EURASIA Journal of Mathematics, Science and Technology Education
ISSN: 1305-8223 (online) 1305-8215 (print)
OPEN ACCESS
2017 13(12):7913-7920
DOI: 10.12973/ejmste/77913

Grey Relational Analysis Method for Group Decision Making in Credit Risk Analysis

Wenshuai Wu [1*]

[1] School of Management, Guangzhou University, Panyu District, Guangzhou, Guangdong, CHINA

Received 17 April 2017 ▪ Revised 18 August 2017 ▪ Accepted 27 September 2017

ABSTRACT

In general, group decision making (GDM) under a multiple criteria environment involves multi-criteria, multi-stakeholders and a finite number of alternatives, which can be interpreted as a multi-criteria group decision making. In this paper, a multi-criteria group decision making model, based on grey relational analysis, is proposed for credit risk analysis. This model can not only deal with the problems involved multi-criteria, multi-stakeholders and a finite number of alternatives, but also reduce individual subjective preference to improve the level of overall satisfaction. In this model, firstly, TOPSIS method is applied for credit risk analysis on four public-listed companies, and the attribute weights are determined by AHP. Then, experts' opinion is integrated on the basis of post-expert information. Finally, grey relational analysis (GRA) method is introduced to gather different experts' opinion to extend the aggregation method for GDM. The empirical results demonstrate and verify that our proposed model for credit risk analysis is feasible and effective, which can be used to support credit risk management and guide business strategy adjustments.

Keywords: Grey relational analysis, GDM, MCDM, credit risk analysis

INTRODUCTION

Credit risk analysis is one of the key problems in modern financial institutions. Since the 1980s, with the development of credit market, there have been many new methods of quantitative analysis models in risk analysis (Jarrow, 1997; Peng et al. 2008; Yang and Zhou, 2013; Kou and Wu, 2014). As a classical multi-criteria decision making (MCDM) method, TOPSIS (Hwang & Yoon 1981) has been used to assess financial risk along with the integration of data mining, machine learning and so on (Wu and Olson, 2006; Wu and Kou 2016). In this research, TOPSIS method is applied to assess credit risk, and the attribute weights are determined by AHP (Saaty, 1980; Peng et al., 2011; Wu et al., 2012; Wu and Kou 2016).

However, decision-making is a complicated interdisciplinary interaction process. In order to make decisions more accuracy and efficiency, decision mistakes caused by a single decision-maker should be reduced. Group decision making (GDM) can synthesize views and information of policy makers, such as brainstorming, to form a common wisdom for making a reasonable choice or sorting and focus on decision-making by post-expert information which can better ensure a good consistency matrix (Hwang and Lin, 1987; Ishizaka and Labib, 2011; Xia and Chen, 2015; Wu and Peng, 2016; Chang, T. C., & Wang, H., 2016). In this work, grey relational analysis (GRA) method is introduced to gather different experts' opinion by post-expert information.

The remaining parts of this article are organized as follows: In Section 2, several MCDM methods are introduced in detail. In section 3, a multi-criteria group decision making model is proposed, based on qualitative and quantitative analysis, MCDM methods and GDM. In section 3, the proposed model is verified by an empirical case on credit risk analysis. Finally, Section 4 concludes the paper.

✉ wenshuai_wu@163.com **(*Correspondence)**

Contribution of this paper to the literature

- The feasibility and effectiveness of the proposed model, which can not only deal with the problems involved multi-criteria, multi-stakeholders and a finite number of alternatives, but also reduce individual subjective preference to improve the level of overall satisfaction.
- The proposed model can be used to support credit risk management and guide business strategy adjustments. In addition, experts' opinion is integrated based on post-expert information by applying MCDM method, rather than simply applying linear weighted geometric mean or arithmetic mean method.
- To calculate grey relational degree in the GDM process, the opinion and preferences of decision-makers can be synthesized to form a common wisdom.

