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Investigating the Efficiency of Teaching Mathematics to Students by Using the Double Ranked Set Sampling

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ABSTRACT

The main objective of this paper is to evaluate the efficacy of double ranked set sampling method in teaching mathematics to the students. The notion of ranked set sampling[1] for estimating the mean of a population and its advantage over the use of a simple random sampling for the sampling is established in the literature. Furthermore, the double ranked set sampling[2] has proven to be even more efficient than RSS. In this research, we review the use of the DRSS to estimate the intercept, the slope, and the standard deviation of the error terms as parameters of a simple linear regression model of teaching mathematics to students, when replications exist at each value of the predictor. Finally, we illustrate the proposed procedure by applying it when the underlying distribution of the error terms is normal or Laplace. Regardless of the assumed number of replications in the experiment, we observe a substantial gain in relative precision while using DRSS procedure over using RSS technique.

Keywords: education, mathematics, regression model, teaching

INTRODUCTION

Given the undeniable impact of faculty empowerment on improving their performance and the effectiveness of the educational system at the community level, educational groups should consider the variety of their roles and the factors affecting it, and take fundamental steps towards the empowerment of faculty members and improve their quality of service. Situations may occur in which precise measurement of the characteristic under study in expensive or difficult, but can be ordered accurately and easily by eye. Professional judgment or any other means relatively easy with negligible cost and not requiring actual quantification. Kaur et al. (1996) suggested a sampling procedure called ranked set sampling (RSS) applied in similar situations to estimate efficiently the mean pasture yield. Nowadays, ranked set sampling procedure is playing a major role in sampling and inference of environmental, agricultural and industrial problems (Taylan et al., 2007). Recently, RSS technique has been widely studied and advanced by various appropriate modifications to nonparametric setups. Lately, many authors who have been working on the subject can be found in the literature: Dell and Clutter (1972), Lam et al. (1994), Stokes (1995), Bohn (1996), Kaur et al (1996), Bhoj (1997), Yu and Lam (1997), Kim and Arnold (1999), Adatia (2000), Chen(2000), Barabesi and Sharaawi (2001), Al-Saleh and Zheng (2002), MacEachern et al. (2002) among others. Meanwhile, studies have been advanced in situations where ranking is not accurate, but a concomitant variable that can be easily used to judgment order the variable of interest exists. Moreover, Kaur et al. (1996), provided a more extensive annotated bibliography for ranked set sampling.

One of the problems today in the field of education is the lack of students' interest in learning, especially in mathematics, such as mathematics (Kozlova & Sakhieva, 2017; Kalimullin & Utemov, 2017; Zelenina & Khuziakhmetov, 2017; Utemov & Masalimova, 2017; Thibaut et al., 2018; Krutikhina et al., 2018; Gluzman et al., 2018). In spite of the importance of all courses, since the elementary period underlying the construction of the scientific personality of students and the basis for creating a positive or negative attitude toward them, especially

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[2] DRSS

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Contribution of this paper to the literature

- This paper help developing the student skills in mathematical statistics as well as developing its mental ability to understand statistical applications.
- Helps students understand how to estimate regression parameters and use them in their research and future studies.
- This research helps to understand the statistic of researchers in interpreting the results of their research.

mathematics, should be interested in a new method. He created this lesson and education, especially in this educational background and in this particular lesson. One of the methods of modern education is the use of information and communication technology. The application of this method in the mathematical curriculum makes students aware of the lesson. Because in this method, teaching is accompanied by beautiful images, and on the other hand, as a student plays a role in learning, it attracts and deepens his learning. Because researchers believe that much of the learning is done through vision, and since information and communication technology is a tool for generating a sense of vision and hearing, it enhances the learning of students. Information and communication technology is also a tool for thinking and action, and it adds to the students' intellectual ability and creativity, and the development of access to quality education. Therefore, its failure to pay attention to its consequences for the education of the country and the quality of its output as a global citizen, which will operate in the not-too-distant future in the interconnected world of economics, business, and culture, will, of course, The country's backwardness in the world of competition and in the international arena will be in the long run. Because its initial period and its quality will play an important role in the process of sustainable development of the country; ICT should be in the training of courses, especially in the first years of education, it will be used. So in this paper we developed a regression model to evaluate its effects on teaching and students learning.

