



Formation of Language Identities in a Bilingual Teaching Intervention on Fractions

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Received 1 May 2016 • Revised 14 January 2017 • Accepted 15 March 2017

ABSTRACT

Students' identities are connected to their productive participation in the mathematics classroom. A bilingual Turkish-German teaching intervention intended to foster the students' conceptual understanding of fractions has to account for the students' identities, since the students' identities within the intervention influence how the students utilize the learning opportunities. To account for the dynamic and interactive nature of identities, positioning theory was applied to reconstruct the students' identities as multilingual mathematics learners in four teaching intervention groups with four different teachers. In each group, students developed different identities as multilingual mathematics learners, ranging from "student in need of help" to "student responsible for mathematics." These identities were differently affected by the Turkish language. The analysis indicates that Turkish becomes a resource in the mathematical conversations when the students collaborate towards a consensual solution and are made responsible for each other's understanding. As a consequence, for developing teaching interventions aiming at building on students' multilingual resources for participating in mathematical discourses, the ways in which students can develop identities must be taken into account in order to enhance productive engagement.

Keywords: multilingual learning, identity, fractions, language, bilingual teaching intervention

INTRODUCTION

While bilingual Turkish-German students use Turkish and German language in their everyday life, Turkish is usually excluded from German mathematics classrooms by most schools' language policies. Establishing the official language as the exclusive language of instruction by not allowing other languages in the classroom has often been criticized (Planas & Setati, 2009), as it contributes to establishing a "language of power" associated with academic success (similar to the English language in South Africa; Setati, 2008).

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State of the literature

- Students' identities influence how they participate in mathematics classrooms and thus their opportunities to learn mathematics.
- There is a need for understanding the specific conditions that support multilingual students in activating their multilingual resources for mathematics learning in a mainly monolingual school system.
- It is an open question as to how the inclusion of home languages in teaching interventions might affect the students' identities as mathematics learners in general and their identity as multilingual mathematics learners specifically.

Contribution of this paper to the literature

- The empirical analysis extrapolates different forms of identities as multilingual mathematics learners. These identities can be shown to be interactively established, with teachers' substantial impact on the process.
- Making students accountable for arriving at a consensual solution seems to enhance the establishment of identities as mathematics learners with responsibility for the mathematics. These patterns occur often in situations where students use both languages from their multilingual repertoire.
- Future multilingual teaching interventions need to address issues of language identities in order to facilitate the activation of multilingual resources.

As a result, many multilingual students in Germany identify themselves as German-speaking mathematics learners. However, this is problematic: First, it is against the official recommendation of the Council of Europe to include students' home languages in subject matter courses (Baecco et al., 2010). Second, empirical studies have shown that when multilingual students feel that their home language "is good enough for learning mathematics" (Noren, 2008, p. 45), their interest in mathematics can increase (Noren, 2008). In particular, when multiple languages are allowed for negotiating the meaning of concepts, student participation has been shown to be promoted (Noren, 2015). Thus, there may be benefits for multilingual Turkish-German students to identify themselves as Turkish speakers in the mathematics classroom.

The use of language shapes the ways in which students identify themselves: "We use language to get recognized as taking on a certain identity or role, that is, to build an identity here and now" (Gee, 1999, p. 18). While in everyday life mixing and code-switching between Turkish and German is the normal way of speaking for many second- and third-generation Turkish-German bilinguals (Auer, 2011), in the mathematics classroom this identification is not possible, as the activation of multilingual resources is usually not allowed (Meyer, Prediger, César, & Norén, 2016). Thus, students have to cope with contrasting language contexts whether they are in class at school or outside of the school in their normal societal environment. This may lead them to developing distinct identities depending on these contexts.

From a sociocultural perspective, student identity has proven a useful construct to capture participation patterns of underprivileged students with a special social status in mathematics classrooms, such as low-performing students (Lange, 2016), language learners, or multilingual students (e.g., Planas, 2011; for an overview on the identity construct, see Bishop, 2012).

This study investigates how a bilingual Turkish-German teaching intervention, intended for fostering students' conceptual understanding of fractions and implemented in 11 groups by four teachers, might support the students' identification with being Turkish mathematics learners.

In particular, the article will

- argue that opportunities to learn depend at least in part on how students identify themselves as multilingual mathematics learners in the ongoing and evolving conversations of the teaching intervention (Sections 2 and 3);
- present a teaching intervention intended to support students' learning of fractions by activating the students' Turkish language resources (Section 4); and,
- by qualitatively analyzing four different groups, show that students' identities develop differently, where different storylines evoke and allow for different personal stories by which students' find their place in the ongoing conversations (Section 5).

IDENTITY IN THE MATHEMATICS CLASSROOM

Identity manifests itself in stories about individuals, either told or held true by individuals themselves or by others (Sfard & Prusak, 2005). It encompasses the individuals' identification with the activities in the mathematics classroom (Cobb, Gresalfi, & Hodge, 2009) in the form of stories that individuals tell about themselves, here regarded as personal stories. At the same time, it encompasses stories told by others about individuals to identify them in certain ways, for example, a teacher identifying a student as multilingual (Reeves, 2009), here more generally referred to as stories. In this study, of particular interest is the students' identity as multilingual mathematics learners.

The different ways of being identified and of identifying oneself in the classroom affect students' opportunities to participate. Students who have been identified as having special needs tend to refrain from participating as they do not want to interfere with the regular classroom, or they see themselves as having nothing to contribute to the mathematics at hand (Civil & Planas, 2004). Therefore, supporting students' identity as "problem solvers, claim makers, and solution reporters" (Empson, 2003, p. 337) in a study on fostering conceptual understanding is one factor that can foster low-achieving students' mathematical success.

Students' identities are interactively established and can be subject to change in ongoing conversations in the mathematics classroom, as the stories that are told about them continually develop in these conversations. Accordingly, both the teacher and peers may influence the student's identity as a multilingual mathematics learner. Teachers who perceive multilingual students as underprivileged immigrants may attribute to them limited

mathematical capacities, which may result in the assignment of different tasks (Planas & Gorgorio, 2004). Perceiving English language-learning students as no different from their peers hinders teachers' ability to see these students' specific additional linguistic resources, resulting in fewer ways to utilize such resources (Reeves, 2009). In contrast, identifying English-learning students as problem solvers and mathematical thinkers has been shown to help students to develop their identity as capable mathematics learners (Turner, Dominguez, Maldonado, & Empson, 2013). Peers can have an equally high impact. In peer-to-peer interactions, students influence how they identify each other in terms of competence, which establishes unequal opportunities to participate. As a result, students may not work cooperatively on cooperative tasks (Bishop, 2012).