MCDM METHODS

AHP

AHP, proposed by Saaty (1980), is an MCDM technique, whose flexibility can facilitate modelling unstructured multi-criteria problems in a realistic and effective way (Winkler 1990). Based on the pair-wise comparison matrix, AHP can combine qualitative analysis and quantitative analysis to obtain a relative priority vector in a suitable ratio scale (Peng et al., 2011). The initial AHP usually consists the following parts: modelisation, valuation, and priorization and synthesis (Levary and Wan, 1998; Moreno-Jiménez et al., 2008; Tiryaki and Ahlatcioglu, 2009; Hsueh and Kuo, 2016). The detailed processes are as follows (Liu and Shih 2005; Escobar and Morenojiménez, 2007):

(1) Modelisation for structuring the hierarchy. A hierarchical construction can be modelled and constructed to reflect all the relevant aspects of the decision problem.

(2) Valuation for constructing a pair-wise comparison decision matrix. The decision matrix can be established through pairwise comparisons according to expert scoring in a 1-9 scale.

(3) Priorisation. Based on the hierarchical composition principle, the local and global priorities can be calculated by the row geometric mean method, the eigenvector method, and so on.

(4) Synthesis. The total priorities can be synthesized by the aggregation procedure: additive or multiplicative, and the synthesis is the existence of a measure for making inconsistent judgments.

TOPSIS

TOPSIS was proposed by Hwang and Yoon (1981) for solving an MCDM problem with crisp numbers (Chen and Hwang 1992). The principle of TOPSIS is the selected alternative should be the one that is nearest to the positive ideal solution (PIS) and farthest from the negative ideal solution (NIS) (Ballı and Korukoğlu, 2009; Zhang and Xu, 2014). In this article, TOPSIS method is applied for credit risk analysis on four public-listed companies. The detailed steps are as follows (Opricovic and Tzeng 2004):

(1) The decision matrix A is normalized:

$$a_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m}(x_{ij})^2}} \quad (1 \le i \le m, \qquad 1 \le j \le n) \tag{1}$$

(2) The weighted normalized matrix D is calculated:

$$D = (a_{ij} * w_j) \quad (1 \le i \le m, 1 \le j \le n) \tag{2}$$

where w_j is the criteria weights and $\sum_{i=1}^{n} w_j = 1$.

(3) The PIS V^* and the NIS V^- are determined:

$$\begin{aligned} V^* &= \{ v_1^*, v_2^*, \cdots, v_n^* \} = \left\{ (\max_i v_{ij} | j \in J), \quad (\min_i v_{ij} | j \in J') \right\} \\ V^- &= \{ v_1^-, v_2^-, \cdots, v_n^- \} = \left\{ (\min_i v_{ij} | j \in J), \quad (\max_i v_{ij} | j \in J') \right\} \end{aligned} \tag{3}$$

(4) The distance of each alternative from PIS and NIS are computed:

$$S_i^+ = \sqrt{\sum_{j=1}^{n}(V_i^j - V^*)^2} \quad (1 \le i \le m, \quad 1 \le j \le n)$$
$$S_i^- = \sqrt{\sum_{j=1}^{n}(V_i^j - V^-)^2} \quad (1 \le i \le m, \quad 1 \le j \le n) \tag{4}$$

(5) The relative closeness can be obtained:

$$Y_i = \frac{S_i^-}{S_i^+ + S_i^-} \quad (1 \le i \le m) \tag{5}$$

When $Y_i \in (0,1)$, Y_i is closer to 1, the alternative is closer to the ideal solution.

(6) The priority can be ranked:

By comparing Y_i values, the priority can be ranked. The larger the Y_i, the better the selected alternative.

