MODESTUM EURASIA Journal of Mathematics, Science and Technology Education, 2026, 22(9), em2902
ISSN:1305-8223 (online)
OPEN ACCESS Research Paper https://doi.org/10.29333/ejmste/19138

From estimation to validation: Operationalizing early-stage modelling in elementary Fermi problems

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Received 03 March 2026 ▪ Accepted 08 August 2026

Abstract

Early modelling processes in elementary classrooms are difficult to observe systematically. This exploratory case study examined two Fermi-problem lessons involving 32 grade 3 students in Japan and operationalized early-stage modelling through four indicators: early assumption articulation, estimation-validation transitions, time allocated to estimation and checking, and validation-oriented discourse. Intra-coder stability for the underlying one-minute modelling activity diagram categories was examined through blind recoding of 43 of 123 units (35.0%; Cohen's κ = .81). Across the analyzed groups, early assumptions occurred in 25.0-50.0%, at least one estimation-validation transition in 66.7-75.0%, and validation-oriented discourse in 50.0-100.0%; estimation-plus-checking ratios were 40.9% and 36.6% across the two tasks. The findings suggest that selected cyclic modelling dynamics may become interactionally visible in grade 3 classrooms. The proposed indicators constitute an exploratory analytical approach whose reliability and transferability require further validation.

Keywords: mathematical modelling, Fermi problems, idealization, classroom discourse, elementary education

INTRODUCTION

Mathematical modelling is widely recognized as a central component of contemporary mathematics education because it connects mathematical reasoning with extra-mathematical situations and supports students' capacity to interpret, structure, and solve real-world problems (Blum & Niss, 1991; Niss et al., 2007). International assessment frameworks have similarly emphasized mathematical literacy as the capacity to formulate, employ, and interpret mathematics in a variety of contexts (OECD, 2013). Modelling involves transforming real-world situations into mathematical structures, working within those structures, and interpreting and validating results in relation to the original context.

Despite this recognition, modelling processes in elementary classrooms remain difficult to observe systematically. This difficulty is particularly salient in elementary classrooms, where students' modelling activity often appears in informal, partial, or emerging forms rather than as complete modelling cycles. Prior studies have suggested that Fermi problems can provide accessible contexts for primary students to engage in estimation, assumption-making, and model construction (Haberzettl et al., 2018; Peter-Koop, 2005). While modelling cycles provide comprehensive theoretical descriptions, empirical research often focuses on complete modelling trajectories or learning outcomes rather than on how modelling processes emerge in their early stages. In particular, the transition from informal estimation to structured validation is rarely examined at the level of observable classroom interaction.

Fermi problems offer a productive context for examining early modelling processes. By design, such problems require students to introduce assumptions, generate approximate values, and evaluate the plausibility of their results under conditions of incomplete information. Through this process, students engage in structuring real-world situations into mathematically workable forms.

In this study, we focus on the early emergence of modelling processes in grade 3 Fermi-problem lessons. We interpret the articulation of assumptions as instances

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Contribution to the literature

- This study operationalizes early-stage mathematical modelling in grade 3 Fermi-problem lessons through four observable classroom indicators: early assumption articulation, estimation-validation transitions, E+Ch engagement ratio, and validation-oriented discourse.
- It provides an exploratory micro-interactional account of how assumption-making, estimation, and validation can become observable and recursive in elementary classroom discourse.
- Methodologically, the study connects modelling theory, idealization, MAD-based temporal coding, and classroom discourse analysis to offer a structured approach for examining early modelling dynamics.

of idealization—understood here as the intentional introduction of assumptions that render a situation mathematically structured (Winkelmann, 2023). Rather than treating idealization as a purely philosophical construct, we examine how it becomes visible in classroom interaction through observable indicators.

To address the difficulty of identifying early-stage modelling processes in practice, this study proposes four theoretically grounded classroom indicators. The purpose is not to evaluate correctness or instructional effectiveness, but to operationalize modelling constructs in a way that allows systematic analysis of estimation-validation transitions in elementary classrooms.

This study is exploratory in nature and addresses the following research questions:

1. How can early-stage modelling be operationalized through observable classroom indicators?
2. How do estimation and validation processes interact in elementary Fermi problem contexts?

