

Extend TOPSIS-Based Two-Sided Matching Decision in Incomplete Indifferent Order Relations Setting Considering Matching Aspirations

Bingwen Yu¹, Qi Yue^{2*}

¹ School of Economics, Jiangxi University of Finance and Economics, Nanchang, CHINA

² School of Information Management, Jiangxi University of Finance and Economics, Nanchang, CHINA

Received 24 June 2017 • Revised 20 September 2017 • Accepted 8 October 2017

ABSTRACT

This paper develops a method for two-sided matching decision in the environment of incomplete indifferent order relations. The two-sided matching decision problem with incomplete indifferent order relations and matching aspirations is firstly described. In order to solve this problem, the incomplete indifferent order relations are converted into the generalized Borda number matrices. The matching aspiration matrix can be determined based on the model calculation on the reciprocal differences of generalized Borda numbers. On this basis, the weighted satisfaction degree matrices are set up. The extended relative closeness matrices are determined by using an extended TOPSIS technique. Moreover, a two-sided matching model is developed. The two-sided matching alternative can be obtained by solving the model. For the purpose of illustration, an example including sensitivity analysis is presented.

Keywords: two-sided matching decision, incomplete indifferent order relation, matching aspiration, extend TOPSIS, model

INTRODUCTION

The two-sided matching decision involves how to match the agents of one side with the agents of the other side based on the preferences of the agents of both sides. The problems of two-sided matching decision exist widely in reality, such as stable marriage assignment (Kümmel et al., 2016; Cseh and Manlove, 2016; Doğan and Yıldız, 2016), college admission (Braun et al., 2014; Chen and Kao, 2014; Liu and Peng, 2015), employee selection (Wang et al., 2011; Mendes et al., 2010; Chen et al., 2016), and personnel assignment (Gallego and Larrain, 2012; Taylor, 2013; Gharote et al., 2015). Therefore, the two-sided matching decision is a hot topic with extensive actual backgrounds.

Gale and Shapley (1962) initially research the problems of college admissions and marriage. In their studies, the concept of stable matching is proposed; then the existence and optimality of stable matching are given; at last, the deferred acceptance algorithm is developed. From then on, various different concepts, theories, techniques and algorithms have been presented with respect to the two-sided matching decision with different formats of information. For example, Li and Fan (2014) propose a stable two-sided matching method considering psychological behavior of agents on both sides to solve the two-sided matching problem with ordinal numbers. Castillo and Dianat (2016) study the truncation strategies in a centralized matching clearinghouse based on the deferred acceptance algorithm. Xu et al. (2015) propose the matching algorithms for one-to-one two-sided dynamic service markets. Chen et al. (2016) point out that the generalized median stable matchings exist in many-to-many two-sided matching markets when contracts are strong replaceable and satisfy the law of aggregate demand. Liang et al. (2015) propose a novel decision analysis method to solve the multiple target of satisfied and stable two-sided matching decision problem considering the preference ordering, where the targets could be satisfied, weak satisfied and stable, α -satisfied and stable, satisfied and stable.

The existing studies enrich the theories of two-sided matching decision, and develop the different algorithms for solving the problems of two-sided matching decision with various formats of information, and expand the actual application background. However, on the one hand, the preferences provided by agents of two sides may

Contribution of this paper to the literature

- Rather than making a contribution to the relevant theories of stable two-sided matching, the focus of this study is to obtain the two-sided matching alternative, which can reflect the matching aspirations of agents.
- This study tries to use the generalized Borda numbers for handling incomplete indifferent order relations.
- The findings of this study can enhance our understanding of an extend TOPSIS technology in two-sided matching decision.

be in the format of incomplete indifferent order relations in some practical problems owing to imprecise source of information containing unquantifiable information and incomplete information. In this case, the classical two-sided matching decision cannot effectively deal with these kinds of problems. On the other hand, the matching aspirations of agents of two sides are seldom considered in the existing studies. Therefore, how to investigate the problem of two-sided matching decision with incomplete indifferent order relations considering matching aspirations is a valuable research topic. In view of this, this paper presents a two-sided matching decision method with incomplete indifferent order relations considering matching aspirations based on an extended TOPSIS (technique for order performance by similarity to an ideal solution) method. It is well known that, the TOPSIS method is first developed by Hwang and Yoon (1981), and is one of the classical multi-attribute decision methods. The basic idea of TOPSIS method is that the selected alternative should have the shortest distance from the positive ideal solution and the farthest distance from the negative ideal solution (Hwang and Yoon, 1981; Yue, 2014). This article intends to apply the idea of TOPSIS into two-sided matching decision with incomplete indifferent order relations information.

The structure of this paper is organized as follows: Section 2 formulates the considered two-sided matching problem. Section 3 presents an extend TOPSIS-based method for two-sided matching decision. Section 4 gives an example. Section 5 concludes this paper.

THE CONSIDERED TWO-SIDED MATCHING PROBLEM

This paper considers the two-sided matching problem, where the preferences provided by the agents of two sides are in the format of incomplete indifferent order relations. And the research angle employed in this paper is matching aspiration. The notation of the considered two-sided matching problem is given as follows.

Let $\partial = \{\partial_1, \partial_2, \dots, \partial_p\}$ ($p \geq 2$) be the set of agents of one side, where ∂_i represents the i th agent of side ∂ , $i \in P = \{1, 2, \dots, p\}$; Let $\wp = \{\wp_1, \wp_2, \dots, \wp_q\}$ ($q \geq p$) be the set of agents of the other side, where \wp_j represents the j th agent of side \wp , $j \in Q = \{1, 2, \dots, q\}$; Moreover, let $\wp_{i_1} > (\sim) \wp_{i_2} > (\sim) \dots > (\sim) \wp_{i_{n_i^\wp}}$ be the incomplete indifferent order relation given by agent ∂_i , where n_i^\wp represents the number of agents of side \wp in $\wp_{i_1} > (\sim) \wp_{i_2} > (\sim) \dots > (\sim) \wp_{i_{n_i^\wp}}$, $\{i_1, i_2, \dots, i_{n_i^\wp}\} \subseteq Q$, $i_1 \neq i_2 \neq \dots \neq i_{n_i^\wp}$; Let $\partial_{j_1} > (\sim) \partial_{j_2} > (\sim) \dots > (\sim) \partial_{j_{n_j^\partial}}$ be the incomplete indifferent order relation given by agent \wp_j , where n_j^∂ denotes the number of agents of side ∂ in $\partial_{j_1} > (\sim) \partial_{j_2} > (\sim) \dots > (\sim) \partial_{j_{n_j^\partial}}$, $\{j_1, j_2, \dots, j_{n_j^\partial}\} \subseteq P$, $j_1 \neq j_2 \neq \dots \neq j_{n_j^\partial}$; Here, symbol " $>$ " or " \sqsupset " denotes "superior to" or "be equivalent to"; Let a_{ij} be the matching aspiration between ∂_i and \wp_j , which usually satisfies the characteristic of non-negativity and normalization.

Remark 1. In above presentation, the matching aspiration a_{ij} is unknown. The determination method will be given in section 3.2.

Remark 2. The concept of two-sided matching can be seen by reference (Yue, 2014). Then we know that a two-sided matching (or a two-sided matching alternative) ρ can be expressed by the union of the set of matching pair ρ_{Two} and the set of single pair ρ_{One} .

Based on the above analysis, the problem researched here is how to obtain the reasonable two-sided matching alternative ρ based on the incomplete indifferent order relations $\wp_{i_1} > (\sim) \wp_{i_2} > (\sim) \dots > (\sim) \wp_{i_{n_i^\wp}}$ ($i \in P$) and $\partial_{j_1} > (\sim) \partial_{j_2} > (\sim) \dots > (\sim) \partial_{j_{n_j^\partial}}$ ($j \in Q$), and the matching aspirations a_{ij} ($i \in P, j \in Q$).

THE PROPOSED TWO-SIDED MATCHING METHOD

Construction of the Normalized Borda Matrices

In order to handle with the incomplete indifferent order relations, the definitions of the generalized Borda numbers are introduced.

Definition 1. Let $g_{ij}^{\partial \rightarrow \wp}$ be the generalized Borda number of ∂_i over \wp_j , then $g_{ij}^{\partial \rightarrow \wp}$ can be calculated as follows:

1) if $j = i_k$, and $j' \neq i_k$ for all $j' \in \{i_1, i_2, \dots, i_{n_i^\wp}\} \setminus \{j\}$, then

$$g_{ij}^{\partial \rightarrow \wp} = n_i^\wp + 1 - k \tag{1a}$$

2) if $Q_{b_1} \sim Q_{b_2} \sim \dots \sim Q_{b_s} \sim Q_{i_k}$, and $Q_{j'} \succ Q_{i_k}$ for all $j' \in \{i_1, i_2, \dots, i_{n_i^\wp}\} \setminus \{b_1, b_2, \dots, b_s\}$, then

$$g_{ib_1}^{\partial \rightarrow \wp} = g_{ib_2}^{\partial \rightarrow \wp} = \dots = g_{ib_s}^{\partial \rightarrow \wp} = n_i^\wp + 1 - k - (s - 1)/2 \tag{1b}$$

3) if $j \notin \{i_1, i_2, \dots, i_{n_i^\wp}\}$, then

$$g_{ij}^{\partial \rightarrow \wp} = \vartheta \tag{1c}$$

where symbol “ ϑ ” represents “it doesn’t exist”.

By Eqs. (1a)-(1c), the incomplete indifferent order relations $\wp_{i_1} > (\sim)\wp_{i_2} > (\sim) \dots > (\sim)\wp_{i_{n_i^\wp}}$ ($i \in P$) can be transformed into the generalized Borda number matrix $G_{\partial \rightarrow \wp} = [g_{ij}^{\partial \rightarrow \wp}]_{p \times q}$.

Due to the different dimension of n_i^\wp ($i \in P$), the generalized Borda number $g_{ij}^{\partial \rightarrow \wp}$ should be normalized (noted as $\bar{g}_{ij}^{\partial \rightarrow \wp}$), which can be calculated by

$$\bar{g}_{ij}^{\partial \rightarrow \wp} = \begin{cases} \frac{g_{ij}^{\partial \rightarrow \wp}}{n_i^\wp}, & j \in \{i_1, i_2, \dots, i_{n_i^\wp}\} \\ \vartheta, & j \notin \{i_1, i_2, \dots, i_{n_i^\wp}\} \end{cases} \tag{2}$$

By Eq. (2), the generalized Borda number matrix $G_{\partial \rightarrow \wp} = [g_{ij}^{\partial \rightarrow \wp}]_{p \times q}$ can be transformed into the normalized Borda matrix $\bar{G}_{\partial \rightarrow \wp} = [\bar{g}_{ij}^{\partial \rightarrow \wp}]_{p \times q}$.

