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How Education Affects Mathematics Teachers' Knowledge: Unpacking Selected Aspects of Teacher Knowledge

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ABSTRACT
It is no surprise that all mathematics teacher education programs attempt to increase future teachers' knowledge, since teachers' knowledge has an effect not only on their teaching but also on their students' achievements. However, measuring the relationship between teachers' knowledge and their education is overly demanding. In this study we unpacked a selection of aspects of Mathematical Knowledge for Teaching (MKT) and created a 72-item survey that would measure teacher educators' and graduated teachers' perceptions of what graduated teachers have learned well or less well during teacher education. The data were collected under the auspices of the University of Eastern Finland. The results show that the teacher educators (N=18) and graduated teachers (N=101) who participated have developed rather similar perceptions of what graduated mathematics teachers learn well or poorly during their teacher education. According to the results, Subject matter knowledge and Pedagogical content knowledge receive similar emphases in the teacher education program, but certain fields of teacher knowledge, such as Common content knowledge, receive a greater emphasis than others. The approach described here provides a simple way of investigating this demanding phenomenon, and hence the limitations and the possibilities of the study will be discussed in detail.
Keywords: mathematical knowledge for teaching, MKT, teacher education, evaluation

INTRODUCTION

Evaluations are usually important for teacher education programs because they can reveal information about the development needs of mathematics teacher education (e.g. König, Blömeke, Paine, Schmidt, & Hsieh, 2011; Qian & Youngs, 2016). The contents of mathematics teacher education are related in various ways to future teachers' knowledge, and future

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State of the literature

- The mathematical and pedagogical knowledge gained by teachers during their teacher education provides a key explanation for the variation in students' test scores in mathematics internationally (e.g., when TEDS-M & TIMSS are compared). The explanation may be regarded as reasonable since the contents of mathematics teacher education are related to future teachers' knowledge, while future teachers' knowledge is related to their students' achievements.
- Developing the contents of mathematics teacher education may provide a great opportunity for increasing teachers' knowledge. However, at the present moment, new approaches are required for the investigation of how the education itself impacts on the teachers' knowledge.

Contribution of this paper to the literature

- Mathematical Knowledge for Teaching (MKT) divides teacher knowledge into six components, three of which are mathematical, while three components represent Pedagogical content knowledge. This study unpacks the six fields of teacher knowledge and investigates how teacher knowledge is emphasized in one mathematics teacher education program.
- According to the perceptions expressed by teacher educators and graduated teachers, Common content knowledge receives greater emphasis than the other five fields of teacher knowledge. How the different fields of teacher knowledge are emphasized in teacher education is individual but fundamental information related to developing the curriculum for future teachers.
- This study contributes by applying MKT in an innovative but also generic way in the gathering of information about mathematics teacher education.

teachers' knowledge is likewise related to students' achievement in mathematics (Hill, Rowan, & Ball, 2005; Monk, 1994; Schmidt, Cogan, & Houang, 2011; Schmidt, Houang, & Cogan, 2011). Hence, the contents or learning opportunities that a mathematics teacher education program can offer to student teachers are important topics in evaluations of the effectiveness of mathematics teacher education (Hsieh et al., 2011; Qian & Youngs, 2016).

A survey can be a useful tool for assessing teacher education (Darling-Hammond, Chung, & Frelow, 2002; Schmidt, Houang, & Cogan, 2011). Surveys may, for example, capture teacher educators' and student teachers' perceptions of the kind of learning opportunities that teacher education has offered, which can be an important indicator for the contents of mathematics teacher education (Schmidt, Cogan, & Houang, 2011; Schmidt, Houang, & Cogan, 2011). The results of surveys investigating graduated teachers' perceptions of what they think that they learned during their own teacher education can be an important indicator of teachers' general preparedness and self-efficacy, i.e., their belief in their own ability to succeed in specific situations or accomplish a task. In addition, the results may help in the development of the contents of teacher education (Darling-Hammond, 2006). Thus, it can be claimed that teacher educators' and graduated teachers' perceptions of the contents or learning can be useful indicators of the development needs in the contents of teacher education in general (e.g. Darling-Hammond, 2006; Schmidt, Houang, & Cogan, 2011).

In light of this claim, to assess one of the teacher education programs used in Finland we created a 72-item survey that would measure teacher educators' and graduated teachers' views concerning what graduated teachers learn during their teacher education. The survey questions were directed at capturing respondents' personal perceptions of graduated teachers' learning, an approach that differs from measuring actual learning. There are some limitations inherent in self-reporting data, such as graduated teachers' perceptions of their own learning may not reflect their actual learning. However, as Darling-Hammond's (2006) study shows, teachers' perceptions of their own learning can reflect their sense of self-efficacy and preparedness, both of which are related to teaching (Darling-Hammond, Chung, & Frelow, 2002; Hill et al., 2005). Hence it can be claimed that preparedness and self-efficacy are also important factors in teachers' professional competence (Darling-Hammond, 2006).

In order to analyze in greater detail teacher educators' and graduated teachers' perceptions of graduated teachers' learning, the survey was developed on the basis of a well-known teacher knowledge framework known as *Mathematical Knowledge for Teaching (MKT*). In this case, "greater detail" refers to the investigation of teacher knowledge within the framework of six subcategories, as developed in the MKT. Teacher educators' and graduated teachers' viewpoints together provide an insight into the nature and extent of the Subject matter knowledge and Pedagogical content knowledge that graduated teachers have gained during their teacher education. The first three research questions were the following:

1. On the basis of their course contents, what mathematical knowledge for teaching do teacher educators believe that their student teachers learn?
2. After completing all of the courses in their mathematics teacher education, what mathematical knowledge for teaching do graduated teachers believe that they have learned?
3. From previous appointed viewpoints, in which ways do teacher educators' and graduated teachers' perceptions converge?

The responses to the first two questions define the nature of the Subject matter knowledge and Pedagogical content knowledge that are emphasized to a greater or lesser extent in the teacher education program under evaluation. The third research question attempts to locate similarities between teacher educators' and graduated teachers' views. When, for example, the teacher educators consider that the graduated teachers have learned something well or less well, it cannot necessarily be claimed that the graduated teachers also think that they have learned about the same topic in the same way.

Curricula in mathematics teacher education seem to have an impact on teachers' knowledge (Schmidt, Houang, & Cogan, 2011; Qian & Youngs, 2016). In the present study, the *mathematics majors* have taken twice as many mathematics courses as have *mathematics minors*. Since the number and also the content of the mathematics courses will have differed, this may well have an impact on graduated teachers' views concerning what they have learned well or less well. Hence, the responses to the fourth research question will indicate whether different curricula in fact affect graduated teacher perceptions of their own learning.

4. How do mathematics majors and minors consider their learning of different topics in the course of their education?

We would claim that the results of this kind of evaluation can be implemented in at least two ways to improve the teacher education program that has been studied. Firstly, knowing how graduated teachers have learned about different fields of teacher knowledge provides an opportunity for us to improve weak parts and to maintain the strong parts of teacher education. Secondly, recognition of the potential differences between mathematics majors' and minors' perceptions of their learning will provide us with an opportunity to develop our future curricula in teacher education.

INTERNATIONAL ASSESSMENT OF MATHEMATICS TEACHER EDUCATION

Teacher Education and Development Study in Mathematics (TEDS-M) was the first international empirical study using large samples to explore the differences between teacher education programs in seventeen countries (Bankov et al., 2013; Brese & Tatto, 2012; Ingvarson et al., 2013; Tatto et al., 2008). Approximately 5 000 teacher educators and 22 000 future teachers from 750 programs in some 500 teacher education institutions in 17 countries[1] participated in the TEDS-M study. Beliefs and views held by teacher educators' and future teachers' were explored to form a coherent picture of the characteristics of teacher education programs with respect to policy, schooling, and social contexts at the national level. Perhaps the most interesting part of the TEDS-M study was the exploration of the differences between future teachers' Mathematics content knowledge and Mathematics pedagogical content knowledge in TEDS-M countries, since in the meantime a lot of research has been published explaining these results (e.g. Blömeke, Hsieh et al., 2011; König et al., 2011; Suhl & Kaiser, 2011; Schmidt, Houang, & Cogan, 2011; Qian & Youngs, 2016).

Based on TEDS-M, a variety of similarities and differences can be found in all teacher education programs. The teacher education programs included three types of courses connected to 1) mathematical knowledge, 2) pedagogical knowledge related to the teaching of mathematics, and 3) general pedagogical knowledge more generally related to instructional practices and schooling (Schmidt, Cogan, & Houang, 2011). However, according to findings published by Schmidt, Cogan, and Houang (2011), these three areas are emphasized differently in the different TEDS-M countries. According to the TEDS-M reports (Ingvarson et al., 2013; Tatto, et al., 2012), a period of teaching practice was included in most programs, but fewer included field experience of learning about issues involving school

[1] The countries participating in the TEDS-M study included Botswana, Canada, Chile, Chinese Taipei, Georgia, Germany, Malaysia, Norway, Oman, the Philippines, Poland, Russia, Singapore, Spain, Switzerland, Thailand, and the USA.

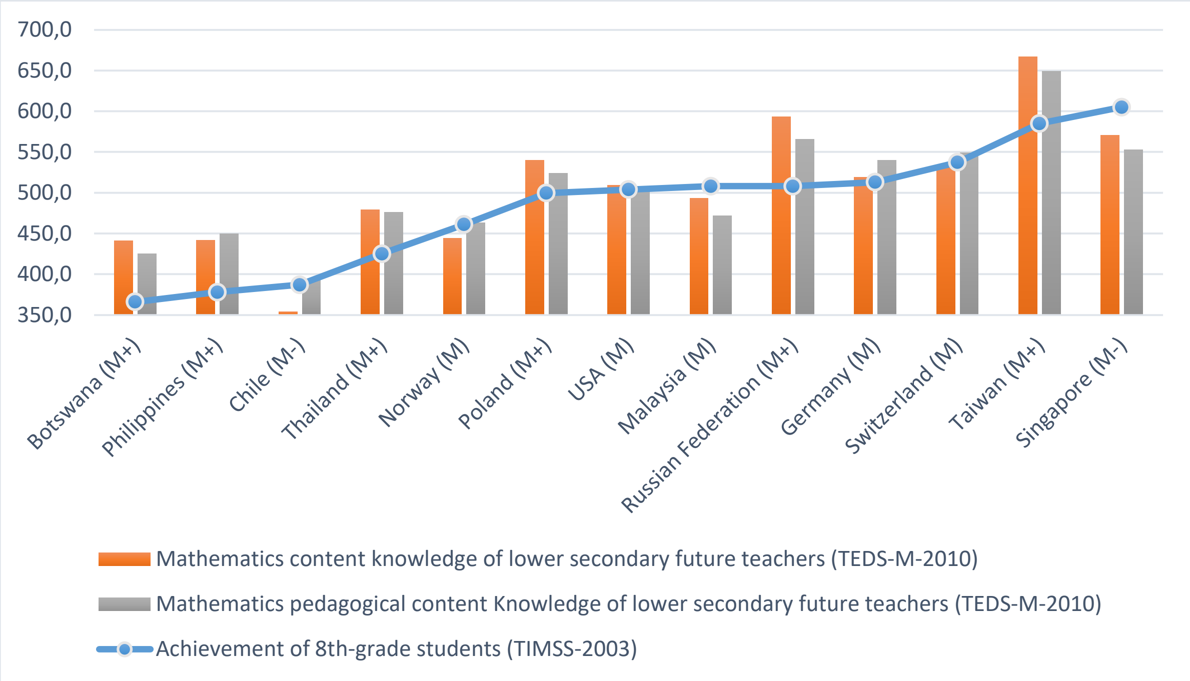

Figure 1. Future teachers' Mathematics content knowledge and Mathematics pedagogical content knowledge (TEDS-M-2010) and students' achievement (TIMSS-2003)

organization and management. In general, programs that focused on preparing teachers to become higher level teachers (lower and upper secondary level) provided opportunities for learning contents in greater depth than teachers preparing for the primary level.