By translating selected modelling constructs into observable analytical units and examining the stability of the underlying activity coding, we aim to explore the feasibility of an analytical approach for investigating early modelling dynamics in elementary education.

THEORETICAL FRAMEWORK

Mathematical Modelling and the Modelling Cycle

Mathematical modelling is commonly described as a cyclic process linking real-world situations and mathematical reasoning (Blum & Borromeo Ferri, 2009; Blum & Leiß, 2007). In this process, learners move from understanding a real situation to simplifying, structuring, mathematizing, working mathematically, interpreting, validating, and communicating results. Prior research has emphasized that these phases are not necessarily linear and that transitions between phases can be a source of difficulty for students (Borromeo Ferri, 2006; Galbraith et al., 2006).

While this cycle provides a comprehensive theoretical account of modelling processes, empirical studies often examine modelling as a complete trajectory or focus on learning outcomes rather than on how modelling processes emerge in early classroom phases. As a result, the initial transition from informal reasoning to structured mathematical modelling remains under-specified at the level of observable classroom practice. The modelling cycle (Blum & Leiß, 2007) was selected as the primary framework because it provides a comprehensive representation of modelling processes applicable across educational levels.

Although the modelling activity diagram (MAD) has been used productively to represent students' modelling activity in Fermi-problem contexts (Albarracín & Ärlebäck, 2019; Ärlebäck, 2009; Pla-Castells et al., 2021), the present study adopts the modelling cycle as the primary conceptual framework for two reasons. First, the purpose of this study is not to reconstruct complete modelling trajectories in detail, but to identify early-stage transitions between assumption articulation, estimation, and validation. Second, the modelling cycle provides a broader conceptual language for linking these transitions to idealization, mathematization, and validation. MAD is therefore treated as a relevant methodological reference, particularly for visualizing modelling sequences, while the modelling cycle serves as the overarching theoretical framework.

Idealization in Early-Stage Modelling

In the early phases of the modelling cycle, students engage in simplifying and structuring a situation in order to make it mathematically tractable (Blum & Leiß, 2007). These operations require selecting relevant quantities, introducing assumptions, and reorganizing contextual information into a form that can be expressed mathematically.

Making assumptions is especially important in modelling tasks with missing or incomplete data, because students must construct a situation model before they can introduce realistic values or simplifying conditions (Surel & Greefrath, 2023). In this sense, assumption-making is not merely a preliminary step but a central modelling action through which students transform an open real-world situation into a mathematically workable representation.

In this study, we interpret these early-phase operations through the lens of idealization (Winkelmann, 2023). Idealization is understood here as the intentional introduction of assumptions that render a situation mathematically structured and workable. For the purposes of this study, idealization refers specifically to the articulation of assumptions that enable subsequent estimation and validation processes.

Recent work on Fermi problems also highlights the central role of assumptions in modelling activities, particularly because solvers must decide which quantities are relevant, which values can be estimated, and which contextual features can be simplified without making the model meaningless (Albarracín & Ärlebäck, 2025).

Although idealization has been discussed in theoretical accounts of modelling, its emergence in elementary classroom interaction has not yet been sufficiently translated into observable classroom indicators. This gap motivates the present study.

Estimation and Validation in Fermi Problems

Fermi problems are widely used to engage students in modelling processes that involve estimation under conditions of incomplete information. Such tasks are typically open, estimation-based problems that require students to decompose a complex situation, introduce assumptions, estimate relevant quantities, and perform approximate calculations (Ärlebäck, 2009; Peter-Koop, 2005). Because they often provide little or no numerical information, Fermi problems can invite students to construct their own models rather than simply apply a given procedure. This structure makes Fermi problems a productive context for examining how estimation and validation interact in classroom activity.

In primary education, Fermi problems have been used to support students' engagement with realistic contexts, collaborative reasoning, and model construction (Albarracín & Ärlebäck, 2019; Brunet-Biarnes & Albarracín, 2024; Haberzettl et al., 2018; Peter-Koop, 2005). These studies suggest that Fermi problems are particularly suitable for observing how young learners negotiate assumptions, develop approximate models, and revise their strategies through interaction.

While prior research has described estimation-validation dynamics at a general level, fewer studies have examined how these dynamics become observable in moment-to-moment classroom interaction, particularly in elementary settings.

