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How middle school students deal with rational numbers? A mixed methods research study

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The aim of this study was to identify 7th grade students' common mistakes and/or misconceptions about representing rational numbers in different forms. Seventy-three 7th grade students participated in this study. We utilized a diagnostic test with 8 questions in order to determine common student mistakes. For identifying misconceptions, we conducted interviews with 25 of the participants purposefully chosen from among the ones who made mistakes more frequently. The data analysis showed that students had difficulties especially in determining rational numbers, relating rational numbers with decimals, representing rational numbers, and rational-number division. Moreover, the current study also revealed students' misconceptions about dividing by zero and division of integers (with + – signs). The results suggest that using concrete materials and multiple representations would be more beneficial for overcoming these difficulties and for a sound understanding of the abstract nature of rational numbers.

Keywords: mixed methods research, rational numbers, common mistakes, misconceptions

INTRODUCTION

The majority of the studies conducted in the field educational research have focused on determination and/or elimination of misconceptions, and students' lack of knowledge in specific domains and the field of mathematics education is no exception (Hiebert & Wearne, 1986; Özmantar, Bingölbali & Akkoç, 2008). Knowledge of mathematics is accumulated on previous knowledge and concepts. Therefore, a conceptual change for students is necessary in order to bring about a sustained meaningful learning (Bell, 1982; Farrell, 1992; Margulies, 1993; Perso, 1992; Küçük & Demir, 2009). To be able provide meaningful learning, teachers need to know students' learning difficulties. Research studies in mathematics education show that the sources of these difficulties are mathematics operations in elementary math, and concepts and symbols learning. Achieving meaningful learning is an ambitious and important but yet a difficult goal. To diagnose student difficulties, to categorize them and become aware of those difficulties is an important factor to be succeeded in this goal (Yetkin, 2003).

While defining rational numbers, Sirotic and Zazkis (2004) assert that an unquestionably accepted certain representation is used and give the definition as '[A] rational number is the number that can be written as "a/b" where "a" is a whole number and "b" is a whole number different from 0.' The topic of rational numbers is among the topics where misconceptions and learning difficulties are very common among pupils. Recent studies show that students have difficulties in understanding the main concepts of rational numbers and performing algebraic operations (Pesen, 2008). Every natural number is an integer. Every natural number and integers are rational numbers with a

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State of the literature

- Rational numbers have four different meanings as verbal, symbolic, solid, and model.
- There exists no study in the literature, which fully coincides in regard to the aims with the current study. Here, we found that students were unable to define the rational numbers conceptually and that was why they were unable to represent them in different formats.

Contribution of this paper to the literature

- The findings of this study illustrate students' difficulties with comprehension problems of the rational numbers.
- In this study differences between students' erroneous responses that cause learning difficulties such as misconceptions and mistakes (e.g., ordering mistakes, representation mistakes, operational mistakes, and mistakes due to negligence) are investigated and examplified.

denominator of one. In other words, rational numbers (or fractions) are obtained by the division of two integers. Although rational numbers are very much similar to the natural numbers and integers, in some respects, they include so many different and complicated features. These differences and complications cause some difficulties in teaching rational numbers (Siegler, Thompson, & Schneider, 2011; Stafylidou & Vosniadou, 2004). The number a/b could indicate one of the followings: (1) the relationship between the piece and the whole entity, (2) measurement, (3) solely a division process, and (4) comparison (ratio) (Alajmi, 2012).

These four different meanings of the rational numbers can be shown as verbal, symbolic, solid and model forms. The transition between these representations is dependent upon the fact that these representations are simultaneously related to each other. However, it was reported that the primary education students are having difficulty to carry out the transitions between the different representations of the rational numbers, wrongly generalized the rules when carrying out the mathematical operations and had trouble in doing comparisons (Haser & Ubuz, 2002; Şiap & Duru, 2004; Tirosh, Tsamir, & Hershkovitz, 2008). At the very foundation of these difficulties lies the frequent use of the traditional education, which urge to people to memorize rather than to learn. Also the overemphasis of piece-whole entity means of the rational numbers and the overuse of the algebraic representation compared to the other possible representations make the conceptual perception of the students' highly difficult (Moseley, 2005). In order to overcome these difficulties it was reported that the use of geometrical models, visual representations, concrete materials, and number lines are very important in their expression (Gürbüz, 2007).