Definition 2. Let $g_{ij}^{\wp \rightarrow \partial}$ be the generalized Borda number of \wp_j over ∂_i , then $g_{ij}^{\wp \rightarrow \partial}$ can be calculated as follows:

1) if $i = j_l$, and $i' \neq j_l$ for all $i' \in \{j_1, j_2, \dots, j_{n_j^\wp}\} \setminus \{i\}$, then

$$g_{ij}^{\wp \rightarrow \partial} = n_j^\wp + 1 - l \tag{3a}$$

2) if $\partial_{a_1} \sim \partial_{a_2} \sim \dots \sim \partial_{a_r} \sim \partial_{j_l}$, and $\partial_{i'} \succ \partial_{j_l}$ for all $i' \in \{j_1, j_2, \dots, j_{n_j^\wp}\} \setminus \{a_1, a_2, \dots, a_r\}$, then

$$g_{a_1 j}^{\wp \rightarrow \partial} = g_{a_2 j}^{\wp \rightarrow \partial} = \dots = g_{a_r j}^{\wp \rightarrow \partial} = n_j^\wp + 1 - l - (r - 1)/2 \tag{3b}$$

3) if $i \notin \{j_1, j_2, \dots, j_{n_j^\wp}\}$, then

$$g_{ij}^{\wp \rightarrow \partial} = \vartheta \tag{3c}$$

where symbol “ ϑ ” represents “it doesn’t exist”.

By Eqs. (3a)-(3c), the incomplete indifferent order relations $\partial_{j_1} > (\sim)\partial_{j_2} > (\sim) \dots > (\sim)\partial_{j_{n_j^\wp}}$ ($j \in Q$) can be transformed into the generalized Borda number matrix $G_{\wp \rightarrow \partial} = [g_{ij}^{\wp \rightarrow \partial}]_{p \times q}$.

Due to the different dimension of n_j^\wp ($j \in Q$), the generalized Borda number $g_{ij}^{\wp \rightarrow \partial}$ should be normalized (noted as $\bar{g}_{ij}^{\wp \rightarrow \partial}$), which can be calculated by

$$\bar{g}_{ij}^{\wp \rightarrow \partial} = \begin{cases} \frac{g_{ij}^{\wp \rightarrow \partial}}{n_j^\wp}, & i \in \{j_1, j_2, \dots, j_{n_j^\wp}\} \\ \vartheta, & i \notin \{j_1, j_2, \dots, j_{n_j^\wp}\} \end{cases} \tag{4}$$

By Eq. (4), the generalized Borda number matrix $G_{\wp \rightarrow \partial} = [g_{ij}^{\wp \rightarrow \partial}]_{p \times q}$ can be transformed into the normalized Borda matrix $\bar{G}_{\wp \rightarrow \partial} = [\bar{g}_{ij}^{\wp \rightarrow \partial}]_{p \times q}$.

Determination of the Matching Aspirations

In order to determine the matching aspiration a_{ij} , the following analysis combined with the absolute difference $|\bar{g}_{ij}^{\partial \rightarrow \wp} - \bar{g}_{ij}^{\wp \rightarrow \partial}|$ is given.

Remark 3. Although the following analysis is given when $|\bar{g}_{ij}^{\partial \rightarrow \wp} - \bar{g}_{ij}^{\wp \rightarrow \partial}| \neq \vartheta$ for all $i \in P, j \in Q$, the similar conclusion can be obtained when some of them don’t exist.

If $|\bar{g}_{ij}^{\partial \rightarrow \wp} - \bar{g}_{ij}^{\wp \rightarrow \partial}|$ becomes greater and greater, then the absolute difference of the satisfaction degree between ∂_i and \wp_j also becomes greater and greater. In this situation, the success rate of matching ∂_i with \wp_j will reduce, in other word, a_{ij} will become smaller. Similarly, the smaller $|\bar{g}_{ij}^{\partial \rightarrow \wp} - \bar{g}_{ij}^{\wp \rightarrow \partial}|$ is, the greater the success rate of matching ∂_i with \wp_j will be. Corresponding, a_{ij} should be greater. Furthermore, in order to enhance the success rate of two-sided matching in reality, the further analysis of reciprocal difference $\frac{1}{|\bar{g}_{ij}^{\partial \rightarrow \wp} - \bar{g}_{ij}^{\wp \rightarrow \partial}|}$ is given below. If $\frac{1}{|\bar{g}_{ij}^{\partial \rightarrow \wp} - \bar{g}_{ij}^{\wp \rightarrow \partial}|}$ is greater (i.e., $|\bar{g}_{ij}^{\partial \rightarrow \wp} - \bar{g}_{ij}^{\wp \rightarrow \partial}|$ is smaller), then the success rate of matching ∂_i with \wp_j will be greater. Corresponding, a_{ij} should be greater, and vice versa. Based on the above idea, the total reciprocal difference (noted as $R_{\partial \leftrightarrow \wp}$) between ∂_i ($i \in P$) and \wp_j ($j \in Q$) can be represented by

$$R_{\partial \leftrightarrow \wp} = \sum_{i=1}^p \sum_{j=1}^q \frac{a_{ij}}{|\bar{g}_{ij}^{\partial \rightarrow \wp} - \bar{g}_{ij}^{\wp \rightarrow \partial}|} \tag{5}$$

Remark 4. In Eq. (5), the case of $|\bar{g}_{ij}^{\partial \rightarrow \wp} - \bar{g}_{ij}^{\wp \rightarrow \partial}| = 0$ may sometimes occur. At this time, Eq. (5) is meaningless.

In order to handle with this case, the denominator $|\bar{g}_{ij}^{\partial \rightarrow \wp} - \bar{g}_{ij}^{\wp \rightarrow \partial}|$ can be replaced by $b^{|s_{a_{ij}} - s_{b_{ij}}|}$, $b > 1$.

According to Remark 3, Eq. (5) can be further expressed by

$$R_{\partial \leftrightarrow \wp} = \sum_{i=1}^p \sum_{j=1}^q \frac{a_{ij}}{b^{|\bar{g}_{ij}^{\partial \rightarrow \wp} - \bar{g}_{ij}^{\wp \rightarrow \partial}|}} \tag{6}$$

Based on the above analysis, the selection of a_{ij} should make $R_{\partial \leftrightarrow \wp}$ greatest. Therefore, the objective function is established as

$$\max R_{\partial \leftrightarrow \wp} = \sum_{i=1}^p \sum_{j=1}^q \frac{a_{ij}}{b^{|\bar{g}_{ij}^{\partial \rightarrow \wp} - \bar{g}_{ij}^{\wp \rightarrow \partial}|}} \tag{7}$$

Moreover, the following linear programming model (M-1) can be constructed, i.e.,

$$(M-1) \begin{cases} \max R_{\partial \leftrightarrow \wp} = \sum_{i=1}^p \sum_{j=1}^q \frac{a_{ij}}{b^{|\bar{g}_{ij}^{\partial \rightarrow \wp} - \bar{g}_{ij}^{\wp \rightarrow \partial}|}}, \\ \text{s. t. } \sum_{i=1}^p \sum_{j=1}^q a_{ij}^2 = 1; a_{ij} \in (0, 1), i \in P, j \in Q \end{cases}$$

Theorem 1. The optimal solution (noted as a_{ij}^*) of model (M-1) is expressed by

$$a_{ij}^* = \frac{1}{b^{|\bar{g}_{ij}^{\partial \rightarrow \wp} - \bar{g}_{ij}^{\wp \rightarrow \partial}|} \sqrt{\sum_{i=1}^p \sum_{j=1}^q \frac{1}{b^{2|\bar{g}_{ij}^{\partial \rightarrow \wp} - \bar{g}_{ij}^{\wp \rightarrow \partial}|}}} \tag{8}$$

Proof. Firstly, let Lagrange function $L = \sum_{i=1}^p \sum_{j=1}^q \frac{a_{ij}}{b^{|\bar{g}_{ij}^{\partial \rightarrow \wp} - \bar{g}_{ij}^{\wp \rightarrow \partial}|}} - \frac{\lambda}{2} (\sum_{i=1}^p \sum_{j=1}^q a_{ij}^2 - 1)$. Then the partial derivatives of function L with respect to variables a_{ij} and λ can be computed, i.e., $\frac{\partial L}{\partial a_{ij}} = \frac{1}{b^{|\bar{g}_{ij}^{\partial \rightarrow \wp} - \bar{g}_{ij}^{\wp \rightarrow \partial}|}} - \lambda a_{ij}$, $\frac{\partial L}{\partial \lambda} = \sum_{i=1}^p \sum_{j=1}^q a_{ij}^2 - 1$. Let $\frac{\partial L}{\partial a_{ij}} = 0$ and $\frac{\partial L}{\partial \lambda} = 0$, then we have

$$a_{ij} = \frac{1}{\lambda b^{|\bar{g}_{ij}^{\partial \rightarrow \wp} - \bar{g}_{ij}^{\wp \rightarrow \partial}|}} \tag{9}$$

$$\sum_{i=1}^p \sum_{j=1}^q a_{ij}^2 = 1 \tag{10}$$

Substitute Eq. (9) into Eq. (10), we obtain

$$\lambda = \sqrt{\sum_{i=1}^p \sum_{j=1}^q \frac{1}{b^{2|\bar{g}_{ij}^{\partial \rightarrow \wp} - \bar{g}_{ij}^{\wp \rightarrow \partial}|}}} \tag{11}$$

Substitute Eq. (11) into Eq. (9), we know Eq. (8) holds. □

Remark 5. Considering the weights usually satisfy the condition of normalization, a_{ij}^* should be normalized.