The results of the TEDS-M suggest that future teachers' knowledge and the content of teacher education are related. Teacher education programs will offer different opportunities for learning that refers to the design of curricula. Qian and Youngs (2016) found that when future teachers acquire mathematical and pedagogical knowledge there is also a relationship between these opportunities to be learned. The content of taken mathematics courses has a more powerful effect on future teachers' Mathematics content knowledge than the number of the courses mastered (Qian & Youngs, 2016). The curriculum in teacher education seems to have an effect on future mathematical and pedagogical knowledge, yet there were differences between the various TEDS-M countries[2], which suggests that for some countries the development of curricula might have even more effect.

The combination of the TIMSS and TEDS-M results reveals that teachers' knowledge and students' achievement are linked. The results describing 8th grade achievements in TIMSS-2003 and the Teacher knowledge scores in TEDS-M-2010 were transformed onto the

[2] Statistically significant differences were not the same for all countries.

same scale[3] (Schmidt, Houang, & Cogan, 2011). A strong correlation (R^2=0.70, P<.0004) between student achievement and teacher knowledge was found, suggesting that when future teachers' Mathematics content knowledge was high, the country was more likely to succeed in the TIMSS. The future teachers' Mathematics content knowledge and Mathematics pedagogical content knowledge and the students' achievement rating are shown in **Figure 1**. **Figure 1** has been constructed from data provided by Schmidt, Houang, and Cogan (2011), to which we added the TEDS-M scores related to Mathematics pedagogical content knowledge (Center for Research in Mathematics and Science Education, 2010).

The ratio of the courses related to Mathematics, Mathematics pedagogy, and General pedagogy seem to play a highly significant role in teacher education. Schmidt, Houang, and Cogan (2011) discovered that when future teachers' knowledge as measured by their TEDS-M scores was either *higher* or *lower* than, or alternatively *in line* with, students' achievement in TIMSS, then the detectable differences were in the ratio of the courses in three areas[4]. Those countries who succeed better in the TEDS-M than in the TIMSS (shown as marked "M+" in **Figure 1**), emphasized especially Mathematics in their curricula (**49%**/31%/21%)[5]. The countries that succeeded better in the TIMSS than the TEDS-M (shown as "M-" in **Figure 1**) emphasized more General pedagogy but less Mathematics (37%/36%/**28%**). The countries with most success in both the TEDS-M and the TIMSS (shown as "M" in **Figure 1**) emphasize was between others (42%/33%/26%). How these three areas are stressed in teacher education seems to bear a relationship to teacher knowledge and student achievement. The results indicate that finding the balance of course-work in these three areas may be the key issue in any attempt to improve existing teacher education programs.

König et al. (2011) introduced two alternative models for interpreting the results of the TEDS-M. In their study they compared future teachers' General pedagogical knowledge scores in the TEDS-M in the contexts of Germany, Taiwan, and the USA. The USA had lower scores in Mathematics pedagogical content knowledge than Germany and Taiwan (see **Figure 1**), and the same was true for future teachers' scores in General pedagogical knowledge. König et al. (2011) found that the USA, when compared with Taiwan and Germany, had weaker scores in items that were measuring *recalling* and *understanding*, but better scores in items measuring *generating*. The second model shows that, when compared with Taiwan and Germany, the USA had similar scores for items of *assessment*, weaker scores for items of *adaptivity*, but better scores for items of *structure* and *management*. These sub-areas, whose General pedagogical knowledge was constructed in the TEDS-M, describe in

[3] These results can be compared since the theoretical frameworks of the TEDS-M and TIMSS were partly similar.
[4] Three areas are connected with 1) mathematical knowledge, 2) pedagogical knowledge related to the teaching of mathematics, and 3) general pedagogical knowledge related to instructional practices and schooling more generally.
[5] Numbers referred to the proportions of Mathematics, Mathematics pedagogy, and General pedagogy.

greater details the areas in which the USA would probably benefit from development. In other words, **Figure 1** is unable to fully describe the required development, but if the Mathematics content knowledge and Mathematics pedagogical content knowledge are split up into their more detailed components, we may nevertheless discover ways in which these respective teacher education programs can be improved. The division of the results into smaller components may also be necessary before the results of the TEDS-M can be employed in the improvement of teacher education programs. This also suggest that this kind of division might be a key factor in other development programs to find the parts need to be improved.

MATHEMATICAL KNOWLEDGE FOR TEACHING

Many models of teacher knowledge (e.g., Ball, Thames, & Phelps, 2008; Ernest, 1989; Fennema & Franke, 1992; O'Meara, 2010; Rowland, Turner, Thwaites, & Huckstep, 2009) reveal the kind of knowledge needed for teaching mathematics, while some of them also suggest ways in which Subject matter knowledge and Pedagogical content knowledge can be divided. *Mathematical Knowledge for Teaching (MKT)* divides teacher knowledge into six components, where three components constitute Subject matter knowledge and three components represent Pedagogical content knowledge. Although the origins of MKT are American, in more recent times the MKT methods of measurement have also been used beyond the American context, for instance in Ireland (Delaney, Ball, Hill, Schilling, & Zopf, 2008), South Korea (Kwon, Thames, & Pang, 2012), Ghana (Cole, 2012), Indonesia (Ng, 2012), Norway (Fauskanger, Jakobsen, Mosvold, & Bjuland, 2012), Iceland (Jóhannsdóttir & Gísladóttir, 2014), and Malawi (Kazima, Jakobsen, & Kasoka, 2016). Interest has also been growing in applying this framework within the Nordic context (Fauskanger et al., 2012; Jankvist, Mosvold, Fauskanger, & Jakobsen, 2015; Jóhannsdóttir & Gísladóttir, 2014; Mosvold, Bjuland, Fauskanger, & Jakobsen, 2011; Mosvold & Fauskanger, 2013).

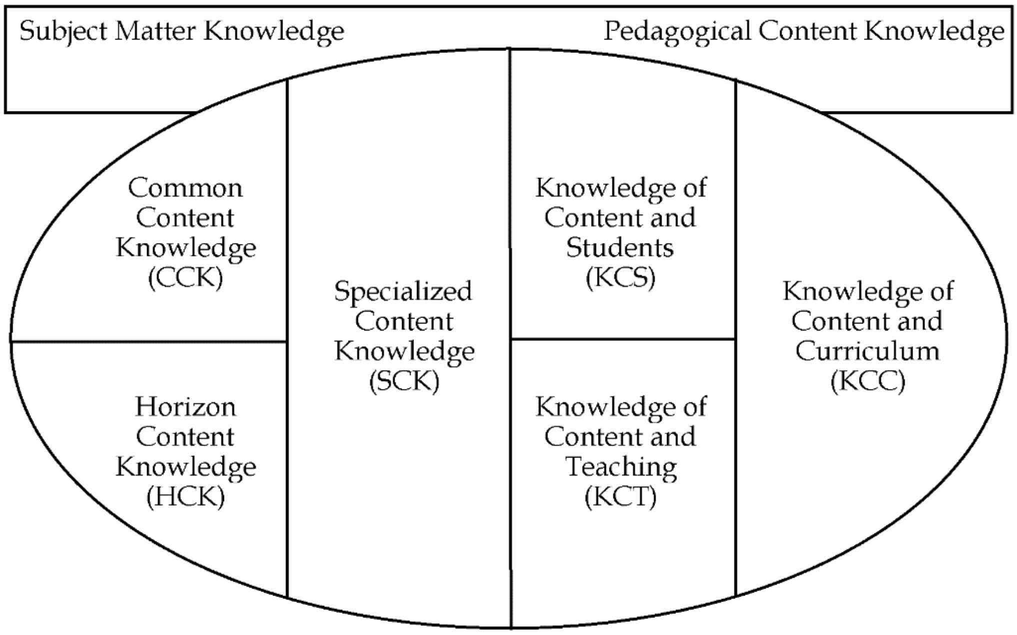

Figure 2. Domains of Mathematical Knowledge for Teaching (MKT) by Ball et al. (2008)

Within this framework, *Subject matter knowledge* and *Pedagogical content knowledge* are divided into three more detailed domains, see **Figure 2**. Subject matter knowledge does not require Pedagogical content knowledge, whereas Pedagogical content knowledge requires Subject matter knowledge (Ball et al., 2008). There are at least two ways to read this. Firstly, Marton and Booth (1997) consider that mathematics is simply mathematics, but we cannot simply teach it because we always teach something. Secondly, if we think about how to organize coursework for future teachers in teacher education, teachers first need at least some understanding of mathematics before they can fully understand Pedagogical content knowledge.

Subject matter knowledge

In many mathematics teacher education programs, student teachers have mathematics courses that are intended to serve both future teachers and also mathematicians. The contents of these courses are usually “pure mathematics”, i.e., concerning knowledge of concepts, results, and proofs in different areas of mathematics. In terms of MKT, these aspects of Subject matter knowledge are referred to as *Common content knowledge (CCK)* for teachers. Hence, CCK can be seen as mathematical knowledge that is not unique to teaching and hence it is also useful in other professions (Ball et al., 2008; Hill, Ball, & Schilling, 2008).

According to Ball et al. (2008), teachers need mathematical knowledge that is unique to teaching, for example, evaluating tasks, designing mathematical problems, and marking exams. Hence, this knowledge has been termed *Specialized content knowledge (SCK)* for teachers. Hill et al. (2008, pp. 377-378) describe SCK as a competence that *“allows teachers to* *engage in particular teaching tasks, including how to accurately represent mathematical ideas, provide mathematical explanations for common rules and procedures, and examine and understand unusual solution methods to problems."* In mathematics teacher education, this could mean that student teachers are prepared, for instance, to formulate, mark, and grade exams. Marking exams is, for example, a specific teaching task but it does not require Pedagogical content knowledge and hence marking exams requires Specialized content knowledge.

Mathematics teachers need to master the structure of mathematics in terms of how concepts are related, how concepts form different topics, and how the structure of mathematics is constructed in relation to the topics. In addition, teachers need to know how concepts can be presented to students at different levels at school. For example, the concept of *function* might be presented to elementary students in the form of an idea. From one school level to another, definitions then become more exact and develop toward more formal versions. This knowledge of mathematical structures and awareness of how mathematical topics are related to each other is known as *Horizon content knowledge (HCK*) for teachers (Ball & Bass, 2009; Ball et al., 2008).

Pedagogical content knowledge

If we understand how people learn, we can understand how to teach better. The idea behind Pedagogical content knowledge in MKT is similar, because knowledge of teaching and of learning are separated. *Knowledge of content and students (KCS)* is more like knowledge of how students learn mathematics: A teacher must be able to anticipate students' difficulties and misconceptions, to hear and respond to students' thinking, and to choose suitable examples while teaching (Ball et al., 2008). Furthermore, a teacher might be better able to motivate and inspire students if s/he knows what students are interested in. Hill et al. (2008) consider that KCS is a primary element in Shulman's (1986) PCK, because one part of Shulman's PCK (1986, p. 9) is "*an understanding of what makes the learning of specific topics easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons*". Thus, KCS is an amalgam of knowledge of students, learning, and mathematics.