In the middle school curriculum the addition, subtraction, division, multiplication, and ordering processes with the rational numbers are taught in step-wise manner in a spiral approach. However, the studies carried on this topic show that the students at every stage of the primary education have great difficulty in understanding the basic concepts and doing the algebraic processes using the rational numbers (Başgün & Ersoy, 2000; Haser & Ubuz, 2001). The reasons for the students having difficulty in the processes carried out with the rational numbers are that they memorize algorithm and the related formulas rather than understanding its essence and perceiving the denominator and numerator of the rational numbers as two different integers (Şiap & Duru, 2004). The decimal numbers as well as the fractions and the percentages are other concepts, which are difficult to comprehend by middle school students (Hart et al., 1998). Researchers draw strong relationships between students' representations and their learning (Lamon, 2001).

The difficulties faced in the learning process of fractions have been the subject of extensive research. It was reported that the students have difficulty in writing the ratios by the use of equal pieces (Haser & Ubuz, 2001), have problems to perceive the concepts related to the ratios at every level (Aksu, 1997) and make mistakes in solving the problems related to them (Başgün & Ersoy, 2000). There is an extensive literature related to this topic (Başgün & Ersoy, 2000; Haser & Ubuz, 2001; Toluk, 2000;). The misconception largely stem from the failure of teaching the concepts and the mathematical skills in an integral manner, the lack of knowledge of the students of the skills, having insufficient knowledge to solve the problems, and carelessness in the solutions (Ersoy & Ardahan, 2003). It was also determined that the misconceptions originate from the perception of the symbolic representation of the fractions as a/b as two numbers with certain values rather than a single entity (Kerslake, 1986; Olkun & Toluk, 2001). Students have difficulties for breaking a whole into its fragments while representing fractions on the number line (Bright, Behr, Post & Wachsmuth, 1988). In order to achieve a conceptual understanding of rational numbers, which is the foundation for the other topics in mathematics, different forms of representations should be given emphasis (Vergnaud, 1983).

In a study carried out on 200 students between 10-16 years of age showed that they think that the increase (or the decrease) the numerator (or the denominator) of a ratio increases (or decreases) the value of it (Stafylidou & Vosniadou, 2004). This belief became apparent during the ordering of the ratios (Şandır, Ubuz, & Argün, 2007).

7th and 8th grade students' misconceptions related to decimal fractions were found as follows (Seyhan & Gür, 2004):

- *the failure of the perception of the meaning of decimal ratio,*
- *the failure of putting the point at a proper place,*
- *the perception of the point as separator between two different numbers,*
- *thinking that the decimals with more decimal places are bigger or smaller than those expressed with less decimal numbers,*
- *disregarding zero as decimal number,*
- *seeing zero as an ineffective element,*
- *failure to define the decimal part of the decimal numbers correctly,*
- *assuming that zero lowers the numbers and failure to perceive the relations between the fractions and the decimal numbers.*

Similarly, 7th grade students' common mistakes and misconceptions about representing of rational numbers on the number line were identified as follows (Yetim & Alkan, 2010):

- *Inability to sort positive or negative rational numbers among themselves,*
- *division a number by zero yielding zero,*
- *failing to recognize that integers (whole numbers), at the same time, are also rational numbers,*
- *confusion with numerators and denominators of the rational numbers,*
- *fail to represent a negative integer fraction on the number line,*
- *when representing the rational number on the number line get confused with operations on the number line,*
- *when representing rational numbers on the number line not being able to divide the spaces according to the denominator.*

In TIMSS-R held in 1999 as repeat of TIMSS (Third International Mathematics and Science Study) there were 38 countries participated in it including Turkey. Among these countries Turkey ranked 31st in the general success in mathematics and 33rd regarding to fractions and sense of numbers (Küçük & Demir, 2009). This low ranking of Turkey in mathematically important topics such as the fractions and the sense of numbers shows that enough attention had not been paid to this topic in the educational system in Turkey. There is no study in the literature that gave the same results with this study. However there are studies which were dealing with misconceptions related to the fractions and decimal numbers. The purpose of this study is to evaluate common mistakes and misconceptions of the 7th grade students in public school in the Keçiören region of Ankara province in showing the rational numbers in different forms and make some proposals based on the results obtained.