Let $a'_{ij} = \frac{a_{ij}^*}{\sum_{i=1}^p \sum_{j=1}^q a_{ij}^*}$, then we obtain

$$a'_{ij} = \frac{1}{b^{|\bar{g}_{ij}^{\partial \rightarrow \wp} - \bar{g}_{ij}^{\wp \rightarrow \partial}|} \sum_{i=1}^p \sum_{j=1}^q \frac{1}{b^{|\bar{g}_{ij}^{\partial \rightarrow \wp} - \bar{g}_{ij}^{\wp \rightarrow \partial}|}} \tag{12}$$

Remark 6. When some of the absolute differences $|\bar{g}_{ij}^{\partial \rightarrow \wp} - \bar{g}_{ij}^{\wp \rightarrow \partial}|$ ($i \in P, j \in Q$) don't exist, the matching aspiration a''_{ij} can be recalculated by Eq. (12), i.e.,

$$a''_{ij} = \begin{cases} \frac{1}{b^{|\bar{g}_{ij}^{\partial \rightarrow \wp} - \bar{g}_{ij}^{\wp \rightarrow \partial}|} \sum_{i=1}^p \sum_{j=1}^q \frac{1}{b^{|\bar{g}_{ij}^{\partial \rightarrow \wp} - \bar{g}_{ij}^{\wp \rightarrow \partial}|}}, & \bar{g}_{ij}^{\partial \rightarrow \wp} \neq \vartheta, \bar{g}_{ij}^{\wp \rightarrow \partial} \neq \vartheta \\ \vartheta, & \bar{g}_{ij}^{\partial \rightarrow \wp} = \vartheta \text{ or } \bar{g}_{ij}^{\wp \rightarrow \partial} = \vartheta \end{cases} \tag{13}$$

By Eq. (13), the matching aspiration matrix $A'' = [a''_{ij}]_{p \times q}$ is set up.

Building of the Extended Relative Closeness Matrices

Based on the normalized Borda matrices $\bar{G}_{\partial \rightarrow \wp} = [\bar{g}_{ij}^{\partial \rightarrow \wp}]_{p \times q}$ and $\bar{G}_{\wp \rightarrow \partial} = [\bar{g}_{ij}^{\wp \rightarrow \partial}]_{p \times q}$, and the matching aspiration matrix $A'' = [a''_{ij}]_{p \times q}$, we consider using the extended TOPSIS method to built the relative closeness matrices.

According to the meanings of Borda numbers, we know that the greater $\bar{g}_{ij}^{\partial \rightarrow \wp}$ (or $\bar{g}_{ij}^{\wp \rightarrow \partial}$) is, the higher the satisfaction degree of agent ∂_i over \wp_j (or the satisfaction degree of agent \wp_j over ∂_i) is. Therefore, based on the normalized Borda matrix $\bar{G}_{\partial \rightarrow \wp} = [\bar{g}_{ij}^{\partial \rightarrow \wp}]_{p \times q}$ and the matching aspiration matrix $A'' = [a''_{ij}]_{p \times q}$, the weighted satisfaction degree matrix $S_{\partial \rightarrow \wp} = [s_{ij}^{\partial \rightarrow \wp}]_{p \times q}$ can be established, where

$$s_{ij}^{\partial \rightarrow \wp} = \begin{cases} a''_{ij} \bar{g}_{ij}^{\partial \rightarrow \wp}, & \bar{g}_{ij}^{\partial \rightarrow \wp} \neq \vartheta, \bar{g}_{ij}^{\wp \rightarrow \partial} \neq \vartheta \\ \vartheta, & \bar{g}_{ij}^{\partial \rightarrow \wp} = \vartheta \text{ or } \bar{g}_{ij}^{\wp \rightarrow \partial} = \vartheta \end{cases} \tag{14}$$

Based on the normalized Borda matrix $\bar{G}_{\wp \rightarrow \partial} = [\bar{g}_{ij}^{\wp \rightarrow \partial}]_{p \times q}$ and the matching aspiration matrix $A'' = [a''_{ij}]_{p \times q}$, the weighted satisfaction degree matrix $S_{\wp \rightarrow \partial} = [s_{ij}^{\wp \rightarrow \partial}]_{p \times q}$ can be established, where

$$s_{ij}^{\wp \rightarrow \partial} = \begin{cases} a''_{ij} \bar{g}_{ij}^{\wp \rightarrow \partial}, & \bar{g}_{ij}^{\wp \rightarrow \partial} \neq \vartheta, \bar{g}_{ij}^{\partial \rightarrow \wp} \neq \vartheta \\ \vartheta, & \bar{g}_{ij}^{\wp \rightarrow \partial} = \vartheta \text{ or } \bar{g}_{ij}^{\partial \rightarrow \wp} = \vartheta \end{cases} \tag{15}$$

With respect to weighted satisfaction degree matrix $S_{\partial \rightarrow \wp} = \begin{matrix} & \wp_1 & \wp_2 & \dots & \wp_q \\ \partial_1 & s_{11}^{\partial \rightarrow \wp} & s_{12}^{\partial \rightarrow \wp} & \dots & s_{1q}^{\partial \rightarrow \wp} \\ \partial_2 & s_{21}^{\partial \rightarrow \wp} & s_{22}^{\partial \rightarrow \wp} & \dots & s_{2q}^{\partial \rightarrow \wp} \\ \dots & \dots & \dots & \dots & \dots \\ \partial_p & s_{p1}^{\partial \rightarrow \wp} & s_{p2}^{\partial \rightarrow \wp} & \dots & s_{pq}^{\partial \rightarrow \wp} \end{matrix} \Big]_{p \times q}$, the positive ideal

point of side ∂ over \wp (noted as $s_{\partial \rightarrow \wp}^+ = (s_1^{+, \partial \rightarrow \wp}, s_2^{+, \partial \rightarrow \wp}, \dots, s_p^{+, \partial \rightarrow \wp})^T$) can be determined, where $s_i^{+, \partial \rightarrow \wp}$ is calculated by

$$s_i^{+, \partial \rightarrow \wp} = \max\{s_{i1}^{\partial \rightarrow \wp}, s_{i2}^{\partial \rightarrow \wp}, \dots, s_{iq}^{\partial \rightarrow \wp}\} = \max_{j \in Q}\{s_{ij}^{\partial \rightarrow \wp}\} \tag{16}$$

the negative ideal point of side ∂ over \wp (noted as $s_{\partial \rightarrow \wp}^- = (s_1^{-, \partial \rightarrow \wp}, s_2^{-, \partial \rightarrow \wp}, \dots, s_p^{-, \partial \rightarrow \wp})^T$) can be determined, where $s_i^{-, \partial \rightarrow \wp}$ is calculated by

$$s_i^{-, \partial \rightarrow \wp} = \min\{s_{i1}^{\partial \rightarrow \wp}, s_{i2}^{\partial \rightarrow \wp}, \dots, s_{iq}^{\partial \rightarrow \wp}\} = \min_{j \in Q}\{s_{ij}^{\partial \rightarrow \wp}\} \tag{17}$$

Moreover, based on the idea of TOPSIS, the positive distance matrix of side ∂ over \wp (noted as $D_{\partial \rightarrow \wp}^+ = [d_{ij}^{+, \partial \rightarrow \wp}]_{p \times q}$) can be determined, where positive distance $d_{ij}^{+, \partial \rightarrow \wp}$ is calculated by

$$d_{ij}^{+, \partial \rightarrow \wp} = \begin{cases} s_i^{+, \partial \rightarrow \wp} - s_{ij}^{\partial \rightarrow \wp}, & \bar{g}_{ij}^{\partial \rightarrow \wp} \neq \vartheta, \bar{g}_{ij}^{\wp \rightarrow \partial} \neq \vartheta \\ \vartheta, & \bar{g}_{ij}^{\partial \rightarrow \wp} = \vartheta \text{ or } \bar{g}_{ij}^{\wp \rightarrow \partial} = \vartheta \end{cases} \tag{18}$$

The negative distance matrix of side ∂ over \wp (noted as $D_{\partial \rightarrow \wp}^- = [d_{ij}^{-, \partial \rightarrow \wp}]_{p \times q}$) can be determined, where negative distance $d_{ij}^{-, \partial \rightarrow \wp}$ is calculated by

$$d_{ij}^{\partial \rightarrow \wp} = \begin{cases} s_{ij}^{\partial \rightarrow \wp} - s_i^{-, \partial \rightarrow \wp}, & \bar{g}_{ij}^{\partial \rightarrow \wp} \neq \vartheta, \bar{g}_{ij}^{\wp \rightarrow \partial} \neq \vartheta \\ \vartheta, & \bar{g}_{ij}^{\partial \rightarrow \wp} = \vartheta \text{ or } \bar{g}_{ij}^{\wp \rightarrow \partial} = \vartheta \end{cases} \quad (19)$$

Furthermore, the extended relative closeness matrix of side ∂ over \wp (noted as $C_{\partial \rightarrow \wp} = [c_{ij}^{\partial \rightarrow \wp}]_{p \times q}$) can be set up, where extended relative closeness $c_{ij}^{\partial \rightarrow \wp}$ is calculated by

$$c_{ij}^{\partial \rightarrow \wp} = \begin{cases} \frac{d_{ij}^{\partial \rightarrow \wp}}{d_{ij}^{+, \partial \rightarrow \wp} + d_{ij}^{-, \partial \rightarrow \wp}}, & \bar{g}_{ij}^{\partial \rightarrow \wp} \neq \vartheta, \bar{g}_{ij}^{\wp \rightarrow \partial} \neq \vartheta \\ \vartheta, & \bar{g}_{ij}^{\partial \rightarrow \wp} = \vartheta \text{ or } \bar{g}_{ij}^{\wp \rightarrow \partial} = \vartheta \end{cases} \quad (20)$$

Obviously, $c_{ij}^{\partial \rightarrow \wp} \in [0,1] \cup \vartheta$, and the greater $c_{ij}^{\partial \rightarrow \wp}$ is, the higher the satisfaction degree of agent ∂_i over \wp_j is.