According to Ball et al. (2008) teachers choose effective strategies, arrange a classroom, and make decisions when they plan their teaching, which all require *Knowledge of content and teaching (KCT)*. Furthermore, if students are asking for the right answer to a given problem, a teacher can hide the answer and try to provide an opportunity for them to discover the answer for themselves. Decisions of this kind require pedagogical thinking: which answers can reasonably be given to students directly and which should reasonably be left hidden for later discovery. Sometimes there are opportunities in a classroom that allow students to make mathematical discoveries by themselves, and thus a teacher needs to decide if he/she is going to depart from the original plan or not. All of these aspects are connected to KCT. In general, KCT is required in both planning and teaching, and it is definitely something that teacher education should prepare students for.

The content of curricula and the requirements of teachers' work are mostly interconnected and hence knowing these aspects is important for teachers. Knowing the content of curricula is referred to as part of *Knowledge of Contents and Curriculum (KCC)* for teachers. A wider perception of KCC also includes knowledge of teaching materials (such as textbooks, other materials, etc.), teaching instruments (blackboards, overhead projectors, etc.), and technology (computers, smart boards, calculators, software, etc.). The use of technology in teaching requires, for instance, amalgam knowledge of mathematics, pedagogy, and technology (Koehler & Mishra, 2009). These aspects of KCC are also something for which teacher education is trying to prepare future teachers.

METHOD

Context

The University of Eastern Finland offers two programs for students of mathematics: one for mathematicians and another for teachers. Both programs are almost identical with respect to mathematics courses, but they differ in terms of minor subjects. In the following, we describe the Mathematics teacher education program.

The Mathematics teacher education program includes the Bachelor's degree (180 cp[6]) and Master's degree (120 cp). Both degrees are required in Finland for the qualification of a mathematics teacher. The program includes mathematical studies (120 cp), pedagogical studies (60 cp), and studies in one or two minor subjects (60 cp each). Most mathematical studies are traditional mathematics courses relevant to both teachers and mathematicians (e.g., calculus, analysis, algebra, differential equations). The pedagogical studies include theoretical studies of teaching and learning (30 cp), the didactics of mathematics (10 cp), and teaching practice (20 cp). Student teachers can study any school subjects as a minor subject, but their typical choices are physics or chemistry, or both. In its entirety, the program provides a qualification to teach mathematics and minor subjects in lower or upper secondary schools and vocational schools.

It is notable that teachers who have studied mathematics as major or minor are regarded as possessing the same competence to teach mathematics at secondary school level, even though their studies are to some extent different. The major difference is in the number of mathematical courses. The number of courses is doubled for future teachers who have studied mathematics as a major (*mathematics majors*) in comparison to future teachers who have studied mathematics as a minor (*mathematics minors*) since mathematics majors take advanced-level mathematical studies (60 cp) in addition to their basic and intermediate mathematical studies (60 cp).

[6] One credit point (cp) is equivalent to 25 hours of study. The recommendation is to take 60 cp per year.

Instrument

Based on the literature, we produced 72 statements about knowledge related to teaching mathematics (e.g., Ball, 2003; Ball et al., 2008; Ball & Forzani, 2009; Ball & Hill, 2008; Hill et al., 2005; Hill, Schilling, & Ball, 2004; Sleep, 2009). However, we faced some challenges in unpacking the conceptualization of MKT theory in practice. For instance, *answering students' Why?-questions* is listed as one type of the mathematical tasks confronted in teaching, which is explicitly classified as Specialized content knowledge (Ball et al., 2008, p. 400). However, we would also suggest that the nature of the knowledge that teachers require in such situations depends on the kind of Why?-questions that students ask. Some of the students' questions are not related to mathematics at all. If a student asks, for example, "Why is this theorem true?" a teacher will probably answer by using Common content knowledge. If the teacher's answer does not satisfy the student, s/he will ask supplemental Why?-questions that will enable them to "*find examples to make a specific mathematical point*", or to "*link representations to underlying ideas and to other representations*" (Ball et al., 2008, p. 400). In that case, the answer will require Specialized content knowledge as presented by Ball et al. (2008). In addition, an understanding of why some topics are central to a discipline and others are not (Ball et al., 2008, p. 391) or an understanding of the structure of mathematics requires an explanation of why some mathematical topics are important for gaining the total picture. Hence, in our opinion, Horizon content knowledge or Knowledge of content and curriculum may also be involved in the search for answers to students' Why?- questions. At present, the domain definitions of MKT are not unambiguous, and hence more attention should be paid to clarifying them and make them more accessible. Relevant examples of this knowledge, its purpose, and its meanings are needed in order to close the gap in the current literature.

In consequence, we put our effort into unpacking the MKT domains as statements about the knowledge required for teaching mathematics. The statements were designed so that half were connected with Subject matter knowledge and half with Pedagogical content knowledge in terms of MKT. The themes and categorization were not presented in the survey, and the placement of items was mixed. It will be pertinent at this stage to describe how these 72 items involving the themes in various ways are related to the kinds of mathematical knowledge that may be needed for teaching mathematics. The numbers in brackets refer to the number of items concerned with a current theme.

Table 1. Themes and items concerned with Common content knowledge

Themes	Content of items
Calculus	Differential calculus
	Integral calculus
Functions	Functions and equations
	Polynomial functions
	Root and logarithmic functions
	Trigonometric functions
Geometry	Analytic geometry
	Geometry
Data and probability	Probability theory
	Statistics
Numbers and vectors	Number sequences
	Vector calculus
Mathematical concepts and notations	Exact use of mathematical notations
	Mathematical concepts
Studying skills	Solving mathematical exercises and problems
	Reading and understanding university-level course materials used in mathematical studies
Mathematical methods	Mathematical calculation methods
	Mathematical reasoning rules (e.g., logic)
	Use of suitable mathematical methods
	Use of figures, diagrams, and models to promote own mathematical thinking

Common content knowledge (see **Table 1***).* A teacher requires, more than anything else, a knowledge of a range of different mathematical topics. In Finland, for example, the mathematical topics are prescribed in the national curriculum for each grade (e.g., The Finnish National Board of Education, 2003). Twelve statements outline the mathematical topics, which cover roughly all of the necessary topics at secondary school level in the country (The Finnish National Board of Education, 2003). The topics are concerned with calculus (2), functions (4), geometry (2), data and probability (2), and numbers and vectors (2). Because Common content knowledge is actually more extensive than a knowledge of the prescribed topics, some of the items are also connected with mathematical concepts, notations (2), and methods (4). Two of the statements are also connected with study skills, since both student teachers and teachers need to possess the requisite skills for solving mathematical problems and also the ability to read mathematical texts (2).

Table 2. Themes and items concerned with Specialized content knowledge

Themes	Content of items
Presenting mathematics	Mathematical representations (verbal, picture, symbolic)
	Use of multiple representations (verbal, picture, symbolic) of the same mathematical entity
	Use of visualizations in mathematics
Formulation and marking assignments	Formulation of mathematical exams
	Formulation of mathematical excercises
	Marking exam responses
	Recognition of straightforward and more problematic mathematical exercises
Influence of mathematics	Mathematical applications in everyday life, science, and technology
	Impact of mathematical development on culture and society
History and philosophy of mathematics	History of mathematics
	Philosophy of mathematics

Specialized content knowledge (see **Table 2***).* In the classroom, teachers present mathematics in a variety of ways, and this process requires a knowledge of mathematical representations and strategies useful in visualizing mathematics (3). Teachers also need to have skills that enable them to formulate new assignments and exams, and to evaluate and grade their students' output (3). Sometimes a teacher in the classroom will attempt to justify the learning of mathematics, and hence it may well be advantageous for the teacher if he or she knows something about the history and philosophy of mathematics (2) or how mathematics has influenced everyday life, culture, and society (2).

Horizon content knowledge (see **Table 3***).* In very general terms, mathematics can be said to be concerned with concepts and with the relationships between them. In consequence, it might be said that mathematics is ultimately a single accurate construction made up of mathematical concepts. In this light it is clear that a teacher needs to possess knowledge of mathematical structures (3). However, experts and novices may well see the structure of mathematics differently, and therefore teachers need to know how the structure of mathematics can be seen differently before teachers can intervene to support the

Table 3. Themes and items concerned with Horizon content knowledge

Themes	Content of items
Mathematical structures	Hierarchy of mathematical concepts (e.g. axioms, definitions, lemmas, propositions)
	Fields of mathematics in general
	Structure of concepts in different mathematical fields (e.g. how mathematical concept of *limit* is related to other concepts)
Learning of mathematical structures	Teaching the structure of mathematics
	Recognition of previous and forthcoming mathematical concepts in teaching mathematics
	Students' actual mathematical know-how at different school levels (e.g., typical issues in the need for revision with students)

Table 4. Themes and Items concerned with Knowledge of content and teaching

Themes	Content of items
Planning of teaching	Planning and defining the learning objectives of individual lessons
	Planning and defining the learning objectives of a course or unit of study
Applying learning theories to teaching	Constructivism and its application to one's own teaching
	Learning theories and their application in the teaching
	Research results of unqualified mathematics and their application in their teaching
Explaining aims of teaching	Underlining the most important aspects of the topic to be learned
	Explaining to students the aims and meanings of the topic under study
	Underlining the significant aspects of mathematical content
Producing learning experiences	Generation of positive experiences in the process of learning mathematics
	Teaching self-evaluation skills to students
Discussion of mathematics	Presenting specific questions to promote learning
	Talking about mathematics
	Answering students why-questions in mathematics
Use of examples	Use of everyday life examples in teaching mathematics
	Formulation of relevant examples for teaching specific topics

understanding of mathematical structures (3).

Knowledge of content and teaching (see **Table 4***)*. A teacher needs knowledge and skills concerned with the planning of his/her teaching (2). To become effective in his/her teaching, a teacher should probably take into account how students generally learn mathematics. Thus, teachers' professional skills may also develop if they apply their knowledge of learning theories to their own teaching (2). In the classroom teachers need access to various skills connected, for example, with discussing mathematics (3) and using examples (2). To achieve the aims of their teaching, teachers may also need to underline the most significant issues and explain the aims of the teaching (3). In general, an effective teacher will endeavor to generate a positive experience of the general process of learning mathematics (2).

Knowledge of Contents and Students (see **Table 5***)*. Teachers need to evaluate students' behavior, and their thinking and speaking at the level of the individual (5). Teachers may be able to identify students' misconceptions of mathematics and also prevent them, if they possess some knowledge of the challenges posed by the learning of mathematics (4). Teachers need to know about supporting and motivating students in their learning (2), because ultimately the teachers are attempting to improve students' mathematical competence (3).

Knowledge of content and curriculum (see **Table 6***)*. In most countries, curricula are regarded as defining specific learning objectives, and hence curricular knowledge will also provide relevant knowledge for a teacher (2). Technology, textbooks, and other equipment will be useful tools for both learning and teaching. Teachers will require this variety of knowledge when selecting and using textbooks in their teaching of mathematics (2). The

Table 5. Themes and Items concerned with Knowledge of content and students

Themes	Content of items
Improving students' competence	Developing students' problem-solving skills
	Developing students' mathematical thinking
Evaluating students' competence	Evaluation of students' competence
	Students' problem-solving and mathematical reasoning skills demonstrating their deep understanding
	Evaluation of the accuracy and coherence of students' conclusions
	Recognition of students' meaningful talk in their learning of mathematics
	Assessment of students' mathematical knowledge, skills, and ability for their future studies
Challenges of learning	Recognition of students' errors and taking them into account in teaching
	Prevention of students' misconceptions
	Recognition of students' misconceptions in mathematics (e.g. multiplication increases and division decreases results)
	Recognition of learning difficulties in mathematics
Motivation and support skills of learning	Motivating students to learn, understand, and know mathematics
	Supporting mixed-level students in learning mathematics
	Supporting students' self-confidence in their mathematical skills

same will be true of knowledge and skills concerned with the use of technology in teaching mathematics (2).

