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Integrating Online Diagnostic Tests in a First Year Engineering Class

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Abstract

The narrative permeating higher education institutions globally is the assimilation of advancing information and communications technology into mainstream Mathematics Education. In this paper we report on a mixed mode case study which explored possible mathematical gaps that created barrier/s when engineering students (n=162) worked with the online diagnostic tests. These engineering students were provided with online tasks on the learner management system (LMS) on factorization, fractions and logarithms. The study was carried out at a University of Technology in South Africa. The data collected from the LMS and written responses of students to quizzes were analysed. Findings emanating from the data analysis indicated that 1) all diagnostic tests logged positive increase in group averages, 2) greater repetition by students invoked better procedural proficiency and 3) better understanding of the basic mathematics leads to greater success in higher mathematics.

Keywords: diagnostic, blended learning, procedural proficiency

INTRODUCTION

Mathematics as a science of patterns in numbers, space, logic or elsewhere is applied in all fields of engineering and is entrenched in all engineering curricula. Mathematics is a requirement for entry into most engineering studies at universities. Carmody (2006, p. 25) contends that ‘the content of an Engineering Mathematics course is a standard entry level Calculus Course’. Mathematics knowledge and its application forms an integral part of engineering courses offered from the first to higher levels and often ‘share a set of compulsory subjects in the first and second years’ (Blanco & Ginovart, 2010, p. 352). Engineering subjects at second and higher levels of study require mathematics as a pre-requisite limiting progression of a student failing at the lower levels and emphasising the importance attached to the inherent cognitive skills acquired when studying the subject. Its relevance and importance is further emphasised by the American Accreditation Board for Engineering and Technology (ABET) which states that ‘engineering is the profession in which knowledge of the mathematical and natural sciences is applied with judgement to develop ways to utilize economically the materials and forces of nature for the benefit of mankind’ (ABET, 2000). Locally the Engineering Council of South Africa (ECSA) links engineering to the economy stating,

‘Engineering is the practice of science, engineering science and technology concerned with the solution of problems of economic importance and those essential to the progress of society. Solutions are reliant on basic scientific, mathematical and engineering knowledge’ (ECSA, 2018). Mathematics is the language for expressing physical and engineering laws and thus mastery of the subject is indispensable for interpreting abstract engineering phenomena that cannot physically be observed. Both bodies list the ability to apply knowledge of mathematics as a first exit level outcome for engineering students.

The rationale for the study was conceptualised when the author acted as the principal researcher of an e-learning project in the Mathematics department at a University of Technology (UOT) initiated by the university. The objective of the preliminary study known as the Pathfinder Project implemented in the second semester of 2014 was primarily to establish the extent to which e-learning was used in all programmes across the university and to provide resources to encourage staff in all departments to launch some form of online presence in their courses. The project laid the

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Contribution to the literature

- This study found that using online diagnostic tests was beneficial in that: They logged a positive increase in group averages for the second and third attempts; The high standard deviation for the fraction diagnostic quiz further justified previous analyses that first year engineering students procedural proficiency in manipulating fractions is a cause for concern; Greater repetition by student attempts led to a consolidation of their procedural proficiency.
- We found that by strengthening the procedural proficiency in the basic maths topics led to more success in problem solving in calculus mathematics topics. We observed this when student S104 demonstrated the technique he gathered from fractions when dealing with a calculus problem in a major test.
- Thus, this study has illustrated how diagnostic testing enforced a positive impact ameliorating gaps in pre-calculus mathematics of first year engineering students including algebraic factorisation, fractions and logarithms.

foundation of this study and hence occupies an integral part of it.

The terms 'blended learning, 'e-learning', 'm-learning', 'online learning' and other related terminology have recently become accepted usage in higher education circles these days. Reid-Martinez and Grooms (2018, p. 2591) recognise that 'these new electronic forms of communication have forced a paradigm shift in education. This move is most avidly seen in distance-learning, where even the terminology has shifted from distance education to words such as online or e-learning'. The internet as a conduit and platform for learning is widely adopted by stakeholders at all levels in the education sector. Schools at the primary and secondary level, tertiary institutions, adult based colleges, distance education colleges, etc. all include some form of internet related content to achieve their educational objectives.

The e-learning regime is vast and expanding at a rapid rate. As a result, educational practitioners have to rethink their teaching strategy from a variety of perspectives. Gikandi (2011) laments, 'Online and blended learning have become common place in 21st century higher education. Larreamendy-Joerns and Leinhardt's (2006) review of the literature "observed two complementary movements in the educational landscape: the merging of online teaching and learning into the stream of everyday practices at universities, and the increasingly salient role of distance programmes in institutions of higher education" (Gikandi, 2011, p. 572). Technology as an enabler of the learning process cannot be ignored in a millennial society that is addicted to the 'internet of things' and social media. The integration of Information Communication Technologies (ICT) in education has over the decades spawned a range of e-learning possibilities ranging from fully online courses to web based interventions that become more important for completing, complementing and supporting the traditional teaching methods.

With this in mind we formulated the following research question:

How can we integrate online diagnostic tests in a first year Calculus class in order to mitigate the relevant mathematical knowledge gaps and skills required for problem solving?

LITERATURE REVIEW

Poor performance in mathematics at all levels of schooling in South Africa has been observed by a mounting body of indicators, assessments and data. The poor performance is attributable to a host of factors. As a result, many students entering higher education are unprepared for the rigorous and abstract mathematical demands encountered at universities. Jaffar, Ng'ambi and Czerniewicz (2007, p. 3) argue that

> 'In a country such as South Africa, for instance, school-leaving certification had a particularly unreliable relationship with higher education academic performance especially in cases where this certification intersects with factors such as mother tongue versus medium of instruction differences, inadequate school backgrounds and demographic variables such as race and socio-economic status'.

Reasons for the under preparedness of mathematical students are numerous including socio-economic woes, low quality and under qualified school mathematics teachers, ill-informed decision making authorities' influence in determining curricular and pressure to meet the social transformation and skills of the new South Africa (Engelbrecht & Harding, 2015). A brief outline of the performance and obstacles that plague mathematics education pertinent to the study will be discussed. This will be accomplished inter alia by considering the various phases that a learner encounters in schooling, mathematics teachers in school, the quality of mathematics taught as well as the guidance given by the education authorities.

Mathematical numeracy begins at an early age when children are exposed to some form of counting principles. Dominated by socio-economic factors in a post-apartheid South Africa, a tale of two streams of learners' education emerge beginning at the pre-school level. Schools are classified into groups known as quintiles according to their socioeconomic status. Van Wyk (2014, p. 149) explains: 'South African schools are divided into five categories (quintiles) based on the socio-economic status of the community in which the school is situated. Quintile 1 schools are the poorest, while quintile 5 schools are the least poor'. The wealthiest government and government subsidised schools appear in the highest classification viz., quintile five. Quintile one represents schools that are situated in areas inhabited by the least fortunate, the population with the lowest income and most challenging social conditions. The difference in lower and higher quintile schools is reflected in the education system. The vast majority of learners attend government aided schools classified in quintiles 1 and 2 (Van Wyk, 2014) situated in both urban as well as rural areas. The impact on mathematics education is tangible. Various studies and national tests confirm the poor mathematics competence of our students from an early level in their schooling. The performance of grades five and nine in the 2015 'Trends in International Mathematics and Science Study' is an indication of poor mathematical knowledge of learners (TIMMS, 2015). This study was conducted internationally and placed South Africa's grade nine learners 38th of 39 countries whilst grade five learners were placed 47th out of the 48 countries that participated (TIMMS, 2015). The respected World Economic Forum (WEF) ranked the quality of South Africa's maths and science education last out of 148 countries (Schwab, 2015). The stagnation and gradual deterioration of the quality of mathematics education in South Africa is systemic. A 2012 study by a local University found that of the 71% of children in grade six who were functionally literate, only 58.6% were considered functionally numerate (Van der Berg et al., 2011). Fixing such a system will require a monumental effort on the part of all stakeholders.