For this article, a teaching intervention was investigated in which 11 different groups were taught by four different teachers. The empirical analysis extrapolates quite different dynamics and opportunities for students to develop their mathematical identity in four of these 11 groups. It was an open question how the inclusion of the students' home languages in this teaching intervention might impact the students' identities as mathematics learners in general and their identity as multilingual mathematics learners specifically.

IDENTITY AS INTERACTIVE AND REFLEXIVE POSITIONING

Students—and teachers—identify themselves in the conversations in the classroom in the form of telling personal stories about themselves in regard to mathematics and using multiple languages. Positioning theory can account for the dynamics of identifying others and oneself in a teaching intervention group based on the actions of the individuals in the conversation, in this case teachers and students. Conversations unfold along storylines, where storylines can be understood as “mutually agreed upon contexts” (van Langenhove & Harré, 1999, p. 9) that establish culturally shared patterns of how a conversation develops. A conversation can revolve around multiple storylines, as the individuals can make reference to moral dilemmas, prototypical characters (the good, the evil, the multilingual), or cultural stereotypes (teacher/student, nurse/patient). The individuals involved understand these storylines differently since they will be based on their own individual previous experiences (see Davies & Harré, 1990). In line with the perceived storyline(s), individuals position themselves and others in the unfolding conversations.

In an ongoing conversation, the participants try to be certain kinds of people (Bucholtz & Hall, 2005). The participating individuals will continually position themselves and others based on the perceived storyline of the conversation in which they are involved (Herbel-Eisenmann, Wagner, Johnson, Suh, & Figueras, 2015) and on the personal stories the individuals tell about who they are in this conversation. “Positioning . . . is the discursive process whereby selves are located in conversations as observably and subjectively coherent participants in jointly produced story lines” (Davies & Harré, 1990, p. 48).

For example, a teacher can take a position (P) of helping a student understand (P1), so the student is positioned as in need of help (P2). These positions allow the teacher to make remarks on the correctness of the student's thinking, while it might relegate the student to ask comprehension questions; these rights and constraints characterize positions P1 and P2.

The teacher's and student's contributions might establish a storyline of tutoring in the eyes of both teacher and student and result in the conversation unfolding in line with this storyline (see van Langenhove & Harré, 1999, p. 17f). There is, however, no pre-determined way to take a position. The teacher might position a student as being in need of help, but the student might resist this positioning and take a different position – individuals can resist a teacher's positioning by means of personal stories that they tell themselves. However, being positioned in a certain way by the teacher or their peers might cause students to actualize or change their personal stories about themselves, for instance, by identifying themselves with the mathematics in a teaching intervention in new ways (Moghaddam, 1999, p. 75). At the same time, the teacher has a certain illocutionary force that makes resisting difficult for students, as teachers have a culturally acknowledged strong position in teacher/student-related storylines (see Davies & Harré, 1990).

The ongoing actualization of personal stories, a student's identity, is based on the dynamics of being continually positioned and of positioning oneself. The former is interactive positioning, here understood as the constraints for action that are interactionally placed upon the student, the expectations that are interactionally established, and the space for actions in which the students are free to act. For example, when a teacher encourages the students to speak in Turkish, the students might change their personal stories because they had previously been forbidden to speak Turkish in the regular classroom. The latter is reflexive positioning, composed of the constraints that students see for themselves, by the expectations they fulfill, and the individual possibilities to act that they see for themselves based on their individual personal stories (Moghaddam, 1999).

By distinguishing between reflexive and interactive positioning, my study reconstructed the ways that individuals' identities – their personal stories – aligned with the affordances of the teaching intervention – the storylines of the unfolding conversations and the general stories held true by the teachers about the students. In the eyes of the students, the teaching intervention might revolve around familiar storylines of teacher-student interactions from regular classrooms, this way suggesting traditional personal stories. However, I assumed that the teachers – with their coercive power to shape the conversation (see Reeves, 2009, for ELLs; also Yoon, 2008) – could act against such traditional storylines and positively influence the students' identities as multilingual mathematics learners in the teaching intervention groups. In summary, there is a complex dynamic of how personal stories develop, and there might even be cases where the personal stories of students have no room in the teaching intervention due to peers and the teacher holding true different stories about an individual.

The theoretical perspective presented here only allows reconstruction of identity as a highly contextualized phenomenon that is dependent upon the specifics of the teaching intervention as well as the notions and activities of the teacher and students.

Research Questions

This study addresses the following research questions:

- Q1. What storylines underlie the interactive positionings, and what personal stories are suggested by the reflexive positionings in the different teaching interventions?
- Q2. What is the spectrum of possible identities that are available to the students in the different teaching interventions, where each teacher might differently contribute to the students' identities?
- Q3. How are these related to the use of students' Turkish home language in the intervention groups?

RESEARCH CONTEXT AND RESEARCH DESIGN

This section introduces the operationalization of individual and normative identities, the research context as given by the teaching intervention of the larger project MuM-Multi (funded by the German ministry BMBF, grant 01JM1403A, held by Prediger, Redder, and Rehbein), its underlying design principles, and the methods of study for case selection and data analysis.

Operationalization: Individual and Normative Identity

The identification of the students' reflexive positionings provides insights into the students' ways of identifying themselves with both the mathematics and the Turkish language. In order to do that, it is important to identify the possibilities "to act" that students see for themselves and the expectations and constraints the need to fulfill. More specifically:

- The possibilities that a student sees for actions in the ongoing conversation are operationalized with the category *initiative*. I assume that students' initiative to contribute to the conversation is equivalent to them intentionally taking a position (van Langenhove & Harré, 1999, p. 22f) and is thus indicative of the personal stories that guide the students' actions. Categories for initiative in educational contexts have been empirically reconstructed by Waring (2011).
- The expectations and constraints that students associate with their positioning are operationalized with the category *participation*. It is operationalized by the length of a contribution, which is dependent on the reflexive positioning of the student. Longer contributions that span two or more sentences are assumed to be instances where students have positioned themselves in line with personal stories that revolve around having an active part in the mathematics in the intervention. Short utterances (a sentence or less) indicate personal stories that have a more passive part in the teacher intervention. In a teaching intervention group where students are given only a few opportunities to participate, short utterances indicate that there are fewer opportunities for the students to act.

