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Integrating mathematical and sign languages for problem-solving in economics

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Abstract
This educational experience describes the articulation of sign language with the learning of fundamental calculus concepts within a mathematics course in the economics program at the University of Quindío (Colombia). The study addresses problem situations involving a student with congenital bilateral hearing loss who is supported by a professional interpreter. Specific mathematical representations and conceptual definitions were developed to facilitate the understanding and integration of calculus foundations and their economic applications. This pedagogical practice contributes to inclusive higher education by supporting the learning processes of deaf students and by promoting a problem-solving approach in which mathematics, sign language, and economics are interconnected. The integration of these domains fosters deeper conceptual understanding and strengthens inclusive strategies within university-level mathematics education.

Keywords: economics, sign language, differential calculus, integral calculus, higher education

INTRODUCTION

The process of inclusion for deaf individuals and students with hearing impairment has undergone significant developments in recent years. This progress has made it possible to integrate vulnerable populations—whose connection to the world is primarily visual and based on sign language—into social, political, economic, and labor spheres (Peña Giraldo & Aldana Bermúdez, 2014). The University of Quindío (Colombia) responds to this call for inclusive education by offering non-hearing students collective and institutional recognition through academic support tools, interpreters, and social welfare services that foster their integration into professional training.

As a result, this scenario generates emerging challenges for educators concerning the epistemological and methodological dimensions of didactic transposition, as well as the design of teacher professional training processes (Rezende Lemos, 2019), classroom methodologies, and strategies that enable the articulation of knowledge between the different languages present in academic disciplines and sign language. In this regard, authors such as Zambrano Steensma (2022) and Carrascosa García (2015) point out the lack of adapted educational materials, the scarcity of didactic strategies, the disconnection of classroom projects across communication systems (mathematical, disciplinary, and sign language), and the general lack of awareness regarding the specific needs of individuals with sensory disabilities.

The teaching and learning of mathematics—and specifically the concepts of differential and integral calculus—are subject to the challenges previously described. The complexity of their structure and specific language poses a dual challenge for deaf students, who must incorporate into their cognitive processes the formal mathematical language adapted to their mode of communication (mathematical signs), and subsequently adapt it to their disciplinary field. In this regard, López-Leyton et al. (2019), in a study situated in higher education, identify how the decontextualization of mathematical knowledge and insufficient attention to teachers' beliefs, attitudes, and pedagogical preparation contribute to a fragmented didactic transposition. When the transformation from scholarly mathematical knowledge to taught knowledge is not critically mediated, abstract calculus concepts tend to be presented as closed formal systems, limiting

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Contribution to the literature

- This article uniquely combines sign language with learning math in an economics course, co-creating signs and definitions that link math concepts with their economic applications and addressing the lack of a consolidated lexicon for economic terms in the deaf community.
- It provides an initial repertoire of signs and a teacher-interpreter mediation guide that articulates disciplinary and sign language, strengthening inclusion and learning in higher education.
- It responds to gaps in the literature on the lack of materials and strategies adapted for students with hearing impairments in mathematics and offers economics teachers a replicable classroom tool.

opportunities for meaningful conceptual construction, particularly for students with disabilities. These conditions directly inform the pedagogical and methodological orientation of the present study. By grounding the instructional design in the theory of didactic situations, this research seeks to restructure the didactic transposition of calculus concepts through carefully designed problem situations that promote action, formulation, and validation. This approach aims to overcome epistemological obstacles by fostering semiotic articulation between mathematical symbolism and sign language, thereby enabling the co-construction of knowledge within inclusive university mathematics classrooms.

In this regard, within the articulation of the mathematical structures of calculus in the context of economic sciences, there is no established sign language lexicon that enables deaf students to adopt and adapt key economic concepts such as demand, supply, income, utility, marginality, among others. In conclusion, this didactic experience seeks to answer the following question: *How can sign language be articulated with the learning of fundamental calculus concepts within the context of a mathematics course in the economics program at the University of Quindío (Colombia)?* To address these educational and communicative challenges, this study draws on problem-based learning (PBL), problem-solving (PS), and the theory of didactic situations as complementary frameworks for designing inclusive mathematical learning environments.

CONCEPTUAL ASPECTS

Problem-Based Learning

The theoretical foundations of PBL are rooted in constructivist theories (such as those of Piaget and Vygotsky). This model is based on how students construct new knowledge by integrating it with their existing experiences and mental frameworks. It also draws on adult learning theory, which emphasizes intrinsic motivation and self-direction (Morales & Landa, 2004).

Research has shown that PBL improves long-term retention, reasoning skills, and motivation when compared to more traditional methods (Poot-Delgado, 2013). However, students may initially feel frustrated by the open-ended nature of the approach and the lack of direct instruction. Therefore, PBL requires expert facilitation by the instructor and proper student preparation to maximize its effectiveness.

The typical PBL process involves several steps (Calero, 2021; Padilla & Flórez, 2022):

1. Students are presented with a problem related to a situation involving an unfamiliar concept. This stimulates initial discussion and critical thinking.
2. They identify and clarify the unknown aspects of the problem. They determine what information is needed to solve it.
3. They generate possible hypotheses and solutions. Students discuss ideas and draw upon existing knowledge.
4. They identify learning objectives based on the unknown aspects of the problem. They engage in independent self-study outside of class.
5. They return to the small group to share and discuss their new information. Findings are synthesized and integrated.
6. They re-evaluate the problem using their new knowledge. Solutions are tested, hypotheses refined, and a resolution is reached.
7. They reflect on the process and the knowledge gained. Concepts and principles learned are identified.

Various studies support the effectiveness of PBL. A meta-analysis by Strobel and van Barneveld (2009) found that students in PBL settings demonstrated better reasoning and PS skills than those educated through conventional methods. Other research suggests that PBL improves motivation and attitudes toward learning (López Cuachayo, 2008).

In conclusion, PBL is a strategy that facilitates the development of fundamental skills and deep understanding. However, it requires careful implementation, lesson preparation tailored to different knowledge levels, and a conceptual breakdown of ideas that are articulated through real-life, contextualized situations as a route for their construction. It also depends on the willingness of teachers to move away from traditional classroom schemes toward learning environments grounded in the challenges faced by

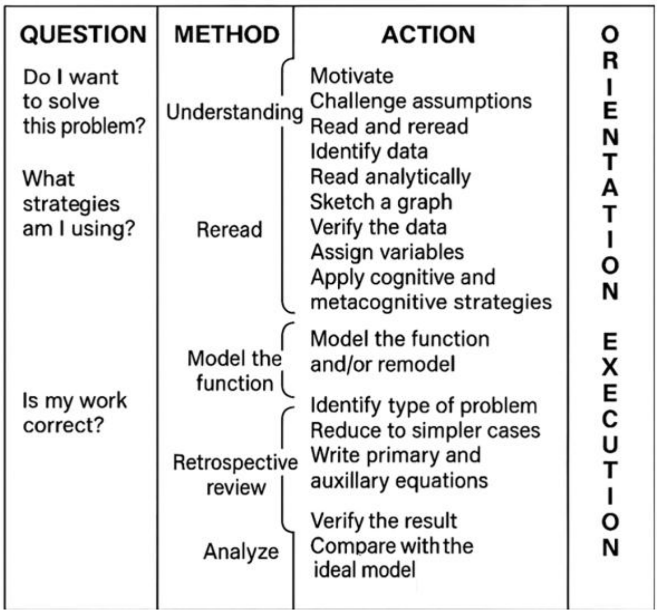


Figure 1. Combination of Schoenfeld's and Polya's methods (Abarca, 2007, p. 26)

societies and communities in achieving educational and scientific development.

The Role of Problem-Solving and Teaching

Research such as that of Abarca (2007) proposes strategies for PS through the combination of methods established by Schoenfeld and Polya, which help students organize their ideas and stimulate critical and analytical thinking. In this regard, a didactic sequence is presented based on Rizo and Campistrous (1999), as cited in Abarca (2007), in which a combination of Schoenfeld's and Polya's methods for PS is implemented as shown in **Figure 1**.

