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Investigating the pedagogical content knowledge of beginning mathematics teachers in teaching Euclidean geometry: A task-based interview approach

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Abstract

Euclidean geometry remains a cognitively demanding component of the South African further education and training curriculum, and persistent learner underperformance raises concerns about the pedagogical readiness of beginning teachers. Pedagogical content knowledge (PCK) is essential for connecting formal geometric knowledge with learners' reasoning, particularly in anticipating misconceptions and supporting proof construction. This qualitative, interpretive study examined the PCK of grade 11 beginning mathematics teachers in two Eastern Cape districts. A geometry test administered to fifteen teachers was used to purposively select six participants across performance levels. Task-based interviews served as the primary data source, focusing on diagnostic reasoning and instructional decision-making. Findings revealed marked variation in conceptual understanding and pedagogical responsiveness. High-performing teachers demonstrated integrated conceptual and diagnostic competence, whereas low-performing teachers relied mainly on procedural instruction. The study proposes Phaliso's classroom knowledge model and underscores the need for targeted professional development in Euclidean geometry instruction.

Keywords: pedagogical content knowledge, beginning teachers, Euclidean geometry, diagnostic competence, task-based interview, Phaliso's classroom knowledge model

INTRODUCTION

Euclidean geometry remains one of the most challenging components of the South African further education and training mathematics curriculum. The topic develops learners' logical reasoning, spatial visualization, mathematical proof, and deductive reasoning (Govender & Amevor, 2025; Mudhefi et al., 2024). However, national assessment reports continue to reveal persistently low learner achievement, particularly in proof-based and problem-solving tasks (Department of Basic Education, 2024; Mlambo & Sotsaka, 2025). These outcomes have intensified concerns about the preparedness of beginning mathematics teachers, whose mathematical and pedagogical knowledge directly influences learners' opportunities to develop conceptual understanding in geometry.

Beginning teachers, unlike pre-service teachers, are qualified educators who have entered the teaching profession and are responsible for classroom instruction. Although both groups are at early stages of professional development, this study focuses exclusively on beginning mathematics teachers. Beginning teachers often face the dual challenge of consolidating their mathematical understanding while simultaneously developing effective pedagogical practices for teaching demanding topics such as Euclidean geometry (Makoa & Segalo, 2021; Sunzuma & Maharaj, 2019b). Pedagogical content knowledge (PCK) is particularly important because it enables teachers to recognize learner misconceptions, sequence instruction effectively, and employ teaching strategies that promote conceptual understanding rather than procedural memorization (Banjo, 2019; Li & Copur-Gencturk, 2024). Previous

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Contribution to the literature

- This study provides evidence on the PCK of beginning teachers in Euclidean geometry within the South African context, a topic with limited empirical research, particularly in the Eastern Cape.
- It demonstrates how teachers at different performance levels vary in conceptual knowledge, ability to diagnose misconceptions, and teaching strategies.
- It introduces the PCKM, which links CK, diagnostic teaching, and PCK, offering a practical tool for teacher development and curriculum planning.

studies have shown that content knowledge (CK) alone is insufficient for effective mathematics instruction; teachers must also understand learners' thinking, anticipate misconceptions, and adapt instruction to diverse learning needs (McConnell et al., 2013; Shing et al., 2018).

Although an extensive body of international research has examined PCK and geometry instruction, comparatively little empirical evidence exists on how beginning mathematics teachers enact these forms of knowledge in South African classrooms, particularly in under-resourced rural districts such as Amathole East and OR Tambo Inland (Maqoqa, 2024; Olabode, 2023). Existing studies have largely concentrated on learner achievement while paying comparatively less attention to the knowledge, instructional practices, and pedagogical decision-making of the teachers responsible for facilitating learning. Consequently, there remains limited understanding of how beginning teachers identify learner misconceptions, interpret learner thinking, and respond instructionally during Euclidean geometry lessons.

Addressing this gap is important because effective geometry teaching requires more than procedural competence. Teachers must integrate CK with pedagogical expertise to interpret learners' mathematical thinking, scaffold conceptual understanding, and promote mathematical reasoning. Without a clearer understanding of how beginning teachers enact these forms of knowledge in classroom practice, it remains difficult to design teacher education programs and professional development initiatives that respond to their specific instructional needs. The present study therefore seeks to contribute empirical evidence on how beginning teachers identify learner misconceptions and apply PCK during the teaching of Euclidean geometry in South African classrooms.

Accordingly, this study was guided by the following research questions:

1. To what extent do beginning teachers identify learners' misconceptions in Euclidean geometry?
2. How do beginning teachers apply their knowledge to address learners' misconceptions in Euclidean geometry?