In this study, we looked for the answers for the following questions:

1. What is the type of mistakes in the representation of the rational numbers in different forms?

2. What are the misconceptions that the students have in the representation of the rational numbers in different forms?

METHODOLOGY

This is a mixed method research study due to its inclusion of both quantitative and qualitative methods (Hesse-Biber, 2010, p.3). Being quantitative in nature, the current study utilized survey method and was also supported with the qualitative data collected through interviews conducted by the researchers. The survey model is a type of study, which describes or defines a situation as it exists. It does not pay attention to the reasons of the survey but it defines the situation thoroughly and carefully. It does not attempt to effect or to change this situation. The thing to be investigated is already resent. The purpose is to define and describe it in correct manner and observe it without making any changes (Karasar, 1999). In the survey method, it is necessary to complete the structure by asking the specific question followed by the general ones (Fraenkel & Wallen, 2006).

Participants

A purposeful sample of a total of 73 students from 7th grade participated in study. The participants were attending to a public middle school located in Keçiören district of Ankara in 2007–2008 academic year.

Data Collection Tools

The quantitative data of the study came from a diagnostic Test (DT) consisted of 8 questions that aimed to determine student mistakes and misconceptions to show the rational numbers in different formats (See Appendix A for the list of the questions included in the study). The questions were chosen from the items employed in the framework of CSMS (Concepts in Secondary Mathematics and Science), NCTM (National Council of Teachers of Mathematics), TIMSS (Third in International Mathematics and Science Study). The selection criterion was the items' accordance with the 6th and 7th grade mathematics curriculum in Turkey. The views of three experts (in the field of mathematics education) were also consulted to approve the items' alignment with the related curricula and their appropriateness for responding the related research questions.

The qualitative data of the study came from student interviews. The interviews were conducted by the researchers in order to interpret the students' answers and to detect their misconceptions and common mistakes, if there is any, regarding representations of the rational numbers data. The interviewees were chose among the students who provided incorrect solutions to the questions.

Analysis of data

The data came from the DT were analyzed by means of descriptive statistics. Table 1 shows the categories and their descriptions used while evaluating students' responses. The categories included misconception, representation mistake, mistake in ordering process, mistake due to carelessness, operational mistake and no answer. We interviewed a total of 25 students who were identified with mistakes. All categories, excluding for misconception category, were evaluated by the researchers based on students' answer sheets.

The term misconception is widely used to describe and explain students' performance in specific subject-matter domains (Eaton, Anderson, & Smith, 1983; Gardner, 1991; Shaughnessy, 1992). Smith, diessa and Roschelle (1993) defines misconceptions as "a student conception that produces a systematic pattern of errors" (p. 119). In order to define a common mistake as a misconception, it is to be constantly repeated and the student should defend it when he/she is challenged (Smith et al., 1993). The category of misconception in this paper was established according to the data obtained from the interviews.

RESULTS AND DISCUSSION

Students' answers for each question are sorted in a table and Table 2 shows the distribution of students' answers for the eight questions included in the Diagnostic Test (see appendix for the questions).

Overall, no students made an ordering mistake regarding rational numbers. Moreover, only a few of the students had difficulty in representing rational numbers (in Question 1) and made a mistake due to their carelessness (in Questions 1 and 8). Operational mistake was the mostly occurred mistake with a percentage range from 21.9 (for Question 8) to 82.2 (for Question 4). Based on the interviews with the students, some misconceptions were evident for the Questions 1, 5 and 8. For any question, less than one-third of the students had provided the correct solution while 31.5% and 57.5% of the students did not have a solution for Question 2 and Question 8 respectively.

Table 1. The employed criteria and their examples

Criteria	Descriptions
Misconception	Recurring mistakes and student's defense for interrogation
Representation Mistake	Confusing the rational numbers with other numbers, Failing to express the rational numbers in representative form
Mistake due to Carelessness	Mistake caused by carelessness such as forgetting placing the negative sign while doing calculations.
Operational Mistake	Mistakes encountered while doing operational algorithms such as division with rational numbers
No Response	The questions left unanswered by the students
Correct Solution	The required answers were given to the questions

Table 2. The distribution of the students' responses to the questions based on coding themes