The reconstruction of interactive positionings provides insight into the stories that the teachers hold true about the students. Due to the teachers' coercive power, these stories frame the potential for the multilingual students to identify themselves with their multilinguality and the mathematics. Here, the focus is on how the students are positioned by the teacher in regard to the use of language, as they indicate the stories that are held true:

- The positionings that constrain and facilitate the use of multiple languages are operationalized with the category *language tasks and how they are accomplished*. This category encompasses on the one hand the assignment of a language task by the teacher and on the other hand the resulting ways in which the teacher takes up how the students accomplish this task.
- The established possibilities for language use in the conversation are operationalized with the category *language support and regulation*. It encompasses three facets: First, the help and support that is given by the teacher in regard to language; second, the praising of utterances; and third, the rejection of utterances in the conversation (see Table 1).

To address the issue of language-specific positionings, that is, whether the positionings are specifically associated with Turkish or German, the above categories were expanded to include the use of Turkish and German: Each attribution of a category was coded with the underlying language use, either Turkish (T), German (G), or, in cases of code-switching and -mixing, both (B). For example, if a language task is assigned in Turkish, then it is coded as T; this might indicate that the teacher acts in line with a storyline where students are continually expected to use Turkish for working on the assigned tasks, as they are interactively positioned as Turkish mathematics learners.

Design Principles for the Teaching Intervention

This study was embedded in a short-term German-Turkish bilingual teaching intervention that is part of the larger project MuM-Multi. The intervention aimed at fostering seventh grade multilingual students' conceptual understanding of fractions in regard to the part-of-

Table 1. Interpretation scheme and its operationalization

Establishing identities in a teaching intervention group	
Indicators for stories behind interactive positionings	Indicators for personal stories behind reflexive positionings
<p>Language tasks and how they are accomplished:</p> <ul style="list-style-type: none"> - Nature of language tasks - Use of language for accomplishing the task 	<p>Initiative:</p> <ul style="list-style-type: none"> - Rephrasing the teacher - Offering the unfitted - Piggybacking - Activating source - Stepping in - Initiating action - Self-selecting for taking turns (Warwick, 2011)
<p>Language support and regulation:</p> <ul style="list-style-type: none"> - Support for language - What contributions are valued and by whom? - What contributions are rejected and by whom? 	<p>Participation:</p> <ul style="list-style-type: none"> - Utterance spanning more than one sentence - Utterance spanning one sentence or less

whole concept, equivalence, and order of fractions. The intervention ran for five 90-minute sessions. In the project, a bilingual Turkish-German intervention was compared with a parallel monolingual intervention and a control group in a mixed methods design with a randomized control trial. The bilingual intervention was designed to foster multilingual students' conceptual understanding by activating their home language, Turkish. Forty-one multilingual students participated in 11 small groups. This study focused on four groups in the bilingual teaching intervention.

The bilingual teaching intervention is an adaption of a German monolingual teaching intervention for fostering students' conceptual understanding of fractions (Prediger & Wessel, 2013). Three main design principles guided the bilingual adaption of the monolingual teaching intervention (see Schüler-Meyer, Prediger, Kuzu, Wessel, & Redder, submitted):

1. Creating opportunities for bilingual communication and Turkish language production: Due to institutionally limited experience in speaking Turkish in schools (Grosjean, 2001), the Turkish language production is fostered systematically by material and teacher (Meyer & Prediger, 2011).
2. Applying the design principles of macro-scaffolding (Gibbons, 2002) and developing the Turkish formal registers: The learning trajectory was sequenced in line with scaffolding mechanisms and by specifically establishing everyday contexts that connect to the students' multilingual out-of-school experiences (Dominguez, 2011). Furthermore, we provided meaning-related words and phrases in those instances where they might be needed for conceptual understanding. For example, the words *Anteil* (German) and *düsen pay* (Turkish), meaning of "part of a whole," were introduced to express fractions (Kuzu, 2014).
3. Relating registers and languages within the relating registers approach: Moving continually upwards and downwards between everyday and formal registers provides learners with possibilities to construct meaning for mathematical language (Prediger, Clarkson, & Bose, 2016). Beyond that, the German and Turkish languages were continually related in the material (so that Turkish becomes a transparent resource; Setati, Molefe, & Langa, 2008) and code-switching was encouraged (Auer, 2011).

Methods for Data Gathering

Within MuM-Multi, students with a low achievement in a pre-test on fractions and a low proficiency in the German language (measured with a C-Test; Grotjahn, Klein-Braley, & Raatz, 2002) were chosen to participate, as these students are especially at risk and might profit most from a language-integrated teaching intervention on fractions. The students' varying proficiencies in Turkish were measured by a Turkish C-Test. Having few previous experiences in Turkish mathematics, the students' Turkish academic or technical language was less developed than their German academic and technical language. All students participated voluntarily in the teaching intervention.

Each session in each of the 11 teaching intervention groups was videotaped (11 groups x 5 sessions). The camera focused on a group of 2 to 3 students, while another 2 to 3 students participated in the same intervention group but were not videotaped. The video material was transcribed and translated by Turkish-German bilingual university students in ways that preserved the meaning of the Turkish utterances as much as possible.

Four teachers implemented the teaching interventions in one to four of the 11 groups. In most groups, the teachers stayed with their group over the course of the intervention. The teachers were trained in a preparation course to implement the teaching intervention in line with the presented principles.

Case Selection for Data Analysis

As discussed above, the teachers had important roles in the students' identity development in the teaching intervention. In order to capture and contrast diverse ways in which identities can be established in a bilingual teaching intervention, the group with the most vivid communication from each teacher was selected. This resulted in four focus intervention groups.

The analysis reported here focuses on the conversations within the first task of the third teaching intervention session. This focus task was based on the context of downloading movies: Four children downloaded movies; each download was presented with its own download bar (see Figure 1). The students were asked to reflect on the idea of the need for a standardized medium for comparing the downloads, namely, a fraction bar with the same length. The task was given in Turkish and German.