Another aspect of teaching and learning that contributes to learner deficiencies in mathematics is the lack of development of all strands of mathematical proficiency (Ally & Christiansen, 2013). Surveys, including TIMMS, underpin the view that teaching of mathematics throughout the world emphasise procedure in mathematics classrooms (Stigler, Gallimore, & Hiebert, 2000). Even at high school level where the formal concepts of algebra, trigonometry and geometry are introduced, there is a preference of procedure over the other strands. Locally, Engelbrecht, Harding and Potgieter (2005, p. 1) claim, 'the general perception is that high school teaching of mathematics in South Africa tends to be fairly procedural'. This was corroborated by Ally and Christiansen's (2013, p. 106) observation of grade 6 mathematics lessons that suggests 'opportunities to develop procedural fluency are common, but generally of a low quality'. They contend that emphasis on procedural skills continues as the main theme in mathematics classrooms especially so in the foundation grades and there is a low prevalence of conceptual understanding and a virtual absence of opportunities to promote adaptive reasoning. These traits that a mathematically proficient learner should possess according to the formulation of 'mathematical proficiency' as termed by Kilpatrick et al (2001) are mostly absent in learners of mathematics in South African schools. Progression through the school system of mathematics learners equipped with limited mathematical reasoning skills constricts their capacity to resolve problems when they engage in cognitively more demanding mathematics. Mechanical learning of rules and algorithms proliferate school mathematics in the United States of America. Studies, including the 2nd International Mathematics Study (McKnight et al., 1987) and the 5th National Assessment of Educational Progress (Mullis, Dossey, Owen, & Phillips, 1991) verify the emphasis of procedures in mathematics classrooms.

Further problems are encountered when learners are required to choose subjects upon entering the senior secondary school phase, viz. grade 10. The attrition rate during this transition from grade 9 to grade 10 is alarming. The number of learners progressing from grade 9 to grade 10 that are allowed to do pure mathematics is reduced significantly amplified by the attrition rate of learners between grade 10 and grade 12. Van Wyk (2014, p. 161) notes, 'for example, there were 1 055 790 Grade 10 enrolments in 2011, but only 528 845 Grade 12 enrolments two years later in 2013 – roughly half'. Despite this selection process learners in the grade 10-12 band still lack fundamental mathematical skills making basic errors and displaying poor conceptual understanding. The diagnostic report of the 2017 exit examination, the National Senior Certificate (NSC), state that candidates show poor algebraic skills, lack fundamental and basic mathematics competencies which could have been acquired in lower grades' (Department of Basic Education, 2018b).

THEORETICAL FRAMEWORK

Hourigan and O'Donoghue (2006, p. 461) argue that 'mathematical proficiency of graduates is increasingly cited as a major factor in a country's economic success and competitiveness, especially graduates emanating from the numerate disciplines such as Science, Engineering, Technology and Business'. The expectation of learners in mathematics classroom is delivery of content that will primarily lead to understanding. Frith and Prince (2009, p. 83) citing Steen (1999) contend that 'the practice of numeracy in people's work, education and daily lives has assumed increasing importance in the last few decades'. Teachers should deliver mathematics lessons of such a nature that learners ultimately achieve these goals. Provision of opportunities that develop mathematical skill should prevail in classrooms. Pursuing effective mathematics learning paved the way for capturing a term that embodies this aspiration. The suggestion of the term 'Mathematical Proficiency' (Kilpatrick, Swafford, & Findell, 2001), incorporating five strands, was proposed capturing the goals of mathematics learning. The characterisation of the term "Mathematical Proficiency, defining it in terms of five interwoven and interdependent strands to be developed in concert" (Kilpatrick *et al*., 2001, p. 106), has become an accepted construct globally. The five intertwining strands include: *Conceptual understanding, Procedural fluency, Strategic competence, Adaptive reasoning and Productive disposition.*

Conceptual understanding refers to a "grasp of mathematical ideas, its comprehension of mathematical concepts, operations and relations" (Kilpatrick *et al*., 2001, p. 115). Merging facts and methods is effortless when mathematical ideas are grasped with deep understanding. Discerning the relationship between ideas and topics is demonstrated in the application, use and representation of concrete and semi-concrete mathematical patterns. Conceptual knowledge forms the foundation of mathematics connecting and relating mathematical ideas. According to Hiebert and Lefevre (1986), conceptual knowledge is achieved in two ways: by "the construction of relationships between pieces of information" or by the "creation of new information that is just entering the system" (Hiebert & Lefevre, 1986, p. 46). Kilpatrick *et al.* (2001) maintain that learners possessing conceptual knowledge preserve mathematical ideas and knowledge without difficulty and recall and remember methods effortlessly.

Procedural knowledge involves the manipulation of mathematical skills to solve problems. Rules, algorithms and formulae are merged with formal language following step by step procedures. Mathematical tasks are completed systematically, choosing the correct rule or algorithm, following steps precisely and accurately resulting in flawless solutions. Kilpatrick *et al.* (2001) conceived the term procedural fluency for this strand of proficiency referring to it as "knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them accurately, flexibly and efficiently" (Kilpatrick *et al.,* 2001, p. 121).

Strategic Competence is described as "the ability to formulate, represent and solve mathematical problems. It is similar to what is generally called problem solving and problem formulation. Students need to encounter situations in which they need to formulate the problem so that they can use mathematics to solve it" (Kilpatrick *et al*., 2001). Students should have exposure to problem solving tasks and construction throughout their school career. When confronted with problems, strategic competence requires students to initiate problem-solving strategies, sift between efficient strategies and thereafter implement the most effective approach to arrive at a solution. An element of conceptual comprehension as well as a degree of procedural fluency is vital in the characteristics of strategic competence.

Adaptive reasoning is a key component of mathematical proficiency holding the others together. The importance of this strand is highlighted by the statement, "In mathematics, adaptive reasoning is the glue that holds everything together, the lodestar that guides learning" (Kilpatrick *et al*., 2001, p. 129). Logical reasoning, illustrations and explanations, justification and argumentation are manifestations of mathematical applications that embody the essence of optimum mathematical thinking. Adaptive Reasoning is the "capacity for logical thought, reflection, explanation, and justification" (Kilpatrick *et al*., 2001, p. 115). Descriptions of procedures, justification of mathematical ideas or reasoning during calculation forms the basis of mathematical understanding and learning. Mathematics is not just an accumulation of facts, rules and formulae. Associating mathematical ideas or representations requires thought, logic, reasoning and justification. These notions of adaptive reasoning connect, supports and holds the other strands together.

The final strand is productive disposition. Motivation is a key factor shaping learners' perspective towards mathematics. Learners who habitually see the need for mathematics in their daily lives develop a positive outlook towards the subject. The application of mathematics in realistic environments fuels the tendency to view mathematics in a positive light. Productive disposition is the "habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy" (Kilpatrick *et al*., 2001, p. 131). Understanding mathematical content is stimulated by learners' desire and willingness to learn. Constructive motivation and perceptions of mathematics as beneficial and worthwhile encourage interested and productive learners. Learners feelings about their ability to accomplish personal goals in mathematics is considered rewarding and purposeful. They exhibit confidence when confronted with the mathematics and appreciation in its application. Personal motivation drives learners to engage with mathematical activity.

