that CAPS provides teachers with a clear understanding of mathematical connections, assisting them in establishing meaningful connections (Brown, 2014; Galant, 2013). However, some participants talked about the inadequacies regarding time allocation compared to the amount of content they were supposed to teach.

T3: The ATP is heavily loaded. There is not enough time to cover certain concepts and use other techniques.

T3 mentioned that using mathematical connections efficiently requires more time, especially given the content that needs to be taught. This view resonates with existing studies regarding insufficient instruction time

(Clark & Linn, 2003; Manouchehri & Goodman, 1998) and the lack of conceptual tasks to use mathematical connections (Remillard & Kim, 2017).

Excerpt 5

> T5: Considering if learners find $\mathbf{5\frac{3}{4}} \div \frac{3}{2}$ too easy, what more complex division problem will I give them? I realised that the ATP does not consider mathematical connections under fractions. Since the learners have learned about squares, square roots, cubes and cube roots, I may create a question to include them. That would be the complex question of fractions to give learners who understand that division problem. So that would be a perfect example of seeing mathematics interconnectedness more often. It would be more interesting for learners had CAPS included such concepts.

The teachers' perceptions suggest that while CAPS provides an order of contents for teachers to use, it leaves them to select and arrange suitable tasks for teaching. The teachers demanded that the curriculum support the teachers with conceptual tasks for teaching.

DISCUSSION

This study aims to understand the JSMTs' perceptions of mathematical connections in the curriculum and teaching to support them in improving the quality of their teaching. This study's participants' perceptions, that mathematical connections foster quality mathematics teaching, align with several prior studies (Basir et al., 2022; Getenet & Callingham, 2017; Pambudi et al., 2020); consequently, enhancing learners' conceptual knowledge and problem-solving skills and enabling them to make informed decisions (Basir et al., 2022). In addition, the teachers highlighted the importance of mathematical connections in building a solid foundation for mathematical concepts (Basir et al., 2022; Koponen et al., 2019). Teachers play a crucial role in promoting mathematical connections during teaching; therefore, investing in professional development can enhance the quality of their classroom practices.

Furthermore, the teachers highlighted the importance of making learners aware of opportunities for connection and how concepts link, consistent with the NCTM. However, these teachers acknowledged that they often neglect to show connections. This study assumes this might be due to the low quality of teacher knowledge (Brijlall et al., 2011; Galant, 2013). Despite South African teachers' awareness of the crucial role of mathematical connections, a notable discrepancy exists in junior secondary learners' ability to recognise and effectively use mathematical connections, as reflected in their poor performance in mathematics. International Mathematics and Science Study (TIMSS) reports prove that teaching mathematics remains challenging in the South African schooling system, and mathematics teachers are experiencing increased stress and anxiety levels (Meeran & Van Wyk, 2022). Nevertheless, Askew et al. (2019) warn that South African teachers must bridge the gap between their awareness of mathematical structure and its implementation.

The teachers in this study attributed the barriers to the effective mathematical connections to the CAPS, which they perceived has failed to provide them with the necessary effective and adequate instructional tools to support the teaching of mathematical connections to their learners. The challenges they perceived align with previous studies. For example, Mokotjo and Mokhele-Makgalwa (2021) found that South African teachers had challenges integrating GeoGebra in teaching mathematics. Galant (2013) opined that a lack of exposure to a variety of cognitive resources and strategies for teaching may hinder effective teaching and result in learners' weak understanding of mathematics. The challenge highlighted in this study regarding insufficient instructional time aligns with Clark and Linn (2003), which revealed that reducing instructional time diminishes learner knowledge integration. This study noted teachers' limited knowledge of teaching with different representations. The participants of this study primarily viewed mathematical connections as a connection between concepts, and approximately 70% could not represent the concept visually. This study assumed that Tshwane's ATP's emphasis on using prior concepts in teaching influenced the teachers' perceptions of mathematical connections, suggesting the need to address the shortcomings of the CAPS curriculum and provide teachers with the necessary resources to teach effectively. In addition, this study suggests investing in professional development programs for mathematics teachers to enhance their content knowledge and pedagogical skills, which becomes crucial to empower JSMTs in the Tshwane region to integrate mathematical connections effectively.