	Ordering mistakes		Misconception		Representation mistakes		Mistake due to negligence		Operational mistake		No Response		Correct solution		Total	
	f	*%*	*f*	*%*	*f*	*%*	*f*	*%*	*f*	*%*	*f*	*%*	*f*	*%*	*f*	*%*
Q1	0	0	6	8.2	9	12.3	4	5.5	29	39.7	1	1.4	24	32.9	73	100
Q2	0	0	0	0	0	0	0	0	38	52.1	23	31.5	11	15.1	73	100
Q3	0	0	0	0	0	0	0	0	53	72.6	8	11	12	16.4	73	100
Q4	0	0	0	0	0	0	0	0	60	82.2	7	9.6	6	8.2	73	100
Q5	0	0	23	31.5	0	0	0	0	30	41.1	2	2.7	18	24.7	73	100
Q6	0	0	0	0	0	0	0	0	64	87.7	3	4.1	6	8.2	73	100
Q7	0	0	0	0	0	0	0	0	59	80.8	4	5.5	10	13.7	73	100
Q8	0	0	7	9.6	0	0	1	1.4	16	21.9	42	57.5	7	9.6	73	100

As a result of student answers to the question 1 that asks to convert a rational number to decimals, it is seen that 12.3% of the students provided a wrong representation, 39.7% of them made an operational mistake, and 32.9% gave correct answers. Moreover, 6 of students among the 25 with common mistakes were found to have misconceptions related to the first questions. Box 1 shows an excerpt from the follow up interview with a student showing a misconception about representation of rational numbers.

Box 1. Excerpt from Student A's responses
Researcher: *It seems that you converted the rational number into a decimal number.*
Student A: *Yes, I think I did*
Researcher: *Okay. How did you do it?*
Student A: *I wrote the sign as it was*
Researcher: *Then?*
Student A: *I wrote the numerator after the sign.*
Researcher: *Then?*
Student A: *Then, I added the point after the numerator to convert it into a decimal number.*
Researcher: *Is that it?*
Student A: *Yes, it is the way to convert it into a decimal number*

As it was the case with student A, six students failed to represent rational numbers in different forms and to convert the compound fraction into the integer fraction. This is compatible with the results of Haser and Ubuz (2002) and Şiap and Duru (2004) who found that students had difficulty in the transition of different representations of the rational numbers and wrongly generalized the rules when they were doing the mathematical operations.

Question 2 asks students to conduct a division of integers and the analysis of the students' responses are shown in Table 1. The findings show that 52.1% of the students made an operational mistake, 31.5% gave no answers and 15.1% gave the right answer. Similarly, students' responses for Question 3 that seeks for the relationship between integers and rational numbers indicated that 72.6% had an operational mistake in doing this, 11% did not provide any answer, only 16.4 % had the correct solution on the sheet, and none of them possessed any misconception related to the Q3, according to the interviews conducted with 25 students who made common mistakes.

Box 2 shows an excerpt from the follow up interview with a student showing an operational mistake related to integers with rational numbers. These common mistakes are due to the lack of knowledge of the student that every integer with "1" in the denominator was a rational number. These results are in good agreement with the results reported by Küçük and Demir (2009) as the students are not aware of the fact that integers are the rational numbers with "1" in the denominator.

Box 2. Excerpt from Student C's responses
Researcher: *why do you think that integers are not rational numbers?*
Student C: *As I stated in the answer: Rational numbers can be either (–) or (+).*
Researcher: *OK how are the integers then?*
Student C: *Integers are also like that. Was there a denominator?*
Researcher: *What do you think that the denominators of integers are?*
Student C: *I think it was 1 or -1*
Researcher: *I that case integers have denominators as well.*
Student C: *Yes, I agree. Integers are rational numbers.*

Question 4 seeks for the relation between the rational numbers and decimals. The results indicated that the ratio of operational mistakes was very high (82.2%) and only 8.2% of the students gave the correct answer (see Table 1). When the common mistakes and misconceptions listed in Table 1 about the use of zero in the denominator of the rational numbers (Question 5) were examined, it was seen that the 31.5% of the students had misconception, 41.1% made an operational mistake, 2.7% left it unanswered and 24.7% gave the right answer.

An example of an interview about Question 5 which was found to be deterministic about the misconception of using zero as denominator of the rational numbers is given below in Box 3;

$\frac{4}{0}$ is a rational number. (….)