Kilpatrick *et al.* (2001, p. 116) posit that learners who possess mathematical proficiency should ideally display evidence of all five strands. Opportunities to develop all aspects of mathematical proficiency should prevail in mathematics classrooms. Exposing learners in classrooms that provide opportunities to develop procedure, encourage conceptual understanding, advance problem-solving skills, reinforce reasoning and justification as well as improve motivation must be promoted. Kilpatrick *et al.* (2001) claim that 'greater understanding of mathematics will be essential for today's school children. Success in tomorrow's job market will require more than computational competence. It will require the ability to apply

Table 1. Pre-Calculus Diagnostic Quiz Test feature information

Quiz	Days open	Questions per level (L).	Time limit (min)	No. of Attempts	Question layout
Factorisation	5	L1 – 10 L2 – 3 L3 -2 Tot - 15	30	3	Shuffled Randomly Deferred Feedback
Algebraic fractions	11	L1 – 5 L2 – 4 L3 - 1 Tot -10	40	3	Shuffle within questions
Logarithms	11	L1 – 10 L2 - 10 Tot–20	45	3	

N.B. Tot – indicates the total number of questions in the quiz test

mathematical knowledge to solve problems… [Students] need to have the mathematical sophistication that will enable them to take full advantage of the information and communication technologies that permeate our homes and workplaces. Students with a poor understanding of mathematics will have fewer opportunities to pursue higher levels of education and to compete for good jobs' (Kilpatick *et al*., 2001). The rapid technological advances in the world with concomitant transformation of employment opportunities require learners to possess the desired mathematical proficiency. The emphasis on procedural mathematics learning at lower levels of schooling affects the cognitive development of students.

This study will refer to the five strands of mathematical proficiency. The nature of the study is such that the incorporation of and the simultaneous advancement of all five strands may be difficult to implement. However, by focussing on developing individual strands, it is envisaged that the eventual outcome would be a student that is mathematically proficient.

METHODOLOGY

The current study embraces mixed method approaches in data collection. Mixed methods research involves the collection of both quantitative and qualitative data and the combination of the strengths of each to answer research questions. (Creswell, 2011). For the quantitative data we have used statistical comparisons from integrated diagnostic tests followed by the capture of data from the online LMS. For the qualitative aspect we used actual student written responses to identify gaps or strengths in their learning of the mathematical concepts in the particular topics.

The introductory segment of traditional university mathematics lectures begins by eliciting the knowledge needed to progress to new topics. It is unclear whether the necessary prior knowledge is understood or recalled. Online diagnostic testing may provide a solution in recalling such knowledge. Additionally, it affords the student the opportunity to ascertain the depth of previous mathematical understanding and necessary pre-requisite skills. Accordingly, details of the instrument used and strategy implemented to obtain quantitative results follow.

DIAGNOSTIC TEST INSTRUMENT

Before embarking on statistical analysis it would be prudent to inspect the features of the quizzes to ascertain the reasons for the alternatives selected. Suitable multiple-choice questions were formulated and placed in the LMS's data bank that could be used for diagnosing first year engineering students' basic mathematical skills and knowledge. The formulation and design of a taxonomy that framed the composition of diagnostic questions of basic mathematics for first year engineering students will be explored in a subsequent paper. Questions were placed in three different levels, namely, level 1 (L1) to level 3 (L3). Timing, pacing and sequencing of the quizzes incorporated into traditional teaching and learning were crucial in making a pedagogical impact. Three pre-calculus diagnostic quiz tests were compiled during the course of the research period. Table 1 displays the information of the diagnostic quizzes indicating the topic tested, number of days open, the number of questions in each test, the number of attempts that were allowed and the number of questions appearing per page of the quiz when opened by the student.

All quizzes in the above table were uncategorised and the highest attempt was selected for grading purposes. In addition quizzes were designed to generate random questions resulting in a positional change every time the question appeared in later attempts. Deferred feedback was the preferred choice. Variation in opening and closing times was due to students' suggestions that emerged during unstructured interviews citing workload and other personal issues that arose at different times in the semester. Test duration of the diagnostic quiz tests varied according to the pedagogical principles envisioned for each quiz as well as the number of questions per level per quiz. The logarithmic quiz, containing ten level-1 and ten level- 2 questions, was set

Table 2. Statistical Comparison of Pre-Calculus Quiz tests

Quiz name	Logarithms	Algebraic Fractions	Factorisation	Basic Mathematics	Assign 02
Number of first attempts	98	90	111	159	157
Total number of complete attempts	169	140	192	159	256
Average grade of first attempts %	73.47	67.00	72.81	77.27	80.35
Average grade of all attempts %	78.64	68.68	75.97	77.27	86.12
Average grade of last attempts %	83.61	72.25	78.69	77.27	95.82
Average grade of highest graded attempts %	83.2	74.01	80.36	77.27	95.94
Median grade %	85.00	70.00	84.44	80.00	100.00
Standard deviation %	15.09	17.51	15.30	14.68	8.19
Skewness	-2.282	-0.181	-0.645	-1.536	-3.159
Kurtosis	9.077	-0.845	-0.538	3.513	12.563

N.B. Median, standard deviation, skewness and kurtosis are calculated for the highest graded

to the longest time of 45 minutes. Three attempts were allowed for all quizzes and the number of questions displayed per page depended on the total number of questions in the quiz which allowed for easy viewing of questions.

CODING

A total of 172 Chemical and Civil Engineering students agreed to participate in the online diagnostic tests. The study was carried out at a South African University of Technology. In this research a chronological coding approach was applied allocating codes to all participants after arranging them alphabetically. S1 was applied to student 1 of the alphabetical list, S2 to student 2 and repeated until the last student. In this way participants were coded from S1 through to S172.

The student coding process adopted facilitated the link to students' performance in quiz tests with the items that they incorrectly selected and their subsequent results on similar items in later attempts. Consequently, the coding assisted in identifying written work in these assessments and linking this to items in the diagnostic test that students needed to apply to successfully answer questions. The coding adopted ensured that collection and dissemination of data proceeded in a manner that protected confidentiality of the data and the identity of the participants.

RESULTS AND DISCUSSION

Diagnostic Test Performance – A Comparative Analysis

Chronicled throughout the research is the emphasis of proficiency in pre-calculus topics and understanding of basic mathematics as the foundation of proficiency in tertiary engineering mathematics. Diagnostic tests are not summative and thus students' performances are not expected to parallel that of first year engineering students in final examinations. On the contrary, elevated performances are expected by all students as a reflection of their mathematics proficiency and understanding. With this tenet as lens of analysis, a discussion of the general statistics for the pre-calculus diagnostic quiz tests initiates this section. Discussion of the cohort as a single group unfolds with a statistical comparison of group performance in the pre-calculus diagnostic quizzes appearing in Table 2.

Two additional quiz tests are included for analysis as part of the pre-calculus online assessments conducted with the research group. The basic mathematics quiz test was a single isolated test with no repeated attempts administered at the commencement of the semester and will feature intermittently in the dissection of the statistics. In the second additional quiz test, assignment 02, participants had a second opportunity to improve their score. The diagnostic ethos of the study precluded allocating a passing mark for any of the tests. Achieving the maximum scores for every quiz was possible if students possessed the mathematical proficiency in the tested areas. The expectation of averages for the graded endeavours was above 80%. This expectation was not achieved on all first attempts.

All diagnostic tests logged positive increases in group averages for the second and third attempts testimony to the constructivist view whereby learners are encouraged to take ownership of the learning process. This view is endorsed by the statement 'learners are conceptualised as active mathematical thinkers, who try to construct meaning and make sense for themselves of what they are doing, on the basis of their personal experience' (Shuard, 1986, p. 179). New concepts are acquired and built or reconstructed from previous ones. Constructivist principles embody previous knowledge, ownership of the learning process, internalisation of concepts and reconstruction of previously erroneous understanding.

