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How do elementary textbooks address equivalence of fractions in the United States and India?

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Abstract

Textbooks are among the primary curricular resources in mathematics education. This study aimed to examine how elementary textbooks (grade 3-grade 6) in the United States and India address equivalent fractions. Three criteria used for comparison were: flow and structure of the concepts, use of relevant and realistic examples, and development of (mathematical) flexibility. Although the overall content covered in both textbook series was largely similar, substantial differences existed in the structure and flow of the equivalent fraction concepts. The US textbooks had more repetition across grades than Indian textbooks. Secondly, US textbooks included fewer real-life examples than Indian textbooks, but both used realistic and relevant contexts for their respective students. Lastly, both textbook series incorporated elements of mathematical flexibility, but their nature and frequency varied. This study's analysis provides crucial insights for curriculum developers, policymakers, and teachers, based on various features of these US and Indian textbook series.

Keywords: mathematics textbook, elementary, equivalent fractions, international, flexibility, real-world contexts

INTRODUCTION

In the United States (USA), mathematical textbooks serve as the primary curricular resources for mathematics instruction, playing three crucial roles in teaching mathematics. First, they help teachers decide the sequence of the mathematics content needed to teach various concepts. Second, they suggest the scope of mathematical content and its depth at a particular grade level. Lastly, textbooks also provide suggestions for instructional ideas and activities that teachers can use when teaching mathematics (Reys et al., 2004). Overall, mathematics textbooks help teachers with planning the lessons (Alajmi, 2012), and "the choice of textbook often determines what teachers will teach, how they will teach it, and how their students will learn" (Reys et al., 2004, p. 61). Mathematics textbooks are also primary curricular resources in India, playing similar roles in mathematics education. Thus, in this study, we aim to review, analyze, and compare two elementary mathematics textbook series, one from the USA and the other from India, to understand how the concept of **equivalent fractions** is addressed in both series across various elementary grades.

There are similarities and differences between the education systems and students in the USA and India. A key similarity between these two countries is the great diversity of students. Diversity in the USA is mainly seen in students' demographics, socioeconomic status, language, and race/ethnicity. In contrast, India's states vary substantially from one another in terms of geographical features, language, culture, demographics, religions, castes, and traditions. These features of diversity are also reflected in the students. This study aims to offer ideas about how each textbook series contributes to its respective group of students. Another similarity between the two countries is that mathematics textbooks are the primary curricular resource in schools, playing a crucial role in shaping the mathematics curriculum, pedagogy, and assessments. In the USA, over 75% of mathematics classes in grade K-grade 5 used commercial mathematics textbooks, primarily to guide the overall structure and content of mathematics (Malzahn, 2013). In India, the Central Board of

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Contribution to the literature

- This study presents a detailed qualitative analysis and comparison of elementary textbooks (grades 3 through 6) from the USA and India to understand how the concepts related to equivalent fractions are presented in both textbook series.
- In this study, both textbook series were compared using three criteria: the structure and flow of equivalent fraction concepts, the presence of relevant and realistic examples, and the presence of opportunities to develop (mathematical) flexibility while learning about fraction equivalence. The analysis highlighted some similarities and many differences across the three comparison criteria.
- This study contributed substantially to the literature by analyzing and comparing the equivalent fraction concepts, which were not extensively studied previously, in two lesser-explored contexts, namely the USA and India, using three comparison criteria that focus on the structure, flow, and certain features of the content.

Secondary Education (CBSE), the national educational board, has mandated the use of textbooks published by the National Council of Educational Research and Training (NCERT), an autonomous body under the Ministry of Education, Government of India. Before this mandate, the schools could choose textbooks from NCERT or private publishers (CBSE, 2024). Therefore, the majority of schools in India affiliated with various educational boards (national, state, or open) use textbooks from public or private publishers.

However, there are also some differences between the two countries. Unlike the USA, India does not participate in large international assessments like the TIMSS or PISA. Previous literature looks at (comparative) analyses and reviews of mathematics textbooks in the USA and other participating countries in such large-scale assessments, such as South Korea, Singapore, China, Japan, etc. (Alajmi, 2012; Ding, 2016; Valverde et al., 2002; Vicente et al., 2020; Wang & Lu, 2018). This research provides one lens for understanding the differences in mathematics performance among students from these countries on international assessments. However, as India does not participate in any of these international assessments, there is no way to understand and compare the features of the mathematics curricula and students' mathematical learning in these two countries. This may be one primary reason for the difficulties encountered while finding existing research that focuses on understanding, reviewing, and comparing various aspects of mathematics textbooks from India and the USA. Thus, these two unexplored countries, India and the USA, have similarities in terms of diversity in their student population and the emphasis given to mathematics in the academic curricula and students' careers, but they have major differences in their education systems as well as in the manner in which mathematics is presented and taught. Due to these reasons, these two countries were chosen for the curricular comparative analysis in this study.

The concept of fractions is an essential area of focus in K-8 mathematical learning. Proficiency in fractions is usually underdeveloped but is also a crucial foundation for algebra, so developing this proficiency is a major goal for K-8 mathematics (U.S. Department of Education, 2008). A major feature in the framework of developing fraction proficiency is the equivalence of fractions (Tsai & Li, 2017), which motivated us to choose this conceptual area as a focus for this study. Thus, this study aims to review and analyze elementary mathematics textbooks from the USA and India. This analysis serves as one of the few lenses to gain deeper insights into the mathematics teaching and learning processes in both countries. Additionally, this study aims to understand the offerings of each textbook to the respective student population and how each textbook can benefit from adopting recommendations from the other country's textbook.

RELATED LITERATURE

Why Textbooks Matter, Especially in Mathematics Education

Textbooks play an important role in education because they are not only a curricular resource for instruction but also reflect culture and values and serve as a means of intellectual development for individual students and, ultimately, for the nation (Fatima, 2021). Textbooks are updated to align with existing educational philosophies and adapted to curricular reform movements (Fatima, 2021). For instance, most textbook publishers revised their elementary school textbooks to illustrate how they integrated the Common Core State Standards for Mathematics into their mathematics curricula after the standards were published and adopted by local and state governments in the USA. Thus, textbooks facilitate efficient and effective teaching and learning processes (Fatima, 2021), as they influence the content teachers teach, their teaching style, and how students learn (Reys et al., 2004). Therefore, all textbooks, including mathematics textbooks, play a crucial role in teaching.

Mathematics as a subject is usually associated with its curricular materials, especially textbooks. Some

probable reasons for this association may be that teachers feel comfortable teaching mathematics using textbooks, and they usually align with societal perspectives on the nature of content presented in them and the methods by which this content is taught or learned (Remillard, 2005). Moreover, mathematics textbooks are more than curricular resources in the process of mathematics teaching and learning. They represent a medium through which readers (students and teachers) interact with the mathematics content, making them a vital *mode of engagement* in the learning process (Remillard, 2011).

Mathematics textbooks determine the content to be taught and the sequence of teaching these concepts, and they suggest various instructional ideas and activities for teachers to engage their students in mathematics. Furthermore, teachers tend to adopt representations and methods from textbooks in their pedagogies (Reys et al., 2004). The micro-scoping review revealed that mathematics textbooks substantially support instructional practices of teachers, new curricula implementation, and existing curriculum maintenance (Fan et al., 2025). Thus, the crucial role of textbooks in mathematics education is uncontested, especially in different countries where they are one of the primary curricular resources used in schools. Therefore, international comparative studies of mathematics textbooks provide additional insights into their offerings to the respective countries and other countries.

International Comparison and Analysis of Textbooks

A vast scope of existing literature describes studies that compare certain concepts in mathematics textbooks from different countries, based on various criteria. Evaluating textbooks is important in the textbook development process, as it helps review and update the quantitative and qualitative aspects of the textbook (Fatima, 2021). Moreover, while reforms in the mathematics curriculum shape textbook design, mathematics textbook research can also play a role in subsequent curricular reforms (Fan et al., 2025). Comparative studies typically analyze mathematics textbooks by examining various aspects, such as curriculum standards, textbook structure, the nature and extent of content, and content quality. This includes evaluating features like cognitive demand, progression, the presence of real-life contexts, etc. These studies aim to highlight the differences in textbooks across countries based on these criteria. The following examples showcase some comparative studies conducted on international textbooks.

Alajmi (2012) compared the concepts of fractions in elementary textbooks from the USA, Kuwait, and Japan, examining the textbooks' physical characteristics, chapter structures, and the nature of the problems. One of the study's findings was that the textbooks from the USA and Kuwait were larger due to their extensive repetition, and they introduced the concept of fractions in grade 1. On the other hand, the Japanese textbooks introduced fractions in grade 3 and approached them differently than the other two countries (Alajmi, 2012). Another study by Shin and Lee (2018) also focused on comparing the progressions of three operations (recursive partitioning, common partitioning, and distributive partitioning) of fractions in one textbook each from the USA and Korea, particularly from the perspective of students' cognitive development. Similar to Alajmi (2012), a higher level of repetition was observed in the US textbooks than in the Korean textbooks. Basically, it was noticed that Korean elementary textbooks had a macro-level spiral structure but not within fractions; however, the US textbooks had a spiral structure within fractions. Moreover, it was also found that textbooks from both countries did not provide sufficient explanations or activities to support the development of these three partitioning operations (Shin & Lee, 2018). Hwang et al. (2021) also compared fractions addition and subtraction in textbooks in the USA and South Korea but used different criteria, namely the sequence of the topic, frequency, contextual features, cognitive demands of the content, and mathematical activities. This study highlighted that the South Korean textbook provided more uniform opportunities for learning fraction addition and subtraction across different criteria than the USA textbook. These studies represent the literature comparing elementary textbooks from the USA and other countries, with a particular focus on fractions. However, other international comparison studies of elementary mathematics textbooks have also focused on number systems, operations, and arithmetic (Chang & Silalahi, 2017). Additionally, Robinson et al. (2023) compared three kindergarten mathematics textbooks from the USA, namely, Go Math!, Eureka Math, and Everyday Mathematics (EM), to compare their instructional grouping, content coverage and sequencing, and representational strategies. The findings highlighted similarities in the frequent use of concrete or pictorial representations, the emphasis on mathematics practices related to number sense and operations, and suggestions for whole-class, teacher-centered instruction. On the other hand, the differences concerned the coverage of content and the emphasis placed on various mathematical topics (Robinson et al., 2023). Thus, this section highlights that the existing literature on elementary textbook comparison studies related to fractions does not primarily focus on equivalent fractions in textbooks from India and the USA, which is the central focus of this study.