Explanation: Zero in the denominator does not have any meaning, 4 is a fraction.

Box 3. Excerpt from Student B's responses
Researcher: Do you think ?
Student B: Yes, I think it is 4.
Searcher: Ok what is ?
Student B: It is 3
Researcher: In other words?
Student B: In other words zero does not have any meaning
Researcher: What do you mean by that?
Student B: It is ineffective

The interviews carried out with 25 students with common mistakes, 23 of them had misconceptions about Q5 (about the effect of zero at the denominator). The results showed that they think that zero does not have effect in division. This is in good accordance with the data reported by Seyhan and Gür (2004) as "thinking that zero has no meaning".

An examination of the students' responses for Question 6 where the relation between the integers and natural numbers was sought indicated that 87.7% of the students made an operational mistake, 4.1% of them left it unanswered and 8.2% of them found the right answer. Similarly, for Question 7 where the representation of integers as rational number was asked, 80.8% of the students made an operational mistake, 5.5% of them left it unanswered and only 13.7% of them found the right answer.

Moreover, the results relating Question 8 where the conversion of decimal numbers into different rational numbers was asked 9.6% of the students had misconception, 1.4% had a mistake due to carelessness, 21.9% made an operational mistake, 57.5% left it unanswered and only 9.6% found the correct solution. An example of the interview with the students about Q8 which was deterministic about common mistakes and misconceptions on the different representations of rational numbers is given below in Box 4:

Box 4. Excerpt from Student D's responses
Researcher: *How did you find this answer?*
Student D: *I found it by dividing 5 to -4.*
Researcher: *Why did you divide it to -4?*
Student D: *There was -4 given in the question*
Researcher: *How can you convert 0.3 into a rational number?*
Student D: *Like 0. 3=3/2.*
Researcher: *Are you sure?*
Student D: *Yes.*

In the interviews carried out with 25 students with a common mistake, 7 of them were observed to have a misconception about Q8. It revealed that the students are not able to convert the decimal numbers into rational numbers. This complies well with the opinion of Moss and Case (1999) as "the conversion of decimal numbers into rational number is an area where the misconceptions are very common".

CONCLUSIONS

The data obtained from a total of 73 students from two different 7th grade groups studying in a public school located in Ankara province showed that the students have numerous common mistakes and misconceptions related to the representation of rational number in different forms and they were not able to develop the understanding of necessary conceptual structures. The major learning mistakes and the misconceptions were as follows;

Learning mistakes

- ✓ *Confusing the rational numbers with other numbers.*
- ✓ *Not knowing that natural numbers and integers are the sub set of rational numbers.*
- ✓ *Not being aware of the fact that integers are rational numbers.*
- ✓ *Failure to represent the rational numbers.*
- ✓ *Not being able to make the operation with zero at the denominator.*
- ✓ *Not being able to distinguish between the rational numbers and the integers.*
- ✓ *Not being able to divide two integers.*
- ✓ *Not being able to carry out the division of the signs of two negative integers.*
- ✓ *Not being able to convert a compound fraction into an integer fraction*
- ✓ *Failure of the conversion of a rational number into a decimal number.*
- ✓ *Failure of the conversion of a decimal number into a rational number.*
- ✓ *Not knowing the relationship between the rational numbers and decimal numbers.*
- ✓ *Not being able to distinguish the integers and the natural numbers.*
- ✓ *Not knowing the fact that every integer with "1" at the denominator is a rational number.*
- ✓ *Not being able to write an appropriate number at the numerator where there was 10 at the denominator during the conversion of a decimal number into a rational number.*

Misconceptions

- ✓ *Division of a number by zero is zero*
- ✓ *-0.5=5/-4*
- ✓ $-\frac{8}{5} = -8{,}5$.
- ✓ *Zero is the identity element in the division of the numbers by zero.*

There was no study in the literature, which fully coincides with this one. In this study, we found that the students were unable to conceptually define the rational numbers and that was why they were unable to show them in different formats. The findings of this study came out from the difficulties of comprehension problems of the rational numbers by the students as mentioned by (Seyhan & Gür, 2004; Stafylidou & Vosniadou, 2004). The facts that student were not able to represent the rational numbers with different formats and not able to distinguish them from the other numbers in the system and also they were not able to convert the compound fraction into an integer fraction

Write true (T) or false (F) next to the following statement.