As shown in **Figure 1**, Abarca (2007) proposes a structured sequence to develop and strengthen students' PS skills in calculus. The process can be summarized as follows:

- **A. Problem comprehension:** Read the problem carefully (at least twice) and analyze its components by identifying the given data, conditions, and unknowns. Represent the situation through a diagram and define the relevant variables.
- **B. Strategy formulation:** Determine the type of problem and establish the necessary primary and secondary equations, decomposing it into simpler sub-problems when required. Discard unsuitable approaches.
- **C. Strategy execution:** Model the function appropriately–often by expressing the main function in terms of auxiliary variables–and apply the corresponding properties or theorems (the first derivative test) to obtain the solution.
- **D. Verification and interpretation:** Check the consistency of units and magnitudes, explore alternative solution methods when possible, and interpret the results within the context of the problem.
- **E. Retrospective analysis:** Reflect on the solution process and consider how the resolved problem may inform the solution of similar problems or applications in related disciplines.

Espinoza and Zumbado (2010), citing Brousseau (1986), present several considerations emphasizing the importance of the student's own intellectual engagement in the teaching and learning process of mathematical concepts. They highlight a process in which students carry out tasks involving formulation, verification, inference, and refutation, and share these with their peers through meaningful situations that enable the intuitive construction of the concepts of limit and derivative as they relate to everyday contexts.

Accordingly, contextualized problem-based activities foster motivation, promote collaborative work, diversify instructional strategies, and create opportunities to manage emotions such as frustration or the satisfaction derived from solving a challenging situation. From the teacher's perspective, it is essential to develop a solid understanding of the theoretical foundations of PS, particularly regarding how questioning strategies guide students' reasoning. This facilitates the creation of discussion spaces in which the teacher acts as a moderator rather than as the provider of answers.

Based on these theoretical foundations, the present study operationalizes these principles through a qualitative case study situated in an inclusive higher education mathematics classroom.

METHODOLOGY

This study follows a qualitative case study design framed within the theory of didactic situations (Brousseau, 1986). The research aims to analyze the processes of conceptual construction that emerge during the implementation of problem-based didactic sequences in an inclusive higher education mathematics classroom. The participant is a university student with congenital bilateral hearing loss enrolled in a regular Economics program course alongside hearing peers. The selection of this case was intentional, given the research objective of examining how didactic situations mediate mathematical learning in inclusive contexts.

Data collection was conducted through multiple qualitative instruments, including:

(a) classroom observations documented in field notes,

(b) the student's written production during the action, formulation, and validation phases, and

(c) construction and systematic photographic documentation of the sign repertoire developed for the mathematical objects conceptualized during classroom instruction.

Data analysis followed a qualitative content analysis approach. Episodes of classroom interaction were coded according to the typology of didactic situations proposed by Brousseau (1986): action, formulation, validation, and institutionalization. Emerging categories related to conceptual understanding, use of representations, and communicative mediation were identified through iterative coding. Triangulation was conducted using observational data, written productions, and interview responses to enhance validity and methodological rigor. In addition, validation sessions were carried out with an expert sign language interpreter in order to refine and consolidate the constructed signs associated with the mathematical objects.

To enhance coding reliability, the analytical process was conducted through iterative category revision and triangulation among multiple data sources, including classroom observations, written productions, and interpreter-mediated interactions. Preliminary coding decisions were reviewed and refined through repeated comparisons across data episodes to ensure consistency in category application. In addition, validation sessions with the professional sign language interpreter functioned as an expert review process, strengthening the credibility and interpretative trustworthiness of the semiotic categories constructed.

Additionally, the design of the intervention was structured as a didactic sequence. According to Popayán and Castillo (2017), a didactic sequence is a set of systematically organized activities centered around an oral or written text genre, aimed at facilitating progressive learning through a succession of interrelated tasks. Based on this perspective, the classroom intervention sequences were conceived as ideal trajectories through which the mathematical concept progressively emerged and was constructed by the student. According to Brousseau (1986), there is a typology of didactic situations that support teaching and learning processes; these can be distinguished as follows:

Action situations: These consist of activities in which the student acts upon a medium—that is, direct action on the problem, context, or materials is prioritized. In such situations, each student develops a personal interpretation of the task, formulates hypotheses, tests them, and draws conclusions regarding the solution to the problem based on their own reasoning.

Formulation situations: These involve peer dialogue. This type of collaborative situation emphasizes student-to-student communication in the development of ideas for solving the problem. It enables the acquisition of explicit language, such as expressions and semiotic representations of mathematical objects.

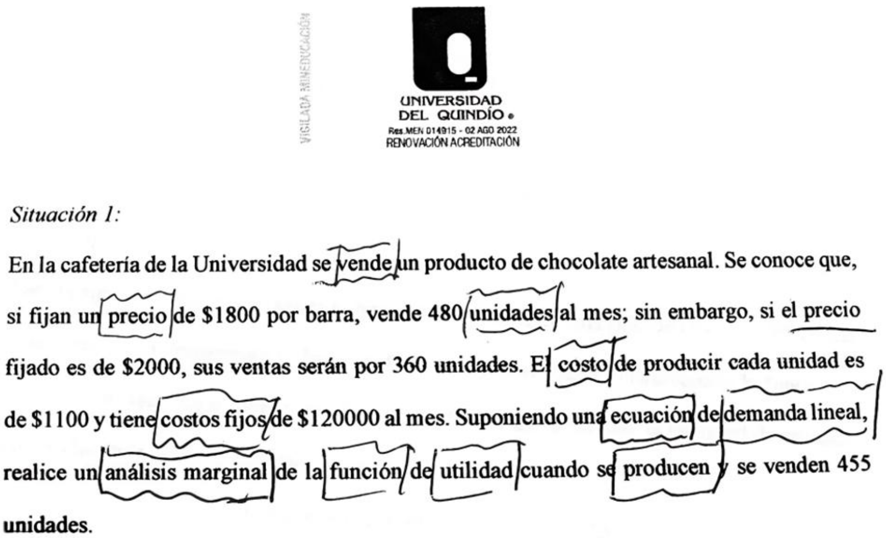
Validation situations: In groups, students can formulate hypotheses and carry them out to test their validity or refute them. In this process, students must use their ideas, justify them, and articulate the procedures that support them. These learning pathways are then presented to other groups, who may accept them, request verification, or challenge them. This dynamic allows each student to apply their mathematical knowledge and share it with others.

Institutionalization situations: These are understood as the final consolidation of knowledge into a language recognized by a scientific community. In this type of situation, the knowledge developed through the previous mathematical activities is acknowledged and formalized. The teacher's role is to connect the students' behaviors and productions with the representations and standards established by the global academic community.

According to Moratalla (1998):

> This research methodology is based on a hypothesis that states that the problems humanity has had to solve in order to reach today's understanding of a given concept are parallel to the problems a current student must overcome to properly grasp that concept. Therefore, the aim is to align the learning process with that historical development, avoiding the mistake of assuming that students follow the same paths taken throughout history or that they must be guided step by step, since present conditions differ greatly from those in the past (p. 234).

In this regard, a lesson plan is developed with a focus on PS to enable the student to rediscover mathematical concepts through personal experience and reasoning within their own processes. The student involved -who is the central focus of this study–is in a regular classroom alongside hearing peers and has congenital bilateral hearing loss. According to Huesca et al. (2022), this condition occurs in approximately one to two out of every thousand live births. It is estimated that a significant proportion–between 60% and 80%–of congenital or prelingual hearing loss cases have a genetic origin. About 70% of these are classified as non-syndromic deafness, characterized by the absence of other clinical findings beyond the hearing impairment. The majority of such cases are caused by pathogenic variants in a single gene, and the inheritance patterns may be autosomal recessive (70-80%), autosomal dominant (10-20%), or X-linked, with an approximate incidence of 1% to 5%. According to Huesca et al. (2022), the complex anatomy of the inner ear involves numerous genes during its development, particularly between the eighth and tenth weeks of gestation. As a result, a wide



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Situación 1:

En la cafetería de la Universidad se vende un producto de chocolate artesanal. Se conoce que, si fijan un precio de $1800 por barra, vende 480 unidades al mes; sin embargo, si el precio fijado es de $2000, sus ventas serán por 360 unidades. El costo de producir cada unidad es de $1100 y tiene costos fijos de $120000 al mes. Suponiendo una ecuación de demanda lineal, realice un análisis marginal de la función de utilidad cuando se producen y se venden 455 unidades.