Other international textbook comparison studies focused on middle and high school topics. Some studies compared US and Chinese textbooks for various mathematical content, such as inverse relations (Ding, 2016) and problems on addition and subtraction of

integers (Li, 2000), and also compared various types of problems from the lower secondary grade level by classifying them in different ways, like traditional/non-traditional, open-ended/close-ended, routine/non-routine, and application/non-application problems (Zhu & Fan, 2006), etc. Wang and Lu (2018) carried out an international comparison among a series of high school textbooks from China, Germany, France, Russia, Japan, the United Kingdom, and the USA to understand the selected content and the nature of its presentation in four major mathematical domains, namely, geometry, algebra, statistics, and probability. Wang and Leung (2025) compared five secondary mathematics textbooks, each from the USA, Australia, Singapore, China, and England, to examine the difficulty levels in terms of connections related to the topic of equations, using social network analysis.

Different comparison criteria are used for various types of international comparisons. Choi and Park (2013) compared the curriculum standards, structure, and geometry items from the textbooks in the USA and Korea. On the other hand, Yang et al. (2017) also compared geometry problems in middle school mathematics textbooks from Taiwan, Singapore, Finland, and the USA using three criteria for comparison: representation forms, contextual features, and response types to understand the role of these geometry problems in the learning experiences of students. Another study comparing geometry-related topics was conducted by Avcu (2019) to analyze and compare the definitions of special quadrilaterals in seven American and seven Turkish textbooks based on various features of definitions like their presentation, mathematical correctness, nature, minimality, and the geometric properties and reference sets used in the definitions.

Some studies focused on international comparisons that excluded the USA. Choy et al. (2020) analyzed three mathematics textbooks from Singapore, Germany, and South Korea to compare how the introductory chapter of gradient was presented in the three countries with respect to related content, context, and instructional variables. Vicente et al. (2020) compared the approaches to solving word problems suggested by primary mathematics textbooks from Singapore and Spain. Another study compared Indian and Singaporean high school textbooks to examine their respective cultivation of higher-order thinking skills (HOTS) using quantitative content analysis and found a significantly higher proportion of HOTS tasks in the Singaporean textbooks compared to the Indian textbooks (Chawla et al., 2026). In their analysis of the Lebanese grade 5 mathematics and science textbooks and the national curricular documents, Antoun et al. (2023) found that, on average, less than 10% of grade 5 mathematics textbooks provided opportunities to address the needs of highly abled students, which indicates that, overall, these textbooks provided limited provisions for addressing the needs of higher ability students. This body of literature presents examples of international textbook analysis and comparison studies that use different comparison criteria, focusing on various mathematical domains. However, very few or almost negligible studies, qualitative and quantitative, were found that compared mathematical topics, especially equivalence of fractions, across textbooks in the USA and India. Thus, this study aims to conduct a qualitative comparison between elementary textbooks in India and the USA.

Importance of Equivalence of Fractions

Fraction concepts are some of the most crucial and form the basis of many subsequent mathematical topics. In the USA, fractions are considered foundational prerequisites for more complex mathematical concepts, such as algebra, percentages, decimals, etc., because students are first introduced to abstraction skills through fractions (Fennell, 2007). Similarly, for Indian students, fractions are considered foundational skills, and students who struggle with fractions often face difficulties with related, more complex concepts in mathematics (de Barros & Ganimian, 2023). Thus, concepts related to fractions are considered very important and foundational in the curricula of the USA and India, which makes it crucial to compare these two countries' curricular resources. According to Tsai and Li's (2017) framework for developing fraction proficiency, five dimensions are identified, namely

(1) five constructs of fractions (part-whole, measure, quotient, operator, and ratio),

(2) the concept related to equivalent fractions,

(3) the conceptual understanding and procedural fluency of fraction operations,

(4) the relationship between fractions, percentages, and decimals, and

(5) the transition between different representations of fractions.

As discussed in the literature review, textbook analysis and comparison studies focus on the overall basics of fractions (Alajmi, 2012) and the operations related to fractions (Hwang et al., 2021; Shin & Lee, 2018). However, not many studies focus primarily on equivalent fractions, which is an important dimension for developing fraction proficiency (Tsai & Li, 2017).

Understanding the concepts related to the equivalence of fractions is considered substantially difficult for elementary or primary students, mainly because it involves the part-whole subconstruct of fractions but also requires an additional multiplicative layer (Ni, 2001, as cited in Pantziara & Philippou, 2012). Along with being fundamental, equivalent fraction concepts are also the building blocks required to solve more complex problems related to finding a common

denominator to compare fractions or solve operations of fractions or converting fractions to decimals, percentages, ratios, etc. That is why understanding how various textbooks address equivalent fractions is important. Therefore, this study focuses on understanding how elementary textbooks in India (NCERT textbooks) and the USA (Everyday Mathematics (EM) textbooks) address concepts related to equivalent fractions.

Educational Philosophies Behind Both the Textbook Series

Everyday Mathematics (EM), a mathematics curriculum for grade K-grade 6, is research-based and field-tested. This curriculum aims to develop skills and understanding in mathematics so that children develop mathematical power. To achieve this goal, the EM curriculum emphasizes the use of meaningful real-life examples to introduce and understand mathematical concepts. Additionally, the curriculum provides ample opportunities to practice computational skills to develop procedural fluency through mathematical problems, verbal exercises, and games. Moreover, this curriculum aims to provide repeated exposure to mathematical content so that children can recall mathematical concepts and skills from their long-term memory. Lastly, developing the ability to use multiple methods and problem-solving strategies is emphasized in this curriculum to support students with different learning styles (Everyday Mathematics, n.d.-c).

The NCERT textbooks were aligned with the curricular reforms recommended by the National Curriculum Framework (NCF) 2005 (Nag Chowdhuri, 2022). The NCF described the narrow aim of school mathematics as developing skills related to various fundamental mathematical concepts. In contrast, the broader aim is to equip a child's mathematical resources to reason and think mathematically, understand the assumptions underlying logical inferences, and develop the skill of abstraction, including an attitude to plan and solve problems. Another important aim of the NCF was to connect school mathematics to students' daily lives (NCERT, 2005). Thus, the chosen textbook series from both countries were comparable to each other on the basis of their curricular focus and educational philosophies as described in the previous paragraphs.

Research Questions and Their Theoretical Significance

The focus of this study is primarily on understanding the flow and structure of concepts related to the equivalence of fractions as addressed in elementary-grade textbooks, the opportunities to learn about the equivalence of fractions using contextual examples, and the opportunities to develop flexibility skills while learning equivalent fractions. The three research questions guiding this study are

(1) How and when are the concepts of equivalence of fractions introduced in the selected elementary textbooks in the USA and India? What is the flow of these concepts in the textbooks in both countries?

(2) How do the selected elementary textbooks in both countries provide opportunities to learn the equivalence of fractions using relevant and contextual examples?

(3) How do the selected elementary textbooks in both countries provide opportunities to develop (mathematical) flexibility while learning the equivalence of fractions?

Both countries' textbooks differ in the timelines and styles of introducing fractions and the equivalence of fractions. Thus, the first research question aims to gain further insights into these differences. Existing literature has indicated that opportunities for applying concepts learned in mathematics to the contexts of real-life situations had a statistically significant and positive influence on secondary school students' mathematics performance in Ghana (Amo-Asante et al., 2025). Thus, using relevant and realistic contexts in mathematics has substantial benefits for students, making it a crucial element of their mathematics learning process. This is the second criterion for comparison in our study. Moreover, it has been observed that most students who are taught mathematics in a traditional manner have a distorted and impoverished perception of the discipline of mathematics, which is significantly different from the nature of mathematics that mathematicians engage in (Boaler, 2015). That is why many scholars believe that students should engage in mathematics in the same way as mathematicians do; thus, they have proposed a set of features of mathematics that are similar to those that mathematicians engage in. Out of these diverse features, we are focusing on two features:

(1) the ability to use mathematics in the context of the real world and to interpret the real world (Boaler, 2015; Fasheh, 1990; National Research Council [NRC], 2001) and

(2) the ability to inculcate flexibility while doing mathematics (Boaler, 2015; Cuoco et al., 1996; NRC, 2001).

Thus, the remaining two research questions focus on these features, respectively.