Explain your reasons

a) Every integer a rational number (F)

Explanation: because the rational numbers can be (-) or (+)

is in good compliance with the fact that they were having great difficulty in the transition between the different representations of the rational numbers and incorrectly generalize the rules when they were doing operation as reported by (Haser & Ubuz, 2002; Moseley, Okamoto, & Ishida, 2007; Şiap & Duru, 2004).

It is necessary to establish a conceptual understanding of the rational numbers before giving some rules to be memorized. In order to do that the conceptual understanding of the rational numbers must be emphasized during education. The active participation of the students in the educational activities must be encouraged. Apart from that students can be motivated to engage in the mathematical activities outside the school such as mathematics clubs. The involvement of the students in the mathematical activities both in and out of the school will be of great benefit for the understanding of the concepts related to representation of the rational numbers.

It is necessary to exert more effort to understand and eliminate the misconceptions of the students and there should be necessary strategies developed for this purpose. In other words the abstract concepts related to the rational numbers such as numerator, denominator, fraction and decimal number should be visualized by giving solid examples from the daily life and the use of educational technologies such as projectors, and intelligent boards must be increased (Cramer & Wyberg, 2009; Guy, 2012; Hudson Hawkins, 2008; Siegler et al., 2010).

As mentioned by Altun (1998) the use of the educational models representing the different expression forms is very important and the use of working sheets or the brain storming may be quite useful when teaching the concepts such as numerator denominator, whole-entity, half-the quarter.

Kieren (1976) says that the emphasis of different forms of representation is very important in teaching the concepts related to rational numbers. It is obvious that the emphasis should be made on the different representation in addition to the traditional method and these representations should be employed as much as possible in the teaching medium. Because models and modeling is very effective method in visualizing the abstract forms in the brain. In addition, the teaching materials should be used and as much as possible and the students must be given the chance of developing their own teaching methods. That is to say, the topic of different representations of rational numbers could be made more concrete by enriching the scope of the course with performance assignments and course activities. The materials such as fraction sticks to visualize concepts such as the nominator and the denominator can be developed together with the students.

The results obtained in study are limited with the number of participants and the questions included in the diagnostic tests. Future research is needed to analyze in detail the possible factors, such as classroom instruction, regarding students' misconceptions of rational numbers. Additional analysis of existing workbooks and activity books used in classrooms as a supplementary aid to textbooks would benefit as well.

REFERENCES

Aksu, M. (1997). Student performance in dealing with fraction. *Journal of Education Research, 90*(6), 375-380.

Alajmi, A. H. (2012). How do elementary textbooks address fractions? A review of mathematics textbooks in the USA, Japan, and Kuwait. *Educational Studies in Mathematics, 79*(2), 239-261.

Altun, M. (1998). *Mathematics teaching in education faculties and primary education pre- service teachers*. Erkam Typography, Bursa, Turkey.

Başgün, M., & Ersoy, Y. (2000). Numbers and arithmetic: The difficulties and misconceptions in teaching fractions and decimal numbers. *IV. Science Teaching Congress*, Symposium Book, Ministry of Education Publication, Ankara, Turkey. pp 604-608.

Bell, A. W. (1982). Diagnosing students' misconceptions. *The Australian Mathematics Teacher, 38*(1), 6-10.

Bright, G.W., Behr, M.J., Post, T.R. & Wachsmuth, I., (1988). Identifying fractions on number lines, *Journal for Research in Mathematics Education. 19(3),* 215-232.

Cramer, K., & Wyberg, T. (2009). Efficacy of different concrete models for teaching the part/whole construct for fractions. *Mathematical thinking and learning, 11*(4), 226–257.

Eaton, J. F., Anderson, C. W., & Smith, E. L. (1983). *Students' misconceptions interfere with learning: Case studies of fifth-grade students* (Research Rep. No. 128). East Lansing, MI: Institute for Research on Teaching.

Ersoy. Y., & Ardahan, H. (2003). Teaching of fractions in primary schools II editing of diagnostic activities. Web: http://www.matder.org.tr/bilim/ioko2tyed.asp?ID=49, 27.04.2005.

Farrell, M. A. (1992). Implementing the professional standards for teaching mathematics: Learning from your students. *The Mathematics Teacher, 85*(8), 656-659.