Guided by these questions, the study explored the interplay between beginning teachers' CK, their understanding of learner thinking, and their instructional practices during Euclidean geometry lessons. In doing so, it contributes empirical evidence to inform teacher preparation and professional development in South African mathematics education.

LITERATURE REVIEW

Research on the teaching of Euclidean geometry has consistently emphasized that effective instruction depends on the interaction between teachers' CK and PCK. Studies conducted across different educational contexts show that teachers with strong conceptual understanding are better able to explain geometric relationships, diagnose learners' misconceptions, and support mathematical reasoning (Sunzuma & Maharaj, 2019a; Zuya & Kwalat, 2015). Nevertheless, the literature also indicates that many beginning and practicing mathematics teachers experience difficulties in integrating these complementary forms of knowledge when teaching geometry, particularly during proof and problem-solving activities. These findings suggest that teacher knowledge remains a critical area requiring further investigation.

Against this background, understanding how beginning teachers integrate mathematical and pedagogical knowledge when teaching Euclidean geometry is essential. This literature review examines four interrelated areas: learner misconceptions in Euclidean geometry, pedagogical approaches for addressing these misconceptions, the role of teachers' CK and PCK in effective geometry instruction, and the contribution of professional development to strengthening beginning teachers' instructional practice. Collectively, these themes provide the conceptual foundation for the present study and establish the context for investigating beginning teachers' PCK in South African classrooms.

Identifying Learners' Misconceptions in Euclidean Geometry

Misconceptions in Euclidean geometry are well-documented and frequently arise from learners' reliance on visual intuition rather than formal deductive reasoning, as well as from an incomplete understanding of geometric definitions, theorems, and proofs (Makhubele et al., 2015; Masilo, 2018; Phaliso, 2022). For example, learners may incorrectly assume that a quadrilateral is cyclic only when a circle is visibly drawn or confuse the properties of different types of triangles when constructing proofs. While Makhubele et al. (2015) and Masilo (2018) primarily attribute these misconceptions to learners' conceptual reasoning, Gardee (2015) argues that teachers' knowledge of learners' thinking is equally important in recognizing and addressing such errors. This perspective is consistent with Hill and Chin's (2018) view that diagnosing misconceptions is a fundamental component of PCK.

Beginning teachers' difficulty in diagnosing misconceptions is widely attributed to limited professional experience, although scholars differ in explaining the underlying causes. Moosapoor (2023) emphasizes limited classroom exposure and CK, whereas Petersen (2017) and Venkat and Spaull (2015) argue that an overreliance on procedural teaching and textbook examples constrains teachers' ability to respond to learners' reasoning. Together, these studies suggest that diagnostic competence develops through the interaction of mathematical knowledge, classroom experience, and responsive instructional practice rather than through CK alone. By contrast, experienced teachers demonstrate greater diagnostic competence by observing learners' reasoning, asking probing questions, and interpreting learners' responses beyond surface-level errors, enabling them to respond more effectively to misconceptions (Kosel et al., 2024). The literature demonstrates that learner misconceptions cannot be understood solely as deficiencies in learners' reasoning. Rather, teachers' ability to recognize, interpret, and respond to learners' misconceptions is equally important, highlighting diagnostic competence as a defining feature of effective classroom practice.

Pedagogical Approaches Addressing Learners' Misconceptions

Once misconceptions have been identified, teachers' ability to respond through appropriate instructional strategies becomes critical. Effective approaches include scaffolded reasoning, guided discovery, and the use of counterexamples to challenge learners' incorrect assumptions (Ingram et al., 2015; Kabadaş & Mumcu, 2024; Komatsu et al., 2017). For example, the misconception that a quadrilateral is cyclic only when a circle is visibly drawn can be addressed by guiding learners to examine angle properties and multiple geometric configurations that satisfy the definition of a cyclic quadrilateral.

Although researchers broadly agree that misconceptions should be addressed through instructional approaches that actively engage learners in reasoning rather than memorization, they differ in the emphasis they place on effective pedagogy. Constructivist scholars advocate guided discovery and learner exploration before formal proof, arguing that these approaches promote deeper conceptual understanding (Ncube & Luneta, 2025; Osei, 2019). In contrast, Wakhata et al. (2022) emphasize the importance of carefully sequenced questioning, scaffolded reasoning, and purposeful task design in supporting learners' transition from empirical or visual reasoning to formal deductive proof. Despite these differences in emphasis, the literature consistently identifies PCK as the mechanism that enables teachers to translate subject knowledge into effective instructional decisions.