METHODS

Data Sources

This study focused on the content area of "equivalence of fractions," as addressed in the elementary textbooks in the USA and India. The Indian

textbooks used were the *Maths Mela* textbook for grade 3 (NCERT, 2024b), *Math-Magic* textbooks for grade 4 (NCERT, 2007) and grade 5 (NCERT, 2008), and the *Ganita Prakash* textbook (grade 6) (NCERT, 2024a), published by the NCERT, as these textbooks include the concepts of equivalence of fractions. The grade 4 and grade 5 textbooks were reprinted multiple times, with the latest reprint in 2024, despite their original publication in 2007 and 2008, respectively. For this study, we used the latest versions, reprinted in 2024, of the grade 4 and grade 5 NCERT textbooks (which were the same editions as the 2007 and 2008 ones, respectively). All the textbooks from the EM textbook series (4th edition) (Bell et al., 2015a, 2015b, 2015c, 2015d) from the USA were used, which included the concepts of equivalence of fractions. These are the most recent print editions of the EM Student Reference Books (student-facing textbooks) available, which were used for analysis in this study. Therefore, we selected the most recent student-facing editions from both countries for this study.

These textbook series were selected because they are commonly used in elementary mathematics curricula in their respective countries. Nearly 220,000 classrooms in the USA use the EM textbooks every year (Everyday Mathematics, n.d.-a), and this textbook series is one of the three most commonly used mathematics textbook series for elementary grades in the USA (Malzahn, 2013). Other empirical studies have used EM series as the USA textbooks for comparison. For example, Ding (2016) used the EM textbooks to compare US and Chinese textbooks, and Hwang et al. (2021) and Shin and Lee (2018) used EM textbooks to compare fraction operations concepts in US and Korean textbooks. Similarly, tens of millions of students use the NCERT textbooks in over 24,000 schools affiliated with the CBSE board in India (Jaswal, 2023).

The NCERT developed the *Math-Magic* textbooks based on the curricular reforms recommended by the NCF 2005 (Nag Chowdhuri, 2022), with an effort to rethink mathematics teaching and learning (Rampal, 2015). Similarly, the EM program is designed for students from pre-K through grade 6. It is a research-based textbook series developed by the University of Chicago School Mathematics Project, aimed at establishing a strong foundation in mathematics (Everyday Mathematics, n.d.-b). Another reason for selecting these two textbook series was their shared philosophy for content development.

This study only included the concepts explicitly related to equivalent fractions. This means that content related to the uses of equivalent fractions in other fraction topics, such as comparing fractions, finding fractions between two fractions, finding common denominators, performing fraction operations, and using equivalent fractions to find decimals, was not included in the analysis of this study. Some subtopics related to the uses of equivalent fractions were included only if they specifically fell under the subtopic of equivalent fractions. Furthermore, the grade 6 EM textbook had a section on equivalent ratios in a separate unit, which was not included in the content of equivalent fractions for analysis. Although this concept used fraction notations and the notion of "equivalence," it aimed to explain the idea of the equivalence of ratios, or proportionality. Therefore, it was not included in this study.

Analytic Framework and Data Analysis

This study employs a comparative case study research design (Stake, 2000), where the two bounded cases being compared are the selected textbooks from grade 3-grade 6 from both textbook series. The comparative case study allows for a thorough comparison of the cases being studied. Additionally, this approach can provide relevant information about the underlying factors that influence the outcomes of each textbook series analysis, improving the overall understanding of this research topic. This study employed content analysis (Krippendorff, 2004) to conduct an in-depth qualitative analysis of the concepts related to equivalent fractions in both textbook series. For the first research question, the flow of content related to equivalence in both textbook series was qualitatively analyzed. For the remaining two research questions, the equivalent fraction concepts were coded qualitatively in both textbook series to present the findings and some relevant descriptive statistics. Although the first author was the primary coder, all the codes and the processes related to coding were corroborated and finalized after a detailed discussion with the second author. After finalizing the codes, the first author completed all the coding to maintain consistency in the coding process across all textbooks in both series, and to minimize any potential subjective bias during coding. The coding for all three criteria was completed within approximately one month to prevent any potential coding drift over time.

To carry out the data analysis for each research question, the content of equivalent fractions in all relevant chapters in both textbook series was divided into "units of analysis." These units of analysis were of various types, as described in **Table 1**. These units of analysis, once identified, were used consistently throughout the three research questions.

To analyze the first research question, these units of analysis were traced and counted. Further, relevant codes (described below) were assigned to these units of analysis for the analyses of the second and third research questions. The scope of the units of analysis in one textbook series was similar across grades, but it differed in length and nature from the scope of the units of analysis in the other textbook series.

Some units of analysis were termed "pseudo-units" if those units were embedded in the main subtopic of

Table 1. Details about the types of units of analysis

Types of a single unit of analysis	Definitions
Question	A single question posed to make students think conceptually.
Statement	A single statement that provides a definition, rule, idea, example, or explanation of a concept.
Statement with a visual representation	A single statement, as defined above, which is accompanied by a visual representation for an explanation.
A story with multiple questions	A contextual or real-life story accompanied by mathematical questions.
One task/activity with multiple questions	A task or activity (mathematical or real-life) accompanied by multiple mathematical questions.
Single problem	A problem with only one mathematical question.
One problem with multiple sub-questions	A mathematical problem with multiple mathematical sub-questions, similar in nature to each other.
Solved example	A mathematical example solved in the textbook with a detailed procedure, explanation, and visual representation (as required).
A table with information or multiple examples	A table with examples or information about specific concepts related to the main topic.
Picture/poster/figure	A visual representation, like a picture, a poster, or a figure.

Table 2. Analytic framework for data analysis (research question 2: RQ2)

The main criterion for comparison	Sub-categories
Use of relevant and contextual real-life examples (RQ2)	Opportunities to understand concepts using real-life examples (2a) Opportunities to apply concepts to real-life scenarios or other contextual situations (2b)

Table 3. Analytic framework for data analysis (research question 3: RQ3)

The main criterion for comparison	Sub-categories
Development of (mathematical) flexibility (RQ3)	Opportunities to explore multiple ways to represent or solve a problem (3a) Opportunities to work on open-ended problems with multiple solutions (3b)

equivalent fractions but were not specifically about equivalent fractions. Generally, these pseudo-units pertain to fraction concepts that prove useful for the subsequent subtopics related to equivalent fractions. Therefore, these pseudo-units were excluded from the textbook analysis because they were not related to equivalent fractions.

Framework for analysis

The analytic framework in **Table 2** was used while analyzing the textbooks for the second research question. This research question examined the opportunities provided by each textbook series from both countries to learn the concepts of equivalence of fractions using relevant and contextual real-life examples. While analyzing both textbook series, every unit of analysis was examined and coded for two sub-categories (2a and 2b). The code for sub-category 2a indicated whether a unit of analysis provided students with an opportunity to understand the concepts through real-life examples. In contrast, the code for sub-category 2b indicated whether the unit of analysis provided students with opportunities to apply their conceptual understanding to real-life, contextual situations. The units of analysis that used real-life contextual examples, such as students eating pizzas or cakes divided among friends, etc., were included. However, the units of analysis that used different fraction models, such as fraction circles or physical tools like paper or fraction strips and rulers, were not accounted for in this analytic criterion.

Lastly, the analytic framework shown in **Table 3** was used while analyzing the textbooks for the third research question. This research question examined the opportunities provided by each textbook series from both countries to develop mathematical flexibility while learning equivalent fraction concepts. While analyzing both textbook series, every unit of analysis was examined and coded for two sub-categories (3a and 3b). The code for sub-category 3a showed whether a unit of analysis allowed students to solve a problem in more than one way or whether the unit of analysis explicitly or implicitly allowed students to choose different ways to solve the problem. The code for sub-category 3b indicated whether the unit of analysis provided students with opportunities to work on open-ended problems with multiple correct answers. Additionally, both code 3a and code 3b were applied to a unit of analysis if it had a mathematical problem that was open-ended with multiple solutions but also allowed for flexibility in solving it.

While analyzing both textbook series for this research question, solved examples that had open-ended questions like "Name a fraction that is equivalent to $\frac{3}{5}$."

Table 4. Details of equivalent fractions across grade 3-grade 6 in EM textbooks

Features	Grade 3	Grade 4	Grade 5	Grade 6
Unit name	Number and Operations - Fractions	Number and Operations - Fractions	Number and Operations - Fractions	The Number System
Number of pages for equivalent fractions	4	7	6	3
Number of units of analysis dedicated to equivalent fractions	12	22	24	12

(Bell et al., 2015b, p. 137) were included, primarily attributing them to code 3b. This attribution is based on the fact that, despite the presence of specific solutions to these problems, there is an opportunity for teachers and students to engage flexibly and develop their own solutions to such open-ended problems, which may or may not be like the solutions provided. For code 3a, mathematical problems that required a specific method for their solutions were excluded. In contrast, closed-ended mathematical problems with a single solution were excluded for code 3b. The sophisticated framework developed from a detailed review of seminal studies by Zeynivandnezhad et al. (2024) highlights six overarching themes for conducting qualitative content analysis of mathematics textbooks. The three comparison criteria of our study are more related to some subthemes within two themes of this framework, namely 'content analysis' and 'visual aids and presentation', than to the remaining themes. But there are some differences in how our comparison criteria are structured and defined compared to this framework (Zeynivandnezhad et al., 2024).