Fraenkel, J. R., & Wallen, N. E. (2006). *How to design and evaluate research in education*. A Business Unit of the McGraw-Hill Companies.

Gardner, H. G. (1991). *The unschooled mind*. New York: Basic Books.

Gürbüz, R. (2007). Students' and their teachers' opinions about the instruction based on the materials on probability subject. *Journal of Kastamonu Faculty of Education, 15*(1), 259-270.

Guy, R. (2012). Overcoming misconceptions in neurophysiology learning: an approach using color-coded animations. *Advances in Physiology Education, 36*(3), 226-228.

Hart, K. M., Brown, M.L., Kuchermann, D.E., Kerslach, D., Ruddoock G., & and Mccartney, M. (1998). Children's

Understanding of Mathematics. In K.M. Hart (Ed.), *The CSMS mathematics team* (pp. 11-16). Antony Rowe Limited.

Haser, Ç., & Ubuz, B. (2001). The conceptual understanding of fractions and processing skills of 5th grade students. *IV. Science Teaching Congress*, Symposium Book, Ministry of Education Publication, Ankara, Turkey, pp: 609-612.

Haser, Ç., & Ubuz, B. (2002). The conceptual and processing skills with fractions. *Education and Science, 27*(126): 53-61.

Hesse-Biber, S. N. (2010). *Mixed methods research: Merging theory with practice.* Guilford Publication.

Hiebert, J., & Wearne, D. (1986). Procedures over concepts: The acquisition of decimal number knowledge. In J. Hiebert (Ed.), *Conceptual and procedural knowledge: The case of mathematics* (pp. 197-223). Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.

Hudson Hawkins, V. (2008). *The effects of math manipulatives on student achievement in mathematics.* (Unpublished dissertation). Minneapolis, MN: Cappella University.

Karasar, N. (1999). *Scientific research methods.* Nobel Typography, Ankara, Turkey.

Kerslake, D. (1986). Fractions: Children's strategies and mistakes. *A Report of the Strategies and Mistakes in Secondary Mathematics Project.* Windsor: NFER-Nelson.

Kieren, T. E. (1976). On the mathematical, cognitive and instructional foundations of rational numbers. In R. A. Lesh (Ed.), *Number and measurement* (pp 101-144). Columbus Publishing.

Küçük A. & Demir, B. (2009). A Study on Some Misperceived Concepts in the Teaching of Mathematics in 6th-8th Grades. *Journal of Dicle University Ziya Gökalp Faculty of Education*, *13*, 97-112.

Lamon, S.J. (2001). Presenting and representing: From fractions to rational numbers.In A. Cuoco (Ed.), Problems of representation in the teaching and learning mathematics (pp. 41-58) Hillsdale, NJ: *Lawrence Erlbaum Associates.*

Margulies, S. (1993). Tips for beginners: Algebraic atrocities. *Mathematics Teacher, 86*(1), 40-41.

Moseley, B. (2005). Student's early mathematical representation knowledge: The effects of emphasizing single or multiple perspectives of the rational number domain in problem solving. *Educational Studies in Mathematics, 60*(1), 37-69.

Moseley, B. J., Okamoto, Y., & Ishida, J. (2007). Comparing US and Japanese elementary school teachers' facility for linking rational number representations. *International Journal of Science & Mathematics Education, 5*(1), 165–185.

Moss, J., & Case, R. (1999). Developing children's understanding of the rational numbers: A new model and an experimental curriculum. *Journal of Research in Mathematics Education, 30*(2), 122-147.

Olkun, S., & Toluk, Z. (2001). *Teaching mathematics in primary education:1-5th grades.* Artım Typography, Ankara, Turkey.

Özmantar, M. F., Bingölbali. E., & Akkoç. H. (Ed.). (2008). *Matematiksel kavram yanılgıları ve çözüm önerileri.* Ankara: Pegem A.

Perso, T. (1992). Making the most of errors. *The Australian Mathematics Teacher, 48*(2), 12-14.

Pesen, C. (2008). Kesirlerin sayı doğrusu üzerindeki gösteriminde öğrencilerin öğrenme güçlükleri ve kavram yanılgıları. *İnönü Üniversitesi Eğitim Fakültesi Dergisi*, *9(15),* 157-168.