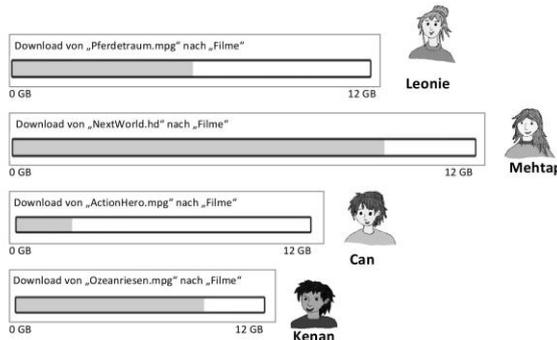
The task was chosen as a focus task for this study for two reasons:

- It is located at a central point of the intended learning trajectory: In Sessions 1 and 2, students had a chance to understand the use of the fraction bar and relevant keywords associated with it. In Session 3, it was intended that the students would internalize the nature of the whole in the part-of-a-whole relationship, for example, that the size of a fraction does not depend on the length of the fraction bar.
- The selected task implements the three design principles: In line with the first design principle, it connects to the students' everyday experiences, in this case the context of downloading movies. This task is also exploratory and encourages collaborative work. It provides room for the students to use both Turkish and German languages while collaboratively working on it. In line with the third principle, the task allows students to work with different representations, in this case the fraction bar and the symbolic representation.

1 Anteile in Fortschrittsbalken vergleichen



Die vier Freunde laden sich ihre Lieblingsfilme auf ihre Rechner:



a) Bearbeitet die Aufgaben in eurer Kleingruppe:

- Wer hat den größten Anteil geladen? Schätzt zuerst.
- Welche Anteile wurden geladen? Schreibt sie links neben die Streifen.
- Wie sieht man die Anteile in den Streifen? Zeichnet Markierungen ein, sodass man die Anteile gut ablesen kann.
- Überträgt die Anteile in die Streifentafel. Warum sieht man an der Streifentafel besser, welcher Anteil größer ist?

b) Schreibt alle Anteile der Größe nach auf. Beginnt mit dem **Größten**.

c) Schreibt noch einmal auf: Warum kann man an der Streifentafel besser vergleichen als an den Bildern oben?

Four friends are downloading movies on their PCs.

a) Work on the tasks in pairs:

- Who has downloaded the biggest share? Estimate first.
- What shares were downloaded? Write them next to the fraction bars.
- How do you see the shares in the fraction bar? Place marks so that you see them better.

- Transfer the shares into the fraction bar board. Why is the fraction bar board better for comparing the shares?

b) Write down the shares according to their size. Start with the biggest.

c) Write again: Why is the fraction bar board better for comparing the shares?

Figure 1. Download task: The first task in teaching intervention Session 3 (given to students in German and Turkish versions)

Data Analysis

The conversations in the focus tasks and the four focus intervention groups were analyzed with regard to interactive and reflexive positionings (see Table 1 for an overview), employing content analysis with the above described categories (Mayring, 2015). Within a category, stories are identified based on frequently occurring phenomena in the material for each group that are then condensed (“reducing procedures,” p. 373) and explicated.

A comparison of stories in each teaching intervention group can indicate the conditions in the teaching intervention groups for students to develop their personal stories, that is, their

identities as multilingual learners of mathematics. Furthermore, it can capture how the students' actual personal stories develop within the conversation, that is, the actual development of the students' identity as multilingual mathematics learners. This gives insight into research question Q2. Comparisons across the groups indicate how the opportunities for developing an identity as a multilingual mathematics learner differ between the groups and can provide insights into storylines in line with research question Q1. The analysis of the use of language within interactive and reflexive positionings will indicate how the students' identity development is related to the use of Turkish and German, addressing research question Q3.

RESULTS

In the following, I compare and contrast two cases. These cases can be read as "extreme cases" with respect to very different opportunities for students to develop their identity. Based on these two cases, hypotheses for mechanisms underlying the construction of a positive identity towards multilingual mathematics learning were generated (Section 5.3). An overview of the results of the analysis for all four analyzed teaching intervention groups is given in Table 2.

Identities in Group E (Teacher: Mr. Flid)

Interactive positionings and related storyline of the conversation

In the following episode from teaching intervention Group E, whose teacher is Mr. Flid, Atiye presents a solution. In reaction to her, Mediha presents a competing solution (Turns 3218-3229). The teacher intervenes in the conversation and asks Atiye to explain how she arrived at her solution. In this way, he positions her as being knowledgeable of the mathematics at hand.

In Turn 3217, Atiye uses the notion of common denominator from the download context—namely that every fictional student downloads 12 gigabytes—to explain her answer, $\frac{7}{12}$. She argues that the grey area has a length of 7 in relation to the length of the fraction bar of 12. Atiye's utterance is followed by the teacher asking Mediha to also present her solution, which is "*Ikide üç?*" ("12 therein 3?") (Turn 3223). After that, the teacher asks Mediha to explain her solution, but she is not able to give an explanation that is understood by the others, so he asks Atiye to explain her solution to Mediha (Turn 3230).

In this episode, the teacher asks all participating students to present their solutions and to verbalize them. The teacher positions the students as being responsible for arriving at a shared understanding, since he asks Atiye to explain her thinking so that Mediha will understand the correct solution. But he also positions the students by requiring that they listen to each other (Turn 3232). While the teacher continually speaks Turkish, he allows the students to answer in German (Turns 3217, 3231), which suggests that his positionings of the students are language independent.

Transcript E1

Turn	Original (Turkish in black, German in grey)	English Translation (from Turkish in red, from German in orange)
3217 Atiye	Wenn man- burası şimdi on iki olsa, dann wird hier ungefähr Sieben.	If one, if this over here would be twelve [points at the Kenan's download bar on the worksheet, see Figure 2], then this would be seven or so.
3218 Flid	Atiye, sence? . . .	[to Mediha, gets the names wrong] Atiye, in your opinion?
3223 Mediha	Ikide üç? . . .	Twelve, therein three? [in Turkish, fractions are expressed "denominator therein numerator"]
3230 Flid	Tamam tamam. Ehm o zaman Atiye Mediha'ya açıklar mısın nasıl on ikide yediği bulduğunu.	OK, OK, um, Atiye, can you then explain to Mediha, how you arrived at the twelve therein seven?
3231 Atiye	Ehm zum Beispiel du hast ja hier Zwölf, ne?	Ehm, for example here you have twelve, haven't you? [points at Kenan's fraction bar]
3232 Flid	Okay, sen de dinle!	[to Okay] Okay, do also listen!