FINDINGS

Flow and Structure of Equivalent Fraction Concepts

The two textbook series differed substantially in their **structure**, **appearance**, **size**, and **intended use**, although both countries use these textbooks as their primary curricular materials. The NCERT textbooks for the respective grades are smaller in terms of the number of pages than the EM textbooks. This study focused on comparing the student-facing versions of both textbook series. The EM textbook series had separate student and teacher versions for every grade. The "Student Reference Book" of each grade was used as the student-facing version of the EM series, as they are mainly used in classrooms to teach mathematical concepts to students. Besides these student reference books, the EM series also has "Student Math Journals," the other student-facing versions with additional practice problems and supplemental material to support classroom instruction. However, this study solely utilized them as a reference, but not for analysis. Unlike this, the NCERT textbooks are the only student-facing versions available for students and teachers. The students usually do not have any other practice workbooks. Similarly, the teachers do not have a separate teacher version of the textbooks, except for some additional resources such as teacher materials and small teacher notes dispersed throughout the textbooks, thus making these NCERT textbooks the primary curricular materials in classrooms.

The EM textbook series introduced fraction concepts in grade 1 and the equivalent fraction concepts in grade 3, which proceeded through grade 6. The EM textbooks introduced the term "equivalent fractions" in grade 3. On the other hand, the NCERT textbooks introduced fraction concepts in grade 3. The idea of equivalent fractions was subtly introduced in grade 3, progressing gradually until grade 6. The NCERT textbooks only introduced the term "equivalent fractions" in grade 6. The following subsections present the details of the flow of the equivalent fraction concepts and their emphases across grades.

Flow and structure: Everyday Mathematics textbook series

In the EM textbook series, from grade 3 through grade 5, the structure of the units related to fractions was similar. Each fraction unit in these grades was divided into subtopics, some of which were explicitly dedicated to concepts about equivalent fractions. However, in grade 6, the unit's structure was slightly different from the other grades, where the larger unit named "The Number System" had multiple topics, one of which was fractions. Some subtopics in the fractions topic were directly associated with equivalent fractions, while others were indirectly related to fraction equivalence. **Table 4** presents the name of the unit that included equivalent fraction concepts, the number of pages dedicated to these concepts, and the number of units of analysis present across all the pages covering equivalent fractions across grade 3 through grade 6. These grades were the focus because they incorporated the equivalent fraction topics in this textbook series.

Figure 1 provides additional details about the flow of the concepts, along with the details presented in **Table 4**. In **Figure 1**, for every grade, each small rectangular box represents a single unit of analysis, and different colors represent different equivalent fraction concepts. As seen in **Figure 1**, the major concepts of equivalent fractions introduced across various grades were

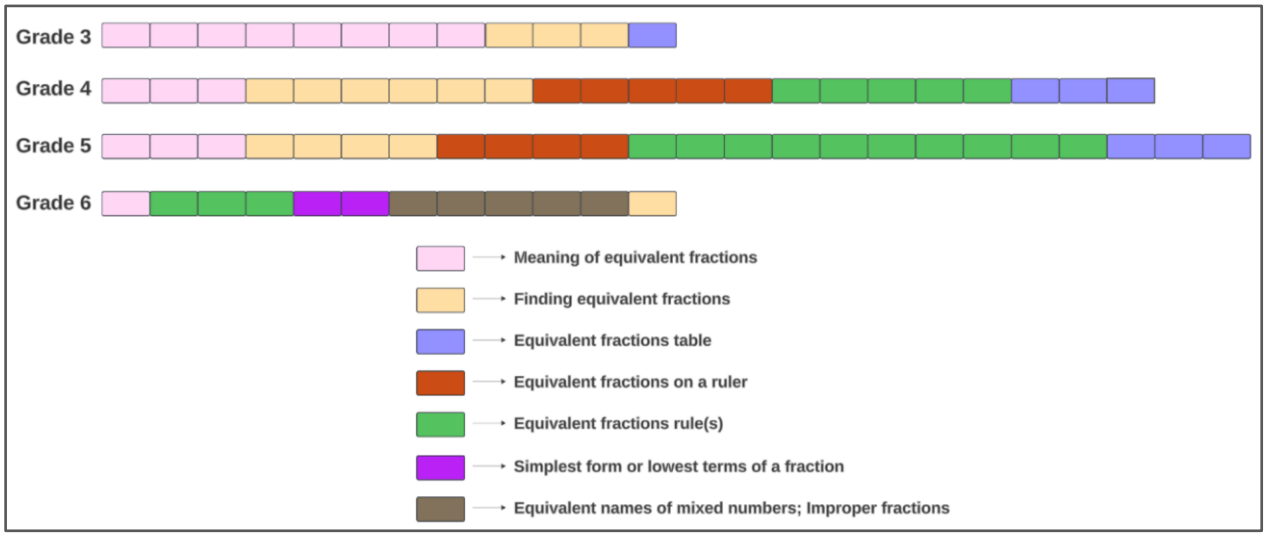

Figure 1. Flow of equivalent fraction concepts across grade 3-grade 6 in EM textbooks. Here, each rectangular box represents a single unit of analysis, while the different colors represent different concepts related to equivalent fractions (Source: Authors' own elaboration).

(1) the meaning of equivalent fractions (description of what it means for two fractions to be equivalent to each other),

(2) finding equivalent fractions (description and illustration of methods to find equivalent fractions using various models),

(3) the equivalent fractions table (a table showing a list of unit fractions with a list of their respective equivalent fractions),

(4) equivalent fractions on a ruler (description and illustration of finding equivalent fractions using a ruler, i.e., the linear fraction model),

(5) equivalent fractions rule(s) (description and illustration of the multiplication and division rules to find equivalent fractions),

(6) simplest form or lowest terms of a fraction, and

(7) converting mixed numbers into (improper) fractions and vice versa and finding equivalent names of mixed numbers.

As seen in **Table 4**, irrespective of the number of pages dedicated to equivalent fractions, the number of units of analysis assigned to the equivalent fraction concepts increased from grade 3 through grade 5 and then decreased in grade 6. Moreover, as shown in **Figure 1**, most units of analysis focused on the "meaning of equivalent fractions" in grade 3, but they also existed in grades 4, 5, and 6, albeit with lower emphasis. Similarly, the units dedicated to "finding equivalent fractions," "equivalent fractions rules," and "equivalent names of mixed numbers or improper fractions" were *majorly* covered in grades 4, 5, and 6, respectively.

As shown in **Figure 1**, the meaning of equivalent fractions was introduced in grade 3 in the EM textbook series by using various real-world examples and fraction models, such as area, set, and linear models. An example then followed, which had an illustration of finding equivalent fractions using a number-line poster, which was followed by a table of equivalent fractions. In grade 4 and grade 5, the meaning and process of finding equivalent fractions was presented in a similar fashion as in grade 3, by using similar models and examples but having more complex fractions. The new subtopics in grade 4 were the illustrations of exploring equivalent fractions (proper and improper) using a ruler and the multiplication rule of finding equivalent fractions. These two new subtopics were also repeated in grade 5 with similar examples and illustrations. The additional subtopic introduced in grade 5 was the division rule of finding equivalent fractions.

As shown in **Figure 1**, equivalent fraction rules were dealt with in more detail in grade 5 than in grade 4. Similar to grade 3, the equivalent fractions section ended with an equivalent fractions table in grade 4 and grade 5. Proper and improper fractions were introduced in all grades except for grade 3, where only proper fractions were used as examples. This is how equivalence of fractions was presented in the EM textbook series across grades.

Overall, the subsequent grades exhibited substantial overlap and repetition of concepts from the previous grades, along with the introduction of some new concepts or additional complexities to the existing content. For example, as described above, only the multiplication rule of finding equivalent fractions was introduced in grade 4. Along with this rule, the division rule for finding equivalent fractions was also introduced in grade 5, along with explanations of both rules.

In grade 6, similar patterns were observed, although there were a few exceptions. For example, there was very little repetition of concepts from the previous grades, but some ideas were newly introduced, like using greatest common factor to find the simplest form of fractions and working with mixed numbers and improper fractions.

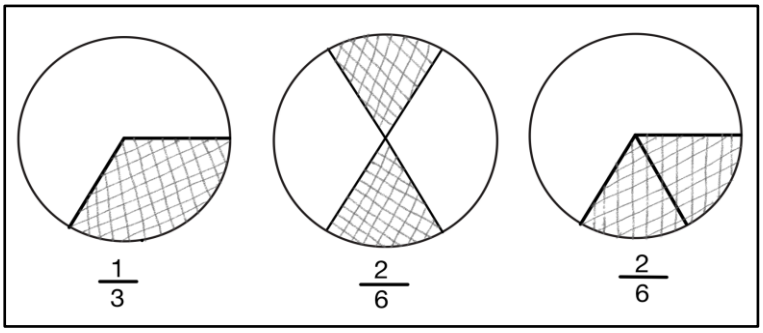

Figure 2. Fraction example in grade 5 adapted from Bell et al. (2015c, p. 166) in the EM textbooks [which is identical in grade 4 (Bell et al., 2015b, p. 137) and has a slight variation in framing in grade 3 (Bell et al., 2015a, p. 152)]. The positions of the second and the third fraction circles in this adapted image are interchanged as compared to the original textbook image.