The pre-calculus quiz tests was not compulsory and not all students attempted them. The fractions diagnostic test recorded the lowest number of attempts where 90 of the 159 students attempted the quiz. 98 and 111 students attempted the logarithm and fractions tests respectively. The lowest average mark of 67% of initial attempts was logged by the quiz testing procedural aspects of fractions. Students' performance was below standard. Algebraic manipulation of fractions is introduced in the early grades of high school. Students are exposed to simplification and algebraic manipulation of expressions containing fractions from grade 10. Despite 50 more tries of the fractions quiz, the difference in averages between first and highest attempts was 7.55% indicating continued difficulty with this quiz. Positive increases between highest graded and first attempts were 9.77%, 7.55%, and 15.19% for the logarithmic, factorisation, and assignment 02 quizzes respectively. Table 2 also displays statistical information for the last and all attempts for each quiz. However, closer inspection of the statistics for the last attempt and the highest graded attempt does not show significant dissimilarity. Analysis of changes between first and last or first and all attempts will only be reviewed where necessary due to the selection of the highest graded attempts for all statistical metrics in Moodle.

Central measure and dispersion metrics included the median and standard deviation. The highest graded attempts documented a median value of 100% in the assignment 02 diagnostic quiz. The median value of 100% demonstrates that half of the highest graded attempts recorded correct answers to all questions in each of these quizzes. The lowest median of 70% was recorded for the highest graded attempts of the fractions quiz. The value consolidates previous analysis of students' mathematical proficiency or lack thereof to navigate simplification and manipulation of algebraic fractions. The basic mathematics diagnostic test posted a median value of 80% indicating that students continue experiencing some difficulty with basic school mathematics concepts. Standard deviation values were reasonable and varied from a low of 8.19% for assignment 02 to a high of 17.51% for the fractions quiz. This value is understandable in the context of the intention when setting this quiz. The researcher intended leveraging online assignments by allowing three attempts where the highest graded would be used in the calculation of a course mark, albeit minimal. Statistics for the test suggest that the intention was achieved with a high mean value, a median value of 100% and the lowest standard deviation where students' marks did not differ significantly from the mean. A t-Test sample for the means of the logarithmic, fractions, factorisation and assignment 02 quiz tests generated a p-value of 0.034, less than the 0.05 significance level, indicating moderate evidence in favour of differences between first and highest attempts. However, the high standard deviation for the fraction diagnostic quiz further justifies previous analyses that first year engineering students procedural proficiency in manipulating fractions is a cause for concern.

The two measures of distortion consolidate and support the view that high marks are expected in these diagnostic tests. The negative values noted for skewness of the data points in the tests indicate that more students are performing better than average. The analysis of the fraction quiz test and its corresponding statistics parallels that of a normal summative class test suggesting that considerable student effort and lecturer intervention is necessary to alter, rectify and improve students' proficiency in the manipulation of fractions. Analysis of the kurtosis values is included for completion and will not be analysed in detail.

Statistical results and analysis continues with the focus per diagnostic quiz test.

Statistical Analysis per Pre-Calculus Diagnostic Test

In this section focus shifts from group performances to individual performances in each diagnostic assessment. The previous analysis contrasted overall statistics for the entire group of participants per diagnostic quiz. Further analysis progresses by applying an adaptation of Ally (2017) approach as a lens for discussion. The recommendation of sifting data and statistics into group and sub-groups followed by finer inspection of individual performances in diagnostic quiz testing analyses is embraced, modified and carried throughout the remainder of the chapter. Sub-groups are compiled according to students' performances in each quiz per number of attempts. The corresponding sub-groups identified included:

a. Three attempts – students that attempted a quiz three times

b. Two attempts – students that attempted a quiz twice

c. One attempt – students that attempted a quiz once only

Results are dissected, analysed and contrasted with the group statistics for each test which is repeated per quiz for methodical analysis throughout this section. Quizzes where only a single try or greater than three tries were set will be analysed appropriately.

Discussion, deliberation and review of students' individual attempts will surface in all the diagnostic quiz tests. However, selection of students' attempts will be done strategically to consolidate, emphasise, debate or examine their solutions relative to the narrative for that particular aspect of the quiz. Viewing responses in all quiz efforts that every student completed would be an undertaking beyond the aim and purpose of the research. Hence, identification of students' attempts will be sort in instances that coalesce with the rationale of the deliberation and analysis. The Moodle data base for the course allowed access of students' performance in all assessments. We now consider the discussion of data topic-wise.

FACTORIZATION

Statistical information for the factorisation quiz appears in Table 3. The raised averages for all the attempts are a feature of the diagnostic nature of this research. Ideally, all students should record maximum marks for basic school factorisation questions. The

Table 3. Factorisation quiz - Three attempts

Student	1st attempt Max 15	%	2nd attempt Max 15	3rd attempt Max 15	Highest	Highest %	Difference
S1	14	93.33	14	15	15	100	1
S2	9	60	10	11	11	73.33	2
S11	8	53.55	15	15	15	100	7
S40	14	93.33	10	11	14	93.33	4
S47	14.33	95.33	14	13	14.33	95.53	-1.33
S55	12	80	15	15	15	100	3
S76	12	80	11	13	13	86.67	2
S97	14	93.3	13	14	14	93.33	1
S99	13.67	91.13	15	14	15	100	1.33
S104	7.67	51.13	7	7.33	7.67	51.13	0.67
S118	14	93.33	14	13	14	93.33	1
S135	13	86.67	14	11	14	93.33	3
S137	9.33	62.2	10.67	12	12	80.00	2.67
S143	13.33	88.87	14	15	15	100	1.67
S164	9	60	10	9.67	10	66.67	1
S172	8.33	55.53	8	11	11	73.33	3
Average	11.6	77.33	12.2	12.5	13.1	87.33	1.5

results suggest that many students initially could not recall all factorisation procedures. Increasing averages for later attempts are indicative of students resolve in improving proficiency in factorisation ability. The average grade of the 1st try was 72.8% increasing incrementally until the highest grade registered an average of 80.36%. The median grade of 84.44% for the highest attempt is encouraging intimating a student cohort in touch with the ability to factorise algebraic expressions.

Table 3 displays the marks obtained by the 16 students who attempted the quiz three times where each quiz had a maximum mark of 15. Results showing decimals indicate that answers in matching type questions each containing 3 sub-questions were incorrectly answered.