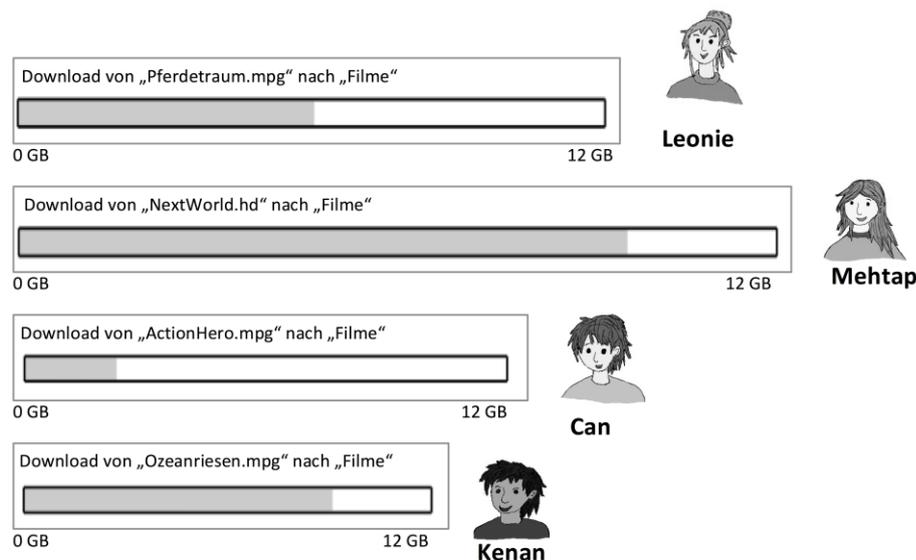


Figure 2. Fraction bars on the worksheet

Table 2.1 Overview of the storylines in the four teaching intervention groups

	Group E	Group D	Group J	Group L
Language tasks and how they are accomplished:	-Teacher positions himself as guide of conversation: Focused, goal-oriented discourse established by repeating task formulations or by adaptively bringing students questions as tasks into the discourse.	-Teacher positions himself as tutor and guide of conversation: Teacher takes responsibility for explanations and observations and for building on the students' utterances	-Teacher positions himself as traditional teacher who allocates speaking rights: -Mathematical correctness of solutions is not relevant; rather, presenting it to the others is more relevant. -Solutions are not negotiated but presented sequentially.	-Teacher positions himself as supporter of mathematical utterances (not conversations): Students are accountable for working in the fraction bar and locating and drawing the fractions, but not for negotiations or building on each other's utterances -No accountability for mathematical correctness
Language support and regulation:	-Teacher continually speaks Turkish. -Students positioned as responsible for task solution: German everyday language is accepted, also for conceptual work; Support in working with the fraction bar (in Turkish).	-Teacher positions himself as tutor and guide of conversation: Expected answers are enforced and unexpected answers are ignored or rephrased so that they fit the expectations -Students positioned as learners in need of help: Short answers allowed, lots of support	-Task solutions have to be presented in Turkish. -Words are treated as vocabulary: not their meanings are negotiated, but their translation Students position themselves as Turkish language learners: When working collaboratively without teacher support, students speak German.	-Teacher positions himself as supporter of mathematical utterances (not conversations): -Verbalizations (e.g., comparing solutions) are supported by the teacher (everybody has to share their solution) Support in using Turkish language.
Storylines that underlie the teacher's positionings of the students	<i>Storyline of quest for a consensual solution, where conversation is guided by the teacher; students positioned as responsible for mathematics</i>	<i>Storyline of working on tasks under guidance of teacher, students positioned to find acceptable answers</i>	<i>Storyline of Turkish language learning in mathematics: Students positioned to give solutions in Turkish (German is language for mathematical thinking among peers)</i>	<i>Storyline of quest for individual solution, where use of representations is supported by the teacher, but students not positioned as responsible for correct mathematics</i>

Table 2.2 Overview of the personal stories and identities in the four teaching intervention groups

	Group E	Group D	Group J	Group L
Initiative:	-Students position themselves as responsible for understanding: Students initiate turns for helping each other or to collaboratively arrive at a solution.	-Students position themselves as learners in IRE pattern or as in need of help: Students initiate conversations with the teacher about expectations in the task or to inquire about the "expected" solution	-Students position themselves interchangeably as Turkish language learners or mathematics learners: -students initiate conversations to arrive at a solution when working in pairs; -Students initiate the translation of words when they need a word in writing down a solution.	-Students position themselves in need of language support: Students initiate turns in order to establish clarity (e.g., how to express gigabyte in Turkish); -Students initiate discourses that are "off-topic"
Participation	-Students position themselves as responsible for solution: -Students are conjecturing, explaining, observing and discursively building on each other's utterances, -fraction bar as an aid for this. Students participate more in translanguaging mode	-Students position themselves within closed conversation pattern and orient their actions accordingly: naming fractions, locating fractions in the fraction bar, in reaction to the closed discourse pattern (both Halim and Hakan)	-Students position themselves as teacher-guided learners: Students participate to question the teacher for explicating the expectations for the solution	-Students position themselves as supervised learners in search of correct solution: Students participate to verbalize solutions with the help of the fraction bar (in German and again in Turkish) to each other and the teacher
Personal stories/identities that the guide the students' actions through reflexive positionings	<i>Stories of collaboratively working towards understanding and solving the task. Students identify themselves as contributors to this endeavor, where their Turkish language is a means to explicate one's understanding</i>	<i>Stories of eager mathematics learner (Halim) or receptive participant (Hakan). Students identify themselves with their part in a traditional conversation pattern of teacher/student interactions in school</i>	<i>Stories of bilingual mathematics learners: Students identify themselves as Turkish language learners while working independently on the tasks</i>	<i>Stories of supervised, material-centered mathematics learning: Students identify with a role of giving a linguistically acceptable task solution (to the teacher) through independent work on the fraction bars</i>

Reflexive positionings and students' personal stories

In the following episode, Mediha shows initiative and steps in to give an explanation as to why $\frac{3}{12}$ is an incorrect solution.

Transcript E2

Turn	Original	English Translation
Person	(Turkish in black, German in grey)	(from Turkish in red, from German in orange)
3144 Flid	Sence?	<i>[addressing the student Okay]</i> In your opinion?
3145 Mediha	Drei Zwölftel geht doch nicht.	Three twelves does not work after all.
3146 Okay	Ist falsch glaube ich.	Is wrong, I believe.
3147 Flid	Mhm mhm.	Mhm mhm <i>[agreeing]</i> .
3148 Mediha	Çünkü mesela sen buraya yapsan, dann kannst du ja nicht mehr weiter machen.	Because, if you do it like that <i>[points at Leonie's download bar on the worksheet, see Figure 2]</i> then you can't do it any further.