Table 5. Details of equivalent fractions across grade 3-grade 6 in NCERT textbooks

Features	Grade 3	Grade 4	Grade 5	Grade 6
Chapter name	Fair Share	Halves and Quarters	Parts and Wholes	Fractions
Number of pages for equivalent fractions	1	1	3	11
Number of units of analysis dedicated to equivalent fractions	1	1	3	30

Before this, the concept of the simplest form or lowest terms of fractions was introduced as a "note" with the equivalent fractions table in grade 4 and grade 5. It was typically observed that the sequence of topics remained the same across grades, despite the addition of new topics or increased complexity, as shown in **Figure 1**. However, in grade 6, the equivalent fractions topic ended with one unit of analysis on "finding equivalent fractions." There was also some repetition in the examples presented in the grade 3 through grade 5 textbooks, as seen in the example shown in **Figure 2**, which was identical in grade 4 and grade 5, while exhibiting a slight variation in grade 3. The mathematical idea presented in this solved example in grade 5 was to show how the fraction 2/6 is equivalent to 1/3 using fraction circles. The example highlights how the shaded piece representing one-third in the first circle and the two shaded pieces together representing two sixths in each of the second and third circles cover the same area of the respective circle (whole), thus making 1/3 and 2/6 fractions equivalent to each other.

The EM textbooks prioritized the use of various models to explain fractions, including area models (primarily fraction circles), linear models (use of rulers and number lines), and set models. Thus, in the EM textbooks, the equivalent fractions are covered from grade 3 through grade 6, with gradual increments in the topics across the grades and a substantial repetition of content.

Flow and structure: NCERT textbook series

The structure of the NCERT textbook series for equivalent fraction concepts differed significantly from that of the EM textbook series. **Table 5** shows that grade 3 and grade 4 each dedicated only one unit of analysis to equivalent fractions. As the chapters on fractions in both these grades focused on introducing specific fractions, such as halves and quarters, equivalent fractions relevant to the context of halves and quarters were introduced. Thus, these grades subtly introduced the concept of equivalent fractions. **Figure 3** and **Figure 4** show the units of analysis related to equivalent fractions introduced in grade 3 and grade 4, respectively.

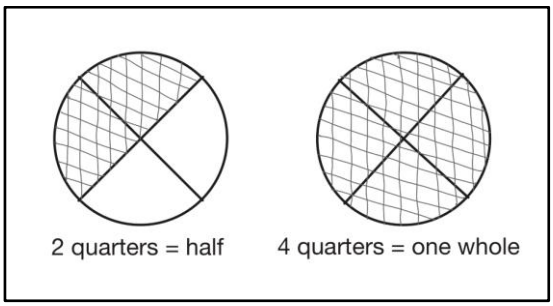

Figure 3. The mathematical idea related to equivalent fractions as introduced in the grade 3 NCERT textbook, adapted from NCERT (2024b, p. 114). In the original textbook image, there are four circles, each divided into four equal parts by a vertical line and a horizontal line, where the first through the fourth circles illustrate one, two, three, and four shaded quarters, respectively. In this adapted image, only the modified versions of the second and the fourth circles from the original textbook image are shown respectively, as they specifically present the mathematical ideas related to equivalent fractions.

As seen in **Figure 3**, the idea of equivalence was introduced using fraction circles by illustrating that two quarters make a half and four quarters make a whole. Similarly, in grade 4, students have to match the fractions in area models (octagons) to their names and numerical versions. This solved example matches the octagon shaded $\frac{2}{4}$ to half ($\frac{1}{2}$), as shown in **Figure 4**.

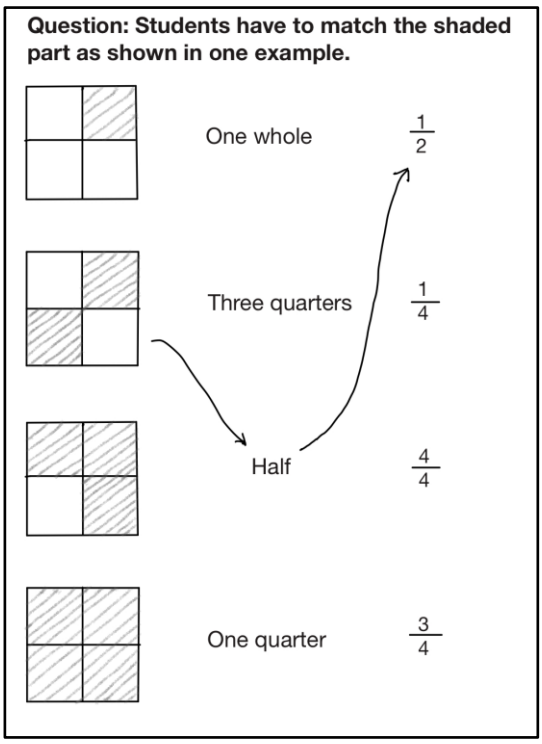

Figure 4. The mathematical idea related to equivalent fractions as introduced in the grade 4 NCERT textbook, adapted from NCERT (2007, p. 102). The original wholes in the textbook (in the first left-most column) are octagons, which have been adapted into squares to illustrate the mathematical idea.

In grade 5, the scope of the fractions chapter broadened to include all fractions, but it still had only three consecutive pages with three units of analysis dedicated to equivalent fractions. It is important to note that although concepts related to equivalent fractions were introduced in these three grades, the term "equivalent fractions" was introduced only in grade 6. Furthermore, the structure of these chapters was quite different from the fractions units in the EM textbooks. In grade 3 through grade 5, the chapter focused on one or more themes throughout the material. The chapters related to fractions were based on specific themes but were not divided into clear subtopics, like in the EM textbooks. The chapter "Fractions" in grade 6 was an exception and was divided into specific subtopics of fractions. Therefore, in grade 6, 11 pages were dedicated to subtopic of equivalent fractions, which included 30 units of analysis, as shown in **Table 5**.

Another vital element of the NCERT textbooks was the limited redundancy of concepts across different grades. The models for fractions used in this textbook series were area (fraction circles and octagons), linear, volume, and set models. **Figure 5** illustrates that the NCERT textbooks across grade 3-grade 6 address the following concepts of equivalent fractions:

(1) the meaning of equivalent fractions (using different fraction models and checking if different fractions are equivalent fractions),

(2) finding equivalent fractions (using various methods like fraction walls or fraction strips, fraction circles, etc. to find equivalent fractions),

(3) using equivalent fractions as a tool to compare fractions (which also included the multiplication rule to find equivalent fractions), and

(4) expressing fractions in their lowest terms or simplest forms by finding highest common factor (HCF), (which also utilized the division rule of finding equivalent fractions).

The subtopics utilizing equivalent fractions for comparison were included as they fell under the 'Equivalent Fractions' subsection in the Fractions chapter. The grade 5 NCERT textbook used three examples to show the meaning of fractions, where the area model, volume model, and fraction strip were used for each example, respectively. In the grade 6 NCERT textbook, various subtopics related to equivalent fractions were presented using many solved examples and mostly area and set models. As described above, the

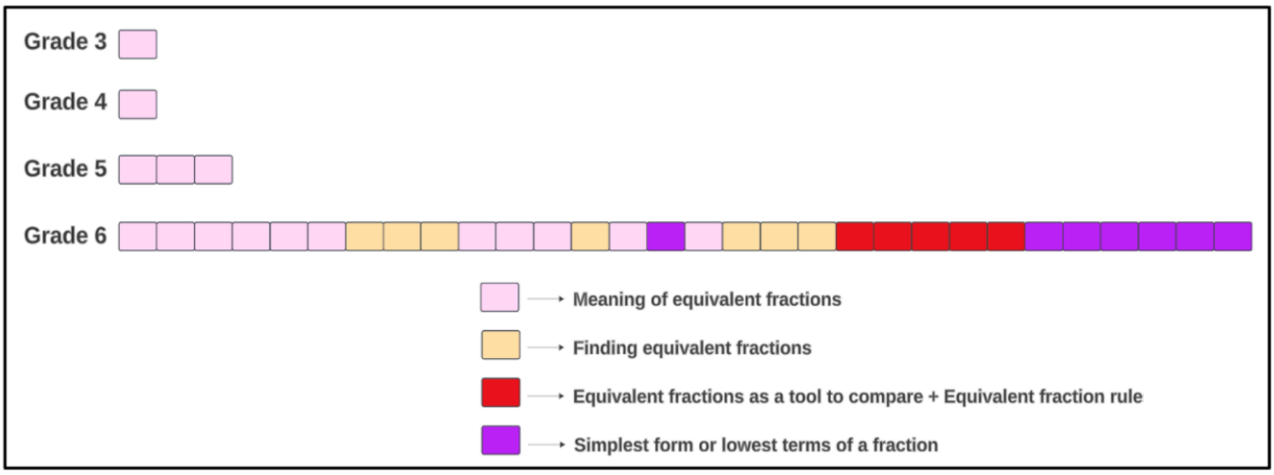

Figure 5. Flow of equivalent fraction concepts across grade 3-grade 6 in NCERT textbooks. Here, each rectangular box represents a single unit of analysis, while the different colors represent different concepts related to equivalent fractions (Source: Authors' own elaboration).

Table 6. Real-life examples present in the EM textbook series

Grades	Code 2a. Understanding with real-life examples	Code 2b. Applying concepts to real-life or other contextual situations	Number of units of analysis with relevant and contextual real-life examples
3	3 examples	1 example	4 (4 out of 12 units)
4	3 examples	None	3 (3 out of 22 units)
5	1 example	1 example	2 (2 out of 24 units)
6	None	None	0 (0 out of 12 units)

multiplication and division rules of finding equivalent fractions were introduced as tools in grade 6 NCERT textbook, but not directly as subtopics, like in EM textbooks. The examples and models used to explain the concept of equivalent fractions were presented differently in the NCERT series compared to the EM series. This is how the equivalence of fractions was presented in the NCERT series.