Similarly, with respect to weighted satisfaction degree matrix $S_{\wp \rightarrow \partial} = \begin{matrix} & \wp_1 & \wp_2 & \dots & \wp_q \\ \begin{matrix} \partial_1 \\ \partial_2 \\ \dots \\ \partial_p \end{matrix} & \begin{bmatrix} s_{11}^{\wp \rightarrow \partial} & s_{12}^{\wp \rightarrow \partial} & \dots & s_{1q}^{\wp \rightarrow \partial} \\ s_{21}^{\wp \rightarrow \partial} & s_{22}^{\wp \rightarrow \partial} & \dots & s_{2q}^{\wp \rightarrow \partial} \\ \dots & \dots & \dots & \dots \\ s_{p1}^{\wp \rightarrow \partial} & s_{p2}^{\wp \rightarrow \partial} & \dots & s_{pq}^{\wp \rightarrow \partial} \end{bmatrix} \end{matrix}$, the

positive ideal point of side \wp over ∂ (noted as $s_{\wp \rightarrow \partial}^+ = (s_1^{+, \wp \rightarrow \partial}, s_2^{+, \wp \rightarrow \partial}, \dots, s_q^{+, \wp \rightarrow \partial})$) can be determined, where $s_j^{+, \wp \rightarrow \partial}$ is calculated by

$$s_j^{+, \wp \rightarrow \partial} = \max\{s_{1j}^{\wp \rightarrow \partial}, s_{2j}^{\wp \rightarrow \partial}, \dots, s_{pj}^{\wp \rightarrow \partial}\} = \max_{i \in P} \{s_{ij}^{\wp \rightarrow \partial}\} \quad (21)$$

the negative ideal point of side ∂ over \wp (noted as $s_{\wp \rightarrow \partial}^- = (s_1^{-, \wp \rightarrow \partial}, s_2^{-, \wp \rightarrow \partial}, \dots, s_q^{-, \wp \rightarrow \partial})$) can be determined, where $s_j^{-, \wp \rightarrow \partial}$ is calculated by

$$s_j^{-, \wp \rightarrow \partial} = \min\{s_{1j}^{\wp \rightarrow \partial}, s_{2j}^{\wp \rightarrow \partial}, \dots, s_{pj}^{\wp \rightarrow \partial}\} = \min_{i \in P} \{s_{ij}^{\wp \rightarrow \partial}\} \quad (22)$$

Based on the idea of TOPSIS, the positive distance matrix of side \wp over ∂ (noted as $D_{\wp \rightarrow \partial}^+ = [d_{ij}^{+, \wp \rightarrow \partial}]_{p \times q}$) can be determined, where positive distance $d_{ij}^{+, \wp \rightarrow \partial}$ is calculated by

$$d_{ij}^{+, \wp \rightarrow \partial} = \begin{cases} s_j^{+, \wp \rightarrow \partial} - s_{ij}^{\wp \rightarrow \partial}, & \bar{g}_{ij}^{\wp \rightarrow \partial} \neq \vartheta, \bar{g}_{ij}^{\partial \rightarrow \wp} \neq \vartheta \\ \vartheta, & \bar{g}_{ij}^{\wp \rightarrow \partial} = \vartheta \text{ or } \bar{g}_{ij}^{\partial \rightarrow \wp} = \vartheta \end{cases} \quad (23)$$

The negative distance matrix of side \wp over ∂ (noted as $D_{\wp \rightarrow \partial}^- = [d_{ij}^{-, \wp \rightarrow \partial}]_{p \times q}$) can be determined, where negative distance $d_{ij}^{-, \wp \rightarrow \partial}$ is calculated by

$$d_{ij}^{-, \wp \rightarrow \partial} = \begin{cases} s_{ij}^{\wp \rightarrow \partial} - s_j^{-, \wp \rightarrow \partial}, & \bar{g}_{ij}^{\wp \rightarrow \partial} \neq \vartheta, \bar{g}_{ij}^{\partial \rightarrow \wp} \neq \vartheta \\ \vartheta, & \bar{g}_{ij}^{\wp \rightarrow \partial} = \vartheta \text{ or } \bar{g}_{ij}^{\partial \rightarrow \wp} = \vartheta \end{cases} \quad (24)$$

Furthermore, the extended relative closeness matrix of side \wp over ∂ (noted as $C_{\wp \rightarrow \partial} = [c_{ij}^{\wp \rightarrow \partial}]_{p \times q}$) can be set up, where extended relative closeness $c_{ij}^{\wp \rightarrow \partial}$ is calculated by

$$c_{ij}^{\wp \rightarrow \partial} = \begin{cases} \frac{d_{ij}^{-, \wp \rightarrow \partial}}{d_{ij}^{+, \wp \rightarrow \partial} + d_{ij}^{-, \wp \rightarrow \partial}}, & \bar{g}_{ij}^{\wp \rightarrow \partial} \neq \vartheta, \bar{g}_{ij}^{\partial \rightarrow \wp} \neq \vartheta \\ \vartheta, & \bar{g}_{ij}^{\wp \rightarrow \partial} = \vartheta \text{ or } \bar{g}_{ij}^{\partial \rightarrow \wp} = \vartheta \end{cases} \quad (25)$$

Obviously, $c_{ij}^{\wp \rightarrow \partial} \in [0,1] \cup \vartheta$, and the greater $c_{ij}^{\wp \rightarrow \partial}$ is, the higher the satisfaction degree of agent \wp_j over ∂_i is.

Development of the Two-Sided Matching Model

Firstly, the 0-1 variable m_{ij} is introduced, where $m_{ij} = \begin{cases} 1, & \mu(\partial_i) = \wp_j \\ 0, & \mu(\partial_i) \neq \wp_j \end{cases}$. Then the matching matrix $M = [m_{ij}]_{p \times q}$ can be set up. Based on the extended relative closeness matrices $C_{\partial \rightarrow \wp} = [c_{ij}^{\partial \rightarrow \wp}]_{p \times q}$ and $C_{\wp \rightarrow \partial} = [c_{ij}^{\wp \rightarrow \partial}]_{p \times q}$, and the matching matrix $M = [m_{ij}]_{p \times q}$, we consider developing a matching model below.

According to the meanings of the extended relative closeness, maximization of the extended relative closeness can be regarded as the objective function. Furthermore, considering the constraint condition of one-to-one two-sided matching, the following two-sided matching model (M-2) can be developed, i.e.,

$$(M-2) \begin{cases} \max O_{\partial} = \sum_{j=1}^q \sum_{i=1}^p c_{ij}^{\partial \rightarrow \wp} m_{ij} \\ \max O_{\wp} = \sum_{j=1}^q \sum_{i=1}^p c_{ij}^{\wp \rightarrow \partial} m_{ij} \\ \text{s.t. } \sum_{j=1}^q m_{ij} = 1, i \in P; \sum_{i=1}^p m_{ij} \leq 1, j \in Q; m_{ij} \in \{0, 1\}, i \in P, j \in Q \end{cases}$$

Solution of the Two-Sided Matching Model

In order to solve model (M-2), the linear weighted method is used. Let w_{∂} and w_{\wp} be the weights of O_{∂} and O_{\wp} respectively, such that $w_{\partial}, w_{\wp} \in [0,1], w_{\partial} + w_{\wp} = 1$, then model (M-2) can be transformed into the following single-objective optimization model (M-3), i.e.,

$$(M-3) \begin{cases} \max O = \sum_{j=1}^q \sum_{i=1}^p c_{ij}^{\wp \leftrightarrow \partial} m_{ij} \\ \text{s.t. } \sum_{j=1}^q m_{ij} = 1, i \in P; \sum_{i=1}^p m_{ij} \leq 1, j \in Q; m_{ij} \in \{0, 1\}, i \in P, j \in Q \end{cases}$$

where $c_{ij}^{\wp \leftrightarrow \partial} = w_{\partial}c_{ij}^{\partial \rightarrow \wp} + w_{\wp}c_{ij}^{\wp \rightarrow \partial}$.

Remark 7. In model (M-3), if $c_{ij}^{\partial \rightarrow \wp} = \vartheta$ or $c_{ij}^{\wp \rightarrow \partial} = \vartheta$, then $c_{ij}^{\wp \leftrightarrow \partial} = \vartheta$. In this case, in order to solve model (M-3), we could suppose $c_{ij}^{\wp \leftrightarrow \partial} = -K$, where K is an enough large positive number.

By solving model (M-3), the optimal matching matrix $M^* = [m_{ij}^*]_{p \times q}$ can be obtained. Based on matching matrix $M^* = [m_{ij}^*]_{p \times q}$, the two-sided matching alternative can be determined.

Determination of the Two-Sided Matching Algorithm

In sum, an algorithm for solving the two-sided matching problem under the conditions of incomplete indifferent order relations considering matching aspirations is given. The steps of the algorithm are provided as follows.

Step 1. Convert the incomplete indifferent order relations $\wp_{i_1} > (\sim)\wp_{i_2} > (\sim) \dots > (\sim)\wp_{i_{n_i}}$ ($i \in P$) into the normalized Borda matrix $\bar{G}_{\partial \rightarrow \wp} = [\bar{g}_{ij}^{\partial \rightarrow \wp}]_{p \times q}$ by Eqs. (1a)-(1c) and (2); Convert the incomplete indifferent order relations $\partial_{j_1} > (\sim)\partial_{j_2} > (\sim) \dots > (\sim)\partial_{j_{n_j}}$ ($j \in Q$) into the normalized Borda matrix $\bar{G}_{\wp \rightarrow \partial} = [\bar{g}_{ij}^{\wp \rightarrow \partial}]_{p \times q}$ by Eqs. (3a)-(3c) and (4).

Step 2. Built the matching aspiration matrix $A'' = [a''_{ij}]_{p \times q}$ based on the normalized Borda matrices $\bar{G}_{\partial \rightarrow \wp} = [\bar{g}_{ij}^{\partial \rightarrow \wp}]_{p \times q}$ and $\bar{G}_{\wp \rightarrow \partial} = [\bar{g}_{ij}^{\wp \rightarrow \partial}]_{p \times q}$ by Eq. (13).

Step 3. Built the weighted satisfaction degree matrix $S_{\partial \rightarrow \wp} = [s_{ij}^{\partial \rightarrow \wp}]_{p \times q}$ based on the normalized Borda matrix $\bar{G}_{\partial \rightarrow \wp} = [\bar{g}_{ij}^{\partial \rightarrow \wp}]_{p \times q}$ and the matching aspiration matrix $A'' = [a''_{ij}]_{p \times q}$ by Eq. (14); Built the weighted satisfaction degree matrix $S_{\wp \rightarrow \partial} = [s_{ij}^{\wp \rightarrow \partial}]_{p \times q}$ based on the normalized Borda matrix $\bar{G}_{\wp \rightarrow \partial} = [\bar{g}_{ij}^{\wp \rightarrow \partial}]_{p \times q}$ and the matching aspiration matrix $A'' = [a''_{ij}]_{p \times q}$ by Eq. (15).