Mediha steps in to give an explanation in Turns 3145 and 3148, where the teacher asks Okay for her opinion. With this, she gives an opportunity to the student Okay, who had mistakenly suggested $\frac{3}{12}$, to explain her false reasoning (Turn 3144). Mediha and Okay agree that the previously suggested solution $\frac{3}{12}$ is not correct (Turns 3145, 3146). The teacher accepts Mediha's initiative.

The students take up multiple positionings in which they stand in for the other and explain their solutions to each other. Here, Mediha stands in for Okay by answering for him (Turn 3148). Furthermore, the students seem to accept each other's positions and cooperatively arrive at a shared understanding. The students take up positionings in the conversation accordingly. Thus, the students identify themselves with the aim that everyone has to understand.

The next episode takes place shortly after the previous one and illustrates how Atiye conjectures about how to determine which download is the largest share.

Atiye observes that the different lengths of the fraction bars in the task do not allow direct comparisons of the grey areas, but that the ratio of the grey area matters. She suggests that one has to shorten all fraction bars to the same length, so that one can compare the grey areas. Her conjecture is that the fraction bars need to have the same length in order to be able to compare the fractions.

Transcript E3

Turn	Original	English Translation
Person	(Turkish in black, German in grey)	(from Turkish in red, from German in orange)
3161-3163 Atiye	Ich glaub Balkenler gleich değil ya. Deswegen kann man das nicht so – çünkü bu kurz olsa dann wäre das ja ungefähr bis da	I believe the bars are not identical. [Flid: # Mhm mhm] That is why one can't like – because if that were short [points at Mehtap's download bar on the worksheet, Figure 2] then that would be till around here.

Atiye engages in a conversation in which the students explain their thinking and question their current, unfinished explanations. Atiye builds on the previous observations of her peers of the fraction bars. It seems that Atiye is positioning herself to be responsible for each of the student's understandings, which is part of the collaborative endeavor to arrive at a shared understanding. Interestingly, this goes hand in hand with translanguaging, that is, with mixing German and Turkish (García, 2009). Hence, the students seemed to identify themselves as multilingual "doers" of mathematics in mathematical activities such as conjecturing.

Relation between interactive and reflexive positionings and the development of the students' identity in intervention Group E

The students' personal stories in intervention Group E developed hand in hand with the teacher-enforced storylines that guided the conversation. Positioning the students as being responsible for the correct solution in a storyline where every student has to understand the others' solutions and explanations is coherent with Atiye and Mediha's reflexive positionings, in which they make themselves responsible for cooperatively arriving at a solution and for the other's understanding. This suggests that the students are able to develop identities as multilingual mathematics learners and that they identify themselves as doers of mathematics across both languages.

Identities in Teaching Intervention GROUP D (Teacher: Mr. Flek)

Interactive positionings and related storyline of the Conversation

The following episode from intervention Group D illustrates how the teacher, Mr. Flek, assigns language tasks and how he evaluates and takes up the students' answers to his tasks. Two students, Halim and Hakan, work together; the teacher is also in charge of three students who work at a separate table.

Halim is working with fraction bars that have 12 as the denominator (Figure 2). In the beginning, when the teacher assigns a task, Halim answers in Turkish by naming the correct

fraction. The teacher positively evaluates Halim’s answer by revoicing it in the same wording.

Transcript D1

Turn	Original	English Translation
Person	(Turkish in black, German in grey)	(from Turkish in red, from German in orange)
3091 Flek	. . . senin bu sıralamana göre Mehtap kaç ne kadar indirdi? Düşen payı ne- ne kadar Mehtap'in?	. . . how much has Mehtap downloaded, according to your ordering? How, how big is Mehtap’s share?
3092 Halim	Ehm on ikide on.	Ehm twelve therein ten.
3093 Flek	On ikide on demi? Kenan'in ne kadar?	Twelve therein ten, isn’t it? How much is Kenan?

The teacher positions himself as responsible for evaluating and building on the students’ utterances when working with Halim and Hakan. In this episode, the teacher takes up Halim’s answer and builds on it by asking a follow-up question (Turn 3093). The interaction positions students as having to answer to the teacher, where short answers will be accepted to “fulfill” these positionings. These positionings might indicate that the teacher and students are together establishing a storyline of tutor and learner in need of support using the Turkish language.

The following episode gives deeper insight into this storyline. The teacher asks Halim to explain the reasoning behind his solution, in which Halim has observed that he has to account for the different lengths of the fraction bars when determining the ratio:

Transcript D2

Turn	Original	English Translation
Person	(Turkish in black, German in grey)	(from Turkish in red, from German in orange)
3089 Flek	Yani, sen ne diyorsun tam olarak?	So [points at Halim], what do you mean, exactly?
3090 Halim	Ja, dass dass die hier größer. Weil das unterschiedliche Balken sind. Also das und dann das. Hier, weil aber obwohl dieser Balken kleiner ist. Aber der hat mehr- mehr- runter- runter- runtergeladen. Das ist das Wichtigste, wer mehr- mehr- runtergeladen hat.	Yes, that that they here bigger [points at Mehtap’s and Can’s download bars, Figure 2]. Because they are different bars. So that [points at Mehtap’s download bar] and then that [points at Kenan’s download bar]. Here, because, but in spite of this bar being smaller [points at Kenan’s bar]. But he has down- downloaded more, more. That is the most important, who downloaded more, more. So #

Transcript D2 continued

Also #

<p>3091 Flek # Sen öyle diyorsun demi? Şimdi sen diyorsun ki ehm Mehtap ve Kenan'ın düşen payı aynı. Du siehst da einen Unterschied. Das ist ja auch völlig ok.</p>	<p>#you mean it like that, don't you? You are saying now that Mehtap and Kenan have the same share. You are seeing a difference there. That is entirely okay, yes.</p>
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Halim explains his thinking in everyday language and by using the fraction bar deictically. He suggests that while the fraction bars are different in length, this does not matter (Turn 3090). Instead, he focuses what has been downloaded in each fraction bar. The teacher frames Halim's utterance by asking for an explanation (Turn 3089) and then by evaluating it (Turn 3091). The teacher takes the position of being responsible for evaluating Halim's answer and interprets its meaning by rephrasing it (Turn 3091). The teacher then evaluates Halim's answer based on this rephrased answer, which mirrors his understanding of what Halim tried to express. As a result, by directing the conversation away from Halim's answer after Turn 3091, the teacher positions Halim as not contributing to the current conversation. As a consequence, Halim might identify himself as not having understood the task and/or the embedded mathematics correctly. It is an open question if this positioning is also connected to Halim's use of German. This interaction indicates, together with the above episode, a storyline of "tutor and learner in need of support" in which the teacher is in a position to frame and evaluate the students' utterances.