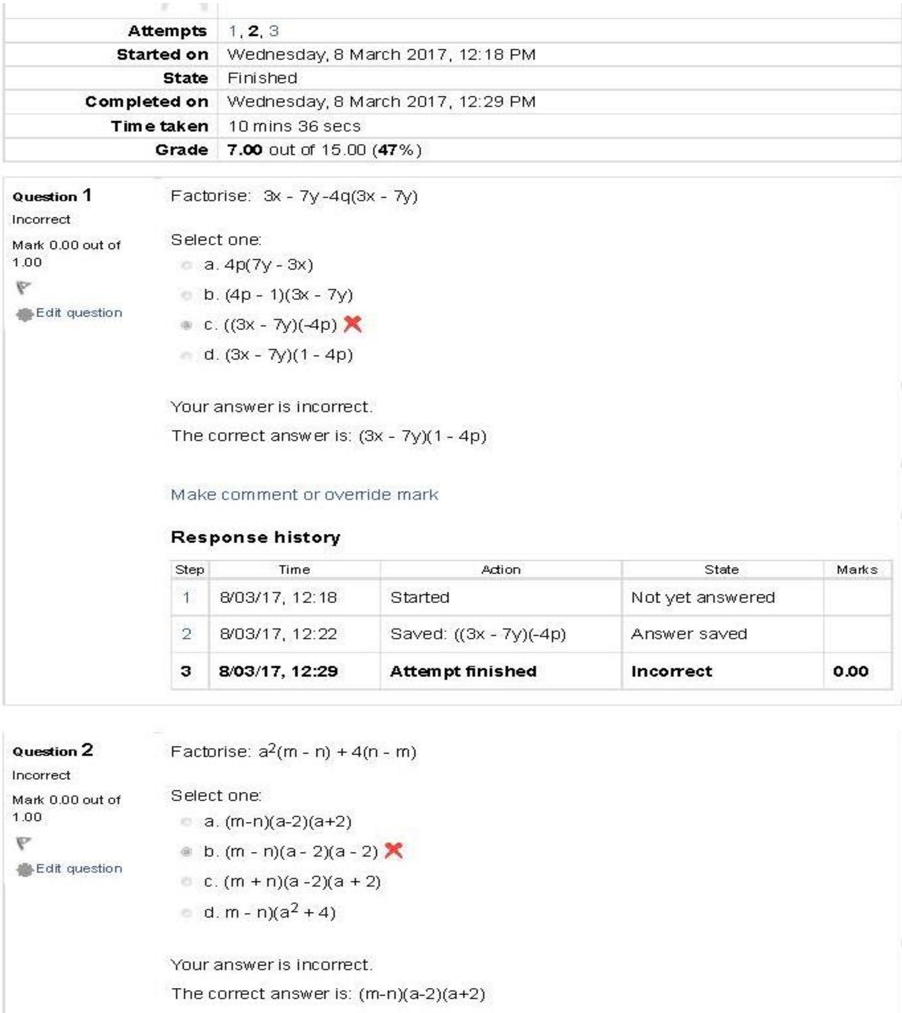
Averages increased from 11.6 at the first attempt to 12.2 in the second and 13.1 for the highest graded attempt and five of the 16 students answered all questions correctly in at least one of their three attempts. The average mark for students that attempted this quiz three times exceeded the group averages in all categories of attempts. For instance, an average of 77.3% was recorded in the first attempt of this sub-group as opposed to 72.8% for the entire group. Six students from this cohort scored below the group average for the first try. For the highest graded attempt, all except four students achieved better than that noted for the average of 80.36% for the entire group.

Individual Reviews

S11 recorded the largest increase initially selecting 8 of the 15 items correct but subsequently achieved 100% for the two later attempts. Many students such as S55 and S76 recorded results above 80% in their first try increasing the number of correct answers in later attempts. S47 posted the only negative difference between 1st and 3rd attempts. Inspection of the results indicates grades in excess of 85% for all efforts. Three students showed minimal increases from lower initial attempts unlike S104 who consistently performed poorly in all three attempts accounting for the smallest increase in this sub-group suggesting a lack of proficiency in basic school factorisation procedures and inability to recall the variety of factorisation methods taught at school. Figure 1 displays questions 1 and 2 of S104's 2nd attempt. In all three tries, the student provided incorrect answers for three of these level-1 type questions significantly affecting the results. Analysis of the selection indicates an algebraic error in question 1 where omission of remaining terms after removal of a common factor occurs and a procedural mistake in question 2 when factorising the difference of square expression after correctly changing a numerical coefficient of a binomial factor illustrated here:

$$a^2(m-n)+4(n-m)$$
$$= a^2(m-n)-4(m-n)$$
$$=(m-n)(a^2-b^2)$$
$$=(m-n)(a-b)(a-b), \text{incorrect factorisation} - \text{DC4}$$

Student S104's inability to correct the errors could be due to a variety of reasons and debating these here would be pure speculation. Suffice it to infer that S104 did not display the commitment and resolve to alter his mathematical processes when confronting similar problems despite obtaining feedback identifying his incorrect choices. It is likely that corrective measures requires external intervention to dynamically change and correct the mathematical construct of sign changing when pivoting binomial terms around negative signs. Analysis of solutions in major tests that S104 rendered, hint at greater awareness and attempts to rectify these mistakes. Figure 2 shows the differentiation of an implicit function provided in a major test. The solution provides evidence of procedural fluency of S104



Attempts	1, **2**, 3
Started on	Wednesday, 8 March 2017, 12:18 PM
State	Finished
Completed on	Wednesday, 8 March 2017, 12:29 PM
Time taken	10 mins 36 secs
Grade	**7.00** out of 15.00 (**47**%)

Question 1
Incorrect
Mark 0.00 out of 1.00
Edit question

Factorise: 3x - 7y -4q(3x - 7y)

Select one:
a. 4p(7y - 3x)
b. (4p - 1)(3x - 7y)
c. ((3x - 7y)(-4p) ✗
d. (3x - 7y)(1 - 4p)

Your answer is incorrect.
The correct answer is: (3x - 7y)(1 - 4p)

Make comment or override mark

Response history

Step	Time	Action	State	Marks
1	8/03/17, 12:18	Started	Not yet answered	
2	8/03/17, 12:22	Saved: ((3x - 7y)(-4p)	Answer saved	
3	**8/03/17, 12:29**	**Attempt finished**	**Incorrect**	**0.00**

Question 2
Incorrect
Mark 0.00 out of 1.00
Edit question

Factorise: a^2(m - n) + 4(n - m)

Select one:
a. (m-n)(a-2)(a+2)
b. (m - n)(a - 2)(a - 2) ✗
c. (m + n)(a -2)(a + 2)
d. m - n)(a^2 + 4)

Your answer is incorrect.
The correct answer is: (m-n)(a-2)(a+2)

Figure 1. S104 - Attempt 2

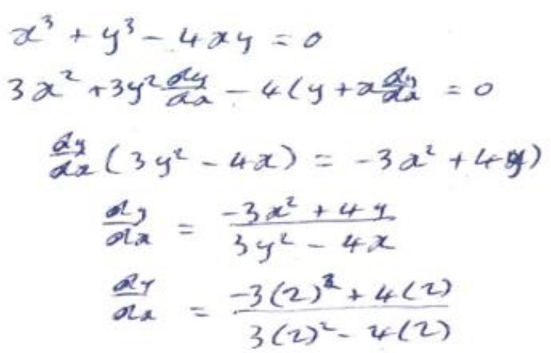


Figure 2. S104 Major test solution

correctly using distribution of terms, removing common factors with negative coefficients and transposing terms efficiently within the context of calculus related topics. The student removed brackets to create the differential $\frac{dy}{dx}$ as a common factor. The tactical approach means that strategic competence (Kilpatrick *et al.*, 2001) played an important role here. The student had the ability to formulate a new situation in order to factorise the expression. This meant that the student acquired knowledge by mental construction and experiences with the e-learning environment. This conforms to the general view of constructivism (Olusegan, 2015).

ALGEBRAIC FRACTIONS

Statistics for the diagnostic quiz testing students' ability in simplifying basic numerical and algebraic fractions is found in Table 2. This test recorded the lowest averages for the various repetitions amongst all diagnostic tests. The average grade of 67% for first attempts is particularly concerning for students since calculus topics such as integration techniques requiring partial fractions or solving differential equations via Laplace transform methods necessitate conceptual understanding and manipulation of algebraic fractions. The exceptionally high value of the error ratio, 71.39%, suggests that the variation in the data is a result of many students guessing answers, a direct consequence of the high standard deviation registered by this quiz. A value of 70% for the median grade of the highest attempts falls far short of the expectations for first year engineering mathematics students. A minimal increase of 5.25% was logged between first and last attempt further indicating lack of conceptual understanding and procedural fluency in this topic.