Table 6. Themes and Items concerned with Knowledge of content and curriculum

Themes	Content of items
Use of textbooks	Use of textbooks in teaching mathematics
	Choosing a suitable textbook or textbook series suited to one's own teaching style
Curricular knowledge	Objectives and contents of the national mathematical curriculum
	Evaluating one's own teaching with respect to the national curriculum
Use of technology	Use of technology in teaching mathematics
	Using technology in one's own teaching of mathematics

Two electronic surveys were produced. The survey given to the graduated teachers was intended for mathematics teachers who had already graduated from the Mathematics teacher education program. The question presented was in the form "What rating would you give to the knowledge / skills[7] that you learned from your teacher education program?" The teachers evaluated the 72 presented knowledge or skills items on a five-point Likert scale (1 = not at all, 2 = poor, 3 = fair, 4 = good, 5 = excellent). The respondents were asked to avoid evaluating any knowledge and skills that they may have gathered since graduation or in the course of their teaching.

The survey for teacher educators was aimed at all educators currently working in the Mathematics teacher education program. The question was in the form "Based on your course contents, what rating would you give to the knowledge / skills that your students have learned?" The educators were asked to evaluate the 72 knowledge or skill issues on the same five-point Likert scale as that used with the graduated teachers (1 = not at all, 2 = poor, 3 = fair, 4 = good, 5 = excellent). The respondents were also asked to take into account all of the courses that they taught.

According to Ihantola and Kihn (2011) pretesting, clear standard instructions, avoiding abstract concepts, the order of questions, and the length of survey should be considered in the design of survey. Therefore, first four independent researchers improved wordings for instructions and seventy-two statements. The order of questions and the length of survey were carefully considered before the survey was piloted. Two persons from the study group and two persons out of the study group piloted the survey. Because the persons out of study group are usually better in evaluating the intelligibility of wordings, and the abstractness of used concepts, they were used as well. All the eight participants gave their opinion about the intelligibility of instructions and statements, and proposed suggestions for developing the survey during pretesting and piloting. Based on pre-testing, only few words were replaced because of their manifold meanings.

[7] Because of the Finnish language, there was need to distinguish between the aspects of knowledge and skills issues, since the term knowledge usually refers to "information" in the Finnish language. Hence, half of the statements referred to knowledge and half to skills. The order of the items was selected so that the respondents would first evaluate teacher knowledge issues and then issues concerned with teacher skills.

Sample

The data were collected in the course of 2012–2013. The survey intended for the graduated teachers was sent to all 187 mathematics teachers who had graduated from the University of Eastern Finland in the period 2002–2012, and our sample (N=101) eventually included 54% of them. Seventy-two per cent (73) of the respondents in the sample had taken mathematics as their major and 28% (28) as their minor. Apart from one respondent, all of them had experience of teaching mathematics at school level or they were currently working as teachers. The survey intended for the teacher educators was sent to all 24 educators in the field of mathematical studies, pedagogical studies, and teaching practice. Our sample (N=18) comprises 75% of all those who were working in the Mathematics teacher education program in 2012. Seventy-eight per cent (14) of the teacher educators were currently teaching mathematics (*Math-Educators*), while and 22% (4) of them were working in pedagogical studies or teaching practice (*PT-Educators*).

Analysis

The data were analyzed as follows. Four mean values for each statement were calculated. The first mean value was calculated from the survey data acquired from the graduated teachers. The mean stands for *the perceptions of all graduated teachers* of how they had personally learned each of the issues mentioned in the survey. In addition, three mean values were calculated for the survey data provided by the teacher educators. These were *the perceptions of all teacher educations, the perceptions of the Math-Educators,* and *the perceptions of the PT educators* of how their students had learned each of the issues. Because the actual learning aims were usually different for the mathematics and PT educators, their views are presented separately even though there were only a handful of educators involved in pedagogical studies and teaching practice.

The statistical difference between those teachers who had studied mathematics as their major or as their minor was explored. A difference was expected because the quantity of mathematical studies for the major-subject student teachers is double that taken by the minor-subject student teachers. The *Mann-Whitney U-Test* was used to explore the statistical differences between the groups (e.g., Sheskin, 2003).

Cronbach Alpha test is widely used for testing the consistency of an instrument. Higher Cronbach Alpha scores denote stronger correlation between tested items (Cronbach, 1951; Tavakol & Dennick, 2011). Since surveyed items were designed in the way that the statements were related to the different MKT domains, the consistency of statements were tested within these categories. The number of graduated teachers' survey responses was fivefold compared to the teacher educators' survey responses, therefore graduated teachers' responses were used for counting the Cronbach's Alpha scores. The Cronbach's Alpha score for the items dealing with Subject matter knowledge was .931, while for those dealing with Pedagogical content knowledge it was .950 (for further details, see **Table 7**). These values

Table 7. Cronbach Alpha scores for the teacher knowledge components in the survey

	Number of items	Cronbach's Alpha (α)
Subject matter knowledge	**37**	**.931**
▪ Common content knowledge	20	.938
▪ Specialized content knowledge	11	.736
▪ Horizon content knowledge	6	.781
Pedagogical content knowledge	**35**	**.950**
▪ Knowledge of content and teaching	15	.885
▪ Knowledge of content and students	14	.921
▪ Knowledge of content and curriculum	6	.668

predict that the items correlated highly within a single component and hence the instrument may be regarded as reliable. In addition, Alpha score .931 in subject matter area gives .13 error variance (random error) and Alpha score .95 in Pedagogical content knowledge gives .10 error variance (see Tavakol & Dennick, 2011).

The dependencies between the perceptions of the teacher educators and the graduated teachers were explored. Their correlation was calculated solely on the basis of the mean values. In another words, the correlation between two mean values was tested for all 72 items. First, a normal distribution of data was tested using *Komogorov-Smirnov* and *Shapiro-Wilk*. The correlation between the mean values for the teacher educators and graduated teachers surveys was then calculated. In the present case, since the data were normally distributed, use was made of Pearson's correlation (Hauke & Kossowski, 2011).

RESULTS

We now present in two parts the results of the survey for the graduated mathematics teachers and the teacher educators involved in the Mathematics teacher education program. Firstly, the results of how the graduated teachers and teacher educators see that graduated teachers have learned different issues during teacher education will be presented. To assist readers to acquire a more structured picture of the results the items detailed in the surveys were categorized by means of the framework supplied by Mathematical Knowledge for Teaching and also on the basis of their different themes. Secondly, because the number and content of mathematical courses were different for those teachers who had taken mathematics as either their major or minor, respectively, analytical attention will be paid to their statistical difference.

How have the graduated teachers learned different issues during their teacher education according to the perceptions of the teacher educators and the graduated teachers themselves?

Perceptions of Common content knowledge

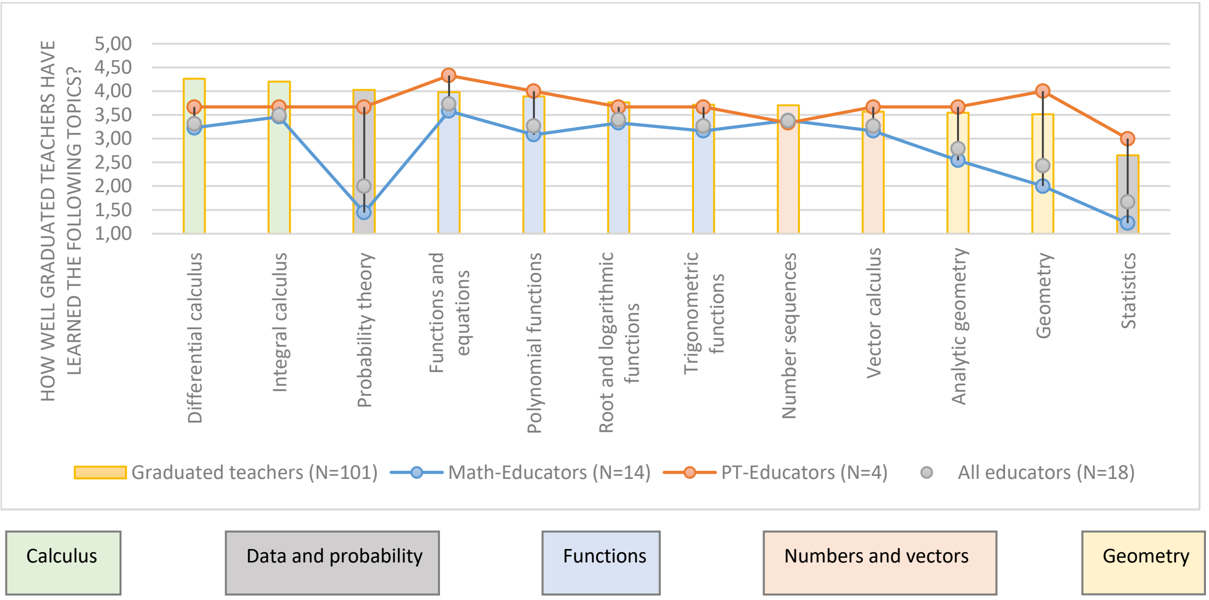

Figure 3. Respondents' perceptions of how the graduated teachers had learned the topics of mathematics (the first part of Common content knowledge, CCK). The sequence chosen shows the graduated teachers' perceptions ranked from the highest to the lowest scores (bars)

Teacher educators' and graduated teachers' responses to items of Common content knowledge is represented in two parts – mathematical topics (**Figure 3**) and a number of general issues, such as using representations, methods, and concepts (**Figure 4**). Apart from theme, data, and probability, the items surrounding themes are close together when the graduated teachers' perceptions opt for a sequence that ranges from the highest to the lowest scores (see **Figure 3**). The graduated teachers considered that they possessed a better knowledge of calculus than they did of numbers and vectors; at the same time, they thought that they possessed a better knowledge of functions than they did of geometry. In terms of theme, data, and probability two items remain remote from any others, suggesting that the graduated teachers considered that they had learned probability theory better than they had statistics.

The views held by the graduated teachers and PT-Educators about learning the topics of mathematics are generally quite similar. In contrast, Math-Educators' perceptions are similar to the graduated teachers' perceptions but differed noticeably for the themes of geometry and of data and probability.