Ranked set sampling procedure consists of choosing n random samples, each of size n, from a population of interest. The n units, in each of the n samples, are ranked by a professional judgment without involving any actual measurement. Afterwards, the smallest ranked unit from the first sample is measured, and then the second smallest ranked unit from the second sample is measured, and so on, this process continues until the largest ranked unit from the n^{th} sample is measured. In fact, we initiated this process with a total of n^2 drawn sample units from a population of interest, but at the end of this process, only n measured units are realized and denoted by (y_{11}, y_{22}, …, y_{nn}). In fact, this procedure can be repeated m times to get m cycles. These n independent but not identically distributed measured observations constitute which was first introduced by (Kaur et al., 1996). Mclntyre (1952) made a further step and extended the latter procedure to a double ranked set sampling (DRSS) and later Al-saleh and Al-omari (2002) advanced to a multistage ranked set sampling. The DRSS is initiated by drawing a total of n^3 units from a population of interest, these units are randomly divided into n sets each of size n^2. In the first stage, in each of the n sets of size n^2. We apply the RSS procedure without any actual measurements to get n sets each of size n representing a RSS. Then, in the second stage, we apply again the RSS procedure and carry out actual measurements, at the end of this process, we realize a total of n independent but not identically distributed measured observations constituting the double ranked set sample (DRSS) labeled by (y_{11}, y_{22}, …, y_{nn}),where y_n is the quantified i th smallest ranked unit the i th set realized in the first stage of the DRSS procedure $y_{ii} = i^{th} \min \{y_{(11)}, y_{(22)}, \ldots, y_{(nn)}\}^{(i)}$

As an illustration, consider the case of n = 2, so we must have a random sample of size 8, 2 subsets with 4 units each $\{x_{11}, x_{12}, x_{21}, x_{22}\}$.

After ranking cash subset by professional judgment, we get:

$$\begin{matrix} X_{(11)} & x_{(12)} \\ X_{(21)} & x_{(22)} \end{matrix}^{(1)} \quad \text{and} \quad \begin{matrix} x_{(11)} & x_{(12)} \\ x_{(21)} & x_{(22)} \end{matrix}^{(2)}$$

Therefore, we have two judgment ranked sets each of size 2:

$S_1 = \{y_{(11)}, y_{(22)}\}^{(1)}$ and $S_2 = \{y_{(11)}, y_{(22)}\}^{(2)}$

$y_{(11)} = \min \{y_{(11)}, y_{(22)}\}^{(1)}$ and $y_{(22)} = \max \{y_{(11)}, y_{(22)}\}^{(2)}$

Finally, $y_{(11)}$ and $y_{(22)}$ constitute a DRSS of size 2.

We should emphasize here that we only quantify n units on the second stage, while on the first stage, ranking is done visually or by using a rough and inexpensive method. As explained by Mode et al. "RSS and therefore DRSS is a two phase sampling procedure that reduces the number of units requiring more expensive measurements, termed costly measurements, by employing experts knowledge or some estimation procedure with negligible cost, termed frugal measurements, to select units. The frugal measurements increase information when DRSS is applied" (Mode et al, 1999: 19). On one hand, the number of quantified units n is small compared to n^3, the number of sampled units, while on the other, all sampled units play a role in adding more information to the quantified units.

McIntyre (1952) obtained the best linear unbiased estimators (BLUEs) for estimating the location and scale parameters and therefore the population mean using RSS. Barreto and Barnett (1999) extended the work of McIntyre (1952) to estimate the parameters of simple linear regression models of teaching Mathematics to Students with replicated observations at each of the values of the predictor variable by the BLUEs using RSS when the response variable is normally distributed and the ordering is assumed perfect. In this article, we extend the works of Barreto and Barnett (1999), and Al-Saleh and Al-Kadiri (2000) to estimate the parameters (slope and intercept, and standard deviation of the error term) of simple linear regression models of teaching mathematics to students (Browder et al., 2008). We assume that replicated observations exist at each of the values of the independent variable. We propose to estimate the mentioned parameters by the BLIEs using the DRSS when the cumulative distribution of the response variable belongs to a location scale family, namely the error distribution with shape parameter y = 1, 2 corresponding respectively to the normal and Laplace distributions (Granato et al., 2014).

BEST LINEAR UNBIASED ESTIMATORS

Let X be a random variable with a cumulative distribution function that belong to a location – scale family F $(\frac{x-\mu}{\sigma})$, where μ is the location parameter and σ > 0 is the scale parameter. Using the order statistics, the best linear unbiased estimator of $\acute{\emptyset}$ = (μσ) is well recognized and established in the literature. For more details, see for example Lloyd (1952).

BLUES OF SIMPLE LINEAR REGRESSION MODEL OF TEACHING MATHEMATICS TO STUDENTS USING RSS

Today, Simple Linear Statistical models for regression are widely used in business administration, economics, engineering, and the social, health and biological sciences. In these situations, the interest is to study the relationship between the independent or predictor variable X and the dependent or response variable Y. For experimental data, at each if the n distinct predetermined values of x ($x = x_i$; i=1,…, n), the experimenter observes the dependent variable y for a certain number of replications m_i , ($y = y_{ij}$: j = 1,…, p_i, i=1, ….., n). McIntyre (1952) proposed estimators of the slope and intercept parameters (x_1, $y_{(11)}$), (x_2, $y_{(22)}$), …., (x_n, $y_{(np)}$) where $y_{(ii)}$ is the i^{th} ordered and measured value of the dependent variable in a sample of size n and x_i is the corresponding value of the independent variable, for i = 1, …, n. Barreto and Barnett (1999) proposed a quite different approach for estimating the parameters of a simple linear regression model of teaching mathematics to students using RSS BLUEs. They considered m_i potential samples each of size m_i potential samples each of size m_i at each prescribed x_i (i = 1,…., n), from which to choose and measure the ranked set sample, this approach require finding a RSS at each value of the independent variable.