Figure 2. Students' notes-1 (the authors' own elaboration)

variety of malformations have been observed that can lead to sensorineural hearing loss. When studying congenital sensorineural hearing loss—whether syndromic or non-syndromic—using imaging techniques such as computed tomography and magnetic resonance imaging, it has been found that 20% to 40% of cases present malformations in both the bony and membranous structures, which contributes to the manifestation of the condition.

ANALYSIS AND DISCUSSION OF RESULTS

To address the analytical dimension of the study, student productions (written notes, graphical representations, and responses to formulation questions) were examined through qualitative content analysis. A coding scheme was developed combining:

(a) conceptual categories derived from the didactic variables (marginal analysis, function behavior, and accumulation) and

(b) theoretical constructs from the theory of didactic situations (action, formulation, validation, and institutionalization).

Additionally, an error analysis was conducted to identify epistemological obstacles and procedural inconsistencies. This triangulation of written productions, classroom observations, and interpreter-mediated interactions strengthened the validity of the interpretations. The following section presents the classroom situations designed for the student.

Sequence name 1: *Marginal analysis at my university*

Didactic variable: *Marginal analysis of cost, revenue, and profit functions*

Objectives: The didactic sequence was designed with the following objectives: to construct linear models based on demand relationships; to interpret the slope as a rate of change within a demand context; to formulate equations representing cost, revenue, and profit; and to apply and interpret the differentiation of revenue, cost, and profit functions in order to determine marginal functions.

Classroom management: To implement this sequence, the teacher-researcher must plan for the use of materials such as paper, a video projector, a computer, GeoGebra software, and an adequately equipped classroom. It is also essential to consider the human component-specifically, the students' prior knowledge and the teacher's readiness to guide them through inquiry-based instruction.

Action situation: The teacher presents the following scenario and instructions to the student (**Figure 2**):

Dear student, please read situation 1 and based on your reading, carry out the following activities:

a. Highlight in the text the mathematical and economic concepts involved in the situation.
b. Together with the teacher and the interpreter, determine the appropriate signs for the concepts identified in the previous step.
c. Propose a possible solution to situation 1.

Situation 1 in **Figure 2** can be translated as follows:

Situation 1:

At the university cafeteria, an artisanal chocolate product is sold. It is known that when a price of $ 1,800 COP is set per bar, 480 units are sold per month; however, if the fixed price is $ 2,000 COP, sales drop to 360 units. The cost of producing each unit is $ 1,100, COP and there are fixed monthly costs of $12,000 COP. Assuming a linear demand equation, perform a marginal analysis of the profit function when 455 units are produced and sold.

In section a, the students discussed and highlighted the key concepts they identified in the problem statement.

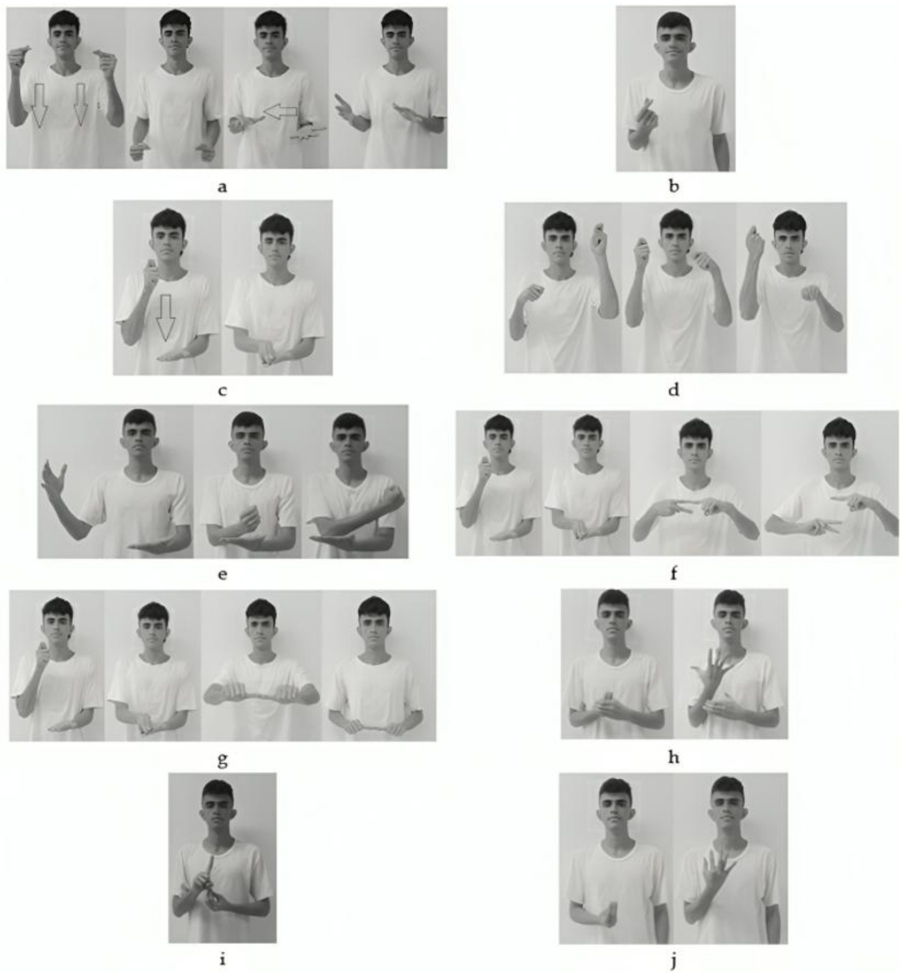


Figure 3. Signs–Situation 1-1 (the authors' own elaboration)

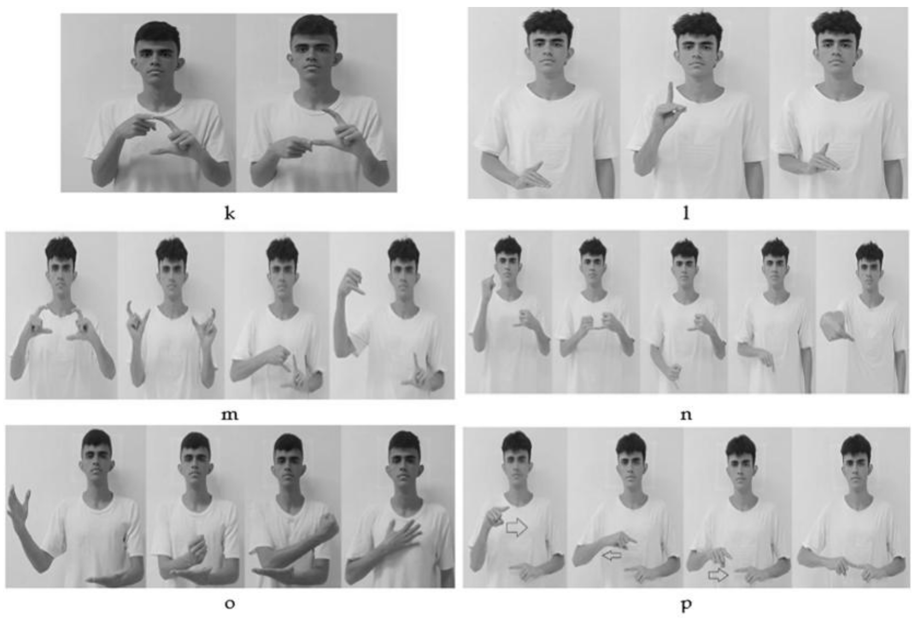


Figure 4. Signs–Situation 1-2 (the authors' own elaboration)

In section b, with the help of the teacher and the interpreter, additional key concepts were identified that required distinctive signs to facilitate communication and reasoning about the situation. The signs in **Figure 3** were established.