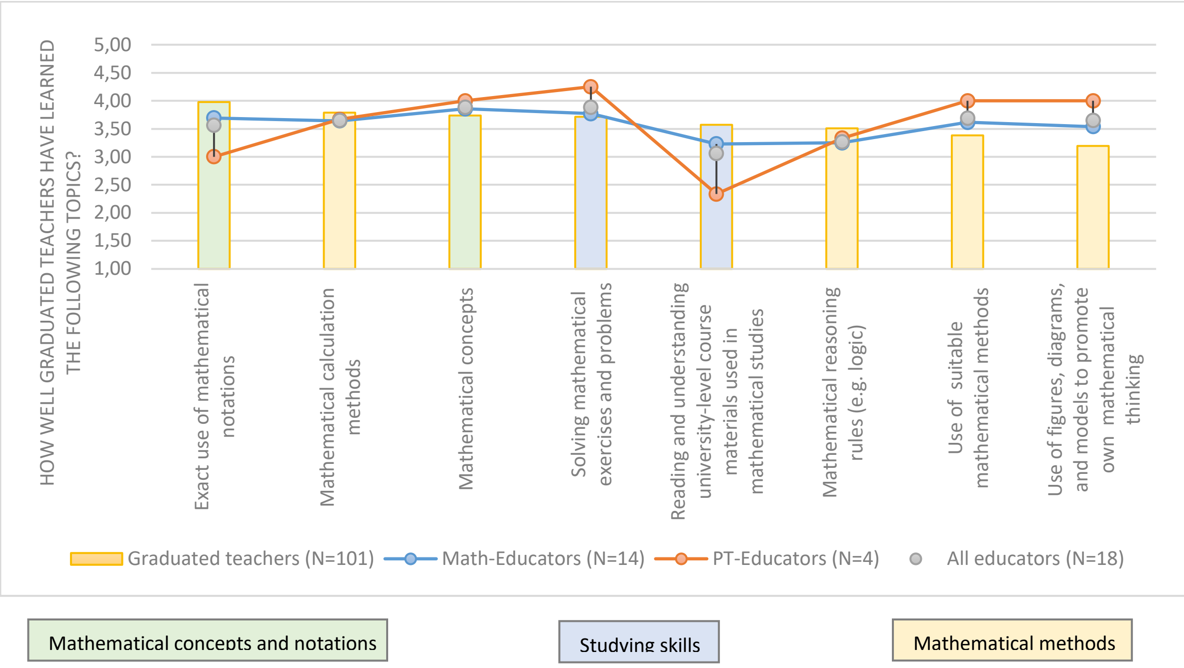

Figure 4. Respondents' perceptions of how graduated teachers have acquired their general mathematical knowledge (the second part of Common content knowledge, CCK). The sequence chosen shows the graduated teachers' perceptions ranked from the highest to the lowest scores (bars)

Figure 4 shows that according to the respondents' perceptions, the graduated teachers have acquired a better knowledge of mathematical concepts and notations than they have of mathematical methods. PT-Educators' and Math-Educators' views intersect in the items labeled Reading and understanding the university-level course materials of mathematical studies and Using mathematical notations correctly. Study skills involving Reading and understanding course materials should be regarded as a prerequisite for learning mathematics at university, but teaching practice may not be the best time for learning this skill. Thus, it seems to be reasonable that the Math- and PT-Educators rate this item more highly than do the PT-Educators. Recognizing the accurate use of mathematical notations may well be easier for PT-Educators, since Math-Educators generally deliver lectures, whereas PT-Educators work more closely with student teachers.

Perceptions of Specialized content knowledge

Items on the themes almost overlap when the chosen sequence concerns the graduated teachers' perceptions ranked from the highest to the lowest scores (Figure 5). The graduated teachers considered that they had a better knowledge concerned with presenting mathematics than with how mathematics has influenced everyday life, culture, and society. In addition, they also suggested that they had a better knowledge of formulating and marking assignments than they did of the history and philosophy of mathematics.

The PT-Educators' opinions concerning the graduated teachers' learning in items related to SCK were generally more positive than those expressed by the Math-Educators or graduated teachers. In addition, the perceptions of all of the teacher educators were similar to those of all of the graduated teachers.

The graduated teachers considered that their learning of the items History of mathematics, Philosophy of mathematics, and Impact of mathematical development for culture and society had been modest during teacher education. The themes of these items are connected with general mathematical knowledge, which can be useful for teachers, but they may not be a prerequisite for actually teaching mathematics. Within the theme of Formulation and marking assignments, the graduated teachers considered that they possessed rather similar skills in formulating mathematical exercises, identifying easy and difficult mathematical exercises, and formulating and marking exams. These skills are connected with the measuring aspect of mathematical competence, which may in fact be more vital knowledge than that connected with those previously considered. The theme of presenting mathematics received the highest scores based on graduated teachers' perceptions. The mathematical representations and skills involved in visualizing mathematics are clearly vital in the teaching of mathematics.

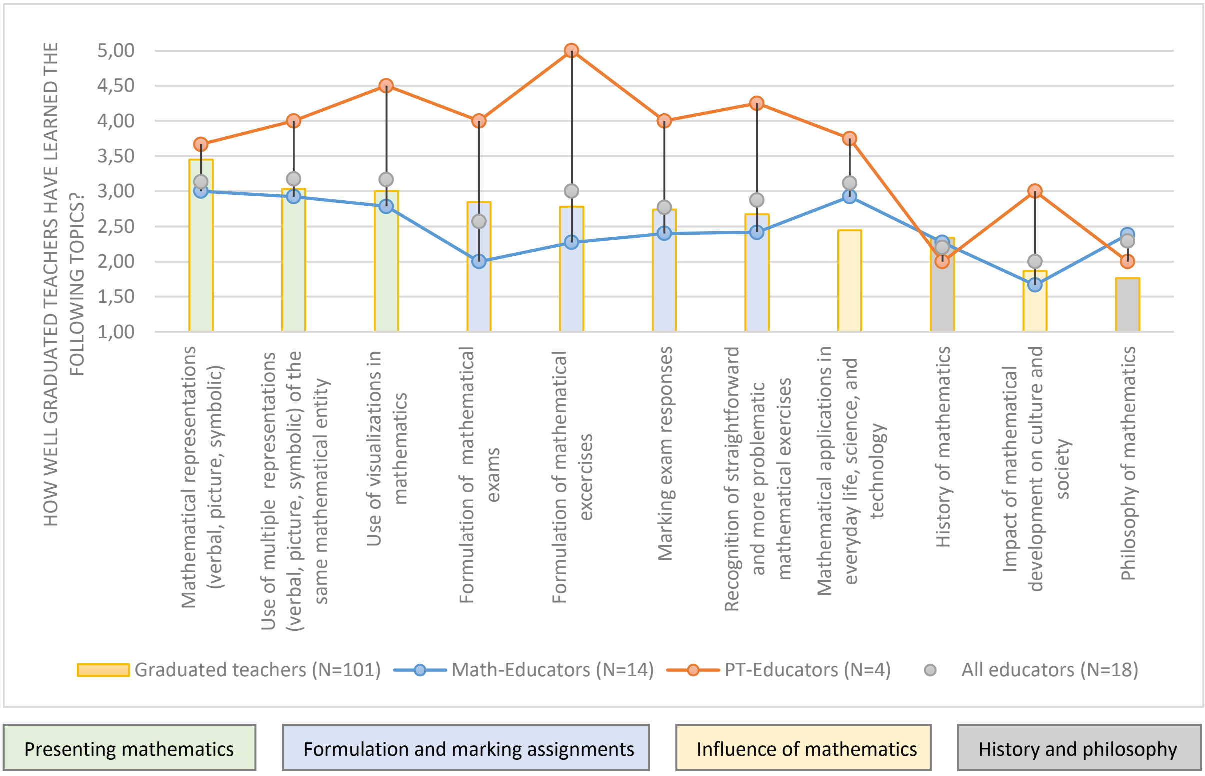

Figure 5. Respondents' perceptions of how graduated teachers have learned the aspects of Specialized content knowledge (SCK). The sequence chosen shows the graduated teachers' perceptions ranked from the highest to the lowest scores (bars)

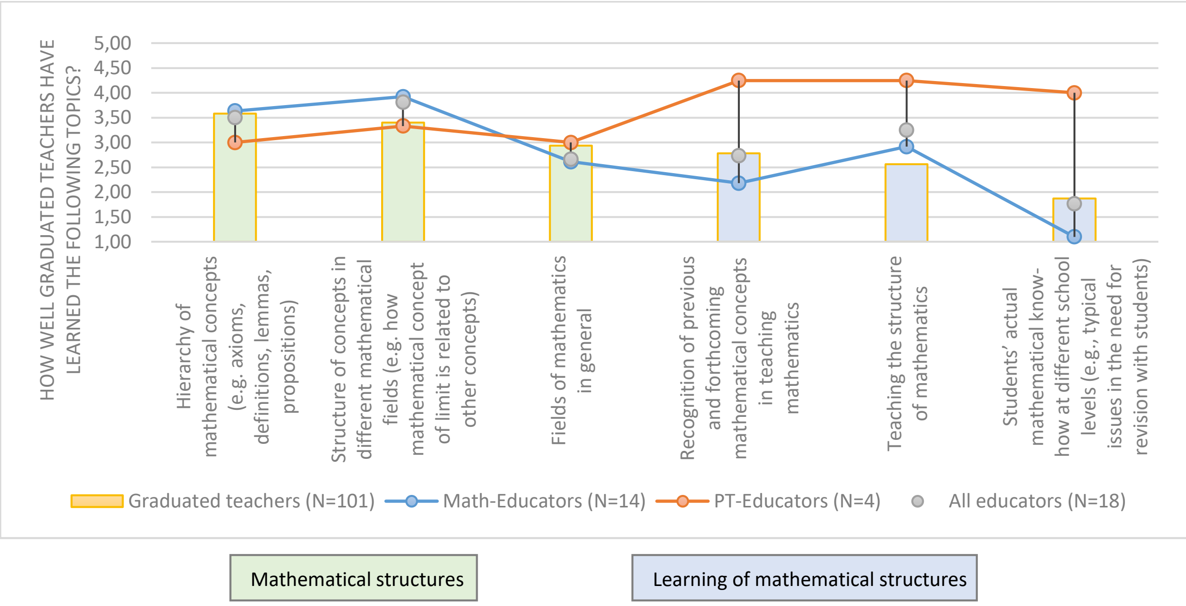

Figure 6. Respondents' perceptions of how graduated teachers have learned issues in relation to Horizon content knowledge (HCK). The sequence chosen shows the graduated teachers' perceptions ranked from the highest to the lowest scores (bars)

When comparing the graduated teachers' perceptions of how they had learned issues related to Common content knowledge and Specialized content knowledge, it may be noted that the mean values are generally considerably lower for specialized content knowledge than for Common content knowledge. In other words, the graduated teachers considered that they had learned pure mathematical issues better that were also useful for other professions, but at the same time they had acquired only a modest mathematical knowledge that was specifically related to teaching mathematics.

Perceptions of Horizon content knowledge

The graduated teachers had experienced more effective learning related to the theme concerning mathematical structures than they had concerning the Learning of mathematical structures. The opinions of the Math-Educators and PT-Educators intersect in the item identified as Fields of mathematics in general (**Figure 6**). Interestingly, the items on the left, to which the Math-Educators had assigned higher values, represent a theoretical knowledge of mathematics, while the items on the right, to which the PT-Educators assigned higher values, are concerned with practical knowledge. These observations seem to be in line with the intention of those particular studies, since mathematical studies do in fact usually have theoretically-oriented aims, while teaching practice has practically oriented aims.