Seyhan, S., & Gür, H. (2004). The Misconception of 7th ad Grades Regarding to Decimal Numbers. Association of Mathematicians,http://www.matder.org.tr/bilim/gshg.asp?ID=76.

Shaughnessy, J. M. (1992). Research in probability and statistics: Reflections and directions. In D. A. Grouws, (Ed.), *Handbook of research in mathematics teaching and learning* (pp. 465-494). New York: Macmillan.

Siegler, R. S. et al. 2010. *Developing effective fractions instruction: A practice guide*. Washington, DC: National Center for Education Evaluation and Regional Assistance, Institute of Education Sciences, U.S. Department of Education.

Siegler, R. S. Thompson, C. A., & Schneider, M. (2011). An integrated theory of whole number and fraction development. *Cognitive psychology, 62*, 273–296.

Sirotic, N. & Zazkis, R. (2004) Making sense of irrational numbers : focusing on representation, *in Proceedings of 28th International Conference for Pyschology of Mathematics Education.*

Smith, J. P., diSessa, A. A. and Roscheile, J. (1993). Misconceptions reconceived: A constructivist analysis of knowledge in transition. *The Journal of the Learning Sciences, 3*(2), 115-163.

Stafylidou, S., & Vosniadou, S. (2004). The development of students' understanding of the numerical value of fractions. *Learning and Instruction, 14*, 503–518.

Şandır, H., Ubuz B., & Argün, Z. (2007). 9th grade students' difficulties in arithmetic operations, ordering numbers, solving equations and inequalities. *Journal of Hacettepe University Faculty of Education, 32*, 274-281.

Şiap, İ., & Duru, A. (2004). Skills of using geometrical models in fractions. *Journal of Kastamonu Education, 12*(1), 89-96.

Tirosh, D., Tsamir, P., & Hershkovitz, S. (2008). Insights into children's intuitions of addition, subtraction, multiplication, and division. In A. Cockburn & G. Littler (Eds.), *Mathematical misconceptions.* New Delhi: Sage Publications.

Toluk, Z. (2000). The process of primary sschool students to relate the division with rational numbers. *Bogazici University Journal of Education, 19*(2), 32-57.

Vergnaud, G. (1983). Multiplicative structures. In R. Lesh and M. Landau (Eds). Acquisition of mathematics concepts and processes. 92-126. *New York: Academic Pres.*

Yetim, S. & Alkan, R. (2010). İlköğretim 7. sınıf öğrencilerinin rasyonel sayılar ve bu sayıların sayı doğrusundaki gösterimleri konusundaki yaygın yanlışları ve kavram yanılgıları (Common mistakes and misconceptions of 7th grade students about the rational numbers and placement of the rational numbers on the number line). *Kırgızistan Türkiye Manas Üniversitesi Fen Bilimleri Dergisi, 2*(11), 87-109.

Yetkin, E., 2003. Student Difficulties In Learning Elementary Mathematics. ERIC Digest, *ERIC Clearinghouse for Science Mathematics and Environmental Education.*

APPENDIX: The Diagnostic Test for Probing Students' Understandings of Rational Numbers

Question 1: Write the given rational number below as a decimal number or as an integer fraction

$-\frac{8}{5}=$..................................

Question 2: What is the result of the $\frac{-24}{-3}$ rational number? Is it 8 or -8? Explain.

Question 3: Write true (T) or false (F) next to the following statement. Explain your reasons

Every integer is also a rational number. (…)

Explanation:…………………………………………………………………………………

Question 4: Write true (T) or false (F) next to the following statement. Explain your reasons.

Every rational number can be written as a decimal number (…)

Explanation:………………………………………………………………………………

Question 5: Write true (T) or false (F) next to the following statement. Explain your reasons.

$\frac{4}{0}$ is a rational number. (…)

Explanation:…………………………………………………………………………………

Question 6: Write true (T) or false (F) next to the following statement. Explain your reasons.

Every natural number is also an integer (…)

Explanation:…………………………………………………………………………………

Question 7: Write true (T) or false (F) next to the following statement. Explain your reasons.

Every integer is a rational number with 1 at the denominator. (…)

Explanation:………………………………………………………………………………

Question 8: Find the numbers to replace the **"?"s**.

$-0.5=\frac{?}{?}=\frac{-4}{?}=-\frac{?}{10}$ ……………………………....