GRA

GRA, developed by Deng (1982), is one of the widely applied MCDM methods, is also a quantitative analysis tool of grey system theory, which can address imprecise and incomplete information (Deng 1988). The principle of GRA is to analyze the similarity relationship between the reference series and alternative series. The chosen alternative series, which has the most closed similarity to the reference series, is the best scheme of the decision problem. The detailed steps of the GRA method are as follows (Hamzaçebi and Pekkaya, 2011; Wu and Peng, 2016). Suppose the initial decision matrix R is as follows:

$$R = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix} \quad (1 \le i \le m, \quad 1 \le j \le n) \tag{6}$$

(1) Get the standardized decision matrix R'. The initial decision matrix R can be standardized as follows:

$$R' = \begin{bmatrix} x'_{11} & x'_{12} & \cdots & x'_{1n} \\ x'_{21} & x'_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ x'_{m1} & x'_{m2} & \cdots & x'_{mn} \end{bmatrix} \quad (1 \le i \le m, \quad 1 \le j \le n) \tag{7}$$

(2) Determine the reference series x'_0:

$$x'_0 = (x'_0(1), x'_0(2), \cdots, x'_0(n)) \tag{8}$$

$x'_0(j)$ is the standardized and largest value in the j^{th} factor.

(3) Calculate the differences $\Delta_{0i}(j)$ between the reference series and alternative series:

$$\Delta_{0i}(j) = \left|x'_0(j) - x'_{ij}\right| \tag{9}$$

$$\Delta = \begin{bmatrix} \Delta_{01}(1) & \Delta_{01}(2) & \cdots & \Delta_{01}(n) \\ \Delta_{02}(1) & \Delta_{02}(2) & \cdots & \Delta_{02}(n) \\ \vdots & \vdots & \vdots & \vdots \\ \Delta_{0m}(1) & \Delta_{0m}(2) & \cdots & \Delta_{0m}(n) \end{bmatrix} \quad (1 \le i \le m, \quad 1 \le j \le n) \tag{10}$$

(4) Compute the grey relational coefficient $r_{0i}(j)$:

$$r_{0i}(j) = \frac{\min_i \min_j \Delta_{0i}(j) + \delta \max_i \max_j \Delta_{0i}(j)}{\Delta_{0i}(j) + \delta \max_i \max_j \Delta_{0i}(j)} \tag{11}$$

where δ is an identification coefficient. In general, the value of δ is set to 0.5 for providing a good stability. The grey relational coefficient is a similarity indicator between the reference series and alternative series.

(5) Calculate the grey relational degree b_i:

$$b_i = \sum_{j=1}^{n} w_j r_{0i}(j) \tag{12}$$

where w_j is criteria weight and $\sum_{i=1}^{n} w_j = 1$. The values of the grey relational degree can be applied to rank the alternatives by the similarity between the reference series and alternative series. The higher the b_i, the better the selected alternative.

PROPOSED MODEL

Based on the company's financial ratio data (Altman, 1968), MCDM are increasingly applied for risk analysis and management (Steuer and Na, 2003). With the social development, decision-making process has also becoming more and more complex. It is difficult to make a scientific and accurate evaluation only by a single decision-maker on complicated practical problems. Thus, it is necessary to gather all the views and preferences of the group members to form a unified view or preference to sort the program and select the most preferred solution. In this paper, GRA method (Deng 1982; Kou and Wu 2014; Wu and Peng 2016; Wu et al., 2016) is introduced to gather different experts' opinion in credit risk analysis in order to extend the aggregation method for GDM. The detailed steps of our proposed model are as follows:

Step 1: **Determine attribute weights**. The attribute weights are determined by AHP method combined with qualitative analysis and quantitative analysis.

Step 2: **Rank the alternative**. The ranking of each alternative is determined by TOPSIS.

Step 3: **Gather post-expert information**. Ishizaka and Labib (2011) indicated that GDM should focus on decision-making by post-expert information, which can better ensure a good consistency matrix.

Step 4: **Determine the experts' weights**. As different experts have different prestige, knowledge, experience, expectations, decision power, and risk preferences, different experts should be assigned with different weights.

Step 5: **Gather experts' opinion**. In this paper, GRA method is introduced to gather different experts' opinion to extend the aggregation method for GDM.

Step 6: **Determine the priorities of alternative**. Determine the priorities of alternative by the grey relational degree for GDM to improve the level of overall satisfaction. The larger the grey relational degree, the larger the values of credit risk.