Perceptions of Knowledge of content and teaching

The themes seem to be mixed in the domain of Knowledge of content and teaching, where the chosen sequence consists of the graduated teachers' perceptions ranked from the

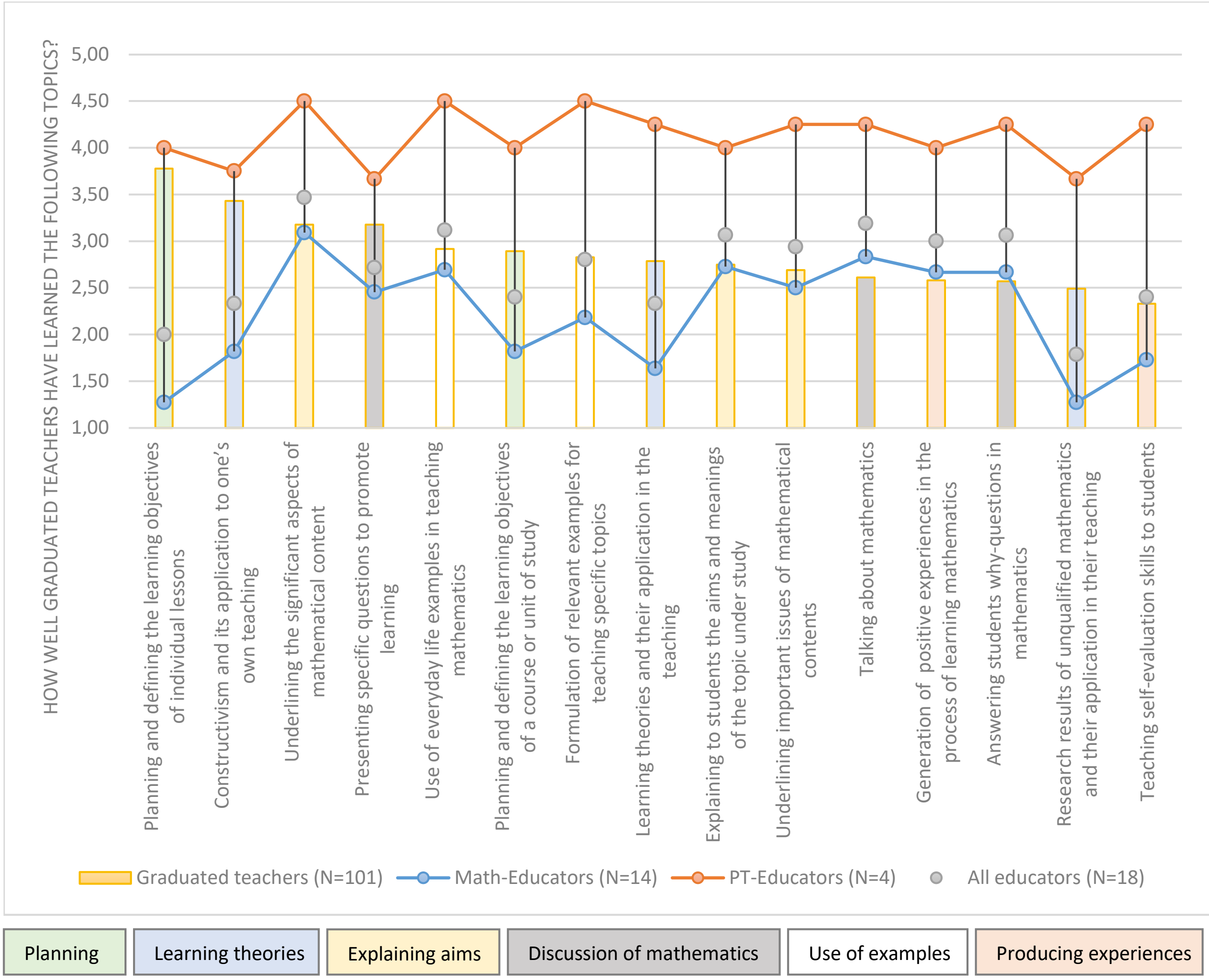

Figure 7. Respondents' perceptions of how graduated teachers have learned issues in Knowledge of content and teaching (KCT). The sequence chosen shows the graduated teachers' perceptions ranked from the highest to the lowest scores (bars)

highest scores to the lowest (see **Figure 7**). In consequence, the themes will be analyzed one by one.

The difference between the views expressed by the Math- and PT-Educators is clear in the domain of Knowledge of content and teaching (**Figure 7**). The PT-Educators' views are more positive than those of the Math-Educators. Since mathematical studies are connected principally with Subject matter knowledge, while pedagogical studies are connected primarily with Pedagogical content knowledge, the results can be regarded as in line with the intentions of these studies.

Analyzing themes one by one in their theme planning, the graduated teachers considered that they had acquired better skills related to planning single lessons than they had with regard to planning complete courses. This finding is logical, since within the

Mathematics teacher education program the student teachers planned individual classes during their teaching practice, but they devoted less time to planning complete courses. The PT-Educators produced high values in relation to the learning of these items, while the values produced by the Math-Educators were low. This is also understandable since PT-Educators teach student teachers to plan lessons whereas Math-Educators do not.

In the context of the theme concerned with Learning theories, the graduated teachers thought that they had a better knowledge of constructivism and its application to their own teaching than they achieved when applying other learning theories. Because constructivism can be seen as the basis of modern learning theory, the result is positive. Nevertheless, research is continuously proceeding further, and new research-based results concerned with learning can be of benefit to teachers. The graduated teachers considered that they possessed the lowest skills when they attempted to apply information gained from research-based results to their teaching. Hence, increasing the contents related to the skills of applying knowledge gleaned from research-based results concerned with learning mathematics to real-life teaching may well be a perspective

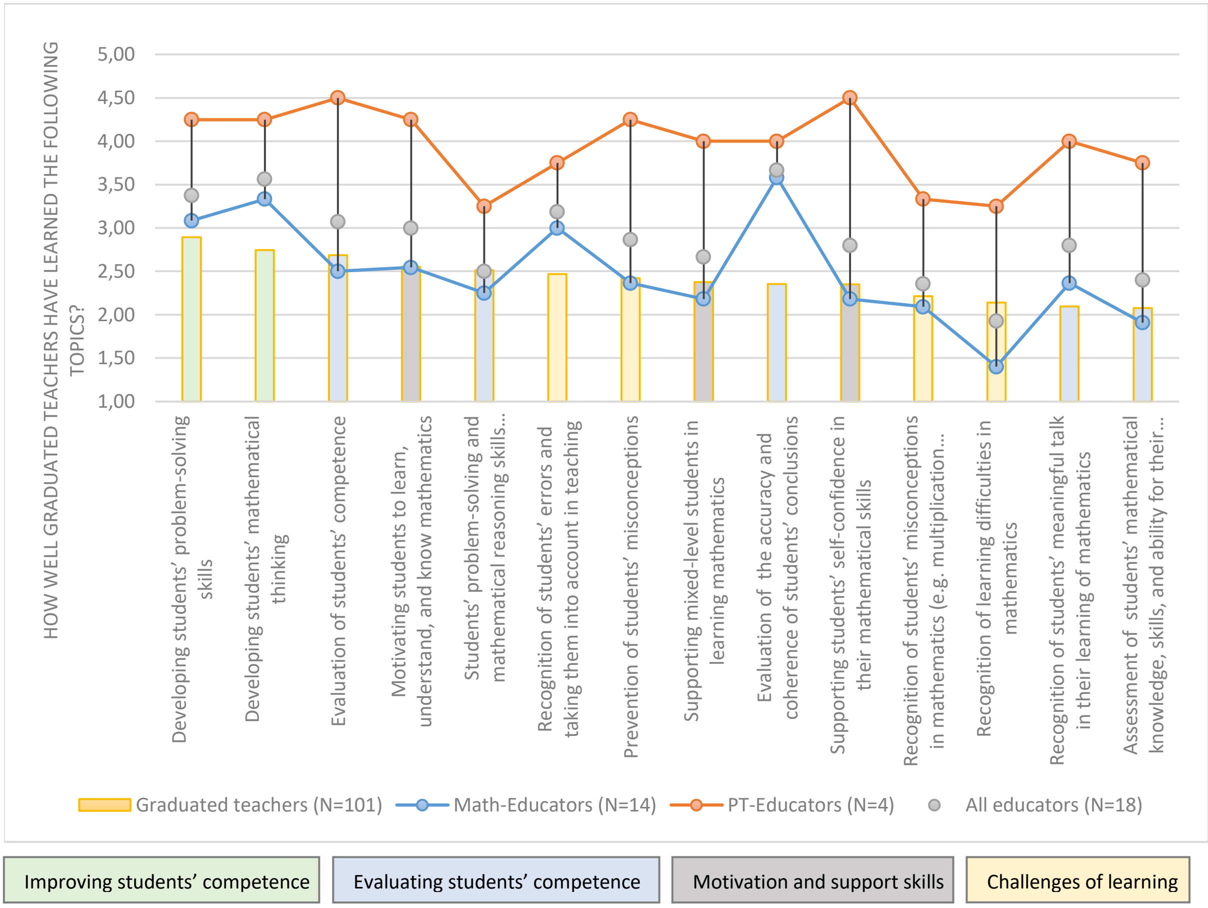

Figure 8. Respondents' perceptions on how graduated teachers have learned issues in terms of Knowledge of content and students (KCS). The sequence chosen shows the graduated teachers' perceptions ranked from the highest to the lowest scores (bars)

that could help in the development of teacher education.

According to the graduated teachers, the mean values for items drawn from other themes, such as explaining the aims of teaching, discussing mathematics, and using examples, are quite similar to each other. The items in these themes are more like inward-oriented action than providing teachers with theoretical knowledge. In addition, producing reports of positive experiences received in the process of learning mathematics relates to working in classrooms. Nevertheless, the scores in this domain are almost the lowest obtained.

Perceptions of Knowledge of content and students

One clearly defined theme–improving students' competence–emerged, whereas other themes in Knowledge of content and students are more mixed (**Figure 8**). In consequence, the themes will be analyzed one by one.

The difference between the views held by the PT- and Math-Educators in this domain is clear. The PT-Educators' views are more positive than those of the Math-Educators. Since mathematical studies are generally connected with Subject matter knowledge, while pedagogical studies are concerned primarily with Pedagogical content knowledge, the results are in line with the aims of these studies. However, the mean values for all of the teacher educators are noticeably higher than those of all of the graduated teachers. The educators seem to have a more positive view of the learning of the graduated teachers than those of the graduated teachers themselves.

Analyzing themes one by one, the graduated teachers considered that they possessed better skills for improving students' competence than they did for other themes. Certainly, ways of developing students' mathematical thinking and problem-solving skills are undoubtedly useful knowledge for mathematics teachers.

In the theme concerned with evaluating students' competence, the graduated teachers considered that they possessed more useful skills for assessing students' competence than they did skills concerned with evaluating students' reasoning, conclusions, or meaningful talk. It is possible that the evaluation of students' competence can be understood generally as simply grading students, but evaluating students' competence on the basis of the students' reasoning, conclusions or meaningful talk may be a more accurate undertaking that can be regarded as in-action knowledge, since that type of evaluation usually occurs in the classroom.

In relation to the theme Challenges of learning the graduated teachers considered that they possessed better skills with regard to recognizing students' mistakes than recognizing students' misconceptions or learning difficulties. On some occasions, recognizing mistakes requires simply Common content knowledge, but a knowledge of misconceptions and learning difficulties in mathematics requires more than simply pure mathematical knowledge (Ball et al., 2008). The results indicate that graduated teachers may have better

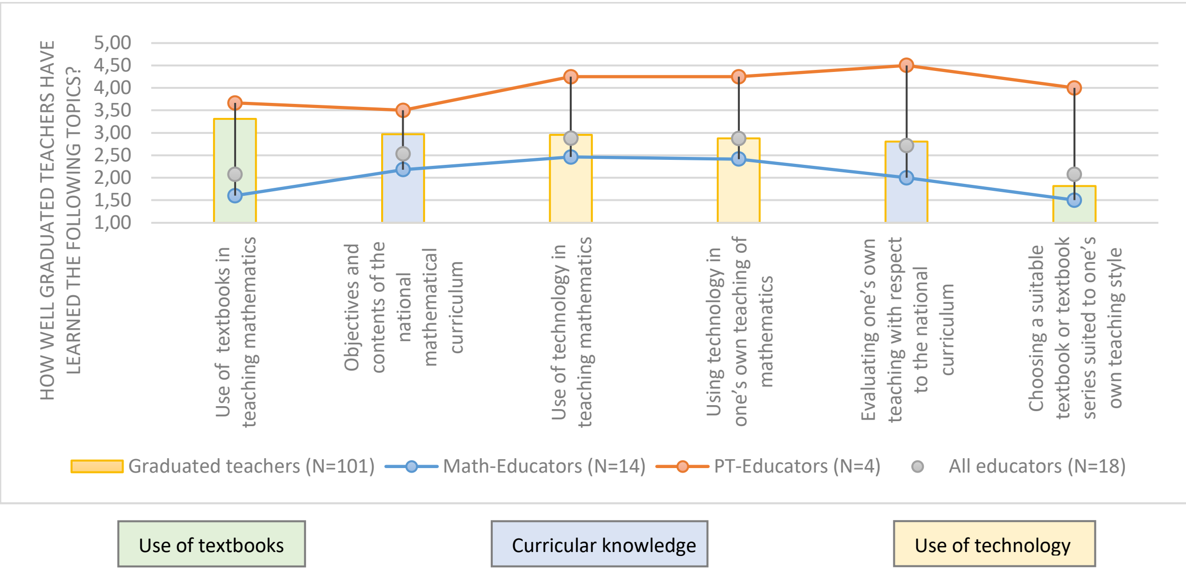

Figure 9. Respondents' perceptions of how graduated teachers have learned issues in Knowledge of content and curriculum (KCC). The sequence chosen shows the graduated teachers' perceptions ranked from the highest to the lowest scores (bars)

skills appropriate for recognizing a wrong answer, but only moderate skills with regard to students' specific challenges in learning.