Figure 5 reveals that concept of equivalent fractions in grade 3-grade 5 and the beginning of grade 6 pertains solely to the meaning of equivalent fractions. In contrast to EM textbooks, which present subtopics mostly in a linear and sequential manner, the grade 6 NCERT textbook exhibits a nonlinear sequence of subtopics, as the concepts of meaning and finding equivalent fractions are revisited multiple times within the chapter.

Summary: Flow and structure

Up until grade 5 in both textbook series, the fraction equivalence was mostly related to proper fractions (numerator is less than denominator), with a few exceptions in the EM series. Grade 4 and grade 5 EM textbooks included equivalent fractions of mixed numbers. Grade 6 NCERT textbook dealt with improper fractions, while grade 6 EM textbook included both mixed numbers and improper fractions. Also, the content of equivalent fractions covered across grade 3-grade 6 in both textbook series was almost similar. Additionally, some pseudo-units were identified in both grade 6 textbooks. Both series used multiple fraction models, but the NCERT textbooks did not use number lines for equivalent fractions. However, the NCERT textbooks employed the unique volume model, a feature absent in the EM textbook series. In addition to the similarities, both textbook series had different chapter structures, repetition levels, and textbook structures.

Using Relevant and Contextual Real-life Examples

Both textbook series included contextual and relevant real-life situations and examples, but the proportions of these elements varied between the two series. In the following sections, we present the details and the nature of the contextual real-life examples existing in each textbook series, and then we describe some patterns observed across both series.

Relevant and contextual real-life situations: Everyday Mathematics textbooks

In the EM textbooks, it was observed that real-life contextual examples were used in various units of analysis in grade 3 through grade 5, but none were used in grade 6. Additionally, the percentage of units of analysis having real-life examples (33.33%, 13.64%, 8.33%, and 0% in grade 3 through grade 6, respectively) and their absolute number decreased sequentially from grade 3 through grade 6. **Table 6** displays these details.

There were some intriguing observations across various grades in the EM textbooks. For example, grade 3 and grade 4 used the same solved example (with slightly different language), which demonstrated that different names, like $\frac{1}{2}$ inch, $\frac{2}{4}$ inch, $\frac{4}{8}$ inch, and $\frac{8}{16}$ inch, represent the length of the same nail. Thus, by using a visual representation of a nail placed in reference to the zero mark on an inch ruler, this example demonstrated how the different lengths of the same nail are equivalent fractions. This example in grade 4 was immediately followed by another solved example, which showed different names for the length of a paper clip on an inch ruler. Since the paper clip has a length greater than 1 inch, this solved example demonstrated the equivalent fractions of mixed numbers, like $1\frac{1}{4}$ inch, $1\frac{2}{8}$ inch, and $1\frac{4}{16}$ inch. The advantage of having both these examples sequentially was that the nail example was a repetition from grade 3, which helped solidify the concept, while the paper clip example introduced the same concept with more complexity. Additionally, both contexts were relevant to students, as they are familiar with a nail and a paper clip. Also, the examples used these contexts with appropriate and reasonable measurements (lengths), which made them relevant and realistic.

Grades 3, 4, and 5 each used a solved example of a set model with students or children in slightly different contexts. For example, in grade 3, children go to a party, while in grade 4 and grade 5, students travel on the school bus. The numbers and fractions become larger and more complex as the grades progress. However, all these solved examples in the three grades used similar patterns of illustrations to demonstrate the notion of equivalent fractions. These contexts of children attending a party or students taking a school bus are relevant to most students. Overall, using contextual and relevant real-life examples in the EM textbooks was limited, but the contexts used were realistic and relevant to students in their respective grades.

Table 7. Real-life examples present in the NCERT textbook series

Grades	Code 2a. Understanding with real-life examples	Code 2b. Applying concepts to real-life or other contextual situations	Number of units of analysis with relevant and contextual real-life examples
3	None	None	0 (0 out of 1 unit)
4	None	None	0 (0 out of 1 unit)
5	1 example*	1 example*	1 (1 out of 3 units)*
6	5 examples	6 examples	11 (11 out of 30 units)

Note. The example with an asterisk (*) had both code 2a and code 2b.

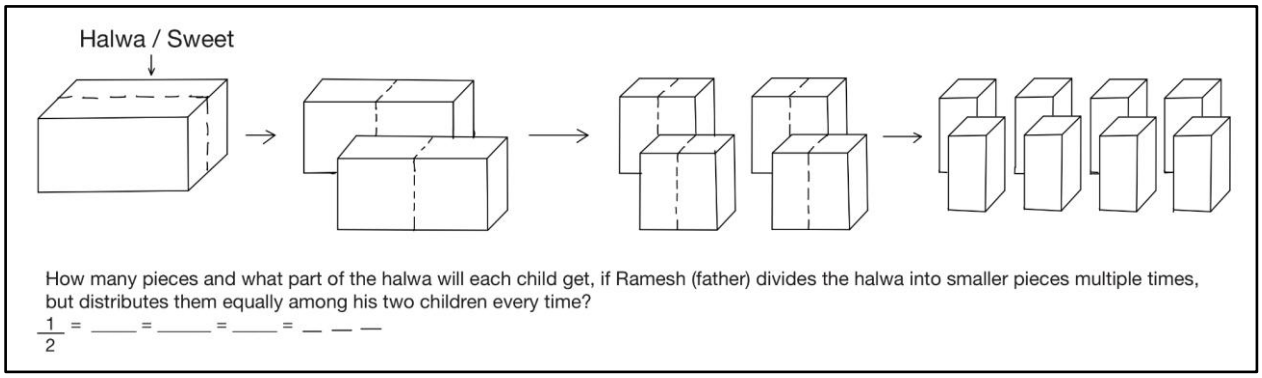

Figure 6. "Cutting the *halwa*" example from the grade 5 NCERT textbook, adapted from NCERT (2008, pp. 63-64). The dotted line shows how every piece is subsequently divided into two equal parts.

Relevant and contextual real-life situations: NCERT textbooks

In contrast to the EM textbooks, the percentages of units of analysis (0%, 0%, 33.33%, and 36.67% in grade 3 through grade 6, respectively) and the absolute count of units that incorporated contextual and real-life examples increased across subsequent grades in the NCERT textbooks. **Table 7** shows these details. Although multiple real-life-related themes were running through the fractions chapters in grade 3 and grade 4, the specific unit of analysis dedicated to equivalent fractions in both these grades did not use any real-life context.

In grade 5, one of the three units of analysis dedicated to equivalent fractions used a real-life context of cutting a *halwa* (an Indian sweet) as a volume model to demonstrate equivalent fractions. **Figure 6** shows an illustration of the volume of the *halwa* being divided into halves multiple times, thus yielding 2, 4 and 8 pieces respectively, after every consecutive division. This example involves a conversation between Ramesh and his two children while Ramesh cuts and distributes the *halwa* between them equally. This conversation includes a series of embedded questions that refer to equivalent fractions. The interesting feature of this problem is that it helps students understand the concept (code 2a) and provides opportunities for students to apply the equivalent fraction concepts in a real-life context (code 2b). This benefit may be attributed to the inclusion of illustrations, demonstrations, and embedded questions, which facilitate students' understanding and application of concepts.

Another intriguing feature is that, in grade 6, there were almost the same number of examples using real-life contexts, which helps students understand the concepts (code 2a) and apply them (code 2b) in real-life situations. The majority of grade 6's real-life examples were related to equal shares. All contexts used in this chapter, such as distributing *rotis* (Indian flatbread), cakes, *chikkis* (Indian brittles), glasses of juice to students or children, and distributing potatoes in bags, were relevant and familiar to the students. Some contexts were repeated multiple times in the same chapter. Like the EM textbooks, the numbers and fractions used in these contexts were reasonable and, hence, realistic. Overall, in the NCERT textbooks, the real-life contextual examples increased with grades and were relevant to the students.

Summary: Using relevant and contextual examples

The contextual real-life examples used in both textbook series were relevant to students, and the use of reasonable numbers and fractions made these contexts realistic. Another trend observed across both series was that, typically, the units of analysis that were solved examples had the code 2a assigned to them, providing opportunities for students to understand equivalent fraction concepts through real-life situations. On the other hand, the units of analysis, which were mathematical problems or questions, were mostly coded 2b, where they provided opportunities to apply the learned concepts in real-life situations. The only exception to this trend was the *halwa* example (**Figure 6**) in the grade 5 NCERT textbook, which was assigned both codes. Lastly, neither textbook used real-life contexts for the simplest or lowest terms of fractions and only used algorithmic and mathematical representations for these concepts.