Table 4. Fractions Quiz - Three attempts

Student	Attempt 1 Max 10	Attempt 2 Max 10	Attempt 3 Max 10	Highest	Difference 1st and 3rd
S1	6	7	9	9	3
S3	9	8	5	9	-4
S11	7	8	10	10	3
S40	7.5	7	6	7.5	-1.5
S52	4	7.5	8	8	4
S70	9.5	9	10	10	1
S76	9	10	8.5	10	1.5
S85	6	5	6	6	1
S122	6	6	3	6	-3
S168	7	6.5	6	7	-1
S121	3.5	7	6	7	3.5
S164	2	6.5	5	6.5	4.5
Average	6.4	7.3	6.9	8.0	2.6

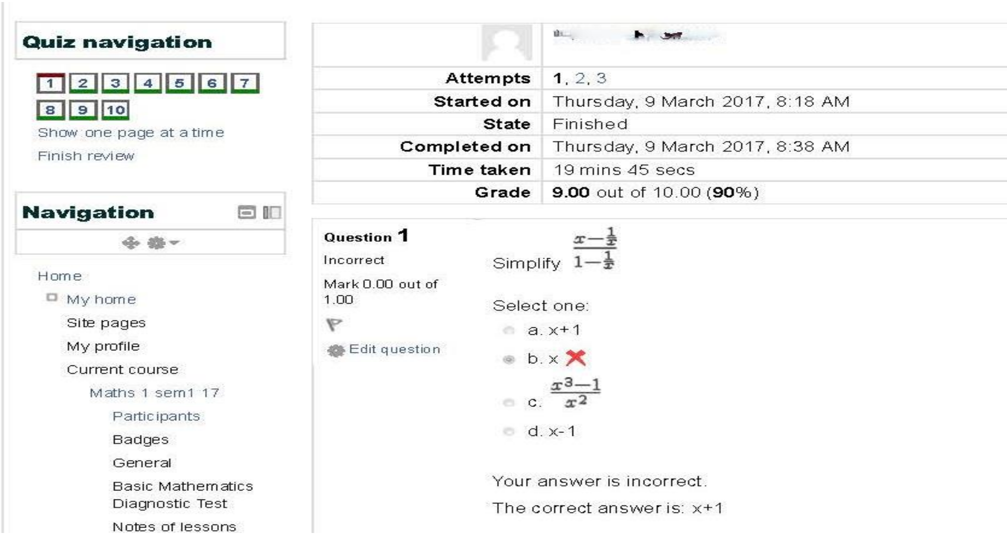


Figure 3. S3 Fractions level 3

Attempts

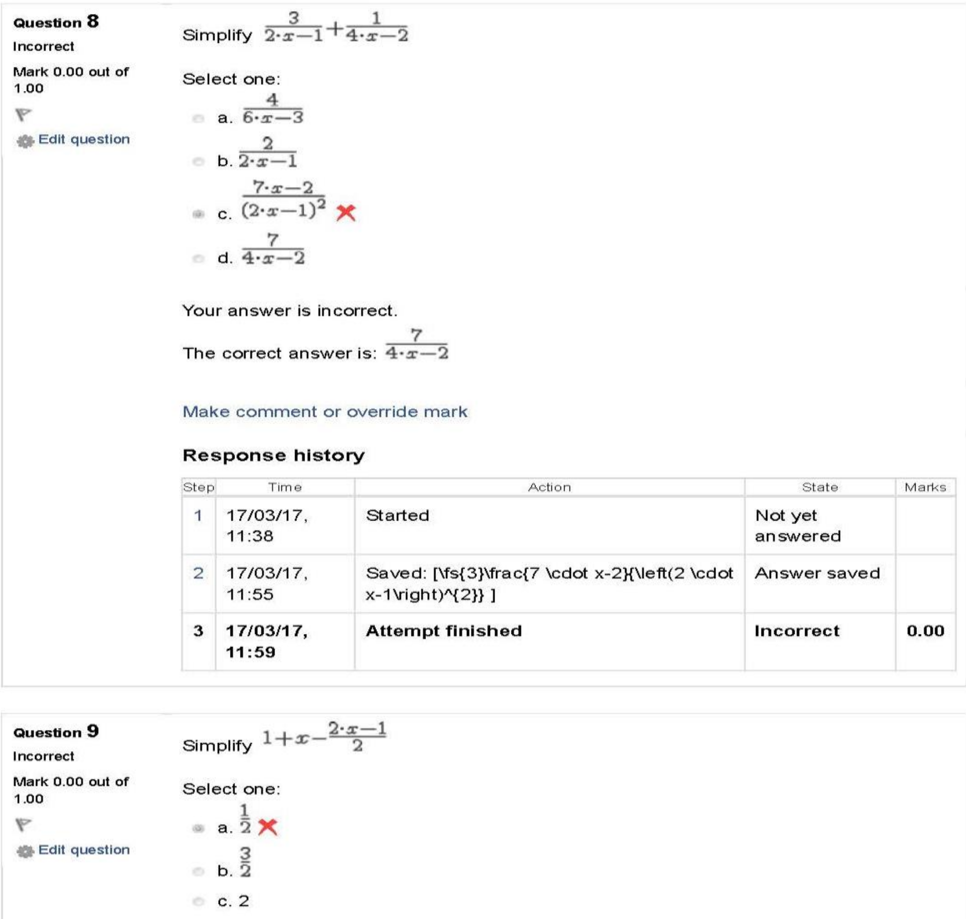
Table 4 shows the students that attempted the fractions quiz three times.

None of the 12 students that attempted this quiz three times scored 100% in their first effort yet only 1 level 3-type question was added to this quiz. The average of 64% for this sub-group was lower than the average for all students that attempted the quiz for the first time, viz. 67%. Although the average of the second and highest attempts exceeds that of the first attempt, many students scored lower marks than their first try, viz. S3, S40, S76, S85, S122 and S168. Student S164 scored the lowest first attempt but together with S52 improved by the largest margin increasing scores from 2 to 6.5 and from 4 to 8 respectively. Although S70 and S76 only increased correct answers by 1 more, they already answered 9 of the ten questions correct in their first attempt. Consequently, together with S11 they scored 100% in their highest efforts.

Individual Review

Discussion of the efforts of S3 whose outcomes between initial and last attempts decreased the most, ensues. Figure 3 displays S3's incorrect answer to question 1 of the first attempt. The simplification of this complex fraction requires multiple steps when numerator and denominator are independently treated. If construed as an entity and multiplied by the identity element for multiplication, viz. $\frac{x}{x}$, the lowest common denominator equivalent of both numerator and denominator, the intermediate fraction obtained is arrived at sooner. Irrespective of method used, identification and factorisation of the difference between squares is required for complete simplification to arrive at the answer (x + 1). However, student S3 made a common mistake where elements of the same kind are grouped. In this case the mathematical fraction $-\frac{1}{x}$ is grouped and cancelled in numerator and denominator leaving x as the possible answer.

Perusal of S3's feedback in the 2nd and 3rd efforts reveal that decreasing results is the inability to answer


Question 8
Incorrect
Mark 0.00 out of 1.00
Edit question

Simplify $\frac{3}{2\cdot x-1}+\frac{1}{4\cdot x-2}$

Select one:
a. $\frac{4}{6\cdot x-3}$
b. $\frac{2}{2\cdot x-1}$
c. $\frac{7\cdot x-2}{(2\cdot x-1)^2}$ ✗
d. $\frac{7}{4\cdot x-2}$

Your answer is incorrect.
The correct answer is: $\frac{7}{4\cdot x-2}$

Make comment or override mark

Response history

Step	Time	Action	State	Marks
1	17/03/17, 11:38	Started	Not yet answered	
2	17/03/17, 11:55	Saved: [\fs{3}\frac{7 \cdot x-2}{\left(2 \cdot x-1\right)^{2}}]	Answer saved	
3	**17/03/17, 11:59**	**Attempt finished**	**Incorrect**	**0.00**