Step 4. Calculate positive ideal point $s_{\partial \rightarrow \wp}^+ = (s_1^{+, \partial \rightarrow \wp}, s_2^{+, \partial \rightarrow \wp}, \dots, s_p^{+, \partial \rightarrow \wp})^T$ based on the weighted satisfaction degree matrix $S_{\partial \rightarrow \wp} = [s_{ij}^{\partial \rightarrow \wp}]_{p \times q}$ by Eq. (16); Calculate the negative ideal point $s_{\partial \rightarrow \wp}^- = (s_1^{-, \partial \rightarrow \wp}, s_2^{-, \partial \rightarrow \wp}, \dots, s_p^{-, \partial \rightarrow \wp})^T$ by Eq. (17).

Step 5. Determine the positive distance matrix $D_{\partial \rightarrow \wp}^+ = [d_{ij}^{+, \partial \rightarrow \wp}]_{p \times q}$ based on the idea of TOPSIS by Eq. (18); Determine the negative distance matrix $D_{\partial \rightarrow \wp}^- = [d_{ij}^{-, \partial \rightarrow \wp}]_{p \times q}$ based on the idea of TOPSIS by Eq. (19).

Step 6. Set up the extended relative closeness matrix $C_{\partial \rightarrow \wp} = [c_{ij}^{\partial \rightarrow \wp}]_{p \times q}$ based on the positive distance matrix $D_{\partial \rightarrow \wp}^+ = [d_{ij}^{+, \partial \rightarrow \wp}]_{p \times q}$ and the negative distance matrix $D_{\partial \rightarrow \wp}^- = [d_{ij}^{-, \partial \rightarrow \wp}]_{p \times q}$ by Eq. (20).

Step 7. Calculate positive ideal point $s_{\wp \rightarrow \partial}^+ = (s_1^{+, \wp \rightarrow \partial}, s_2^{+, \wp \rightarrow \partial}, \dots, s_q^{+, \wp \rightarrow \partial})$ based on the weighted satisfaction degree matrix $S_{\wp \rightarrow \partial} = [s_{ij}^{\wp \rightarrow \partial}]_{p \times q}$ by Eq. (21); Calculate the negative ideal point $s_{\wp \rightarrow \partial}^- = (s_1^{-, \wp \rightarrow \partial}, s_2^{-, \wp \rightarrow \partial}, \dots, s_q^{-, \wp \rightarrow \partial})$ by Eq. (22).

Step 8. Determine the positive distance matrix $D_{\wp \rightarrow \partial}^+ = [d_{ij}^{+, \wp \rightarrow \partial}]_{p \times q}$ based on the idea of TOPSIS by Eq. (23); Determine the negative distance matrix $D_{\wp \rightarrow \partial}^- = [d_{ij}^{-, \wp \rightarrow \partial}]_{p \times q}$ based on the idea of TOPSIS by Eq. (24).

Step 9. Set up the extended relative closeness matrix $C_{\wp \rightarrow \partial} = [c_{ij}^{\wp \rightarrow \partial}]_{p \times q}$ based on the positive distance matrix $D_{\wp \rightarrow \partial}^+ = [d_{ij}^{+, \wp \rightarrow \partial}]_{p \times q}$ and the negative distance matrix $D_{\wp \rightarrow \partial}^- = [d_{ij}^{-, \wp \rightarrow \partial}]_{p \times q}$ by Eq. (25).

Step 10. Develop the two-sided matching model (M-2) based on the extended relative closeness matrices $C_{\partial \rightarrow \wp} = [c_{ij}^{\partial \rightarrow \wp}]_{p \times q}$ and $C_{\wp \rightarrow \partial} = [c_{ij}^{\wp \rightarrow \partial}]_{p \times q}$, and the matching matrix $M = [m_{ij}]_{p \times q}$.

Step 11. Transform model (M-2) into model (M-3) by using the linear weighted method.

Step 12. Determine the two-sided matching alternative by solving model (M-3).

ILLUSTRATED EXAMPLE

In this section, an example is used to illustrate the application of the proposed extend TOPSIS-based two-sided matching decision.

Suppose an oversea venture-capital company plans to invest a cell-phone company in Nan Chang of China. In order to enable the new cell-phone company to run smoothly, the manager intends to assign experienced staffs to vacant positions in the new factory. Each position in the new cell-phone company is held by one staff, and each staff is assigned to only one position. There are five vacant positions, which consist of a purchaser (∂_1), a material handler (∂_2), a production planner (∂_3), a technician (∂_4) and a quality inspector (∂_5). Now seven experienced staffs \wp_1, \wp_2, \dots , and \wp_7 who have multiple skills apply to the five positions. The decision makers from five position departments evaluate the staffs from four perspectives: personality characteristics, technical skill, previous experience, and human relationship skill. Seven staffs evaluate the positions from three perspectives: salary and welfare, development space, and work environment. The incomplete indifferent order relations $\wp_{i_1} > (\sim) \wp_{i_2} > (\sim) \dots > (\sim) \wp_{i_n}$ ($i \in P = \{1, 2, \dots, 5\}$) and $\partial_{j_1} > (\sim) \partial_{j_2} > (\sim) \dots > (\sim) \partial_{j_n}$ ($j \in Q = \{1, 2, \dots, 7\}$) are given below.

In order to enhance the level of operating efficiency, the intermediary who specializes in human resource allocation is employed to show the two-sided matching alternative.

$$\begin{aligned}
 \partial_1: & \wp_3 > \wp_7 \sim \wp_6 \sim \wp_4 > \wp_5 \sim \wp_1, \\
 \partial_2: & \wp_6 > \wp_3 \sim \wp_1 > \wp_5 > \wp_2 \sim \wp_7 \\
 \partial_3: & \wp_4 \sim \wp_5 > \wp_1 \sim \wp_7 > \wp_6 > \wp_2 \\
 \partial_4: & \wp_2 > \wp_7 \sim \wp_5 > \wp_6 > \wp_1 > \wp_3 \sim \wp_4 \\
 \partial_5: & \wp_7 \sim \wp_1 \sim \wp_5 > \wp_6 > \wp_4 \sim \wp_2 > \wp_3 \\
 \wp_1: & \partial_2 > \partial_4 > \partial_3 \sim \partial_1 \\
 \wp_2: & \partial_4 > \partial_3 > \partial_1 \sim \partial_5 \\
 \wp_3: & \partial_3 \sim \partial_5 > \partial_1 > \partial_4 \\
 \wp_4: & \partial_1 > \partial_4 \sim \partial_3 > \partial_2 \\
 \wp_5: & \partial_1 > \partial_4 > \partial_2 \sim \partial_3 > \partial_5 \\
 \wp_6: & \partial_4 \sim \partial_2 > \partial_5 > \partial_1 \sim \partial_3 \\
 \wp_7: & \partial_5 > \partial_1 \sim \partial_3 > \partial_4 \sim \partial_2
 \end{aligned}$$

To solve the above problem, the proposed two-sided matching decision is used and the procedure is given as follows.

Step 1. According to the incomplete indifferent order relations $\wp_{i_1} > (\sim) \wp_{i_2} > (\sim) \dots > (\sim) \wp_{i_n}$ ($i \in P$), the normalized Borda matrix $\bar{G}_{\partial \rightarrow \wp} = [\bar{g}_{ij}^{\partial \rightarrow \wp}]_{p \times q}$ can be built by Eqs. (1a)-(1c) and (2), which is shown in **Table 1**. According to the incomplete indifferent order relations $\partial_{j_1} > (\sim) \partial_{j_2} > (\sim) \dots > (\sim) \partial_{j_n}$ ($j \in Q$), the normalized Borda matrix $\bar{G}_{\wp \rightarrow \partial} = [\bar{g}_{ij}^{\wp \rightarrow \partial}]_{p \times q}$ can be built by Eqs. (3a)-(3c) and (4), which is shown in **Table 2**.

Table 1. The normalized Borda matrix $\bar{G}_{\partial \rightarrow \wp} = [\bar{g}_{ij}^{\partial \rightarrow \wp}]_{5 \times 7}$

$\bar{g}_{ij}^{\partial \rightarrow \wp}$	\wp_1	\wp_2	\wp_3	\wp_4	\wp_5	\wp_6	\wp_7
∂_1	0.25	ϑ	1	0.6667	0.25	0.6667	0.6667
∂_2	0.75	0.25	0.75	ϑ	0.5	1	0.25
∂_3	0.5833	0.1667	ϑ	0.9167	0.9167	0.3333	0.5833
∂_4	0.4286	1	0.2143	0.2143	0.7857	0.5714	0.7857
∂_5	0.8571	0.3571	0.1429	0.3571	0.8571	0.5714	0.8571

Table 2. The normalized Borda matrix $\bar{G}_{\wp \rightarrow \partial} = [\bar{g}_{ij}^{\wp \rightarrow \partial}]_{5 \times 7}$

$\bar{g}_{ij}^{\wp \rightarrow \partial}$	\wp_1	\wp_2	\wp_3	\wp_4	\wp_5	\wp_6	\wp_7
∂_1	0.375	0.375	0.5	1	1	0.3	0.7
∂_2	1	ϑ	ϑ	0.25	0.5	0.9	0.3
∂_3	0.375	0.75	0.875	0.625	0.5	0.3	0.7
∂_4	0.75	1	0.25	0.625	0.8	0.9	0.3
∂_5	ϑ	0.375	0.875	ϑ	0.2	0.6	1

Table 3. The matching aspiration matrix $A'' = [a''_{ij}]_{5 \times 7}$

a''_{ij}	\wp_1	\wp_2	\wp_3	\wp_4	\wp_5	\wp_6	\wp_7
∂_1	0.0388	ϑ	0.0299	0.0336	0.0251	0.0328	0.0413
∂_2	0.0356	ϑ	ϑ	ϑ	0.0423	0.0395	0.0409
∂_3	0.0366	0.0282	ϑ	0.0346	0.0317	0.0413	0.039
∂_4	0.0339	0.0423	0.0413	0.0318	0.0419	0.0337	0.0302
∂_5	ϑ	0.0418	0.0255	ϑ	0.0268	0.0415	0.0383