Classroom Discourse and Observable Indicators

Classroom discourse plays a central role in shaping and revealing modelling processes. In modelling tasks, students' assumptions, estimations, and validations often become visible through questioning, justification, disagreement, and negotiation. Prior studies on mathematical argumentation have emphasized that classroom interaction can support students in making reasoning public, examining the validity of claims, and coordinating individual and collective mathematical meanings (Conner et al., 2014; Krummheuer, 1995; Yackel, 2002).

Such interaction is particularly important in the early phases of modelling, where assumptions must be articulated and evaluated. Validation is therefore not limited to a final checking stage after calculation. Rather, validating activity can occur throughout the modelling process as students evaluate the adequacy of assumptions, models, and results in relation to the real-world situation (Czocher, 2018; Solar et al., 2022). This perspective is especially relevant for early-stage modelling, where validation may appear as brief questions, challenges, or recalculations rather than as a fully developed evaluative phase.

However, translating theoretical constructs such as estimation, validation, and idealization into systematically identifiable classroom indicators remains a methodological challenge. To address this challenge, the present study proposes four observable indicators derived from modelling theory and discourse analysis, aiming to make early-stage modelling processes visible in elementary Fermi-problem lessons. This need becomes particularly salient in elementary contexts.

Although modelling has been widely investigated in secondary and upper-secondary contexts, comparatively fewer studies have examined how modelling processes emerge in elementary classrooms, particularly at the level of micro-interactional discourse. Existing research on Fermi problems has primarily focused on problem-solving strategies or estimation accuracy, while less attention has been paid to how assumption articulation and validation transitions become interactionally visible in early phases of modelling. Consequently, there remains a need for analytical frameworks that can capture modelling dynamics as they unfold in elementary classroom dialogue.

Conceptual Relationships Among the Four Indicators

To clarify the theoretical role of the four indicators, this study distinguishes among assumption articulation, simplification, idealization, estimation, and validation. These concepts are closely related but analytically distinct in modelling research (Blum & Leiß, 2007; Borromeo Ferri, 2006; Czocher, 2018; Surel & Greefrath, 2023). Assumption articulation refers to students' explicit verbalization of a condition that is not directly given in the task. Simplification refers to the reduction of contextual complexity, such as treating variable quantities as uniform. Idealization is understood as a modelling move in which such simplifications are used intentionally to construct a mathematically workable representation of a real-world situation. Estimation refers to the generation of approximate numerical values under conditions of incomplete information, whereas validation refers to the checking of the plausibility or adequacy of these values in relation to the original context.

The four indicators were selected because they correspond to theoretically important transitions in early-stage modelling. Assumption articulation captures the emergence of idealization; E↔Ch transitions capture movement between estimation and validation; the E+Ch engagement ratio provides a descriptive measure of the extent of modelling-oriented interaction; and validation-oriented discourse captures the social and argumentative dimension through which students question, justify, or refine emerging models. Thus, the indicators are not intended to represent the entire modelling cycle, but to make visible selected early-stage modelling dynamics that are often difficult to observe in elementary classroom interaction.

METHODS

Research Design

This study adopts an exploratory case study design to examine how early-stage modelling processes become observable in elementary classroom settings. The aim is not to generalize findings statistically, but to investigate whether theoretically derived indicators can meaningfully capture the emergence of estimation and validation in real classroom interaction.

An exploratory approach is appropriate because the proposed indicators have not previously been applied in elementary Fermi-problem contexts. The study therefore focuses on identifying observable patterns, testing interpretability, and examining reliability rather than establishing causal claims.

Participants and Context

The study was conducted in one grade 3 elementary classroom in Japan. The class comprised 32 students aged 8-9 years. Students worked in eight pre-existing seating groups based on the classroom's regular seating arrangement during both lessons. The two Fermi-type tasks were implemented in separate 45-minute lessons, with task A preceding task B.

Each lesson followed the same broad sequence: approximately 5 minutes for task introduction, 25 minutes for small-group exploration, 10 minutes for whole-class sharing, and 5 minutes for reflection. The classroom teacher introduced each task, monitored the groups during exploration, and facilitated the whole-class sharing and reflection. The teacher had prior experience implementing modelling-oriented tasks. Permission to conduct the study was obtained from the participating school, and informed consent was secured from the students' guardians. Under the applicable institutional procedures, formal ethics committee review was not required because the study was conducted as part of regular classroom instruction and involved only anonymized classroom interaction data.