Table 8. Mathematical flexibility present in the EM textbook series

Grades	Code 3a. Multiple ways to represent/solve a problem with a single solution	Code 3b. Open-ended problems with multiple solutions	Examples with both code 3a and code 3b	Number of units of analysis with mathematical flexibility
3	2 examples	0 examples	0 examples	2 out of 12 units
4	3 examples	5 examples	2 examples	6 out of 22 units
5	2 examples	7 examples	0 examples	9 out of 24 units
6	1 example	6 examples	1 example	6 out of 12 units

Table 9. Mathematical flexibility present in the NCERT textbook series

Grades	Code 3a. Multiple ways to represent/solve a problem with a single solution	Code 3b. Open-ended problems with multiple solutions	Examples with both code 3a and code 3b	Number of units of analysis with mathematical flexibility
3	0 examples	0 examples	0 examples	0 out of 1 unit
4	0 examples	0 examples	0 examples	0 out of 1 unit
5	1 example	1 example	0 examples	2 out of 3 units
6	8 examples	7 examples	4 examples	11 out of 30 units

Development of (Mathematical) Flexibility

Both textbook series offered opportunities, in differing degrees, to develop mathematical flexibility by allowing engagement with different problem-solving methods or open-ended problems with multiple solutions, or a combination of both. The following sections describe these features related to equivalent fractions and patterns observed across both textbook series.

Development of (mathematical) flexibility: Everyday Mathematics textbooks

The EM textbooks showed a sequential increase in the percentages of mathematically flexible units of analysis from grade 3 through grade 6, which were 16.67%, 27.27%, 37.5%, and 50%, respectively. Also, there was a fluctuation in the absolute number of units of analysis, which had mathematical flexibility across these grades. **Table 8** presents these details.

Along with these trends in the EM textbooks, interesting examples were present, providing flexibility in various ways. For example, in the grade 4 EM textbook, the problem states, "Use the table [of equivalent fractions] to find three fractions that are equivalent to $\frac{1}{5}$." (Bell et al., 2015b, p. 142) was not allotted to any of the codes related to mathematical flexibility. In contrast, the subsequent problem "Find two more fractions equivalent to $\frac{1}{5}$ that are not in the table." (Bell et al., 2015b, p. 142) received code 3a and code 3b. The first (excluded) problem had no flexibility because the only way to find equivalent fractions was using the equivalent fractions table. Moreover, this was not an open-ended problem because its solution was the limited set of equivalent fractions of $\frac{1}{5}$ that were in the table. Both of these requirements were addressed in the second (included) problem, as it had the flexibility element in choosing the solution method and had multiple existing solutions.

In another example, fraction circles were used to find an equivalent fraction of $\frac{1}{3}$, which is $\frac{2}{6}$. In this process, this example demonstrated two ways to represent the fraction $\frac{2}{6}$ in the fraction circle. Therefore, this example was included as multiple ways of representing a solution. This example was present in grades 3, 4, and 5, with slight variations, and it is also shown in **Figure 2**. Lastly, one unique example present in grade 5, is a "single problem with multiple sub-questions" type of unit of analysis. This problem is: "1. True or false? a. 1/3 = 5/15; b. 4/4 = 8/8; c. 6/10 = 15/25; d. 5/8 = 40/72" (Bell et al., 2015c, p. 170). Here the students are given multiple pairs of equivalent fractions, and they have to decide if the equations are true or false. This example was coded only as 3a because here students can decide to use any method to come up with an answer, but the solution to each of the sub-questions is unique. Thus, each of the four grades in the EM textbook series included examples that develop students' mathematical flexibility, but they are present in different proportions across grades.

Development of (mathematical) flexibility: NCERT textbooks

In the NCERT textbooks, the absolute number of units of analysis having mathematical flexibility increased from grade 3 through grade 6, with no units of analysis in grade 3 and grade 4 having mathematical flexibility. The percentages of units of analysis with mathematical flexibility are 0%, 0%, 66.67%, and 36.67%, respectively, which shows fluctuations in percentages across grades. **Table 9** presents these details.

As observed in **Table 9**, in grade 5 and grade 6, there were almost equal numbers of units of analysis that were coded 3a and 3b, respectively, which means that, in both these grades, there were similar numbers of examples

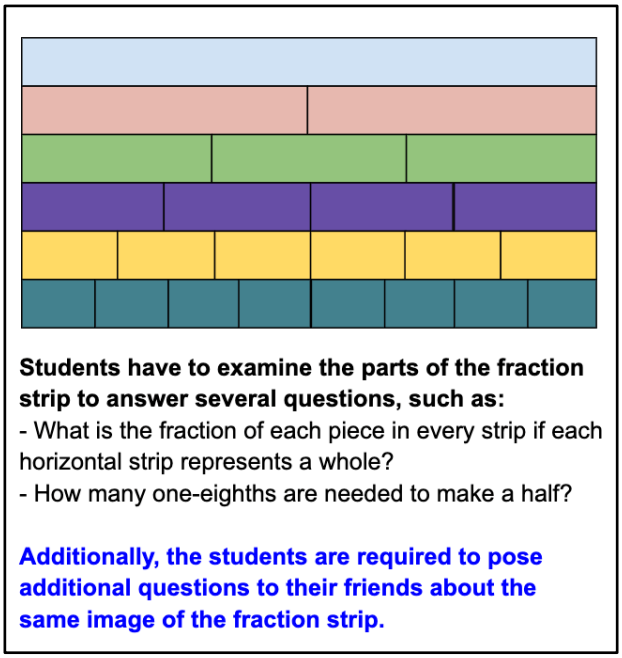

Figure 7. "Parts of the strip" example from the grade 5 NCERT textbook, adapted from NCERT (2008, p. 64). In this example, students have to first answer questions, and then they are expected to pose some additional questions about the fraction strip to their friends.

that had flexibility in the problem-solving methods and the examples that were open-ended problems. Moreover, in grade 6, four examples were coded both 3a and 3b, which was the highest number of examples in one grade having both codes, across both series.

In the NCERT textbooks, some examples provide opportunities for flexibility in different ways. For example, in grade 6, there is a question: "Do you notice anything about the relationship between the numerator and denominator in each of these fractions?" (NCERT, 2024a, p. 168). This open-ended question has the potential to encourage discussion among students, which can help introduce the subsequent concepts. Another interesting example was seen in the grade 6 textbook, where the solution to the problem of finding the simplest fraction of $\frac{36}{60}$ was shown using two methods. The first method showed dividing both the numerator and denominator by any common factor that is quickly apparent and repeating this process multiple times until the lowest term is reached. The other method demonstrated finding the HCF first and then dividing the numerator and denominator by the HCF to directly determine the simplest fraction. This example made it clear that both approaches work and that students could choose either method based on the problem.

In the grade 5 textbook, the unit of analysis shown in **Figure 7** is another unique example that has the flexibility element because students are asked to pose their own problem. The questions asked in this example are not open-ended, but after students have answered these straightforward questions, they have to ask more questions based on **Figure 7**, which is an open-ended activity in itself. Through this activity, students understand the concept better and think about it more flexibly. Thus, the grade 5 and grade 6 NCERT textbooks had many examples that provided opportunities to develop mathematical flexibility in diverse ways, some of which are described above.

Summary: Development of (mathematical) flexibility

The overall analysis of both the textbook series demonstrated that, from grade 3 through grade 6, students got opportunities to learn different ways to represent or solve problems. For example, they learned to apply different models to figure out and identify equivalent fractions, such as the area model, number line (linear model), set model, volume model, etc. Students could also use different fraction tools like equivalent fraction tables, fraction circles, fraction strips, fraction walls, inch rulers, and matrix representations of the set model to find equivalent fractions. All of these factors contributed to the development of mathematical flexibility skills by exploring different tools and methods for solving problems. Additionally, the inclusion of a substantial number of open-ended problems in both textbook series provided opportunities for students to become acquainted with mathematical problems that have multiple solutions, which also contributed to the development of mathematical flexibility.

In summary, it was found that both textbook series had a considerable number of examples that foster and develop mathematical flexibility in students. Both textbook series included the two key elements of mathematical flexibility, namely, multiple examples of using various methods to solve a problem (code 3a) and the presence of open-ended problems with multiple solutions (code 3b). It was also observed that the NCERT textbooks had almost a similar number of examples of both these elements. In contrast, in a particular grade, the EM textbooks had more examples of the second element than the first. Both textbook series had valuable examples of developing mathematical flexibility, with the EM textbooks having examples spread across all four grades, while the NCERT textbooks had such examples only in grade 5 and grade 6, but they were almost equally spread across both elements of flexibility.

DISCUSSION

This study employed three analytic criteria to review, analyze, and compare how equivalent fraction concepts were addressed in two textbook series: Everyday Mathematics from the USA and NCERT from India. The three criteria for analysis focused on understanding the structure and flow of equivalent fractions, the use of contextual and relevant real-life examples, and the development of mathematical flexibility in both textbook series. Especially, the second and third criteria are

features that support students in engaging with mathematics in a manner similar to that of mathematicians (Boaler, 2015; Cuoco et al., 1996; Fasheh, 1990; NRC, 2001). Although with varying numbers and types of examples, both textbook series demonstrate the presence of these two criteria, which helps expose students to the holistic and enriched nature of mathematics (Boaler, 2015). Moreover, the respective structure and flow of the textbooks prove conducive in helping each textbook provide enriched mathematical learning experiences to its respective group of students.

The curricular frameworks of both textbook series explicitly highlight the importance of connecting students' real-world contexts to school mathematics (Everyday Mathematics, n.d.-c; NCERT, 2005), thereby encouraging the use of realistic and relevant contexts while teaching mathematics. Additionally, NCERT (2005) aims to develop students' mathematical concepts while developing their skills and mathematical resources to plan for problem-solving in multiple ways. Similarly, the EM curriculum explicitly emphasizes the development of conceptual understanding alongside procedural fluency and the ability to employ multiple methods and strategies in problem-solving (Everyday Mathematics, n.d.-c), which involves becoming flexible in problem solving or developing mathematical flexibility. Thus, the analytic criteria for understanding the flow of concepts, using contextual examples for learning concepts, and developing mathematical flexibility were valuable for this textbook analysis, as they aligned with the educational philosophies of both textbook series.