Question 9
Incorrect
Mark 0.00 out of 1.00
Edit question

Simplify $1+x-\frac{2\cdot x-1}{2}$

Select one:
a. $\frac{1}{2}$ ✗
b. $\frac{3}{2}$
c. 2

Figure 4. S3 Fractions 3rd attempt

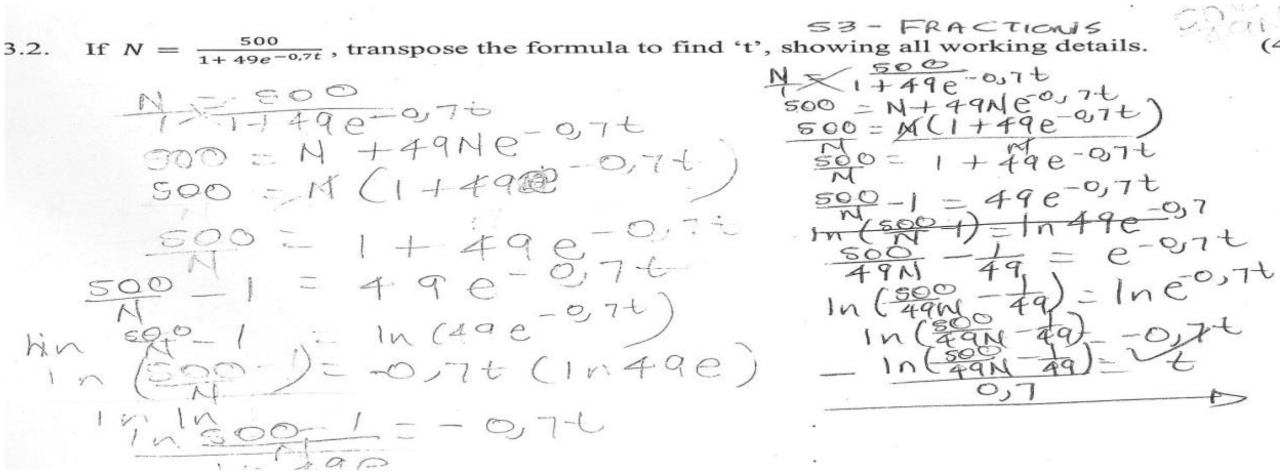


Figure 5. S3 exam solution

questions in level-2 and 3 highlighting the lack of proficiency in manipulating more complex algebraic fractions. Figure 4 is included to emphasise the level-2 questions that S3 could not answer correctly. Question 8 signifies the student's lack of understanding when confronted with the manipulation of algebraic fractions. This type distractor induced similar incorrect selections of questions 3 and 4 in the third attempt. Question 9's wrong choice was due to an algebraic oversight when finding the equivalent numerator of a negative fraction containing more than one term, presented here:

$1+x-\frac{2x-1}{2}=\frac{2+2x-2x+1}{2}=\frac{3}{2}$ correct solution

However, the answer chosen was $\frac{2+2x-2x-1}{2}=\frac{1}{2}$

The perceived student interaction with the diagnostic quiz in a cyclical analytic pattern, would assist students in diagnosing basic gaps in pre-calculus topics and rectifying these while performing calculus related questions. One way of demonstrating this is to analyse students written responses in examinations. Figure 5 contains a copy of S3's solution to the question which requires the manipulation of fractions and the

Table 5. Logarithms Quiz Three attempts

Student	1st attempt	2nd attempt	3rd attempt	Highest	Difference
S3	16	18	18	18	2
S8	13	20	17	20	7
S30	11	15	19	19	8
S72	17	19	20	20	3
S76	15	17	15	17	2
S78	16	17	19	19	3
S89	12	15	18	18	6
S92	15	16	18	18	3
S104	12	15	15	15	3
S107	14	19	16	19	5
S118	16	18	15	18	3
S133	15	17	20	20	5
S135	19	17	19	19	2
S137	12	18	18	18	6
S168	14	16	19	19	5
S172	14	18	18	18	4
Average	14.3	16.9	17.6	17.6	3.3

introduction of logarithms in an equation which is fluently completed by the student.

S3's solution in the examination containing simplification and manipulation of fractions supports the student's self-regulating practice to master procedural fluency in pre-calculus topics.

LOGARITHMS

The diagnostic quiz on logarithms contained 10 level-1, 10 level-2 questions and no level-3 type problems. Statistics for the group (Table 2) indicate a higher level of achievement than in the previous fractions diagnostic quiz. A possible factor in this anomaly is explicated in the education authority's curriculum design practices. In the South African School Mathematics curriculum , grade 12 learners are taught to solve exponential equations appearing in financial mathematics using law 3 indicated below without teaching the conceptual connection to exponents and without knowledge of any of the other laws of logarithms.

Law 1: $log_t\,AB = log_t\,A + log_t\,B$

Law 2: $log_t\,\frac{A}{B} = log_t\,A - log_t\,B$

Law 3: $\log_t A^r = r\,\log_t A$

Consequently, we included logarithms in the preliminary pre-calculus section of the first year engineering Mathematics 1A course and is taught in the first two weeks of the semester together with other fundamental pre-calculus topics. Hence, students received instruction in all the laws and concepts of logarithms just before the quiz was taken.

Observation of the metrics for the logarithmic quiz test in Table 2 reveals an increase of approximately 10% between averages of the first and highest attempts and a reasonable standard deviation of 15.09%. The high value for the coefficient of internal consistency suggests the quiz can be a reliable diagnostic quiz source for future cohort of first year engineering mathematics students. The median grade of 85% suggest that more than half of all the efforts scored above this mark insinuating that many students are proficient in the application of the definition and laws of logarithms.

Attempts

Table 5 displays the performances of students who attempted the logarithm quiz test three times.

Several aspects of students' grasp of logarithmic understanding are revealed in the analysis of attempts appearing in Table 5. Throughout the study emphasis on the pedagogical implementation of online content permeated the narrative. The first already referred to is ascertaining the level of mathematical understanding through diagnostic testing shortly after discussion and teaching of a topic. Secondly, repeated attempts with immediate feedback are essential in developing self-regulating skills. 16 students had 3 trials and all except S8 and S107 reported increases from 1st through to 3rd attempts. S8 and S107 increased from 1st to 2nd attempt but showed declines from the 2nd to 3rd attempt. However, it is noted that S8 was one of only three students who answered all 20 questions in any attempt correctly achieving this mark in the second venture. S72 outperformed in all attempts recording higher scores relative to his peers in this sub-group. S30 scored 55% in the first attempt but showed determination scoring 75% and 95% in consecutive retries logging the largest increase in the group. Averages for the first and highest attempts of the group increased from 14.3 to 17.6 of a maximum of 20.

Individual Attempts

The performance by S8 in three attempts highlights the researcher's view that multiple attempts of a quiz are necessary to identify as many gaps in basic mathematics pre-calculus topic. Scores of 13, 20 and 17 in the three