Tasks

Two Fermi-type tasks were implemented. Task A asked, "How much time do we spend brushing our teeth in one week?" Task B asked, "How many students are needed to surround the classroom while holding hands?" Task A involved estimating a quantity related to everyday activity, whereas task B involved estimating a spatial quantity using body-based approximations. Both tasks were designed to elicit assumption articulation, estimation, and validation processes.

Fermi problems were selected because their open structure requires students to introduce assumptions, generate approximate values, and evaluate plausibility. This structure provides a productive context for examining early-stage modelling processes.

Data Collection

Classroom interaction was recorded and transcribed in Japanese. Two complementary coding units were used. First, transcript-based discourse segments, bounded by a change in speaker or a shift in activity focus, were used to identify assumption articulation and validation-oriented discourse. Overlapping speech was attributed to the dominant speaker, and short silences were included within the preceding segment. Second, one-minute intervals were used for temporal MAD activity coding (read, model, estimate, compute, check, and summarize), from which E↔Ch transitions and E+Ch ratios were derived. Teacher-initiated utterances were identified separately so that student-initiated discourse could be distinguished from instructional prompts.

All eight groups participated in both lessons. For the temporal analysis, however, only group records with sufficiently complete recordings and observation sheets to support minute-level MAD coding were included. This yielded six analyzable groups for task A and four for task B. Records with missing or insufficient information for assigning MAD activity categories were excluded. The resulting percentages therefore describe the analyzed groups rather than the entire class.

All excerpts were translated from Japanese to English for reporting purposes.

Development of the Indicators

The four indicators proposed in this study were derived through a process of theoretical translation from modelling theory, Fermi-problem research, and classroom discourse analysis into observable classroom units. The temporal coding of classroom activity was informed by prior protocol-analytic approaches to mathematical problem solving and by the use of MADs

Table 1. Coding scheme for the four indicators

Indicator	Theoretical basis	Inclusion criteria	Exclusion criteria	Example
Early assumption articulation	Idealization/ simplification	An explicit assumption articulated within 3 minutes of task presentation that restricts, simplifies, or fixes a real-world condition to make the problem mathematically workable	A numerical answer without an explicit assumption; an assumption articulated after 3-minute window; off-task talk; teacher explanation only	"Let's assume everyone's arm span is the same."
Estimation–validation transition (E↔Ch)	Movement between mathematical result and real situation	At least one transition from estimate to check or from check to estimate within a group sequence; recursive cases could include E→Ch→E	Linear calculation without plausibility checking; correction of an arithmetic error only	"This is only an average. It might be different in reality."
E+Ch engagement ratio	Descriptive time allocation to estimation and validation	One-minute MAD units coded as estimate or check/validation	Units coded as read, model, compute, or summarize; managerial, measuring-only, or off-task interaction	Percentage of analyzable minute-level units coded as E or Ch
Validation-oriented discourse	Social validation/ classroom discourse	A student question, refutation, challenge, or justification directed at plausibility or model adequacy	Simple agreement; repetition; teacher confirmation without student reasoning	"Should we include brushing at school?"

Table 2. Summary of results across tasks

Task	Early assumption articulation (%)	E↔Ch transition (%)	Validation-oriented discourse (%)	E+Ch engagement ratio (%)
Task A (n = 6)	50.0	66.7	50.0	40.9
Task B (n = 4)	25.0	75.0	100.0	36.6

in Fermi-problem research (Albarracín & Ärlebäck, 2019; Ärlebäck, 2009; Schoenfeld, 2016). Rather than being constructed inductively from data alone, the indicators were informed by the conceptual framework.

Specifically, early-phase modelling was interpreted through the lens of idealization as assumption articulation and structuring. From this theoretical basis, four observable dimensions were identified:

1. Early assumption articulation
2. Estimation-validation transitions (E↔Ch)
3. E+Ch engagement ratio
4. Validation-oriented discourse

Each indicator was defined operationally to enable systematic coding of classroom interaction. The operational definitions of each indicator are summarized in **Table 1**.