The findings of this study emerged inductively from a thematic analysis; hence, teachers' perceptions were not viewed from an ETC's perspective. The teachers recognised mathematical connections as a feature of mathematics and pedagogical practices involving linking mathematical concepts (intra-mathematical) and mathematics with other school subjects or real-life situations (extra-mathematical) which is supported by prior studies (Cantillo-Rudas et al., 2024; Ledezma et al., 2024; Rodríguez-Nieto et al., 2023, 2025). Teachers understand these connections as relating concepts, meanings, and representations (Ledezma et al., 2024; Rodríguez-Nieto et al., 2023, 2025). Furthermore, they view mathematical connections as a cognitive process that facilitates the integration of concepts, other disciplines and real-life contexts (Cantillo-Rudas et al., 2024; Rodríguez-Nieto et al., 2023). The integration of mathematical connections proposed by the teachers is evident in the STEAM approach, highlighting the inherent relationship between mathematics and science, technology, engineering, and art (Rodríguez-Nieto & Alsina, 2022; Rodríguez-Nieto & Escobar-Ramírez, 2022). Mathematical modelling is a prime example of extra-mathematical connections that bridge students' mathematical knowledge with real-world problem-solving (Ledezma et al., 2024). While the teachers believe mathematical connections in the curriculum and the accompanying instructions influence how they integrate mathematical connections in teaching, empirical evidence to substantiate this claim remains limited. However, existing studies highlight the need for detailed analysis and research to address challenges of integrating mathematical connections and improve the teaching and learning of mathematical concepts through collaborative research (Cantillo-Rudas et al., 2024) and curricular approach (Olivero-Acuña et al., 2025).

CONCLUSION

The study revealed that teachers' understanding of mathematical connections is limited; it lacks the inclusion of different representations of a concept. The teachers' perceptions that mathematical connections involve scaffolding and result in improved mathematical knowledge are consistent with previous studies. They opined that the CAPS impedes their ability to implement mathematical connections to achieve the desired goal. Past studies have shown that teachers' personal beliefs often create a gap between their curriculum knowledge and teaching practices (Amirali & Halai, 2010; Stols et al., 2015). This study found that teachers' knowledge, perceptions, and access to curriculum resources are the three main factors that enable effective mathematical connections in teaching among the JSMTs. These findings align with the previous (Schoenfeld, 2020) emphasising how teachers' knowledge and beliefs shape their teaching practices. This study agrees that teachers' beliefs are dynamic and can be changed to accommodate the required knowledge and practices (Manouchehri & Goodman, 1998) and recommends that teacher professional development programs include these factors.

This study experienced some limitations. First, the data primarily relies on semi-structured interviews of a small sample size of only six teachers, and the context is confined to fraction concepts. Second, the data collection process was inconsistent due to COVID-19 constraints, which made some participants being interviewed virtually on WhatsApp rather than in-person. These factors may have impacted the quality of the data collected, thus restricting the generalisability of the findings to other contexts or populations. This study anticipates that future research on teachers' perceptions of mathematical connections should expand the sample size, include teachers from diverse environments and explore mathematical concepts beyond fractions. In addition, future research should employ a variety of data collection sources to provide a better understanding of teachers' perceptions on the topic. Lastly, future research should delve into teachers' perceptions of mathematical connections as there is currently a scarcity of this kind. This would help provide valuable insights into how teachers integrate mathematical connections in their teaching practices.

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Ethical statement: The authors stated that both the participating professors and the university were informed that this article is for educational purposes, not economic or political purposes, and that they agreed to participate voluntarily. Furthermore, the first author holds the perception data available to them. The second author is the doctoral advisor of the first author and is an expert in the subject. Furthermore, Luneta was supervisor of the main project. The third author is an expert in mathematical connections who validates the work. The authors of this work are authorized to conduct research with students to improve their understanding of mathematical concepts.

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