The signs in **Figure 4** represent a concept within the context of economic sciences, and they are defined in **Table 1**.

Table 1. Constructed meanings–Situation 1

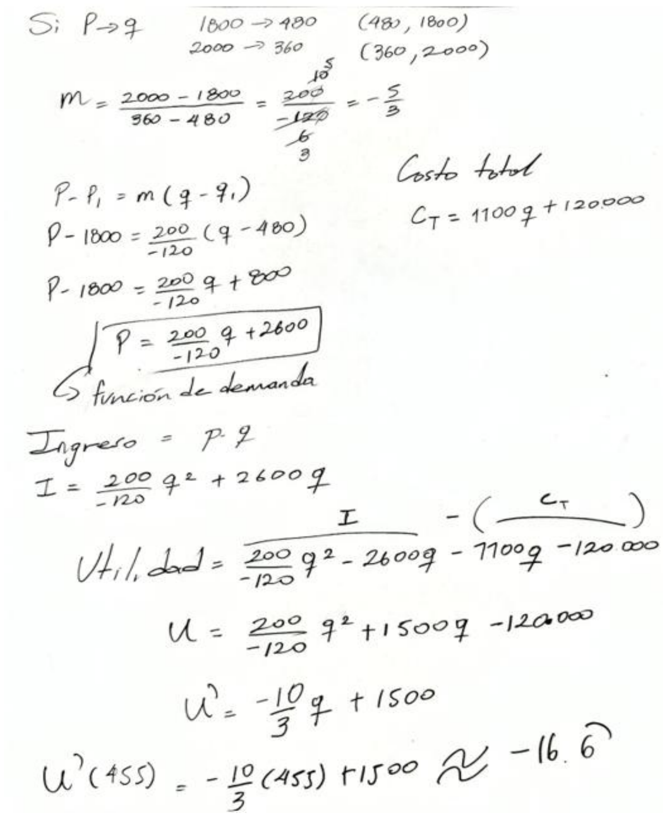
Sign	Definition
a. Average	The term "average" refers to a statistical measure obtained by summing a group of values and dividing the total by the number of elements in that group. The average provides a central numerical indication of the data set, facilitating a general understanding of the typical magnitude of the values.
b. Price	The term "price" refers to the monetary value associated with a good, service, or asset in the market. It constitutes the amount of money required or that one is willing to pay to obtain, use, or access a specific product or service.
c. Cost	The term "cost" refers to the monetary value of the resources that an entity or individual must give up or invest in to produce a good, service, or carry out a particular action.
d. To sell	The term "to sell" is the process through which a seller transfers the ownership of a good, service, or product to a buyer in exchange for compensation, generally in the form of money.
e. Revenue	The term "revenue" refers to the total amount of money that a person, company, or nation receives within a specific period. Revenue originates from various sources and is classified into categories according to its origin.
f. Variable costs	The term "variable costs" refers to the expenses associated with the production of goods or services that change in direct proportion to the level of production. These costs fluctuate according to the number of units produced.
g. Fixed costs	The term "fix cost" refers to expenses that remain constant regardless of the quantity of goods or services produced by a company in the short term. These costs do not vary, even if the company is not generating any production. Typical examples of fixed costs include facility rent, administrative staff salaries, insurance payments, and the depreciation of assets such as machinery
h. To produce (production)	The term "production" refers to the process in which economic resources are combined to create goods and services that meet the needs and desires of society. This process involves the transformation of inputs such as raw materials, labor, and capital into final products or services that have economic value.
i. Unit	The term "unit" is used to refer to an individual quantity or single element of a good or service.
j. Quantity	The term "quantity" is used to describe the measure or extent of goods, services, or resources. This measure can relate to various aspects, such as the number of units produced of a specific good, the number of services provided, or the amount of a resource available in a given context.
k. Equation	The term "equation" is a mathematical expression that establishes a quantitative relationship between different economic variables. These variables may represent different aspects of economic activity, such as revenue, costs, prices, demand, supply, among others.
l. Demand	The term "demand" refers to the quantity of a good or service that consumers are willing to purchase at different prices and conditions, within a specific time period.
M. Linear function	The term "linear function" refers to the mathematical relationship between two variables that produces a straight line on a graph and can be used to model relationships between magnitudes such as revenue, costs, or demand.
n. Marginal	The term "marginal" is used to describe the additional changes or effects associated with one extra unit of a particular variable. This concept is essential in various areas, such as cost, revenue, utility, and productivity.
o. Utility	The term "utility" refers to the measure of satisfaction or benefit that a person experiences when consuming a good or service. Utility is subjective and may vary among individuals as well as in different situations. This concept is crucial in economic theory, especially in microeconomics, where it explores how consumers make purchasing decisions in order to maximize their utility.
p. Simplify	The term "simplify" refers to the process of reducing a mathematical expression to a simpler or more straightforward form, while preserving its equivalence.

In section c, with the assistance of the teacher and through collaborative work among students, the following process for solving situation 1 was established (**Figure 5**).

The student's written production was coded under the category "interpretation of slope as rate of change." The response–"for every 200-peso increase in price, 120 fewer units are sold"–demonstrates a correct covariational interpretation of the derivative in a discrete context. This indicates conceptual understanding rather than procedural reproduction, as the student reformulates the slope numerically and economically.

Furthermore, the distinction made between accumulated profit and marginal profit was classified under the category "differentiation between state function and rate function." This suggests a transition from operational manipulation to structural comprehension of functions, aligning with the didactic variable of marginal analysis.

Formulation situation: At this point in the sequence, several questions are proposed for students to resolve in groups through the argumentation of their processes:


Si P→q   1800 → 480   (480, 1800)
2000 → 360   (360, 2000)

$m = \frac{2000-1800}{360-480} = \frac{200}{-120} = -\frac{5}{3}$

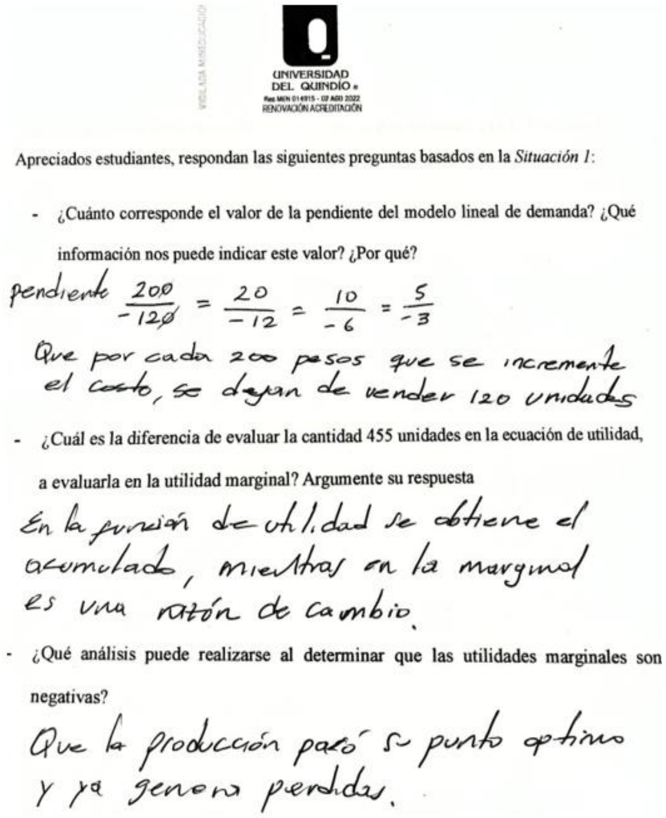
$P - P_1 = m(q - q_1)$
$P - 1800 = \frac{200}{-120}(q - 480)$
$P - 1800 = \frac{200}{-120}q + 800$
$P = \frac{200}{-120}q + 2600$
↳ función de demanda

Costo total
$C_T = 1100q + 120000$

Ingreso = P·q
$I = \frac{200}{-120}q^2 + 2600q$

Utilidad = $\frac{200}{-120}q^2 - 2600q - 1100q - 120000$
$U = \frac{200}{-120}q^2 + 1500q - 120000$
$U' = -\frac{10}{3}q + 1500$
$U'(455) = -\frac{10}{3}(455) + 1500 \approx -16.6$

Figure 5. Students' notes-2 (the authors' own elaboration)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Apreciados estudiantes, respondan las siguientes preguntas basados en la *Situación 1*:

- ¿Cuánto corresponde el valor de la pendiente del modelo lineal de demanda? ¿Qué información nos puede indicar este valor? ¿Por qué?

pendiente $\frac{200}{-120} = \frac{20}{-12} = \frac{10}{-6} = \frac{5}{-3}$
Que por cada 200 pesos que se incremente el costo, se dejan de vender 120 unidades

- ¿Cuál es la diferencia de evaluar la cantidad 455 unidades en la ecuación de utilidad, a evaluarla en la utilidad marginal? Argumente su respuesta

En la función de utilidad se obtiene el acumulado, mientras en la marginal es una razón de cambio.