Intra-Coder Coding Stability

The coding-stability analysis concerned the one-minute MAD activity categories that underlay the temporal indicators. The categories were read, model, estimate, compute, check, and summarize. E↔Ch transitions and E+Ch ratios were derived from this minute-level coding. Assumption articulation and validation-oriented discourse, by contrast, were identified from transcript-based discourse segments and were not included in the kappa calculation.

A total of 123 valid minute-level coding units were identified across the six group observation sheets used for the stability check. After a washout period, the same coder blindly recoded 43 units from two group records without access to the initial coding decisions. These 43 units corresponded to 35.0% of the analyzable minute-level units. Disagreements were examined only after the recoding had been completed and were used to refine the interpretation of the coding criteria.

Cohen's κ was calculated for agreement between the initial coding and the blind intra-coder recoding of the minute-level MAD activity categories. The overall Cohen's κ was .81. This value provides evidence of intra-coder coding stability for the underlying MAD categories; it should not be interpreted as a separate reliability estimate for each of the four focal indicators. E↔Ch transitions and E+Ch ratios were derived from temporal sequences and proportions, while separate reliability coefficients were not calculated for the transcript-based assumption-articulation and validation-oriented-discourse codes. This limitation is acknowledged later.

RESULTS

Table 2 shows the summary of results across tasks.

Early Assumption Articulation

The first indicator examined the timing of assumption articulation during the lesson. In task A, 3 out of 6 groups (50%) articulated explicit assumptions within three minutes of task presentation. In task B, 1 out of 4 groups (25%) did so.

In task A, students frequently differentiated contextual conditions in order to restructure the situation into manageable subcases. For example, one group proposed separating school days and non-school days before determining daily brushing frequency. These utterances were coded as early structuring assumptions.

In task B, the group meeting the three-minute criterion began by discussing variability in arm span and a simplifying assumption:

"When people stretch out their arms, it is different for each person. Some are longer and some are shorter, right?"

"So, we can just think of everyone as having the same height, right?"

"But no one has exactly the same height, right?"

The original Japanese transcript uses the word "height" (身長), rather than "arm span." We therefore retain "same height" as the literal translation. The surrounding utterances show that this statement arose during the group's discussion of variability in arm span. Because this exchange occurred within the three-minute window, it was included in the early-assumption indicator.

To illustrate how assumption articulation could also become more elaborated later in the task, we present a second excerpt from another group.

A later, more elaborated instance of assumption articulation was observed in another group working on the "classroom perimeter" task. This episode occurred after the students had measured the classroom and the arm spans of their group members and was therefore not included in the three-minute early-assumption indicator. One student proposed measuring all group members and calculating an average:

"1 m 25 cm, 1 m 20 cm, 1 m 33 cm, 1 m 30 cm … The average is about 1 m 28 cm."

The student then proceeded to divide the total classroom perimeter by this average value to estimate the number of students required.

This later moment illustrates idealization as an intentional structuring move. Rather than attempting to model individual variation precisely, the group introduced a representative value to render the situation mathematically manageable. The explicit reference to the result as "just an average" further indicates an emerging awareness of the simplificatory nature of this assumption.

Assumption articulation
↓
Estimation (E)
↓ ↑
Checking/validation (Ch)
↓
Re-estimation (E)
↓
Validation-oriented discourse

Figure 1. Representative recursive estimation–validation sequence in task B (the author's own elaboration)

Taken together, these examples show that assumption articulation could occur both early and later in the task, while only utterances within the specified three-minute window contributed to the early-assumption indicator.

Across the analyzed episodes, assumption articulation sometimes served as a transition from contextual description to mathematical structuring. These assumptions provided a basis for subsequent estimation and validation activity in some groups. In this limited sense, idealization was observable as a possible entry point into the modelling process.

Estimation-Validation Transitions and Recursive Sequences (E↔Ch)

To make the recursive character of the E↔Ch movement visible, **Figure 1** presents a representative interactional sequence from task B. The upward return arrow between checking and estimation represents the renewed estimation observed after contextual checking.

The sequence illustrates that validation did not occur only after calculation was completed. Rather, checking triggered renewed estimation and refinement, suggesting a recursive movement between mathematical results and contextual plausibility.