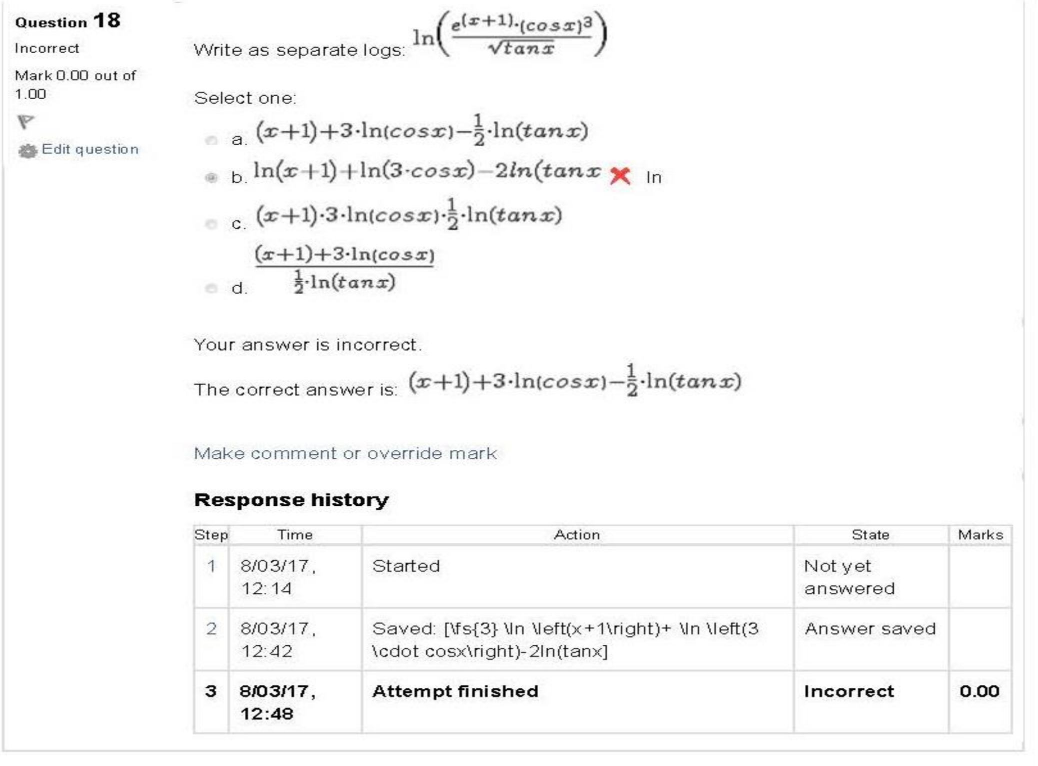


Figure 6. S8 1st attempt question 18

attempts shows the decrease in performance between the 2nd and 3rd attempts. Notwithstanding the maximum mark in the second attempt the student attempted the quiz a third time recording a lower mark. On completion of a quiz the mark obtained is displayed and students then have time to view their responses and act on the feedback provided. Question 18 of S8's 1st effort entails application of the laws of logarithms. The definition of a logarithm, viz. if $a^x = y \Rightarrow \log_a y = x, a > 0, a \neq 1$, can be used as a basis to prove all logarithmic laws. Application of the laws however, principally becomes procedural as in question 18 in Figure 6. Efficient separation of logarithms is essential when differentiating functions requiring this technique. S8 executed a procedural error when applying the three basic logarithmic laws in question 18. The incorrect answer, indicated by a 'X', suggests the student also made an error since the natural logarithm of Euler's number 'e' is equal to one, i.e. $lne^{x+1} = (x+1)lne = (x+1)$, since $lne = 1$.

A similar discussion could be applied to the 1st and 3rd attempts by student S89 whose marks rose from 12 in the 1st attempt to 18 in the 3rd attempt. In the 1st effort an incorrect choice to question 18 which required students to write as separate logarithms, viz. $ln\left(\frac{5^{x+1}.(e^x-1)}{\sqrt{cosx}}\right)$ was selected. A similar question appeared in the students 3rd attempt, viz. write the following as separate logarithms: $ln\left(\frac{e^{x+1}.(cosx)^3}{\sqrt{tanx}}\right)$. The correct answer was chosen for this question. Other questions of similar form in the 3rd attempt were also correctly selected. The discussion changes course and an analysis of the students solution in a major test is observed in Figure 7. The question informs students to follow the logarithmic technique of differentiation. Logarithmic laws are accurately applied in the first and second step. Unfortunately the differentiation of composite functions is not completed effectively. Perusal of the question is similar in form and level to that in the 1st and 3rd efforts of S89 demanding separation of the function into the sum and difference of logarithmic terms before the process of differentiation continues.

Despite not achieving full marks for the question, the initial application of logarithmic laws, which S89 had problems with in the first logarithmic quiz test attempt, was correctly applied demonstrating the positive effect of repeating diagnostic quiz tests.

DISCUSSION

Identifying and enhancing basic mathematical knowledge and skills of first year engineering students at a South African University of Technology, via Information Communication Technology (ICT), was the primary driver of the research. Students entering first year tertiary studies are not adequately prepared for the mathematics taught and attributable to a host of factors of which instruction received in school is the primary reason (Anthony, 2000; Tolley, 2012). These misconceptions, or alternative conceptions, are latent in students entering tertiary studies and are difficult to change (Driver, 1981). Identifying these fundamental

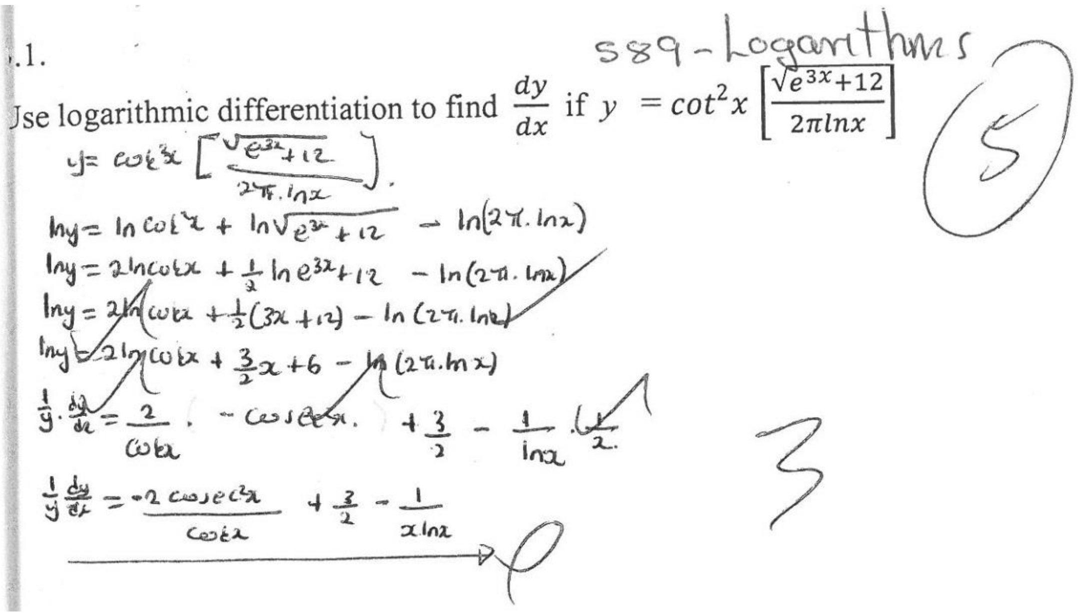


Figure 7. S89 Major Test solution

mathematical flaws, correcting and consolidating these mathematical skills via diagnostic quiz tests, both on and off campus via the Moodle LMS, are reported in this study. The impact and extent that the online diagnostic assessments influenced student gains in basic mathematical knowledge incorporates the consequential improvement in application and utilisation of this knowledge in first year pre-calculus analysed in this paper.

A number of aspects during the research surfaced that impeded and hindered the flow of the study. These included creation of the question data bank, time restrictions, technological issues as well as participants' resource limitations. Despite these impediments, the overwhelming interest and motivation observed in students' participation, has propelled the researchers to pursue and develop a model of online assistance that reinforces traditional lectures in a meaningful way.

CONCLUSION

Concluding remarks begin with the author emphasising that the role of information and communication technologies to support learning and teaching is becoming the norm rather than the exception. Many institutions have systems in place to ensure that a substantial portion of the academic programme is available online. The manner in which academics utilize technologies to enhance learning is dependent on the academics' desire to incorporate this technology in a meaningful way. Although pedagogy in ICT cannot be ignored, it may be limited in extent by the learner management system used' (Ally, 2017, p. 64). We noted that Mathematics is a requirement for entry into all engineering disciplines. In this study we provided opportunity for first year engineering students to participate in online activity so as to strengthen their procedural proficiency. This study focussed on the topics of factorisation, fractions and logarithms.

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