- ¿Qué análisis puede realizarse al determinar que las utilidades marginales son negativas?

Que la producción pasó su punto óptimo y ya genera pérdidas.

Figure 6. Students' notes-3 (the authors' own elaboration)

- What is the value of the slope in the linear demand model? What information can this value provide? Why?
- What is the difference between evaluating a quantity of 455 units in the profit equation and evaluating it in the marginal profit function? Justify your answer.
- What analysis can be made when determining that marginal profits are negative?

Figure 6 can be translated as follows:

Dear students, please answer the following questions based on situation 1:

- What is the slope value of the linear demand model? What does this value tell us? Why?

Slope: $\frac{200}{-120} = \frac{20}{-12} = \frac{10}{-6} = \frac{5}{-3}$. *This means that for every 200-peso increase in price, 120 fewer units are sold.*

- What is the difference between evaluating 455 units in the profit function versus evaluating it in the marginal profit? Explain your answer.

In the profit function, the result represents the accumulated profit, while the marginal profit is a rate of change.

- What analysis can be made when marginal profits are found to be negative?

It indicates that production has passed its optimal point and is now generating losses.

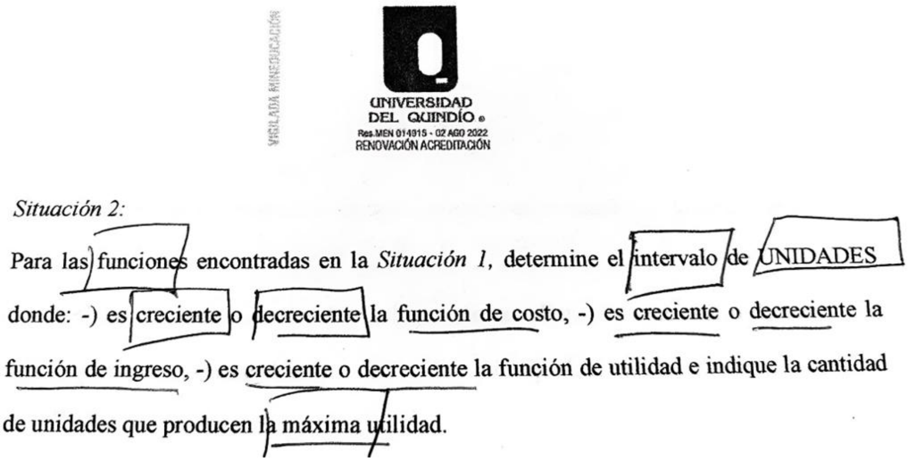
In **Figure 6**, it is possible to determine that the student understands the slope as a rate of change, which, when simplified, conveys the same information in a different measure. Additionally, the student recognizes the difference between two functions-one representing an accumulation and the other a marginal rate of variation. Finally, it is concluded that, in general terms, the student understands that production has generated a maximum profit and is subject to a law of diminishing returns.

Name of sequence 2: *Profit maximization*

Didactic variable: *Analysis of the behavior of cost, revenue, and profit functions*

Objectives: The purpose of this didactic sequence was to guide students in determining a cost function derived from a demand relationship and in analyzing the behavior of cost, revenue, and profit functions. In particular, students were expected to identify intervals of increase and decrease using the first derivative test, determine critical points of a function, and establish maximum or minimum values through the application of the second derivative test.

Class management: For the development of this sequence, the teacher-researcher must anticipate the use of materials such as paper, video projector, computer, GeoGebra software, and a classroom equipped for this purpose. Human resources must also be considered, based on the students' prior knowledge and the willingness of the teacher to guide them through questions.



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Situación 2:

Para las funciones encontradas en la *Situación 1*, determine el intervalo de UNIDADES donde: -) es creciente o decreciente la función de costo, -) es creciente o decreciente la función de ingreso, -) es creciente o decreciente la función de utilidad e indique la cantidad de unidades que producen la máxima utilidad.

Figure 7. Students' notes-4 (the authors' own elaboration)

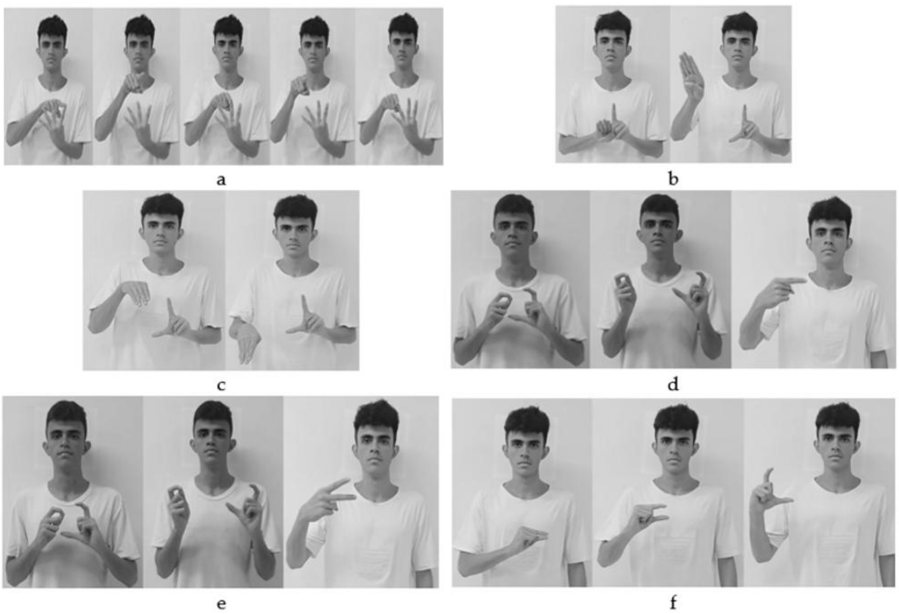


Figure 8. Signs–Situation 2 (the authors' own elaboration)

Action situation: The teacher presents the following situation and instructions for the student:

Dear student, please read situation 2 carefully and, based on your reading, carry out the following activities:

a. Highlight the (mathematical and economic) concepts involved in the situation.
b. Together with the teacher and interpreter, determine the signs (sign language representations) of the concepts identified in the previous step.
c. Proceed to establish a possible solution for situation 2.

Situation 2 in **Figure 7** can be translated as follows:

For the functions obtained in situation 1, determine the interval of UNITS where: -) the cost function is increasing or decreasing, -) the revenue function is increasing or decreasing, -) the profit function is increasing or decreasing, and indicate the number of units that yield the maximum profit.

In item a (**Figure 8**), the students discussed and highlighted the concepts they identified in the statement of the situation:

In item b, with the assistance of the teacher and the interpreter, key concepts were identified that required distinctive signs to enable effective communication and reasoning about the situation.

The signs in **Figure 8** represent a concept from the field of economic sciences, defined in **Table 2**.

In item c, with the guidance of the teacher and through collaborative work among the students, the solution process for *situation 2* was established as shown in **Figure 9**.