In task A, 4 out of 6 groups (66.7%) exhibited at least one estimation-validation (E↔Ch) transition. In task B, this increased to 3 out of 4 groups (75%).

In task B, after calculating the required number of students based on average arm span, one student remarked that the result was "just an average" and might differ in reality. This reflective statement prompted recalculation and further checking, constituting a clear E↔Ch transition.

After computing the number of students by dividing the classroom perimeter by the average arm span, one student stated:

"But this is only an average. It might be different in reality."

This utterance marked a shift from estimation (E) to checking (Ch). The student did not simply accept the numerical result but explicitly questioned the adequacy of the modelling assumption underlying it. The reference to "only an average" indicates awareness of representational limitations.

Importantly, this checking move was not terminal. The group returned to recalculating and reconsidering the division procedure. Thus, the sequence followed an E→Ch→E pattern rather than a linear progression toward closure.

Such transitions suggest that validation in early-stage modelling operates as a recursive mechanism embedded within estimation activity. Rather than appearing only at the end of the modelling cycle, checking functions as a trigger for renewed estimation.

Comparable E↔Ch dynamics were also observed in task A. For example, when one student questioned whether the weekly multiplication factor should be ×5 or ×7, the group reconsidered the contextual conditions (school days versus weekends) before revising its calculation. This episode illustrates how checking could prompt model refinement, although the small number of tasks and groups does not support a claim that the pattern is task-independent.

In terms of the modelling cycle, these E↔Ch sequences can be interpreted as micro-level movements between a mathematical result and its meaning in the real context. Within this classroom, the sequences are consistent with the possibility that aspects of cyclic modelling can become visible when tasks invite assumption articulation and public reasoning.

These patterns suggest that validation operated iteratively rather than as a terminal stage of the modelling process.

E+Ch Engagement Ratio

The proportion of classroom interaction coded as estimation and validation (E+Ch) was 40.9% in task A and 36.6% in task B. These values are reported descriptively to indicate the extent to which classroom interaction was devoted to estimation and validation activities.

In this study, the E+Ch ratio was not used as a threshold-based measure or as an evaluative standard for determining whether modelling had occurred. Rather, it was used as a descriptive indicator of the relative presence of modelling-oriented interaction within each task. This distinction is important because the purpose of the study was exploratory: the aim was to examine how early-stage modelling processes became interactionally visible, not to establish a general benchmark for modelling engagement.

The slightly lower E+Ch ratio in task B may be interpreted in relation to the structure of the task. Task B involved physical measurement activities, such as using body-based approximations and measuring classroom dimensions. These activities temporarily shifted interaction toward procedural coordination and measurement. Nevertheless, estimation and validation remained visible within the task, particularly when students questioned the adequacy of average arm span as a modelling assumption and returned to recalculation or checking.

Thus, the E+Ch ratio should be interpreted as one complementary descriptive indicator, alongside early assumption articulation, E↔Ch transitions, and validation-oriented discourse. It does not by itself establish the presence or quality of modelling, but it helps describe how much interactional space was occupied by estimation and validation processes during the observed lessons.

Validation-Oriented Discourse

Questioning or refutation occurred in 3 out of 6 groups (50%) in task A and in all 4 groups (100%) in task B.

In task A, questions such as "Should we include brushing at school?" were observed, though often without extended follow-up discussion. In task B, refutations more frequently triggered remeasurement or recalculation (e.g., "Maybe we should measure again"), contributing to iterative refinement.

These discourse features indicate that validation was not limited to individual cognitive reflection but was socially mediated through questioning and peer challenge. The interactional visibility of such moves supports the interpretation that modelling processes are co-constructed within classroom dialogue.

DISCUSSION

Early-Stage Modelling as an Observable Process

The findings suggest that selected early-stage modelling processes, particularly assumption articulation and transitions between estimation and validation, can be made analytically visible through structured indicators. In the analyzed groups, assumption articulation occurred during initial exploration, supporting a tentative interpretation of idealization as a structuring mechanism in this classroom context.

The observed E↔Ch transitions further suggest that validation was not always an endpoint but sometimes functioned iteratively within the modelling process. This is consistent with theoretical accounts of modelling as cyclical and provides illustrative micro-level evidence from one elementary classroom.