The assessment flow chart is presented in **Figure 1**.

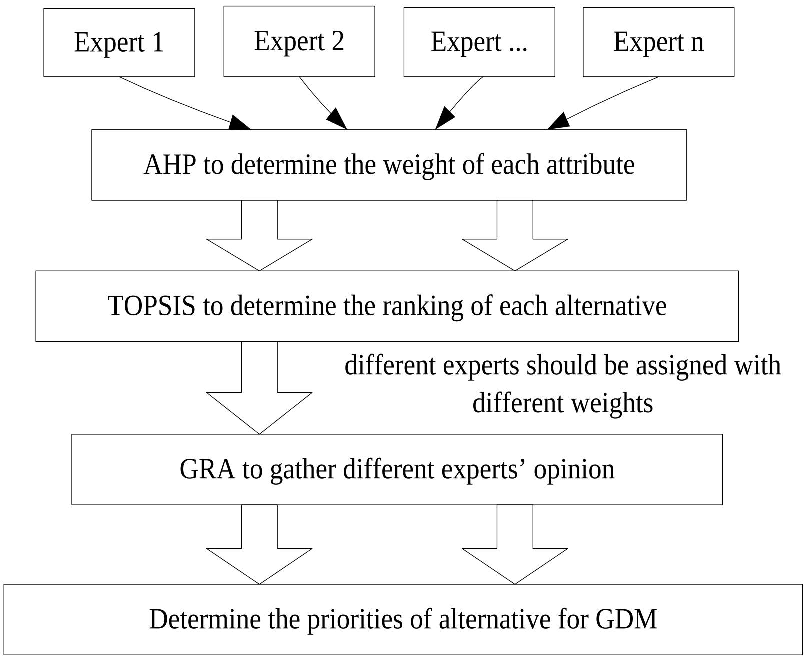

Figure 1. The Assessment Flow Chart

EMPIRICAL ANALYSIS

In this section, an empirical case on credit risk analysis is tested to verify the proposed model for credit risk analysis.

Index System and Data Sample

The selected data and indicators are collected from the public-listed company's financial data in the first half year of 2008 (Lu and Wu, 2010). Four companies A, B, C and D are selected as the assessment object. The data of financial indicators are obtained and further to be pre-processed to get the standardized data, as presented in **Table 1** (Lu and Wu, 2010).

Table 1. Index system and data sample (Lu and Wu, 2010)

Attributes	Company A	Company B	Company C	Company D
Quick ratio	0.0013	0.1872	0.2054	0.6061
Current ratio	0.1228	0.4122	0.0350	0.4300
Asset-liability ratio	0.1714	0.5889	0.2375	0.0022
Property net profit ratio	0.2267	0.0420	0.0560	0.6754
Net assets returns ratio	0.2502	0.0191	0.0764	0.6542
Net profit ratio	0.2309	0.0400	0.0909	0.6382
Account receivable ratio	0.1967	0.0211	0.2013	0.5809
Total assets return ratio	0.1293	0.0155	0.0546	0.8005
Total assets increasing ratio	0.3309	0.0104	0.0007	0.6580
Net capital increasing ratio	0.7183	0.0936	0.1872	0.0009

Empirical Process

We consulted three domain experts for credit risk analysis on four public-listed companies. The procedures are as follows:

(1) Determine attribute weights and consistency test. The consulted three experts include a risk neutral expert k_1, a risk-preference expert k_2, and a risk-averse expert k_3. The judgment matrix of the consulted three experts is collected respectively to determine each attribute weight value by AHP method.

$$w_{k_1} = (0.112, 0.041, 0.195, 0.039, 0.066, 0.124, 0.112, 0.097, 0.032, 0.182)$$
$$w_{k_2} = (0.072, 0.116, 0.072, 0.161, 0.168, 0.228, 0.072, 0.042, 0.024, 0.045)$$
$$w_{k_3} = (0.139, 0.025, 0.025, 0.059, 0.092, 0.038, 0.203, 0.091, 0.038, 0.290)$$

The consistency test results of the three matrices are $CR_{k_1} = 0.055 < 0.1$, $CR_{k_2} = 0.023 < 0.1$ and $CR_{k_3} = 0.021 < 0.1$ respectively. The pair-wise comparison matrices of the three experts are all satisfied the condition of the consistency ratio, indicating that the opinion of each expert is reasonable and effective.