Table 4. The weighted satisfaction degree matrix $S_{\partial \rightarrow \wp} = [s_{ij}^{\partial \rightarrow \wp}]_{5 \times 7}$

$s_{ij}^{\partial \rightarrow \wp}$	\wp_1	\wp_2	\wp_3	\wp_4	\wp_5	\wp_6	\wp_7
∂_1	0.0097	ϑ	0.0299	0.0224	0.0063	0.0219	0.0275
∂_2	0.0267	ϑ	ϑ	ϑ	0.0212	0.0395	0.0102
∂_3	0.0213	0.0047	ϑ	0.0317	0.0291	0.0138	0.0227
∂_4	0.0145	0.0423	0.0089	0.0068	0.0329	0.0193	0.0237
∂_5	ϑ	0.0149	0.0036	ϑ	0.023	0.0237	0.0328

Table 5. The weighted satisfaction degree matrix $S_{\wp \rightarrow \partial} = [s_{ij}^{\wp \rightarrow \partial}]_{5 \times 7}$

$s_{ij}^{\wp \rightarrow \partial}$	\wp_1	\wp_2	\wp_3	\wp_4	\wp_5	\wp_6	\wp_7
∂_1	0.0146	ϑ	0.015	0.0336	0.0251	0.0098	0.0289
∂_2	0.0356	ϑ	ϑ	ϑ	0.0212	0.0356	0.0123
∂_3	0.0137	0.0212	ϑ	0.0216	0.0159	0.0124	0.0273
∂_4	0.0254	0.0423	0.0103	0.0199	0.0335	0.0303	0.0091
∂_5	ϑ	0.0157	0.0223	ϑ	0.0054	0.0249	0.0383

Step 2. According to the normalized Borda matrices $\bar{G}_{\partial \rightarrow \wp} = [\bar{g}_{ij}^{\partial \rightarrow \wp}]_{5 \times 7}$ and $\bar{G}_{\wp \rightarrow \partial} = [\bar{g}_{ij}^{\wp \rightarrow \partial}]_{5 \times 7}$, the matching aspiration matrix $A'' = [a''_{ij}]_{p \times q}$ is established by Eq. (13), which is shown in **Table 3**.

Step 3. Based on the normalized Borda matrix $\bar{G}_{\partial \rightarrow \wp} = [\bar{g}_{ij}^{\partial \rightarrow \wp}]_{5 \times 7}$ and the matching aspiration matrix $A'' = [a''_{ij}]_{5 \times 7}$, the weighted satisfaction degree matrix $S_{\partial \rightarrow \wp} = [s_{ij}^{\partial \rightarrow \wp}]_{5 \times 7}$ is built by Eq. (14), which is shown in **Table 4**. Based on the normalized Borda matrix $\bar{G}_{\wp \rightarrow \partial} = [\bar{g}_{ij}^{\wp \rightarrow \partial}]_{5 \times 7}$ and the matching aspiration matrix $A'' = [a''_{ij}]_{5 \times 7}$, the weighted satisfaction degree matrix $S_{\wp \rightarrow \partial} = [s_{ij}^{\wp \rightarrow \partial}]_{5 \times 7}$ is built by Eq. (15), which is shown in **Table 5**.

Step 4. Based on the weighted satisfaction degree matrix $S_{\partial \rightarrow \wp} = [s_{ij}^{\partial \rightarrow \wp}]_{5 \times 7}$, positive ideal point $s_{\partial \rightarrow \wp}^+ = (s_1^{+, \partial \rightarrow \wp}, s_2^{+, \partial \rightarrow \wp}, \dots, s_p^{+, \partial \rightarrow \wp})^T$ is calculated by Eq. (16), i.e., $s_{\partial \rightarrow \wp}^+ = (0.0299, 0.0395, 0.0317, 0.0423, 0.0328)^T$; the negative ideal point $s_{\partial \rightarrow \wp}^- = (s_1^{-, \partial \rightarrow \wp}, s_2^{-, \partial \rightarrow \wp}, \dots, s_p^{-, \partial \rightarrow \wp})^T$ is calculated by Eq. (17), i.e., $s_{\partial \rightarrow \wp}^- = (0.0063, 0.0102, 0.0047, 0.0068, 0.0036)^T$.

Step 5. Based on the weighted satisfaction degree matrix $S_{\partial \rightarrow \wp} = [s_{ij}^{\partial \rightarrow \wp}]_{5 \times 7}$ and the positive ideal point $s_{\partial \rightarrow \wp}^+ = (0.0299, 0.0395, 0.0317, 0.0423, 0.0328)^T$, the positive distance matrix $D_{\partial \rightarrow \wp}^+ = [d_{ij}^{+, \partial \rightarrow \wp}]_{5 \times 7}$ is determined by Eq.

Table 6. The positive distance matrix $D_{\delta \rightarrow \wp}^+ = [d_{ij}^{+, \delta \rightarrow \wp}]_{5 \times 7}$

$d_{ij}^{+, \delta \rightarrow \wp}$	\wp_1	\wp_2	\wp_3	\wp_4	\wp_5	\wp_6	\wp_7
δ_1	0.0202	ϑ	0	0.0075	0.0236	0.008	0.0024
δ_2	0.0128	ϑ	ϑ	ϑ	0.0183	0	0.0293
δ_3	0.0104	0.027	ϑ	0	0.0026	0.0179	0.009
δ_4	0.0278	0	0.0334	0.0355	0.0094	0.023	0.0186
δ_5	ϑ	0.0179	0.0292	ϑ	0.0098	0.0091	0

Table 7. The negative distance matrix $D_{\delta \rightarrow \wp}^- = [d_{ij}^{-, \delta \rightarrow \wp}]_{5 \times 7}$

$d_{ij}^{-, \delta \rightarrow \wp}$	\wp_1	\wp_2	\wp_3	\wp_4	\wp_5	\wp_6	\wp_7
δ_1	0.0034	ϑ	0.0236	0.0161	0	0.0156	0.0212
δ_2	0.0165	ϑ	ϑ	ϑ	0.011	0.0293	0
δ_3	0.0166	0	ϑ	0.027	0.0244	0.0091	0.018
δ_4	0.0077	0.0355	0.0021	0	0.0261	0.0125	0.0169
δ_5	ϑ	0.0113	0	ϑ	0.0194	0.0201	0.0292

Table 8. The extended relative closeness matrix $C_{\delta \rightarrow \wp} = [c_{ij}^{\delta \rightarrow \wp}]_{5 \times 7}$

$c_{ij}^{\delta \rightarrow \wp}$	\wp_1	\wp_2	\wp_3	\wp_4	\wp_5	\wp_6	\wp_7
δ_1	0.1441	ϑ	1	0.6822	0	0.661	0.8983
δ_2	0.5631	ϑ	ϑ	ϑ	0.3754	1	0
δ_3	0.6148	0	ϑ	1	0.9037	0.337	0.6667
δ_4	0.2169	1	0.0592	0	0.7352	0.3521	0.4761
δ_5	ϑ	0.387	0	ϑ	0.6644	0.6884	1

Table 9. The positive distance matrix $D_{\wp \rightarrow \delta}^+ = [d_{ij}^{+, \wp \rightarrow \delta}]_{5 \times 7}$

$d_{ij}^{+, \wp \rightarrow \delta}$	\wp_1	\wp_2	\wp_3	\wp_4	\wp_5	\wp_6	\wp_7
δ_1	0.021	ϑ	0.0073	0	0.0084	0.0258	0.0094
δ_2	0	ϑ	ϑ	ϑ	0.0123	0	0.026
δ_3	0.0219	0.0211	ϑ	0.012	0.0176	0.0232	0.011
δ_4	0.0102	0	0.012	0.0137	0	0.0053	0.0292
δ_5	ϑ	0.0266	0	ϑ	0.0281	0.0107	0

(18), which is shown in **Table 6**. Based on the weighted satisfaction degree matrix $S_{\delta \rightarrow \wp} = [s_{ij}^{\delta \rightarrow \wp}]_{5 \times 7}$ and the negative ideal point $s_{\delta \rightarrow \wp}^- = (0.0063, 0.0102, 0.0047, 0.0068, 0.0036)^T$, the negative distance matrix $D_{\delta \rightarrow \wp}^- = [d_{ij}^{-, \delta \rightarrow \wp}]_{5 \times 7}$ is determined by Eq. (19), which is shown in **Table 7**.

Step 6. Based on the positive distance matrix $D_{\delta \rightarrow \wp}^+ = [d_{ij}^{+, \delta \rightarrow \wp}]_{5 \times 7}$ and the negative distance matrix $D_{\delta \rightarrow \wp}^- = [d_{ij}^{-, \delta \rightarrow \wp}]_{5 \times 7}$, the extended relative closeness matrix $C_{\delta \rightarrow \wp} = [c_{ij}^{\delta \rightarrow \wp}]_{5 \times 7}$ is set up by Eq. (20), which is shown in **Table 8**.

Step 7. Based on the weighted satisfaction degree matrix $S_{\wp \rightarrow \delta} = [s_{ij}^{\wp \rightarrow \delta}]_{5 \times 7}$, positive ideal point $s_{\wp \rightarrow \delta}^+ = (s_1^{+, \wp \rightarrow \delta}, s_2^{+, \wp \rightarrow \delta}, \dots, s_q^{+, \wp \rightarrow \delta})$ is calculated by Eq. (21), i.e., $s_{\wp \rightarrow \delta}^+ = (0.0356, 0.0423, 0.0223, 0.0336, 0.0335, 0.0356, 0.0383)$; the negative ideal point $s_{\wp \rightarrow \delta}^- = (s_1^{-, \wp \rightarrow \delta}, s_2^{-, \wp \rightarrow \delta}, \dots, s_q^{-, \wp \rightarrow \delta})$ is calculated by Eq. (22), i.e., $s_{\wp \rightarrow \delta}^- = (0.0137, 0.0157, 0.0103, 0.0199, 0.0054, 0.0098, 0.0091)$.