By rendering early-stage modelling processes analytically visible, the present study addresses a persistent gap between theoretical modelling cycles and classroom-based observation. While modelling theory provides detailed conceptual descriptions, these processes often remain implicit or distributed across interactional episodes in elementary classrooms. The proposed indicators functioned as analytical lenses that made transitional moments—such as assumption articulation and estimation-validation shifts—empirically identifiable.

Importantly, making early-stage modelling observable does not require treating the modelling cycle as a set of isolated components. Rather, the indicators can help trace how cyclic movement emerges through micro-level interaction. In this classroom, some modelling trajectories were dynamically negotiated through discourse and shared reasoning rather than proceeding as fixed linear sequences.

This finding is consistent with prior research suggesting that modelling processes often involve non-linear movement across phases rather than a fixed sequence (Borromeo Ferri, 2006; Galbraith et al., 2006). It also extends research on Fermi problems by showing how such movement can be identified in elementary classroom discourse at a micro-interactional level (Ärlebäck, 2009; Brunet-Biarnes & Albarracín, 2024; Peter-Koop, 2005).

Thus, early-stage modelling may be understood not only as a simplified precursor to more developed modelling, but also as a potentially meaningful phase in which selected modelling dynamics begin to emerge.

The Role of Idealization

The results illuminate a possible role of idealization in early-stage modelling. In this study, idealization was interpreted as the articulation of assumptions that restructured a situation into a mathematically workable form. The early appearance of such assumptions in some groups suggests that students in this classroom could engage in intentional structuring rather than merely respond to given conditions.

Importantly, idealization did not appear as a single isolated act but was intertwined with validation-oriented discourse. For example, references to averages were often followed by questioning or reconsideration, indicating awareness of the representational limits of the model. This interplay highlights the dynamic character of early modelling activity.

The present findings invite a cautious reconsideration of idealization in elementary modelling contexts. Rather than assuming that idealization is visible only among advanced learners, the analyzed episodes suggest that some young students can use intentional simplification to render a complex situation tractable. Assumptions such as treating arm spans as uniform or introducing an average function as modelling moves in these episodes rather than as incidental remarks. This interpretation aligns with research emphasizing assumptions in modelling tasks with missing data and Fermi problems (Albarracín & Ärlebäck, 2025; Surel & Greefrath, 2023). The present study adds an exploratory illustration of how such assumptions may become visible in spoken classroom interaction.

Furthermore, idealization appeared intertwined with validation. Students not only introduced simplifying assumptions but also reflected on their limitations ("It is only an average"). This coupling of simplification and critical reflection indicates that idealization operates within a dynamic regulatory process, where models are both constructed and monitored.

From a theoretical perspective, these episodes are consistent with viewing idealization as an interactionally observable practice rather than only as a philosophical concept. Operationalizing idealization through assumption articulation offers one possible way to connect abstract modelling theory with empirical classroom analysis.

Methodological Contribution

A central contribution of this study lies in translating theoretical constructs of modelling into observable classroom indicators. While modelling theory offers detailed conceptual descriptions, these processes are often difficult to identify systematically in classroom data.

Beyond providing descriptive evidence, the present study explores how abstract theoretical constructs can be operationalized as analytically tractable units. The proposed indicators specify observable interactional markers—such as assumption articulation, E↔Ch transitions, and validation-oriented discourse—that can be systematically examined. In this respect, the study complements prior methodological work using MAD to represent modelling activity over time (Albarracín & Ärlebäck, 2019; Ärlebäck, 2009; Pla-Castells et al., 2021), while shifting attention toward theoretically derived indicators of early-stage modelling in elementary classroom discourse.

This operationalization addresses a recurring challenge in modelling research: the gap between comprehensive theoretical models and the practical analysis of classroom interaction. The clearly defined indicators and the intra-coder stability check for the underlying minute-level MAD categories (Cohen's κ = .81) suggest that the temporal activity coding can be applied consistently within this dataset. They do not, however, establish indicator-specific reliability or validity beyond the present case.

Importantly, the indicators are not intended as evaluative checklists or normative benchmarks. Instead, they function as exploratory methodological tools for tracing modelling dynamics at a micro-interactional level. The approach may be potentially transferable to other contexts, age groups, or task types, but such transfer requires further empirical testing and indicator-specific validation.