(2) Determine the experts' weights. The value of the experts' weights should reflect consistency of the results between one expert and group experts in the decision activities. That is to say, the more concordant of decision results between one expert with a majority of group experts, the greater the decision power of the expert, and the larger the experts' weights. So the weights of three experts are assigned as 0.1, 0.3, 0.6 respectively.

(3) Determine the final priorities of alternative according to the Step 2-6. First, TOPSIS method is used to determine the priority of each alternative. The evaluation results are presented in **Table 2**. Then, experts' opinion is gathered by GRA method. Finally, the grey relational degree of GDM is calculated to improve the level of overall satisfaction, as shown in **Table 2**.

Table 2. Alternative Ranking

Company	Expert 1 relative close degree	Expert 2 relative close degree	Expert 3 relative close degree	Grey relational degree	Credit risk Ranking
Company A	0.4906	0.3294	0.6041	0.9262	2
Company B	0.3943	0.2276	0.1437	0.7895	4
Company C	0.2721	0.1339	0.2575	0.7926	3
Company D	0.4657	0.8033	0.4583	0.9487	1

Results Analysis

When assessing credit risk, different experts focus on different financial indicators. The analysis of attribute weights presents that the risk neutral expert focuses on the asset-liability ratio, so the asset-liability ratio has the largest weight of 0.195. The risk-preference expert concerns about the net profit ratio, so the net profit ratio has the largest weight of 0.228. The risk-averse expert concerns about net capital increasing ratio, so the net capital increasing ratio has the largest weight of 0.290. There is significant inconsistency among decision-makers in the judge of the same decision.

From **Table 2**, Company D's credit risk value is the highest, followed by Company A and Company C, and Company B's credit risk value is the lowest, its credit is best. While Lu and Wu (2010) shows that the credit risk level of the four companies is D>A>C>B. The results of the assessment are consistent, which illustrates that our proposed model for credit risk analysis is feasible and effective.

According to the weight analysis of AHP, we found that financial institutions should pay more attentions on the impacts of asset-liability ratio, net profit ratio and net capital increasing ratio by the dynamic changes of index value to guide decision-making, adjust business strategy and reduce the credit risk level.

CONCLUSION

This article conducts a multi-criteria group decision making model, on the base of grey relational analysis, which combined MCDM and GDM theory to analyze credit risk on four public-listed companies. The empirical results demonstrate the feasibility and effectiveness of the proposed model, which can not only deal with the problems involved multi-criteria, multi-stakeholders and a finite number of alternatives, but also reduce individual subjective preference to improve the level of overall satisfaction. The results further indicate that the proposed model can be used to support credit risk management and guide business strategy adjustments. In addition, experts' opinion is integrated based on post-expert information by applying MCDM method, rather than simply applying linear weighted geometric mean or arithmetic mean method. Furthermore, by calculating grey relational degree in the GDM process, the opinion and preferences of decision-makers can be synthesized to form a common wisdom.

ACKNOWLEDGEMENTS

This research has been partially supported by Grants from Fund for Less Developed Regions of the National Natural Science Foundation of China(#71761014), Research start-up Fee Project (#2003414092, #2700050384), Youth Science and Technology Project of Department of Education of Jiangxi Province (#GJJ150512), Major Project of the Education Science "Thirteen Five" Plan of Jiangxi Province (#16ZD019), 2016 Soft Science Research Project of Jiangxi Province (#20161BBA10081), 2016 National Statistical Science Research Project (#2016416), China Postdoctoral Science Foundation (#160448).

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