A review of the existing literature on international textbook comparison studies revealed that very few, if any, studies compare the textbooks of India and the USA. Moreover, few studies focus on understanding how equivalent fractions are presented in different textbooks, which are important but challenging mathematical concepts for students to grasp (Pantziara & Philippou, 2012). Thus, this study makes a unique contribution to the larger literature by conducting a qualitative analysis and comparing textbooks from India and the USA, specifically examining how they explain the idea of equivalent fractions using three criteria that analyze the structure, flow, and certain features of the mathematical content.

While analyzing the flow and structure of the chapters related to equivalent fractions, it was observed that more pages were allotted to the equivalent fraction concepts in the EM textbooks across grade 3-grade 6. Additionally, the EM textbooks featured more repetition of concepts and mathematical examples across different grades than the NCERT textbooks. This characteristic of ample repetition and the larger size of US textbooks compared to those in other countries was also noted in other studies (Alajmi, 2012; Shin & Lee, 2018). Repetition across grades may initially appear negative due to the increased size of textbooks. The EM curriculum developers intentionally included repetition in the EM curriculum so that the repeated exposure to the same mathematical content could help students recall these concepts from their long-term memory (Everyday Mathematics, n.d.-c). Moreover, this approach in the EM textbooks ensures that students build a strong foundational understanding of fractions before moving on to more complex concepts. By revisiting these topics, the curriculum reinforces learning and helps students develop confidence in their mathematical abilities (Everyday Mathematics, n.d.-c).

However, whether or not the repetition of mathematical content benefits students depends on their context, as a trade-off is usually present. For example, most students in India have to carry their textbooks and notebooks back home after school, as they do not have lockers in schools like many students in the USA. For these students, having smaller textbooks (in terms of their page count) makes sense. However, the Indian curriculum should ensure that students are provided with additional practice resources to compensate for less repetition. Additionally, curriculum developers in India should consider how to condense the content into a more concise format without compromising its educational value. On the other hand, due to repetition, the progression in US textbooks is slower compared to that in the Indian ones. This approach may suit some students but may also bore others, especially students who are mathematically talented and work on mathematical problems with exceptional accuracy and speed (Rotigel & Fello, 2004). This suggests that excessive repetition may limit how textbooks address or support the needs of some students with higher mathematical abilities, similar to the study by Antoun et al. (2023), which indicated that only limited provisions were found in the Lebanese national policy documents and in mathematics and science textbooks to support the needs of students with higher abilities in these subjects. That is why the US teachers should have something to keep high-ability students engaged.

This study focused on the NCERT textbooks and the "Student Reference Books" from the EM series, both of which are considered student-facing versions. The NCERT textbooks are the only major student-facing versions in India.

In contrast, along with the Student Reference Books, the EM series also has the supplemental "Student Math Journals" to provide additional practice to students. While it is beneficial to have supplemental resources for students to practice mathematical problems, it may place an additional burden on teachers to develop a set of practice problems if some US school districts do not purchase the entire textbook package.

The second analysis criterion indicated that both textbook series used multiple examples of relevant and

realistic contexts to work with the equivalent fraction concepts. This is not an easy task to accomplish in diverse countries like India and the USA, where students in different regions of each country may have diverse contexts relevant to them. Despite this, the EM textbooks have utilized real-life contexts, such as children attending a party and students traveling on a school bus, which are relevant to most US students. Similarly, in the NCERT textbooks, examples such as *halwa* and *rotis* are included, which are relevant to the daily lives of most Indian students, irrespective of the different cultures and states they belong to.

Lastly, both textbook series had examples related to the third criterion of mathematical flexibility. Moreover, both series introduced multiple methods and models to find equivalent fractions, which contributes to developing mathematical flexibility. This holistic approach inspires pupils to explore various solutions and fosters their confidence in tackling complex issues. By integrating diverse teaching methodologies, educators can substantially encourage the development of mathematically adaptable thinkers who are equipped to confront the challenges of a constantly changing environment. However, it is also important to acknowledge that occasionally, a lack of flexibility can be advantageous when students are expected to master a specific method. Having said that, flexibility offers multiple pathways for students with different learning styles to grasp the concepts at a fundamental level.

CONCLUSION AND IMPLICATIONS

In this study, the Everyday Mathematics textbook series from the USA and the NCERT textbook series from India were compared to understand how each addressed the concepts related to equivalent fractions using three analytic criteria. It was found that there were differences in the textbook structures, chapter structures, their flow, and the manner in which various equivalent fraction concepts were presented. Both textbook series had examples that used real-world contexts, which were realistic and relevant to the students. Lastly, both textbook series featured various examples that showcased mathematical flexibility in diverse ways. Overall, both textbook series contribute towards providing valuable mathematical content and, thereby, holistic and enriched learning experiences to students. We will now present detailed conclusions pertaining to each criterion.

In both textbook series, the concept of equivalent fractions was introduced in grade 3; however, the NCERT textbooks provided only a brief introduction to the idea of fraction equivalence in grade 3 and grade 4. In contrast, these concepts were introduced to a larger extent in the EM textbooks. Also, the NCERT textbooks generally had a lower level of repetition across grades. Despite these differences, the overall sequence of the concepts and the total extent of content related to equivalent fractions were similar in both textbook series. The EM textbooks had limited real-life examples compared to the NCERT textbooks. However, both series used realistic contexts relevant to their respective groups of students. Furthermore, the numbers and fractions associated with these contexts were reasonable, which added to their realistic nature. Lastly, both textbook series included numerous examples related to the two elements of mathematical flexibility: opportunities to use multiple methods to solve problems and opportunities to work with open-ended problems that have multiple solutions. The NCERT textbooks had almost a similar number of examples of both elements of mathematical flexibility, whereas the EM textbooks had more open-ended problems than the first element.

This study only focuses on the curriculum of both countries. For future research, it will be advantageous to understand the perspectives of teachers and students from both countries regarding their experiences with these textbooks. This understanding could contribute to triangulating the findings. For the scope of this study, we have qualitatively analyzed one textbook series from each country for all the elementary grades that cover equivalent fractions, based on the three comparison criteria. Although both these chosen textbook series were comparable in terms of their curricular focus and educational philosophies, for future studies, we will plan to incorporate more diverse textbook resources to provide additional insights into the presentation of equivalent fractions in these two countries' curricula. Additionally, in future studies, we will try to examine whether and how the mathematics textbooks in India and the USA meet the needs of their respective higher-ability students, similar to Antoun et al. (2023).

Additionally, for this study, the first author was the sole researcher responsible for coding throughout the entire study, although the codes were finalized after discussing with the second author. Having only one researcher responsible for the coding may introduce a degree of bias or limit the diversity of perspectives in the analysis. To reduce the impact of this limitation, the first author completed all the coding in approximately one month to ensure consistency and minimize any coding drift over time. This was also done to minimize any potential subjective bias in the coding process that may arise due to one coder. To address this limitation, future studies can involve multiple researchers involved in the coding process to enhance reliability and validate the findings through triangulation, thus strengthening the trustworthiness of the findings. Having said that, having a single researcher can also result in a more cohesive and consistent coding approach, as this researcher possesses a profound understanding of the data.

Finally, the findings from this study can provide important insights to curriculum developers, teachers, and policymakers about recommendations of each

textbook from each country that would help provide rich mathematical learning experiences to diverse groups of students. Curriculum developers and policymakers from the USA may explore the nature of the limited repetition in the content of the Indian textbooks, along with the structure of the textbooks and chapters. Conversely, the curriculum developers and policymakers in India may explore the advantages of incorporating repetition in the mathematical content across different grades. While doing that, they may also contemplate the various benefits of having limited repetition in their current textbooks and ensure that the educational value of the content is not lost while developing textbooks with limited repetition across grades. Moreover, the curriculum developers from the USA may use some beneficial features of the Indian NCERT textbooks as inspirations to incorporate more relevant and real-life examples, as well as examples that allow students more flexibility in using multiple methods while solving problems. Similarly, the curriculum developers from India may consider introducing equivalent fractions extensively in earlier grades, as seen in the US textbooks. Also, they may include some interesting features from the EM textbooks, such as using number-lines in worksheets or posters for teaching and learning fractions in their own textbook series, as well as include more open-ended problems in their textbooks.

At the same time, teachers from each country may explore the textbooks from the other country along with their own and incorporate any features that they feel would add value to their pedagogical processes. For example, for the US teachers, these additions may be in the form of incorporating mathematical examples with storylines that are contextual to their students, similar to the examples in the Indian textbooks. On the other hand, the Indian teachers may consider using number-lines or rulers to teach fractions, similar to the US textbooks.

In conclusion, the inferences from this study will contribute to the field and the sparse existing literature by providing a lens to understand the mathematics curriculum and learning processes in the USA and India. Additionally, this study will offer further insights into the contributions of each textbook and the learning opportunities it offers to its target student population. Each textbook series has unique features related to every criterion, but it can also benefit from the other textbook series by addressing its gaps while keeping in mind that the context of students is different in both countries. Thus, the findings of this study are relevant to the current discussions about textbook analysis.

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