By bringing modelling theory into dialogue with classroom discourse analysis, the study may contribute to the empirical examination of early modelling in elementary education.

Limitations

Several limitations should be acknowledged. First, the study was conducted in a single classroom context, and the findings should be interpreted as illustrative rather than representative. In addition, the numbers of analyzable group records differed between task A and task B because only records with sufficiently complete recordings and observation sheets were retained for minute-level MAD coding (six groups for task A and four for task B). This difference in analyzable records was not experimentally controlled and limits direct comparison between tasks. Therefore, cross-task differences should be interpreted with caution.

Because the study was situated in one Japanese classroom, local instructional routines and discourse norms may have shaped the observed interaction. These contextual features were not independently examined in the present study. Transfer to classrooms with different curricular, cultural, or instructional conditions should therefore be treated as an empirical question rather than assumed.

Second, the indicators were developed within Fermi-type tasks; further research is needed to examine their applicability across different modelling contexts and age groups. Continued refinement of the operational definitions may also enhance clarity and transferability. A further methodological limitation concerns reliability. Cohen's κ was calculated for the underlying minute-level MAD activity categories rather than separately for each of the four indicators. E↔Ch transitions and E+Ch ratios were derived from temporal sequences and proportions, while assumption articulation and validation-oriented discourse were coded from transcript segments. Future studies should examine indicator-specific reliability on larger datasets, including separate stability or agreement estimates for transcript-based, transition-based, count-based, and ratio-based indicators.

Implications

Despite these limitations, the study suggests that early-stage modelling processes can be examined at a micro-interactional level through structured observation. For researchers, this approach offers a methodological pathway for analyzing modelling in elementary settings. For teachers, the indicators may support reflective awareness of moments when students articulate assumptions or engage in validation-oriented discourse.

CONCLUSION

By operationalizing modelling theory through four classroom indicators—early assumption articulation, estimation-validation transitions (E↔Ch), E+Ch engagement ratio, and validation-oriented discourse—the study suggests that selected foundational modelling dynamics can become interactionally visible in a grade 3 classroom.

In this dataset, idealization was visible through articulated assumptions during initial exploration, and validation sometimes operated recursively rather than only as a terminal step. These patterns are consistent with the possibility that aspects of cyclic modelling may emerge among young learners when tasks invite assumption-making and public reasoning.

Methodologically, the study offers an exploratory structured approach for translating selected modelling constructs into observable analytical units. The proposed indicators are intended to trace early modelling dynamics within classroom discourse rather than to replace or fragment the modelling cycle. Their validity and transferability require further investigation.

Although limited to a single classroom and two tasks, the approach provides a candidate basis for further empirical investigation of modelling processes across age groups, task types, and educational settings. Future research should refine the operational definitions, test the indicators with larger and more diverse datasets, and examine indicator-specific reliability and validity.

By foregrounding the observability of early-stage modelling, this study invites renewed attention to how modelling theory can be grounded in elementary classroom practice.

Funding: This study was supported in part by the 2026 Research Grant of Yamanashi Gakuin Junior College.

Acknowledgements: The author would like to thank Principal Shunsuke Ishida and Asuka Shirato of Warabi Higashi Elementary School for their generous cooperation in implementing the classroom lessons for this study. The author would also like to thank the participating students and all those involved in the study for their cooperation.

Ethical statement: Permission to conduct the study was obtained from the participating school, and informed consent was secured from the school and guardians. The study was conducted as part of regular classroom instruction and involved analysis of anonymized classroom interaction data. Under the applicable institutional procedures, formal ethics committee review was not required. Accordingly, no ethics committee name, approval code, or approval date is applicable. No personally identifiable student information is reported in the manuscript.

AI statement: OpenAI ChatGPT was used during manuscript preparation and revision to assist with English-language editing, clarity, organization, and the preparation of responses to reviewer comments. The manuscript was prepared and revised over an extended period during which multiple ChatGPT model versions were used, and the exact version used at each stage was not consistently recorded. ChatGPT was not used to generate empirical data or independently conduct the study's analysis. The author independently verified all analyses, interpretations, citations, references, and final text and takes full responsibility for the content.

Declaration of interest: No conflict of interest is declared by the author.

Data sharing statement: Data supporting the findings and conclusions are available upon request from the corresponding author.

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