In **Figure 9**, the students determine, through the use of inequalities, the intervals where the revenue function increases or decreases. For the cost function, they do not reach a conclusion, as the scenario arises in which the first derivative does not have the necessary structure to solve the inequality. Regarding profit, although a situation similar to that of the cost function occurs, the

Table 2. Constructed meanings–Situation 2

Sign	Definition
a. Interval	The term "interval" refers to a set of real numbers located between two specific values, known as the endpoints of the interval. Interval notation is used to represent these sets and can be closed or open, depending on whether the endpoints are included or excluded.
b. Increasing	The term "increasing" is used to describe a function or sequence that shows an upward trend or increase in its values as the independent variable or index increases. A function or sequence is considered "increasing" if, as one moves along the domain or the sequence, the corresponding values increase.
c. Decreasing	The term "decreasing" is used to describe a function or sequence in which the values decrease as the independent variable or index increases. In other words, a function or sequence is classified as "decreasing" when the corresponding values decrease as one moves along the domain or the sequence.
d. First-order derivative	The first-order derivative of a function represents the instantaneous rate of change of that function with respect to its independent variable, which allows for identifying intervals where the function increases or decreases, as well as its critical points. Put simply, it indicates how the function changes in response to extremely small changes in the independent variable.
e. Second-order derivative	The second-order derivative of a function is obtained by differentiating the first-order derivative. It allows for determining whether a critical point of the original function is maximum or minimum.
f. Maximize	The process of finding the critical point that leads to the maximization of a function; that is, in the context of economic sciences, identifying the values of quantities and prices that yield the highest profit.

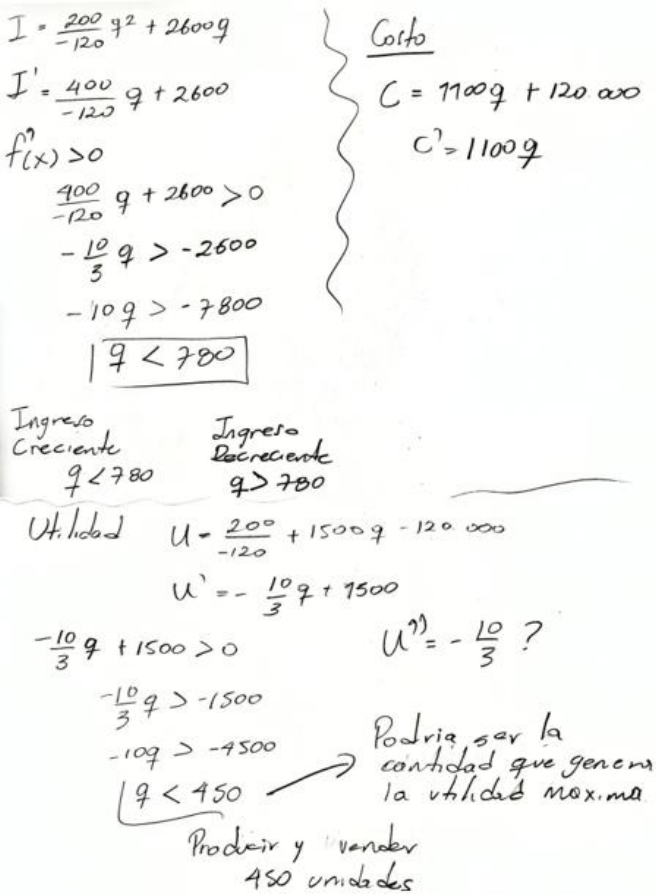


Figure 9. Students' notes-5 (the authors' own elaboration)

students infer that the boundary point causing the shift from increasing to decreasing behavior results in a maximum profit. In situation 2, the students justify some of their processes; however, it becomes necessary to engage in the formulation process to encourage reflection on their procedural reasoning.

Formulation situation: At this stage of the sequence, several questions are posed for students to address in groups by reasoning through their processes:

- What is the result of the first derivative of total costs, and how would you interpret it?
- What is the result of the second derivative of the profit function, and how would you interpret it?
- Could you use another process to find the maximum of the profit function?

Figure 10 can be translated as follows:

Dear students, please answer the following questions based on situation 2:

- What is the result of the first derivative of the total cost function, and how would you interpret it?

$C' = 1{,}100q$ *I believe that in this case, it is not possible to determine the behavior using the first derivative test.*

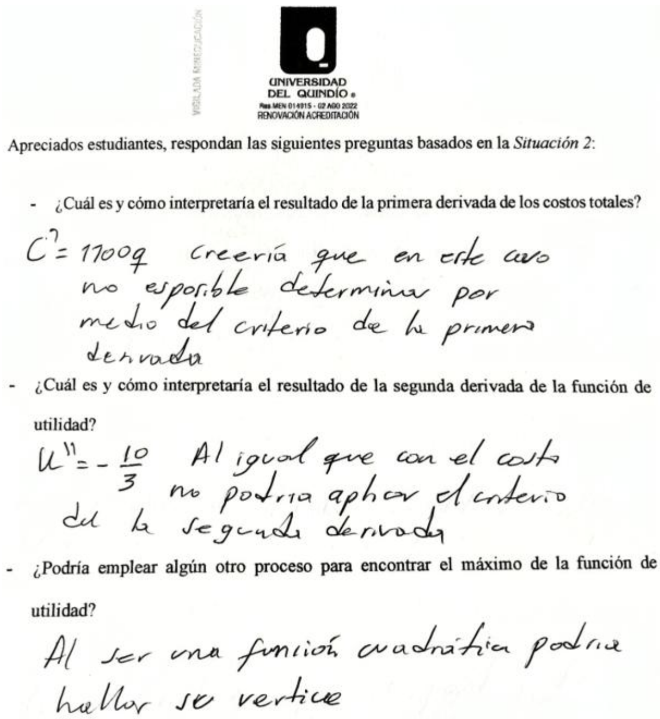
- What is the result of the second derivative of the profit function, and how would you interpret it?

$U'' = -\frac{1}{30}$ *As with the cost function, the second derivative test may not be applicable here.*

- Could any other method be used to find the maximum of the profit function?

Since it is a quadratic function, its vertex can be used to find the maximum.

In **Figure 10**, it is evident that the student, lacking an explicit equation, tends to believe that the criteria do not apply. This case was discussed during the institutionalization process of the first derivative criterion. In the following question, the student displays the same difficulty. Finally, in the last question, the student draws on prior knowledge by recognizing that the maximum of a quadratic function can also be found using its vertex.



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Apreciados estudiantes, respondan las siguientes preguntas basados en la *Situación 2*:

- ¿Cuál es y cómo interpretaría el resultado de la primera derivada de los costos totales?

$C' = 1100q$ creería que en este caso no es posible determinar por medio del criterio de la primera derivada

- ¿Cuál es y cómo interpretaría el resultado de la segunda derivada de la función de utilidad?

$U'' = -\frac{10}{3}$ Al igual que con el costo no podría aplicar el criterio de la segunda derivada

- ¿Podría emplear algún otro proceso para encontrar el máximo de la función de utilidad?

Al ser una función cuadrática podría hallar su vertice

Figure 10. Students' notes-6 (the authors' own elaboration)

The student's assertion that the first derivative test "is not applicable" was coded under the category "misinterpretation of symbolic generality." This response reveals an epistemological obstacle: the student associates derivative criteria exclusively with explicit algebraic expressions, failing to recognize that qualitative analysis can be conducted from structural properties of functions. This difficulty evidences a partial institutionalization of the derivative as an analytical tool, reinforcing the need to strengthen the didactic variable related to functional behavior analysis.

Name of sequence 3: *Revenue projection*

Didactic variable: *Applications of the integral in economic sciences*

Objectives: The objective of this didactic sequence was to promote students' understanding of rates of change and accumulation processes within a calculus framework. Specifically, students were expected to identify functions representing rates of change, analyze the effects of accumulation on such rates, interpret the area under a curve as an accumulation function, and determine the integral of a rate-of-change function as a representation of accumulated quantity.