Reflexive positionings and students' personal stories

In the following episode Halim and Hakan engage in the mathematical conversation by initiating questions directed to the teacher.

Transcript D3

Turn Person	Original (Turkish in black, German in grey)	English Translation (from Turkish in red, from German in orange)
3062 Flek	çubuğa aktardığınız düşen payları ehm büyüklüğüne göre en büyüğü ile başlayarak şuraya yazın.	[points at the symbolic fractions on Halim's worksheet, Figure 3] these shares that you transferred to the bar according to their size [points at task 1b] beginning with the biggest, write them down there.
3063 Hakan	Die beiden sind gleich.	These two are equal [points at two download bars]
3064 Flek	Tamam. Güzel.	Ok. Nice.

4228

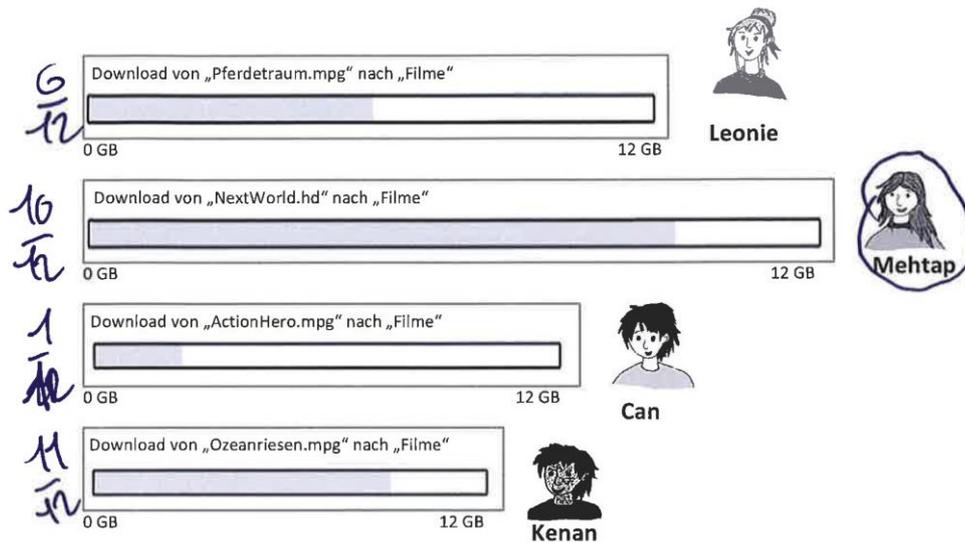


Figure 3. Halim's worksheet

The teacher assigns a task to Halim and Hakan (Turn 3062). The nature of the task involving writing something down suggests that the teacher intends the students to work alone on this task, without assistance. However, Hakan steps in and gives an answer where it is not expected (Turn 3063). The teacher values the utterance from Hakan, which breaks up the individual work and reestablishes the assistance given by the teacher (Turn 3064 and following).

Hakan's stepping in with an answer or a clarifying question after the teacher has assigned a task is a usual pattern in this group. Usually it leads the teacher to give more explanations, in this instance by fragmenting the larger task into smaller tasks. By stepping in this way, the students might position themselves as being in need of help, and they usually receive help after stepping in. Hence, Hakan's actions seem to connect to personal stories where he sees himself as a receptive participant in the conversation and positions himself as being in need of help; in the course of the conversation, Hakan participates in Turkish, which suggests that he has actualized his personal stories into the Turkish language.

The following episode D4 illustrates how Halim tends to participate in the conversation.

Previous to this episode, the teacher assigned tasks from the worksheet to the students. This confuses Halim (Turn 3019) and the teacher comes to Halim (Turn 3022). Then, Halim engages in the conversation with longer than usual utterances to express what irritates him (Turn 3023). In the next Turn (3024), this exchange is ended by an explanation from the teacher that results in the assignment of a more specific task.

Transcript D4

Turn	Original	English Translation
Person	(Turkish in black, German in grey)	(from Turkish in red, from German in orange)
3019 Halim	Hää? Hat wieviel hat? Wie? Ehm, ich verstehe das nicht.	Heee? Has how much has? How? Ehm [<i>looks at the teacher</i>] I do not understand this. . . .
3022 Flek	Mhm mhm! Tam olarak neyi anlamiyorsun?	Mhm mhm! What exactly do you not understand?
3023 Halim	Ja! Das. So also das hier, das hier Also aber ich weiß nicht. Das sind doch Anteile. Das sollen doch also hier ist es doch sowas wie Anteil	[<i>points at his worksheet</i>] Yes. That. So, thus that one, that one [<i>points at the download bars</i>] But I don't know. These are parts of a whole. That here should be something like part of a whole [<i>points at the fraction bar board</i>]

Halim engages in the conversation to clarify the assigned task. This way, Halim might position himself as – most likely – a German mathematics learner like in regular classrooms, which usually involves working on assigned tasks in order to learn mathematics. At the same time, he positions the teacher as being responsible for how the task is meant to be solved. This suggests that Halim is acting in line with personal stories of being an eager mathematics learner who works thoroughly on assigned tasks in order to learn under the guidance of the teacher. These stories might have been transferred from the regular mathematics classroom and are thus told in German. This seems to be the usual way for Halim to participate in the conversation in this task.

Relation between interactive and reflexive positionings and the development of the students' identity in intervention Group D

In teaching intervention Group D, the teacher establishes a storyline of “tutor guiding students who are in need of help” while working on the task. Halim and Hakan position themselves differently in this storyline: Hakan as learner who is in need of assistance and Halim as learner who works thoroughly on the mathematical tasks in order to learn under the guidance of the teacher. Accordingly, the teacher and both students contribute to perpetuating this storyline in which the teacher is the guide/helper.