Be the ranked set sample for each of the independent variable $x = x_i$, i = 1, …, n. The conditional mean and variance of y are E (y / X = x) = α + β_x and var (y / x) = σ^2 respectively and the reduced order statistics at each value of x is $z_{i(j)} = \frac{y_i(j) - \alpha - \beta x_i}{\sigma}$.

When the underlying distribution is

f $(\frac{y-\mu}{\sigma})$ and the sample sizes at each value of the independent variables are the same, i.e. $p_i = m$, for all i, then the reduced order statistics $z_{i(j)}$ will have the same means and the same variances v_j at each value x_i, therefore, the BLUEs of α, β, and σ can be expressed as follows.

If in addition the underlying distribution is symmetrical F $(\frac{y-\mu}{\sigma})$ then thus the above formulae will be reduced.

BLUES OF SIMPLE LINEAR REGRESSION MODEL OF TEACHING MATHEMATICS TO STUDENTS USING DRSS

When using double ranked set sampling, the formulae of the BLUEs from Section 3 remain, we compute the latter expected value and variance from the continuous cumulative distribution function of the corresponding reduced order statistics z_u derive when using the double ranked set sampling procedure. These z_u derived when using the double ranked set sampling procedure. These z_u are independent but not identically distributed.

THE RELATIVE PRECISION OF THE DRSS – BLUES VERSUS RSS – BLUES

When the underlying distribution is symmetrical continuous cumulative F $(\frac{y-\mu}{\sigma})$ and the sample sizes at each of the values of the predictor variable are the same, i.e. $p_i = p$, for all i, then the relative precision of the proposed estimators irrespective of the values of x_i and n can be written as follows. We illustrate the procedure, by computing

Table 1. The relative precisions of the DRSS – BLUEs versus RSS – BLUEs

	p = 2	p =3	p = 4	p = 5
RP (α) or Rp (β)	1.2169	1.4218	1.6132	1.7906
RP (σ)	1.6815	1.6851	1.7120	1.7721

Table 2. The relative precisions of the DRSS – BLUEs versus RSS – BLUEs

	p = 2	p =3	p = 4	p = 5
RP (α) or Rp (β)	1.1477	1.5427	1.8072	12. 0624
RP (σ)	1.5252	1.4475	1.5226	1.5966

the relative precision of the DRSS – BLUEs versus the RSS – BLUEs when the error distribution is the underlying distribution of the response variable with shape parameter y = 1,2 corresponding respectively to the normal and Laplace distributions with the number of replications being p = 2,3,….5.

Table 1 shows the relative precisions of the DRSS – BLUEs versus RSS – BLUEs when the underlying distribution is normal.

While, **Table 2** shows the relative precisions of the DRSS – BLUEs versus RSS – BLUEs when the underlying distribution is Laplace.

CONCLUDING REMARKS

The results showed that the role of technology or technology in changing the attitude, stability and stability of the content, the skill of reasoning, and the power of creativity, and ultimately the active absorption of mathematical lessons. Usually, when using the RSS procedure, the experimenter is advised to use a small sample size, so that the error in ranking is minimalized, Recall also that if we need a sample consisting of five units, then we need to start the procedure with 25 sample units, in the case of DRSS technique, if we need to achieve a sample consisting of five units, then we need to start the process with 125 sample units, Since DRSS involves two stages of ranking, this will involve surely more errors in ranking. For that reason, the experimenter is seriously advice to use a minimal sample size not exceeding 5 to reduce the errors involved with the imperfect ranking. We can observe from both tables that the DRSS- BLUEs when estimating simple linear regression model of teaching mathematics to students outperform in all cases the RSS – BLUEs. For example in the case of the normal distribution, when estimating α and β, DRSS-BLUEs perform 42.18 % and 79.06 % better than RSS-BLUEs when the number of replications is respectively 3 and 5. While, when estimating, DRSS- BLUEs perform 68.51 % and 77.21 % better than RSS- BLUUEs when the number of replications is respectively 3 and 5. Moreover, in the case of Laplace distribution, when estimating α and β, DRSS-BLUEs perform 80.72 % and 106.24 % better than RSS- BLUEs when the number of replications is respectively 4 and 5. Finally, from these results when comparing them with the ones of Barreto and Barnett (1999), we can see that the efficiency of DRSS- BLUEs when estimating α and β with m = 4is achieved by RSS – BLUEs only when m is between 7 and 8. The survey conducted among students showed us the following results: 99% of respondents agreed that it is important for students to learn21stcentury skills. 80% of respondents agreed with the idea that children need to be taught differently today, than20years ago. 6 out of 10respondentsconfirmedthat, the schools today do not meet the new requirements. Respondents gave priorities to such skills, as critical thinking, ethical and social responsibility, teamwork and communication. 78% of respondents believe that the balance between basic skills and 21stcentury skills should be equally presented in the curriculum.

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