Class management: For the development of this sequence, the teacher-researcher must anticipate the use of materials such as paper, video projector, computer, GeoGebra software, a classroom equipped for this purpose, and human resources aligned with the students' prior knowledge and the willingness to guide them through questioning.

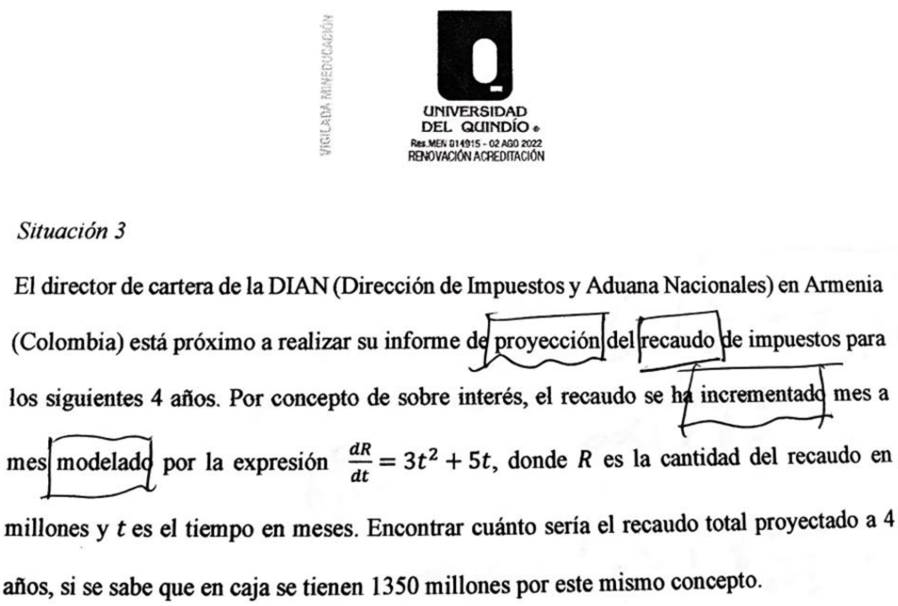
Action situation: The teacher presents the following situation and instructions for the student:

Dear student, read situation 3 carefully and, based on your reading, carry out the following activity:

d. Highlight in the text the concepts (both mathematical and economic) involved in the situation.
e. Together with the teacher and the interpreter, identify the signs corresponding to the concepts found in the previous step.
f. Proceed to establish a possible solution to situation 3 in collaboration with the teacher.

In item a, the students discussed and highlighted the concepts they had identified in the statement of the situation.

Figure 11 can be translated as follows:



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Situación 3

El director de cartera de la DIAN (Dirección de Impuestos y Aduana Nacionales) en Armenia (Colombia) está próximo a realizar su informe de proyección del recaudo de impuestos para los siguientes 4 años. Por concepto de sobre interés, el recaudo se ha incrementado mes a mes modelado por la expresión $\frac{dR}{dt} = 3t^2 + 5t$, donde R es la cantidad del recaudo en millones y t es el tiempo en meses. Encontrar cuánto sería el recaudo total proyectado a 4 años, si se sabe que en caja se tienen 1350 millones por este mismo concepto.

Figure 11. Students' notes-7 (the authors' own elaboration)

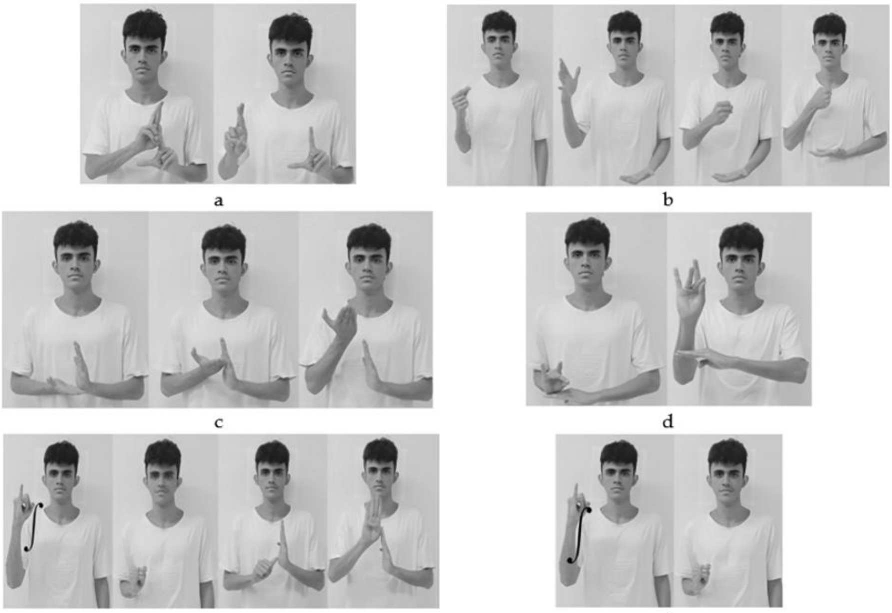

Figure 12. Signs–Situation 3 (the authors' own elaboration)

Table 3. Constructed meanings–Situation 3

Sign	Definition
a. Projection	The term "projection" refers to the estimation or forecast of economic variables based on analysis and modeling. Economic projections are essential tools used by economists, analysts, and planners to anticipate the likely behavior of various variables in the future.
b. Revenue collection	Refers to the total income obtained through tax collection or other funding sources by a government. It represents the amount of money a government—whether at the federal, state, or local level—receives from various sources to fund its expenditures and programs.
c. Increase	The term "increase" refers to the rise or growth in the value of an economic variable or quantity. It can be applied in various contexts, such as growth in the production of goods and services, employment levels, or price levels (inflation), among others.
d. Modeling	Refers to the process of using mathematical structures and concepts to represent, describe, and analyze real-world phenomena. The purpose of mathematical modeling is to create a model—a mathematical abstraction or symbolic representation that captures the essential characteristics of a system or situation.
e. Definite integral	In economics, the definite integral is a mathematical tool used to calculate accumulation or the total amount accumulated over a specific interval.
f. Integral	The term "indefinite integral" suggests the idea of continuous accumulation of certain economic quantities over time, although its use does not necessarily imply the formal mathematical structure associated with an indefinite integral in calculus.

The portfolio director of DIAN (National Directorate of Taxes and Customs) in Armenia (Colombia) is about to present his tax revenue projection report for the next four years. Due to interest surcharges, tax revenue has increased month by month, modeled by the expression $\frac{dR}{dt} = 3t^2 + 5t$, where R is the amount of revenue in millions, and t is time in months. Determine he total projected revenue over four years, knowing that 1,350 million pesos are currently available under this category.

In item b in Figure 12, with the support of the teacher and the interpreter, additional necessary concepts were identified, each requiring a distinctive sign to enable effective communication and reasoning regarding the situation.

The signs in Figure 12 represent concepts from the context of economic sciences, which are defined in Table 3.

In item c, with the guidance of the teacher and through collaborative work among the students, the following solution process for situation 3 was established (Figure 13).