In relation to the theme of motivation and supporting skills, the graduated teachers considered that they possessed better skills for motivating students to learn mathematics than they did for handling mixed-level students' learning. Indeed, motivation skills are important for a teacher, but in the classroom there are always students with good or poor mathematical skills, and hence it is obviously important that the teacher should know how to handle mixed-level students.

With respect to this component of teacher knowledge, the mean values are under three for every item within Knowledge of content and students according to the graduated teachers' observations. This particular aspect of the study is clearly lower in comparison with other components in the area of MKT. In developing mathematics teacher education, all of the issues connected with Knowledge of Content Students may well be worth considering.

Perceptions of Knowledge of content and curriculum

The differences between the PT- and Math-Educators' views are the highest in this domain (Figure 9). Curricular knowledge and the use of textbooks seem to increase the difference between them. Math-Educators normally use their own teaching materials rather than textbooks, and hence it is reasonable that their views are lower. Issues related to national curriculum are generally dealt with within the framework of pedagogical studies or teaching practice, a detail that helps to explain the difference.

Within the theme of the use of textbooks, the graduated teachers considered that they possessed better skills related to using a textbook in their teaching, compared with the requisite skills involved in choosing a textbook best suited to their teaching style. The usual textbooks are used in teaching practice, but choosing the right textbook is also an important skill, since in Finland teachers have the right to choose the textbooks/series that they wish to use. Thus it is reasonable to argue that this decision requires knowledge about the differences between the different textbooks. Choosing the right textbook is the first thing that a graduated teacher needs to do after graduation. Hence, exploring the differences between textbooks or studying textbooks with an eye on teaching at school may well be something to consider when developing the teacher education program.

With respect to the other themes, the graduated teachers apparently consider that their knowledge of technology is at approximately the same level as their skills related to using technology in their teaching. Knowledge of national curricula and of the skills involved in evaluating teaching within the context of the national curriculum have similar mean values. Technology and its use in teaching is growing strongly nowadays at school level and hence it would be worth considering whether this is sufficient.

Overall results: Differences between the components of teacher knowledge

According to the perceptions of the graduated teachers, the average of the mean values in the field of Subject matter knowledge is 3.24, while that of the mean values in the field of Pedagogical content knowledge is 2.67. These results indicate that the graduated teachers thought that they had learned about the various issues in Subject matter knowledge slightly better than in Pedagogical content knowledge. However, the more accurate components of MKT reveal that the graduated teachers felt they had learned the strongest issues in the context of Common content knowledge. The mean values for each component in MKT are presented in **Figure 10**. The next best values are related to issues in Knowledge of content and teaching. Almost the same average values pertain for the domains of Horizon content knowledge and Knowledge of content and curriculum. Graduated teachers evaluated they had learned the modest issues in Specialized content knowledge and in Knowledge of content and students.

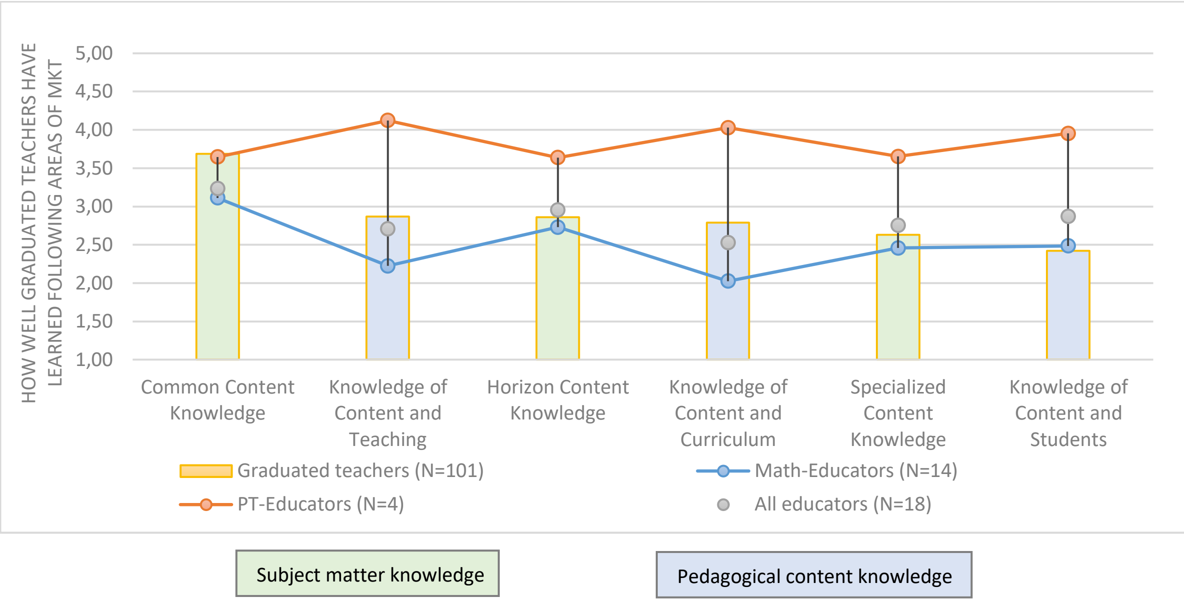

Figure 10. Overall results of respondents' perceptions of how graduated teachers have issues learned were related to components of Mathematical Knowledge for Teaching (MKT). The sequence chosen shows the graduated teachers' perceptions ranked from the highest to the lowest scores (bars)

The PT-Educators' perceptions are generally more positive than those expressed by the Math-Educators (**Figure 10**), but if we take into account the number of teacher educators in these two areas, it can be seen that the mean value for all of the teacher educators seems to be more in line with the perceptions of the graduated teachers (**Figure 11**). The Pearson correlation test indicates similar conclusion. For all 72 of the items, there was significant positive linear correlation ($r=.502$, $p< .000007$). This result means that both the teacher educators and the graduated teachers have quite similar perceptions of what graduated teachers have learned well and what they have learned less well in the course of their teacher education.

Comparisons between the domains of MKT reveal that the role played by the different types of knowledge is not included in this teacher education program. Rather, some of the MKT components are stressed more than others, while there remain some components that would deserve greater emphasis. Thus, if we wish to change how these fields of teacher knowledge are emphasized in the future, it will be important to know how these components of teacher knowledge are currently handled.

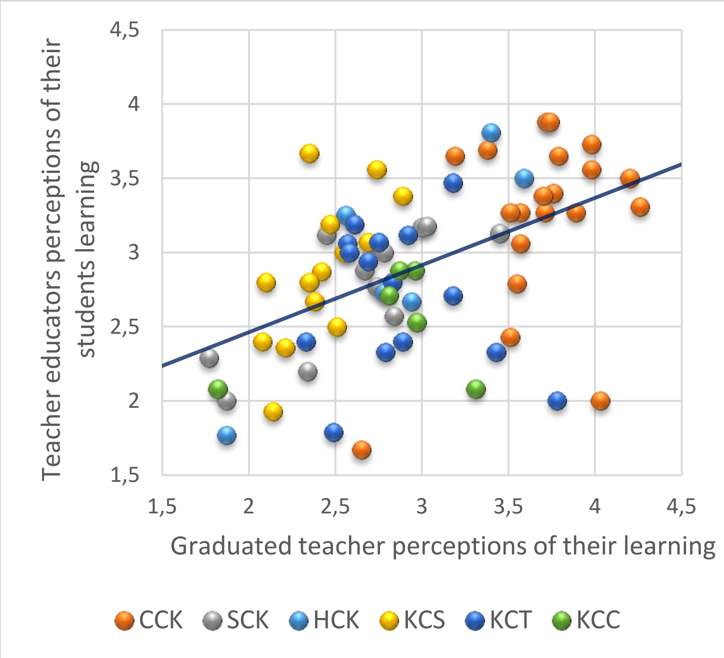

Figure 11. Scatter lot of the mean values for the 72 items based on teacher educators and graduated teachers' perceptions

Ways in which mathematics majors or minors thought about what they had learned about different issues during their education

The *Mathematics majors* had taken twice as many mathematics courses during their teacher education as had the *mathematics minors*. Because the number and contents of the mathematics courses were significantly different for the mathematics majors and minors, the statistical differences were explored with the aid of the Mann-Whitney U-test. The results show that the mathematics majors considered that they had learned the following issues better than the mathematics minors (**Table 8**).

The mathematics majors felt that they had learned thirteen items better than had the mathematics minors. One of them was related to Pedagogical content knowledge and twelve to Subject matter knowledge. Most of the items fell into Common content knowledge.

The mathematical courses labeled *Geometry*, *Number sequences* and *History of mathematics* are optional in the studied teacher education program, but in our experience, most mathematics majors take them. These single special courses seem to have had an effect on the graduated teachers' views. The differences between the contents of the mathematics courses that the mathematics majors and minors had taken explain the observed differences. However, by reading course materials and using mathematical methods or notations, a knowledge of the structure of mathematics may well prove to be knowledge and skills components that will improve if the individual student takes more courses involving university mathematics. Hence, it can be claimed that the number of mathematics courses could explain these differences.

Interestingly, apart from one item, all of the differences occurred in Subject matter knowledge, and especially in the domain of Common content knowledge. In addition, no differences occurred in the context of Knowledge of content and students or Knowledge of content and teaching. Thus, results of this kind help to provide a picture of the kind of courses that their teacher education has offered these teachers. In other words, the results indicate the impact of the different curricula on mathematics majors and minors: a wider mathematics curriculum of seems to have a greater impact on the graduated teachers' views concerning their learning Subject matter knowledge and less concerning their learning Pedagogical content knowledge.

Table 8. Statistical differences show that mathematics majors considered that they have learned the following issues better than had the mathematics minors

Significance	**Common content knowledge**	**Specialized content knowledge**	**Horizon content knowledge**	**Knowledge of content and curriculum**
p<.001	• Mathematical concepts • Reading and understanding university-level course materials used in mathematical studies	• History of mathematics	• Hierarchy of mathematical concepts (e.g., axioms, definitions, lemmas, propositions)	• Choosing a suitable textbook or textbook series suited one's own teaching style
p<.01	• Mathematical reasoning rules (e.g., logic) • Geometry		• Structure of concepts in different mathematical fields (e.g., how mathematical concept of *limit* is related to other concepts) • Fields of mathematics in general	
p<.05	• Number sequences • Analytic geometry • Use of suitable mathematical methods • Exact use of mathematical notations			

DISCUSSION

In this study we have explored graduated mathematics teachers' perceptions of how they have learned different issues during their teacher education, and also teacher educators' perceptions of how they view the ways in which graduated teachers have learned the same issues during their teacher education. In order to examine these components of teacher knowledge more precisely, the present survey was developed by using the teacher knowledge framework widely known as Mathematical Knowledge for Teaching (MKT).