Step 8. Based on the weighted satisfaction degree matrix $S_{\wp \rightarrow \delta} = [s_{ij}^{\wp \rightarrow \delta}]_{5 \times 7}$ and the positive ideal point $s_{\wp \rightarrow \delta}^+ = (0.0356, 0.0423, 0.0223, 0.0336, 0.0335, 0.0356, 0.0383)$, the positive distance matrix $D_{\wp \rightarrow \delta}^+ = [d_{ij}^{+, \wp \rightarrow \delta}]_{5 \times 7}$ is determined by Eq. (23), which is shown in **Table 9**. Based on the weighted satisfaction degree matrix $S_{\wp \rightarrow \delta} = [s_{ij}^{\wp \rightarrow \delta}]_{5 \times 7}$ and the negative ideal point $s_{\wp \rightarrow \delta}^- = (0.0137, 0.0157, 0.0103, 0.0199, 0.0054, 0.0098, 0.0091)$, the negative distance matrix $D_{\wp \rightarrow \delta}^- = [d_{ij}^{-, \wp \rightarrow \delta}]_{5 \times 7}$ is determined by Eq. (24), which is shown in **Table 10**.

Table 10. The negative distance matrix $D_{\wp \rightarrow \partial}^- = [d_{ij}^{-, \wp \rightarrow \partial}]_{5 \times 7}$

$d_{ij}^{-, \wp \rightarrow \partial}$	\wp_1	\wp_2	\wp_3	\wp_4	\wp_5	\wp_6	\wp_7
∂_1	0.0009	ϑ	0.0047	0.0137	0.0197	0	0.0198
∂_2	0.0219	ϑ	ϑ	ϑ	0.0158	0.0258	0.0032
∂_3	0	0.0055	ϑ	0.0017	0.0105	0.0026	0.0182
∂_4	0.0117	0.0266	0	0	0.0281	0.0205	0
∂_5	ϑ	0	0.012	ϑ	0	0.0151	0.0292

Table 11. The extended relative closeness matrix $C_{\wp \rightarrow \partial} = [c_{ij}^{\wp \rightarrow \partial}]_{5 \times 7}$

$c_{ij}^{\wp \rightarrow \partial}$	\wp_1	\wp_2	\wp_3	\wp_4	\wp_5	\wp_6	\wp_7
∂_1	0.0411	ϑ	0.3917	1	0.7011	0	0.6781
∂_2	1	ϑ	ϑ	ϑ	0.5623	1	0.1096
∂_3	0	0.2068	ϑ	0.1241	0.3737	0.1008	0.6233
∂_4	0.5342	1	0	0	1	0.7946	0
∂_5	ϑ	0	1	ϑ	0	0.5853	1

Table 12. The coefficient matrix $[c_{ij}^{\wp \leftrightarrow \partial}]_{5 \times 7}$

$c_{ij}^{\wp \leftrightarrow \partial}$	\wp_1	\wp_2	\wp_3	\wp_4	\wp_5	\wp_6	\wp_7
∂_1	0.1029	ϑ	0.7567	0.8093	0.2804	0.3966	0.8102
∂_2	0.7379	ϑ	ϑ	ϑ	0.4502	1	0.0438
∂_3	0.3689	0.0827	ϑ	0.6496	0.6917	0.2425	0.6493
∂_4	0.3438	1	0.0355	0	0.8411	0.5291	0.2857
∂_5	ϑ	0.2322	0.4	ϑ	0.3986	0.6472	1

Step 9. Based on the positive distance matrix $D_{\wp \rightarrow \partial}^+ = [d_{ij}^{+, \wp \rightarrow \partial}]_{5 \times 7}$ and the negative distance matrix $D_{\wp \rightarrow \partial}^- = [d_{ij}^{-, \wp \rightarrow \partial}]_{5 \times 7}$, the extended relative closeness matrix $C_{\wp \rightarrow \partial} = [c_{ij}^{\wp \rightarrow \partial}]_{5 \times 7}$ is set up by Eq. (25), which is shown in **Table 11**.

Step 10. Based on the extended relative closeness matrices $C_{\partial \rightarrow \wp} = [c_{ij}^{\partial \rightarrow \wp}]_{5 \times 7}$ and $C_{\wp \rightarrow \partial} = [c_{ij}^{\wp \rightarrow \partial}]_{5 \times 7}$, and the matching matrix $M = [m_{ij}]_{5 \times 7}$, the two-sided matching model (M-2) is developed, i.e.,

$$(M-2) \begin{cases} \max O_{\partial} = \sum_{j=1}^7 \sum_{i=1}^5 c_{ij}^{\partial \rightarrow \wp} m_{ij} \\ \max O_{\wp} = \sum_{j=1}^7 \sum_{i=1}^5 c_{ij}^{\wp \rightarrow \partial} m_{ij} \\ \text{s.t. } \sum_{j=1}^7 m_{ij} = 1, i \in P; \sum_{i=1}^5 m_{ij} \leq 1, j \in Q; m_{ij} \in \{0, 1\}, i \in P, j \in Q \end{cases}$$

Step 11. Suppose $w_{\partial} = 0.6, w_{\wp} = 0.4$, then model (M-2) is transformed into model (M-3) by using the linear weighted method, i.e.,

$$(M-3) \begin{cases} \max O = \sum_{j=1}^7 \sum_{i=1}^5 c_{ij}^{\wp \leftrightarrow \partial} m_{ij} \\ \text{s.t. } \sum_{j=1}^7 m_{ij} = 1, i \in P; \sum_{i=1}^5 m_{ij} \leq 1, j \in Q; m_{ij} \in \{0, 1\}, i \in P, j \in Q \end{cases}$$

where coefficient matrix $[c_{ij}^{\wp \leftrightarrow \partial}]_{5 \times 7} = [0.6c_{ij}^{\partial \rightarrow \wp} + 0.4c_{ij}^{\wp \rightarrow \partial}]_{5 \times 7}$ is shown in **Table 12**.

Step 12. By solving model (M-3), the matching matrix $M^* = [m_{ij}^*]_{5 \times 7}$ can be determined, which is shown in **Table 13**.

Table 13. The matching matrix $M^* = [m_{ij}^*]_{5 \times 7}$

$c_{ij}^{\rho \leftrightarrow \partial}$	\wp_1	\wp_2	\wp_3	\wp_4	\wp_5	\wp_6	\wp_7
∂_1	0	0	0	1	0	0	0
∂_2	0	0	0	0	0	1	0
∂_3	0	0	0	0	1	0	0
∂_4	0	1	0	0	0	0	0
∂_5	0	0	0	0	0	0	1

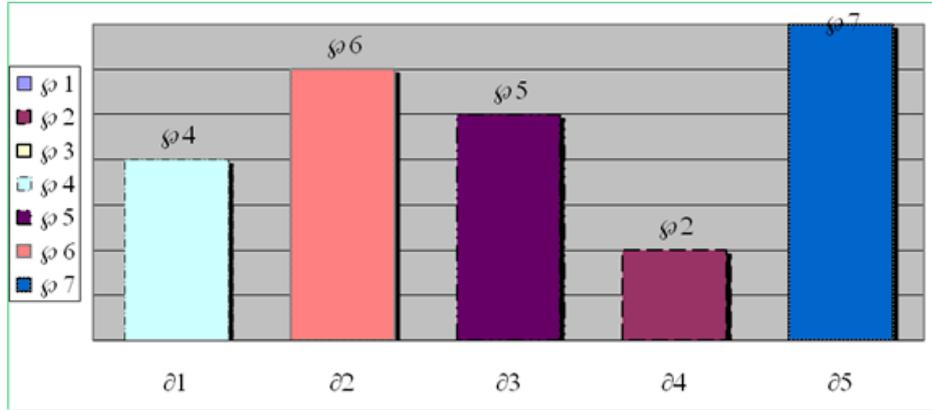


Figure 1. The two-sided matching alternative

Table 14. The coefficient matrix $[c_{ij}^{\rho \leftrightarrow \partial}]_{5 \times 7}$

$c_{ij}^{\rho \leftrightarrow \partial}$	\wp_1	\wp_2	\wp_3	\wp_4	\wp_5	\wp_6	\wp_7
∂_1	0.1132	ϑ	0.8175	0.7775	0.2103	0.4627	0.8322
∂_2	0.6942	ϑ	ϑ	ϑ	0.4315	1	0.0329
∂_3	0.4304	0.062	ϑ	0.7372	0.7447	0.2661	0.6537
∂_4	0.3121	1	0.0414	0	0.8146	0.4849	0.3333
∂_5	ϑ	0.2709	0.3	ϑ	0.4651	0.6575	1

Based on the matching matrix $M^* = [m_{ij}^*]_{5 \times 7}$, the two-sided ρ^* matching alternative ρ^* can be obtained, i.e., $\rho^* = \rho_{Two}^* \cup \rho_{One}^*$, where $\rho_{Two}^* = \{(\partial_1, \wp_4), (\partial_2, \wp_6), (\partial_3, \wp_5), (\partial_4, \wp_2), (\partial_5, \wp_7)\}$, $\rho_{One}^* = \{(\wp_1, \wp_1), (\wp_3, \wp_3)\}$. In other word, purchaser ∂_1 matches with staff ∂_4 , material handler ∂_2 matches with staff \wp_6 , production planner ∂_3 matches with staff \wp_5 , technician ∂_4 matches with staff \wp_2 , quality inspector ∂_5 matches with staff \wp_7 , staffs \wp_1 and \wp_3 are not matched, which is vividly shown in **Figure 1**.

In the following, we discuss the influence of weights w_∂ and w_\wp towards the two-sided matching alternative.

Case I. If $w_\partial = 0.7, w_\wp = 0.3$, then model (M-2) is transformed into model (M-3) by using the linear weighted method, where coefficient matrix $[c_{ij}^{\rho \leftrightarrow \partial}]_{5 \times 7} = [0.7c_{ij}^{\partial \rightarrow \wp} + 0.3c_{ij}^{\wp \rightarrow \partial}]_{5 \times 7}$ is shown in **Table 14**.

By solving the above model, the matching matrix $M^* = [m_{ij}^*]_{5 \times 7}$ can be determined. Based on the matching matrix $M^* = [m_{ij}^*]_{5 \times 7}$, the two-sided matching alternative ρ^* can be obtained, i.e., $\rho^* = \rho_{Two}^* \cup \rho_{One}^*$, where $\rho_{Two}^* = \{(\partial_1, \wp_3), (\partial_2, \wp_6), (\partial_3, \wp_5), (\partial_4, \wp_2), (\partial_5, \wp_7)\}$, $\rho_{One}^* = \{(\wp_1, \wp_1), (\wp_4, \wp_4)\}$.