Figure 13 can be translated as follows:

Problem-Solving Strategy

- Identification of given data
- Revenue projection over 4 years (or 48 months)
- Revenue modeled by $\frac{dR}{dt} = 3t^2 + 5t$
- 1,350 billion in reserve

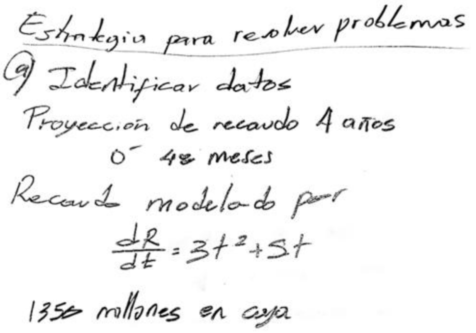

Figure 13. Students' notes-8 (the authors' own elaboration)

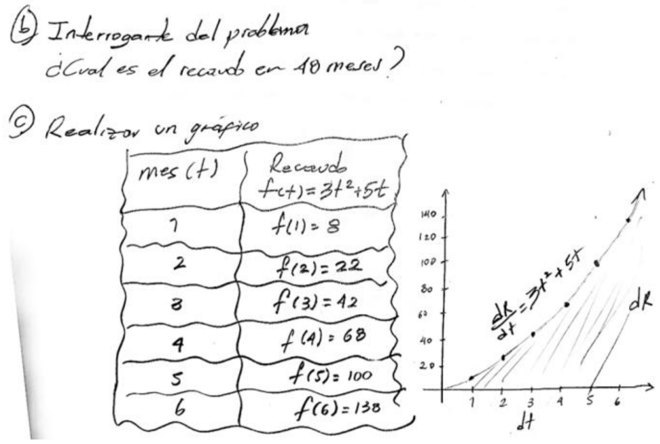

Figure 14. Students' notes-9 (the authors' own elaboration)

Students' solution strategies in **Figure 14** and **Figure 15** were analyzed under the category "integration of rate and accumulation reasoning." The explicit identification of 48 months as the integration interval and the interpretation of the integral as total revenue accumulation indicate a conceptual shift from interpreting *R(t)* as a static value to understanding it as a rate of change. In the case of the deaf student, collaborative interactions mediated through newly constructed signs were analyzed as instances of semiotic negotiation. The emergence of shared signs for "definite integral" and "projection" evidences the co-construction of mathematical meaning within a bilingual (sign-written language) environment.

To enhance methodological rigor and make explicit the analytical procedure followed in this study, a system of categories was constructed based on the didactic variables of each sequence and the theoretical constructs of the theory of didactic situations. Student written productions, PS strategies, and responses during formulation phases were coded according to these categories. **Table 4** summarizes the main analytical categories, the corresponding empirical evidence extracted from student data, and the interpretative inferences derived from the analysis. This framework makes explicit the connection between classroom data, theoretical constructs, and claims regarding student learning outcomes.

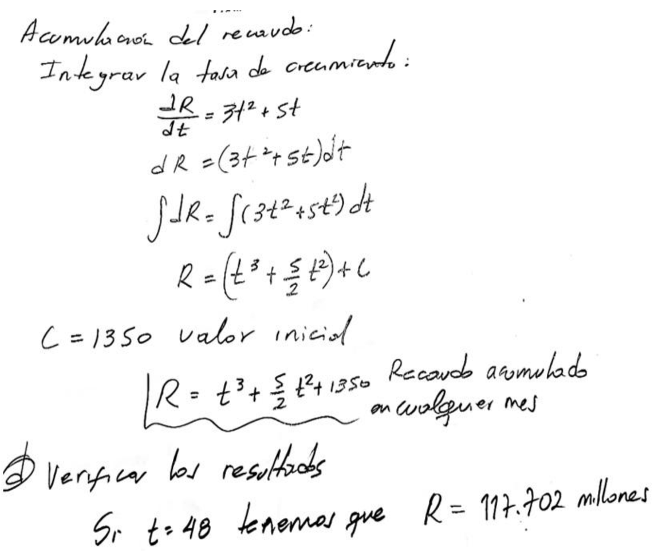

Figure 15. Students' notes-10 (the authors' own elaboration)

CONCLUSIONS

This study contributes to the field of mathematics education by empirically demonstrating that the theory of didactic situations can effectively support inclusive mathematics learning in higher education economics contexts. The findings provide evidence of conceptual development across action, formulation, validation, and institutionalization phases, particularly in relation to marginal reasoning, functional behavior, and accumulation processes, while also identifying epistemological obstacles, especially in the generalization of derivative criteria, that clarify how didactic variables shape mathematical meaning construction. In this way, the study responds to the needs identified by Zambrano Steensma (2022) and Carrascosa García (2015).

From a theoretical perspective, the study extends the application of the theory of didactic situations by integrating semiotic mediation within a bilingual (sign-written language) learning environment. The negotiated construction of disciplinary signs functioned as a structuring element of the didactic milieu, revealing that inclusion is not merely an accessibility measure but a transformation of the epistemic configuration of the classroom.

Methodologically, this research offers a structured analytical framework linking didactic variables, phases of the didactic situation, qualitative coding categories, and empirical evidence of learning. By positioning contextualized PS as a mechanism through which students inductively construct conceptual understanding grounded in their immediate environment, the study reinforces the approach proposed by López-Leyton et al. (2024) and strengthens the interpretative rigor of case-based research in inclusive mathematics education.

Table 4. Qualitative content analysis framework

Didactic variable	TDS phase	Analytical category	Empirical evidence	Type of analysis	Interpretation
Marginal analysis	Formulation	Interpretation of slope as rate of change	“For every 200-peso increase … 120 fewer units are sold”	Conceptual coding	Demonstrates covariational reasoning and economic interpretation of derivative
Marginal analysis	Formulation	Differentiation between accumulated and marginal function	Written comparison of profit vs marginal profit	Conceptual coding	Indicates structural understanding of function vs. rate
Function behavior	Validation	Misinterpretation of derivative test	“It is not possible to determine …”	Error analysis	Reveals epistemological obstacle regarding symbolic generality
Profit maximization	Institutionalization	Use of alternative strategy (vertex)	Reference to quadratic vertex	Transfer analysis	Shows activation of prior knowledge beyond procedural routine
Accumulation (integral)	Action/ formulation	Recognition of integral as accumulation	Identification of 48-month interval	Conceptual coding	Evidence of shift from static to dynamic interpretation of function
Semiotic mediation	Action	Construction of new mathematical signs	Creation of sign for “definite integral”	Semiotic analysis	Demonstrates co-construction of meaning in bilingual environment

Overall, the findings suggest that inclusive mathematical modeling in economics education can be understood not only as a pedagogical adaptation, but as a reconfiguration of the didactic system itself.

Future research should extend this line of inquiry both theoretically and methodologically. From a longitudinal perspective, it would be valuable to examine the durability and transferability of marginal and accumulation reasoning beyond the immediate didactic intervention. Tracking students overtime could provide insight into whether the institutionalization of derivative and integral concepts results in stable structural understanding rather than context-dependent performance. Comparative research involving both deaf and hearing students may further illuminate the role of semiotic mediation in mathematical meaning-making. In particular, future studies could investigate whether the negotiated construction of disciplinary signs not only facilitates accessibility but also generates epistemic advantages in conceptual abstraction. Such research would contribute to ongoing discussions on epistemic justice in mathematics education by examining how bilingual (sign-written language) environments reshape the didactic milieu.

From a methodological perspective, subsequent studies could adopt mixed-methods designs to complement qualitative coding with quantitative measures of conceptual change. Pre- and post-intervention assessments, covariational reasoning tests, or modeling competence scales would allow for triangulation and strengthen claims regarding learning outcomes. Expanding the sample across multiple institutions would also enhance the external validity of the analytical framework proposed in this study. Additionally, the epistemological obstacles identified–particularly the misinterpretation of symbolic generality in derivative criteria–warrant focused investigation. Experimental studies manipulating didactic variables (e.g., explicit structural analysis versus procedural emphasis) could examine how instructional design influences students' ability to generalize functional behavior analysis.

Finally, extending this framework to more advanced mathematical contexts -such as multivariable optimization, dynamic economic models, or differential equations–would test the robustness of the proposed didactic configuration and explore how inclusive mathematical modeling evolves in increasingly complex epistemic settings.

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Ethical statement: This study reports a classroom-based educational experience conducted within the regular teaching activities of the economics program at the University of Quindío (Colombia). As it did not involve medical, clinical, or psychological interventions, formal approval by a bioethics committee was not required. Participation was voluntary, and written informed consent was obtained from the participant for the use and publication of photographs, classroom productions, and educational materials. All information was handled confidentially, and appropriate measures were taken to protect the participant's privacy.

AI statement: Generative AI tools were used exclusively for language editing, grammar correction, and improvement of writing style. The authors reviewed, validated, and assumed full responsibility for all the content presented in this manuscript.

Declaration of interest: No conflict of interest is declared by the authors.

Data sharing statement: Data supporting the findings and conclusions are available upon request from the corresponding author.

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