Differences can be found in the smaller fields of teacher knowledge (König et al., 2011), and therefore breaking Subject matter knowledge and Pedagogical content knowledge down into their smaller components can reveal some unexpected issues.

Based on the results, the teacher educators offering pedagogical courses generally have a more optimistic perception of how the graduated teachers have learned different issues during their teacher education compared with the perceptions expressed by the teacher educators of mathematical courses. However, the views of all of the teacher educators and graduated teachers are quite similar at a general level (see 5.1.7). In consequence, all of the teacher educators and graduated teachers can be referred to using the term *both groups*.

These two perceptions of the two groups have informed our results concerning the ways in which Subject matter knowledge and Pedagogical content knowledge are emphasized in one particular teacher education program in Finland. The results indicate that in this teacher education program Subject matter knowledge and Pedagogical content knowledge are emphasized almost equally. However, when we divided teacher knowledge into its more detailed components it was interestingly revealed that smaller fields of teacher knowledge are stressed differently.

Strengths and challenges in Subject matter knowledge

Based on the results, Common content knowledge is clearly the most strongly emphasized component of Subject matter knowledge in the studied teacher education program. According to the perceptions of both groups, the graduated teachers had learned pure mathematical issues better however, there were some exceptions, such as statistics, which does not belong to the present teacher education program.

Our results also indicate that issues related to Specialized content knowledge seem to be emphasized less than those concerned with Common content knowledge in the present teacher education program. Both groups considered that the graduated teachers had generally learned issues in Common content knowledge better than they had issues concerned with Specialized content knowledge. In the context of Specialized content knowledge, both groups felt that the graduated teachers had learned issues concerned with presenting mathematics, formulating and marking assignments more proficiently than issues related to the history and philosophy of mathematics or issues dealing with the ways in which mathematics has influenced everyday life and society.

Common content knowledge is undoubtedly an area of fundamental knowledge for teachers, and it is a fact that without knowledge of this kind teaching mathematics is impossible. A study published by Copur-Gencturk and Lubienski (2013) indicates that mathematics content courses have an impact on future teachers' Common content knowledge, but at the same time courses of this kind may not actually make the Subject matter knowledge the most relevant for effective teaching. Their results indicate, rather, that Specialized content knowledge is also needed for effective teaching. Copur-Gencturk and

Lubienski (2013, p. 219) claim rather convincingly that "*perhaps specialized content knowledge should actually be considered a subset of common knowledge if it simply adds a teaching-specific story line around the everyday mathematics content*."

The narrative context of mathematics might be presented, for example, if a teacher knows something about the history of mathematics and how mathematics has influenced the development of culture, society, or everyday life. Our results indicate that it is precisely this area of knowledge that remains less emphasized than knowing about mathematics content. In order to improve the present teacher education program, one procedure might be to lay greater stress on mathematical contents that take into account the special characteristics of teachers' work and can be referred to under the heading of Specialized content knowledge. Perhaps designing a new hybrid course where the fields of teacher knowledge are mixed would be a possible option for developing this aspect of the present teacher education program. Hybrid courses seem to have a good effect on future teachers' Common content knowledge and Specialized content knowledge, whereas the content courses only impact on Common content knowledge (Copur-Gencturk & Lubienski, 2013).

Strong aspects and challenges in Pedagogical content knowledge

Based on our results, Knowledge of content and teaching is the component of Pedagogical content knowledge that is most strongly highlighted in the present teacher education program. As the results show, items scored differently within the themes in Knowledge of content and teaching, which suggests that such items should be investigated, theme by theme. In Knowledge of content and teaching, both groups thought that the graduated teachers had, for example, learned more about planning single lessons than about planning complete courses. Similarly, they appeared to have acquired a better knowledge of constructivism than of other learning theories, and about presenting questions than answering them. These results are not dramatic, since planning a course usually includes planning several lessons. Constructivism might also be considered more important than other learning theories, and presenting questions is generally considerably easier than answering them, since presenting can be planned in advance, but answering cannot. Despite this, the overall results indicate that Knowledge of content and students has received less emphasis than Knowledge of content and teaching. In Knowledge of content and students, both groups thought that the graduated teachers possessed a better knowledge concerning the improvement of students' problem-solving skills and mathematical thinking but weaker knowledge concerned with recognizing students' misconceptions or learning difficulties.

Teachers do, of course, sometimes recognize which mathematical exercises are difficult for students, but they do not consistently know *why* these exercises are difficult for their students (Hill et al., 2008). However, if a teacher does know why some students are experiencing difficulties in specific exercises; it might be possible to approach these students differently. Indeed, teachers require knowledge related to both teaching and learning, since these knowledge types are closely connected in many classroom situations. Observation of students' learning can be an effective starting point for changing one's teaching style, or knowing how students learn mathematics, or understanding the kinds of challenges that they might face at the individual level may very well provide some of the keys for teaching more effectively. In the present state of teacher education, knowledge concerning the ways in which students learn mathematics seems to less emphasized than knowledge about teaching mathematics per se, and hence increasing Knowledge of content and students may well be worth considering.

The impact of a wider mathematics curriculum

Based on our results, the mathematics majors considered that they had learned some issues better during their teacher education than had the mathematics minors (see 5.2). Statistical differences occurred in thirteen of the seventy-two items. These differences appeared, for example, in knowledge of geometry, number sequence, and the history of mathematics. Generally speaking, the mathematics majors had completed courses in the studied teacher education program referred to as Geometry, Number sequence, and the History of mathematics, whereas the minors had rarely done so, which may help to explain these statistical differences.

Qian and Youngs (2016) found that the contents of mathematical courses have an effect in TEDS-M countries on future teachers' knowledge. Their results indicate that the content of mathematics content courses impacts on future teachers' Mathematics content knowledge. However, the number of mathematics courses taken also has an effect on future teachers' Mathematics content knowledge. Their study indicates that it is important not only how many courses a teacher education program offers but also what kind of contents these courses concentrate on. If teachers have studied a wider or different mathematics curriculum in their teacher education, this may also have an effect on their Mathematics content knowledge.

Based on our results, it was interesting to note that all except one of the differences occurred in Subject matter knowledge, particularly in Common content knowledge. The curricular difference between mathematics majors and minors may be a contributing cause of these differences. A wider curriculum for mathematics majors seems to have had a greater effect on how graduated teachers felt that they had learned Subject matter knowledge but less effect on their sense of what they had learned about Pedagogical content knowledge. We consider that these results help to form a picture of the *kind of course contents* that mathematics majors have acquired more than mathematics minors. Because the curriculum can have an effect on future teachers' knowledge (Qian & Youngs, 2016), there is opportunity for improving teacher education by reforming the curriculum for future teachers.

Reassessing the study and context

External validity refers to the generalizability and transferability of research findings. It is especially concerned with generalizing results by applying them to other samples, time periods, and settings (Ihantola & Kihn, 2011). In the present study, the teacher educator and practicing teacher sample sizes were comprehensive and the samples were representative. All of the respondents originated from a single teacher education program and hence these findings primarily describe the actual circumstances of the program that was investigated. In the present study the mean values of the graduated teachers' perceptions suggest that there are topics that are learned well and less well. Interestingly, the mean values related to the teacher educators' perceptions indicate the same conclusion. The positive correlation between the teacher educators' and graduated teachers' perceptions suggests that the two distinct study groups evaluated the graduated teachers' learning in similar ways, resulting in a strong significance ($r=.502$, $p<.001$). Since the two distinct study groups felt that the graduated teachers may have learned some topics differently than others, our actual findings indicate that at least some potential breakdowns must have occurred in the teaching and learning. However, further research is needed to discover how suitable the present course contents actually are for future teachers. Earlier studies have shown that the perceptions of teachers and their students regarding their teaching and learning may exist in a complex set of relations. Furthermore, these perceptions may also be reflected in the teaching outcomes and students' learning (see Trigwell, Prosser, & Waterhouse, 1999).

The internal validity of the study is closely connected with the logic linking research and existing theory and with valid conclusions drawn on the basis of the study (Ihantola & Kihn, 2011). In our opinion, MKT provides a highly useful teacher knowledge framework for evaluating mathematics teacher education, but more research attention needs to be directed toward clarifying the descriptions of the six MKT domains. Missing still is a clear sense of which items are likely to provide good representations for each domain. In this vein, Ball et al. (2008, p. 403) have pointed out that "How to capture the common and specialized aspects of teacher thinking, as well as how different categories of knowledge come into play in the course of teaching, needs to be addressed more effectively in this work." Greater attention also needs to be paid to the ways in which different kinds of knowledge can be accurately classified in terms of the six MKT categories requires as well. We tend to agree with Markworth, Goodwin, and Glisson (2009) that there are pros and cons when MKT is applied in the investigation of the principal characteristics of teacher knowledge. In order to evaluate the kind of knowledge learned by student teachers during their teaching practicum course, Markworth et al. (2009) used the domains of MKT to encode their interview responses and conversational topics. By using MKT in their analysis, they were able to encapsulate information in rather greater detail related to the kind of knowledge that the student teachers had gained during the course. Markworth et al. (2009, p.70) have suggested, however, that MKT "categories are static and do not reflect their use in practice and that the boundaries between categories are sometimes fuzzy". This is the so-called boundary problem of MKT, which has also been noted by Ball et al. (2008): "Related to this is a boundary problem: It is not always easy to discern where one of our categories divides from the next, and this affects the precision (or lack thereof) of our definitions. We define common content knowledge as the mathematical knowledge known in common with others who know and use mathematics, but we do not find that this term always communicates well what we mean.” (p.403). Thus, if the theory does not communicate effectively to the research community, researchers may perceive the various conceptualizations rather differently. According to Hoover, Mosvold, Ball, and Lai (2016), this has indeed occurred in a number of studies.

In our opinion, MKT provides a highly useful teacher knowledge framework for evaluating teacher education programs in a Finnish context, since similar contents can be recognized in the Finnish teacher education coursework compared with the components of MKT. In Finnish teacher education, purely mathematical courses are usually intended for both future teachers and mathematicians. On the other hand, pedagogical mathematics courses are intended primarily for future teachers, and the content of these courses takes the special characteristics of teachers’ work more into account. This division is similar to that in MKT between Common content knowledge and Specialized content knowledge, where the first part is useful for any profession requiring mathematics, while the second part is specialized, making it more appropriate for mathematics teachers (Ball et al., 2008).

In Finnish teacher education some of the contents of pedagogical courses focus on mathematics teaching, while others focus more on the process of learning mathematics. Thus, for example, some contents focus on learning mathematics, i.e., learning difficulties, and learning theories, while other contents focus on teaching mathematics in terms of such aspects as the different teaching methods. All of these contents in Finnish teacher education can be seen to possess a similar point of view in terms of Knowledge of content and students and Knowledge of content and teaching in MKT. In addition, curricular knowledge and also the use of equipment in teaching mathematics are usually concentrated in pedagogical courses, while the mathematical contents related to the structure of mathematics are covered in mathematics courses. All of the components of MKT contain viewpoints that are similar to the contents of mathematics teacher education in Finland. Based on these perspectives, we think that MKT fits into the Finnish context well, especially as the main background theory for evaluating mathematics teacher education.

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