Halim and Hakan's identities might develop on different pathways: Hakan might develop an identity as multilingual learner who specifically needs assistance – coherent with being placed in a teaching intervention intended to foster his understanding – while Halim might develop an identity similar to his mathematical identity in the regular classroom, where he

perceives himself as a mathematics learner who needs to thoroughly work on tasks under the guidance of the teacher in order to learn.

Comparison of the Four Cases

The four cases presented in Table 2, from which the two in-depth cases presented above have been taken, differ in the overarching storylines that are established in each. These differences are a product of the teachers trying to implement the Turkish language into the intervention group in line with the design principles of this study. For example, the Turkish language can become a medium for mathematical talk, such as conjecturing and observing as in Group E, or it can become an aspect of the content to learn, as in Group J. In the former, the Turkish language possibly enriched the mathematical conversations, but in the latter, the Turkish language probably did not directly contribute to the mathematical conversation. This suggests that the extent to which the Turkish language contributed to the mathematical conversation depended to a large degree on the established storyline within the teaching intervention group. Here, the storylines were usually teacher centered because the teacher's illocutionary force allowed him to guide the conversations and the intervention was designed to be teacher centered.

The storylines in the teaching intervention groups provide the stage for the students to develop their personal stories. Hence, in each group there are different opportunities for the students to develop the stories they tell about themselves, that is, their identities. As shown above, in intervention Group E, Atiye's and Mediha's personal stories of being responsible for the solution and each other's understanding connect to "collective" positionings: Atiye and Mediha positionings are characterized more by being part of the group than by being an individual in the conversation. This is quite different from Group D, where the students for some reason work individually despite sitting at the same table. Halim and Hakan position themselves individually, and this results in different opportunities to engage in the conversation. Hence, not only do the storylines differ between the teaching intervention groups, the students also position themselves differently within the teaching intervention groups. As a result, there is a large spectrum of possibilities for identity development in the bilingual teaching intervention in the larger project MuM-Multi. At the same time, however, this spectrum might be limited by the students' previous identities from the regular classrooms that the students import into the teaching intervention. For example, Hakan's personal stories might connect to the regular German-dominated classroom, where he also might continually seek assistance from the teacher.

SUMMARY AND DISCUSSION

With respect to research question Q1, we observe that in the four analyzed teaching intervention groups, a specific storyline was established in each that guided the conversation. This storyline was teacher centered in these groups and relatively stable over

the course of the analyzed focus task presented here. These storylines opened a stage for the personal stories to develop, resulting in personal stories that were clearly connected to the storyline of the conversation.

In regard to research question Q2, we see that each of the four groups developed unique storylines that set the stage for unique ways for students to develop their personal stories. At the same time, the storylines revolved in some way around relating mathematics and the Turkish language, and thus were relatively similar. Within an intervention group, the spectrum of identities was limited by the room the established storyline provided for students to develop their personal stories.

The students' personal stories—their identities as multilingual mathematics learners within the teaching intervention—were connected to the use of Turkish in the four analyzed intervention groups, just as the storylines in the intervention were a product of the multilingual nature of the teaching intervention (Q3). Accordingly, students were able to include Turkish as a part of their mathematical identity when they engaged in Turkish in activities such as conjecturing and explaining (Group E). The Turkish language also became the language to ask for and to receive help and could in this way connect to an identity of “needing assistance in mathematics” (Hakan in Group D). Turkish also became the language of correct solutions, resulting in identities of being Turkish language learners of Turkish mathematical language (Group J).

This study has focused on the comparison of students' identities in four different teaching intervention groups led by four different teachers and how they were influenced by the storyline of the conversations generated by one task that occurred during the third session of a five-session teaching intervention. It has not, however, given insights into the development of the students' identities over the course of the five sessions. Furthermore, only four out of the 11 groups were analyzed. Nevertheless, assuming that the teachers acted similarly in all five sessions, the students may have developed relatively stable identities. A cursory examination of the other seven groups suggests that in other groups, the same teacher may have established very different storylines; thus, in these groups there were different opportunities for students to develop their identities.

In this study, I have used the identity construct to investigate identities of individuals. At the same time, I have used the construct of storylines to characterize the conversations in the different intervention groups in order to assign a form of normative identity to each group. This, however, can be problematic in conversations where multiple storylines are enacted and guide the individuals in developing their personal stories (see Herbel-Eisenmann et al., 2015).

Elsewhere it has been shown that teaching interventions that segregate students from their regular mathematics classrooms can affect their participation in mathematics, mediated by their identification with having specific needs (Civil & Planas, 2004). In this study, the initial positioning of the students was similar, as the students were asked to participate in a teaching intervention in addition to their regular mathematics classroom. However, this study shows that students can profit from a teaching intervention in regard to their identity as long as an adequate storyline is established that guides the conversations in the

intervention. For example, the students in Group E developed an identity that allowed them to be comfortable using Turkish to explain, conjecture about, or observe mathematical phenomena. For this to be successful, a “quest for a consensual solution” storyline might be necessary in which students are positioned as responsible for the mathematics at hand. Such an accountability for consensual understanding (Greeno, 2006) led the students to use Turkish while exercising authorship of mathematical ideas and agency (“taking up room”; Hand, 2012).

This teaching intervention was built on research on multilingual mathematics learning, which has suggested that, under certain conditions, multilingualism is a resource for mathematics learning. These studies focus on the mathematical side of such interventions: mathematics that connects to the students’ everyday experiences, which allows them to initiate a multilingual, everyday mathematical discourse (e.g., Dominguez, 2011; Moschkovich, 2015). However, the results presented here suggest that it might not be sufficient that the material and conversation are multilingual. Instead, it seems that students need to identify themselves as multilingual in storylines where they are made responsible for the mathematics at hand: In teaching intervention group D, students likely fell back to storylines from their regular classroom and thus to monolingual use of language, whereas in Group E, Turkish became a resource. The results presented here suggest that a teaching intervention aiming to build on students’ multilingual resources for participating in mathematical discourses has to carefully consider the ways in which students can develop identities as multilingual mathematics learners in ways that students do not fall back on monolingual identities from regular classrooms.

ACKNOWLEDGEMENT

I thank the anonymous reviewers for their valuable feedback. I thank Susanne Prediger and our project partners within the project “MuM-Multi: Fostering Language in multilingual mathematics classrooms”, which is funded by the German ministry BMBF (grant 01JM1403A to Prediger, Redder, Rehbein) and conducted with T. Kuzu, J. Wagner, M. Çelikkol.

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