Case II. If $w_\partial = 0.8, w_\wp = 0.2$, then model (M-2) is transformed into model (M-3) by using the linear weighted method, where $c_{ij}^{\rho \leftrightarrow \partial} = 0.8c_{ij}^{\partial \rightarrow \wp} + 0.2c_{ij}^{\wp \rightarrow \partial}$. By solving the model, the two-sided matching alternative ρ^* can be obtained, i.e., $\rho^* = \rho_{Two}^* \cup \rho_{One}^*$, where $\rho_{Two}^* = \{(\partial_1, \wp_3), (\partial_2, \wp_6), (\partial_3, \wp_4), (\partial_4, \wp_2), (\partial_5, \wp_7)\}$, $\rho_{One}^* = \{(\wp_1, \wp_1), (\wp_5, \wp_5)\}$.

Case III. If $w_\partial = 0.9, w_\wp = 0.1$, then model (M-2) is transformed into model (M-3), where $c_{ij}^{\rho \leftrightarrow \partial} = 0.9c_{ij}^{\partial \rightarrow \wp} + 0.1c_{ij}^{\wp \rightarrow \partial}$. Similarly, through solving the model, the two-sided matching alternative ρ^* can be obtained, where $\rho_{Two}^* = \{(\partial_1, \wp_3), (\partial_2, \wp_6), (\partial_3, \wp_4), (\partial_4, \wp_2), (\partial_5, \wp_7)\}$, $\rho_{One}^* = \{(\wp_1, \wp_1), (\wp_5, \wp_5)\}$.

Case IV. If $w_\partial = 0.9, w_\wp = 0.1$, then model (M-2) is transformed into model (M-3), where $c_{ij}^{\rho \leftrightarrow \partial} = 0.9c_{ij}^{\partial \rightarrow \wp} + 0.1c_{ij}^{\wp \rightarrow \partial}$. Through solving the model, the two-sided matching alternative ρ^* can be obtained, where $\rho_{Two}^* = \{(\partial_1, \wp_3), (\partial_2, \wp_6), (\partial_3, \wp_4), (\partial_4, \wp_2), (\partial_5, \wp_7)\}$, $\rho_{One}^* = \{(\wp_1, \wp_1), (\wp_5, \wp_5)\}$.

Table 15. The comparative analysis

Weight		Two sided matching alternative ρ^*	
w_δ	w_\wp	ρ_{Two}^*	ρ_{One}^*
1	0		
0.9	0.1	$\{(\delta_1, \wp_3), (\delta_2, \wp_6), (\delta_3, \wp_4), (\delta_4, \wp_2), (\delta_5, \wp_7)\}$	$\{(\wp_1, \wp_1), (\wp_5, \wp_5)\}$
0.8	0.2		
0.7	0.3	$\{(\delta_1, \wp_3), (\delta_2, \wp_6), (\delta_3, \wp_5), (\delta_4, \wp_2), (\delta_5, \wp_7)\}$	$\{(\wp_1, \wp_1), (\wp_4, \wp_4)\}$
0.6	0.4		
0.5	0.5		
0.4	0.6	$\{(\delta_1, \wp_4), (\delta_2, \wp_6), (\delta_3, \wp_5), (\delta_4, \wp_2), (\delta_5, \wp_7)\}$	$\{(\wp_1, \wp_1), (\wp_3, \wp_3)\}$
0.3	0.7		
0.2	0.8		
0.1	0.9	$\{(\delta_1, \wp_4), (\delta_2, \wp_6), (\delta_3, \wp_7), (\delta_4, \wp_2), (\delta_5, \wp_3)\}$	$\{(\wp_1, \wp_1), (\wp_5, \wp_5)\}$
0	1	$\{(\delta_1, \wp_4), (\delta_2, \wp_6), (\delta_3, \wp_7), (\delta_4, \wp_5), (\delta_5, \wp_3)\}$	$\{(\wp_1, \wp_1), (\wp_2, \wp_2)\}$

Case V. If $w_\delta = 1, w_\wp = 0$, then model (M-2) is transformed into model (M-3), where $c_{ij}^{\delta \leftrightarrow \wp} = c_{ij}^{\delta \rightarrow \wp}$. Through solving the model, the two-sided matching alternative ρ^* is obtained, where $\rho_{Two}^* = \{(\delta_1, \wp_3), (\delta_2, \wp_6), (\delta_3, \wp_4), (\delta_4, \wp_2), (\delta_5, \wp_7)\}$, $\rho_{One}^* = \{(\wp_1, \wp_1), (\wp_5, \wp_5)\}$.

In conclusion, the comparative analysis on the influence of weights w_δ and w_\wp towards the two-sided matching alternative is shown in **Table 15**. From **Table 15**, we know that the two-sided matching alternative may be changed when weights w_δ and w_\wp are changed. Therefore, weights w_δ and w_\wp play an important role in determining the two-sided matching alternative.

CONCLUSIONS

This paper presented a decision method for solving the two-sided matching problem with incomplete indifferent order relations considering matching aspirations. The incomplete indifferent order relations were transformed into the generalized Borda number matrices, and the matching aspirations were calculated based on model calculation. Based on this, the weighted satisfaction degree matrices were built. Then, the extended relative closeness matrices can be determined by using an extended TOPSIS method. Furthermore, a two-sided matching model can be constructed. By solving the proposed model, the two-sided matching alternative was determined. An example with sensitivity analysis is also given to illustrate the effectiveness of the presented method.

Comparing with the existing research, the main contribution of this paper is as follows: (1) the generalized Borda numbers were adopted to handle incomplete indifferent order relations, which is a new idea; (2) the research angle was matching aspirations, and hence the obtained two-sided matching alternative could reflect the matching aspirations of agents; (3) an idea of extend TOPSIS was firstly introduced into two-sided matching decision, which was a novel idea; (4) the presented method developed the theory and method for two-sided matching decision with incomplete indifferent order relations.

The main limitation of this paper is that it only discussed the two-sided matching problem with less incomplete indifferent order relations. And the related theory of stable matching under the condition of incomplete indifferent order relations was not studied.

Hence, the following two aspects could be further studied. First, if the complete two-sided matching alternative cannot be obtained based on the information of incomplete indifferent order relations, then this kind of two-sided matching decision problem should be further investigated. Second, the theory and property of stable matching with incomplete indifferent order relations should be further probed.

ACKNOWLEDGEMENTS

This work was partly supported by the National Natural Science Foundation of China (Project No. 71261007), the Natural Science Fund of Jiangxi Province (Project Nos. 20171BAA208003, 20161BAB201025), and the National Social Science Foundation of China (Project No. 15BGL005).

REFERENCES

Braun, S., Dwenger, N., Kübler, D., & Westkamp, A. (2014). Implementing quotas in university admissions: An experimental analysis. *Games and Economic Behavior*, 85, 232-251.

Castillo, M., & Dianat, A. (2016). Truncation strategies in two-sided matching markets: Theory and experiment. *Games and Economic Behavior*, 98, 180-196.

- Chen, B., Fujishige, S., & Yang, Z. F. (2016). Random decentralized market processes for stable job matchings with competitive salaries. *Journal of Economic Theory*, 165, 25-36.
- Chen, P., Egesdal, M., Pycia, M., & Yenmez, M. B. (2016). Median stable matchings in two-sided markets. *Games and Economic Behavior*, 97, 64-69.
- Chen, W. C., & Kao, Y. C. (2014). Simultaneous screening and college admissions. *Economics Letters*, 122(2), 296-298
- Cseh, Á., & Manlove, D. F. (2016). Stable Marriage and Roommates problems with restricted edges: Complexity and approximability. *Discrete Optimization*, 20, 62-89.
- Doğan, B., & Yıldız, K. (2016). Efficiency and stability of probabilistic assignments in marriage problems. *Games and Economic Behavior*, 95, 47-58.
- Echenique, F. (2008). What matchings can be stable? The testable implications of matching theory. *Mathematics of Operations Research*, 33, 757-768.
- Gale, D., & Shapley, L. (1962). College admissions and the stability of marriage. *American Mathematical Monthly*, 69, 9-15.
- Galleo, F., & Larrain, B. (2012). CEO compensation and large shareholders: Evidence from emerging markets. *Journal of Comparative Economics*, 40, 621-642.
- Gharote, M., Patil, R., & Lodha, S. (2015). Assignment of trainees to software project requirements: A stable matching based method. *Computers & Industrial Engineering*, 87, 228-237.
- Hwang, C., & Yoon, K. (1981). *Multiple Attribute Decision Making: Methods and Applications*. Berlin: Springer-Verlag.
- Kümmel, M., Busch, F., Wang, D. Z. W. (2016). Taxi Dispatching and Stable Marriage. *Procedia Computer Science*, 83, 163-170.
- Li, M. Y., & Fan, Z. P. (2014). Method for stable two-sided matching considering psychological behavior of agents on both sides. *Systems Engineering - Theory & Practice*, 34, 2591-2599.
- Liang, H. M., Jiang, Y. P., & Kong, D. C. (2015). Decision method on multiple targets of satisfied and stable two-sided matching considering the preference ordering. *Systems Engineering - Theory & Practice*, 35, 1535-1546.
- Liu, Q. J., & Peng, Y. P. (2015). Corruption in college admissions examinations in China. *International Journal of Educational Development*, 41, 104-111.
- Mendes, R., van den Berg, G. J., & Lindeboom, M. (2010). An empirical assessment of assortative matching in the labor market. *Labour Economics*, 17, 919-929.
- Taylor, L. A. (2013). CEO wage dynamics: Estimates from a learning model. *Journal of Financial Economics*, 108, 79-98.
- Wang, G. A., Atabakhsh, H., & Chen, H. (2011). A hierarchical Naïve Bayes model for approximate identity matching. *Decision Support Systems*, 51, 413-423.
- Xu, X. K., Wang, C., Zeng, Y., Deng, X., Wang, H., Barklon, D., & Thibault, D. (2015). Matching Service Providers and Customers in Two-Sided Dynamic Markets. *IFAC-Papers OnLine*, 48(3), 2208-2213.
- Yue, Z. L. (2014). TOPSIS-based group decision methodology in intuitionistic fuzzy setting. *Information Sciences*, 277, 141-153.

<http://www.ejmste.com>