However, studies also indicate that beginning teachers frequently struggle to implement these approaches because of limited teaching experience and a tendency to rely on teacher-centered demonstrations rather than learner-centered reasoning (Venkat & Spaull, 2015). Consequently, the effectiveness of instructional interventions depends not only on teachers' knowledge of geometry but also on their ability to select, adapt, and sequence pedagogical strategies that respond to learners' misconceptions and promote meaningful conceptual understanding.

The Role of Content Knowledge and Pedagogical Content Knowledge

Ball et al. (2008) distinguish between common content knowledge (CCK) and specialized content knowledge (SCK), arguing that effective teaching requires more than the ability to solve mathematical problems. Their framework is complemented by studies conducted by Seherrie and Mawela (2022) and Venkat and Spaull (2015), which demonstrate that weaknesses in SCK constrain teachers' ability to explain concepts and respond effectively to learners' misconceptions. Extending this perspective, knowledge of content and teaching (KCT) highlights how mathematical knowledge is translated into instructional decisions, suggesting that effective geometry teaching depends on the interaction rather than the isolation of these knowledge domains.

While Ball et al. (2008) distinguish between the domains of mathematical knowledge for teaching (MKT), Wakhata et al. (2023) demonstrate how these domains operate in classroom practice. They argue that teachers who connect learners' prior knowledge with new concepts, anticipate misconceptions, and adapt instructional strategies exemplify the interaction between SCK and KCT. Together, these perspectives show that high-quality geometry teaching depends on teachers' ability to integrate mathematical understanding with responsive pedagogical decision-making, enabling them to interpret learners' mathematical thinking and select instructional strategies that promote conceptual understanding.

Professional Development and Support for Beginning Teachers

Studies consistently identify structured professional development as an important mechanism for strengthening beginning teachers' capacity to recognize and address learner misconceptions. However, researchers differ in the forms of professional learning they consider most effective. Geletu (2024) emphasizes collaborative approaches such as lesson study and mentoring because they promote reflection on classroom practice, whereas other studies advocate task-based professional learning that focuses explicitly on strengthening teachers' pedagogical reasoning. Despite these differences, the literature consistently suggests that sustained, practice-based professional development is more effective than one-off training workshops, particularly for beginning teachers working in under-resourced contexts.

Existing South African studies have largely examined either learners' misconceptions or the practices of experienced teachers (Danso, 2020; Makhubele et al., 2015; Masilo, 2018; Nojiyeza, 2019; Phaliso, 2022). Consequently, limited empirical evidence is available on how beginning grade 11 mathematics teachers integrate CK, pedagogical reasoning, and diagnostic competence while teaching Euclidean geometry in authentic classroom settings. This gap is particularly evident in under-resourced rural districts such as Amathole East and OR Tambo Inland, where contextual challenges may further influence instructional practice. Accordingly, the present study examines how beginning teachers identify learner misconceptions and enact PCK during Euclidean geometry instruction.

Theoretical Framework

The theoretical framework for this study draws primarily on Ball et al.'s (2008) model of MKT, which illustrates the structure and interrelationships of teachers' professional knowledge. The model represents CK as the foundational layer, encompassing teachers' understanding of mathematical concepts and procedures. Embedded within this foundation is PCK, which includes specialized knowledge used for teaching, such as KCT and knowledge of content and students (KCS). The egg-shaped structure emphasizes that PCK does not exist independently of CK but is developed through the transformation of CK for instructional purposes. In this study, the model provides a lens for examining how beginning teachers draw on their geometry CK to interpret learner thinking, select instructional strategies, and respond to misconceptions. The emphasis on KCT aligns with the model's focus on the practical use of knowledge in classroom decision-making.

Ball et al. (2008) distinguish between CCK, which refers to the mathematical knowledge used in general contexts, and SCK, which refers to the knowledge required to teach mathematics effectively. SCK includes the ability to identify and address potential learner difficulties, design examples that reveal underlying concepts, and explain complex ideas clearly. In teaching Euclidean geometry, SCK enables beginning teachers to anticipate errors and misconceptions, a critical aspect of responsive instruction (Hoth et al., 2022). In addition to CK, PCK content plays a central role in the conceptual framework guiding this study. Shulman (1986) originally introduced PCK to describe the integration of subject matter knowledge and pedagogy that enables teachers to transform mathematical ideas into forms that are accessible to learners. In this study, PCK is conceptualised through three interrelated sub-domains selected to align closely with the research questions and objectives. The first sub-domain, knowledge of explanations, captures teachers' ability to articulate geometric concepts, theorems, and proofs in ways that support learner understanding, including the use of scaffolding strategies that facilitate progression from procedural demonstration to conceptual understanding (Petersen, 2022). This sub-domain is closely associated with KCT, as effective explanations require informed instructional decision-making grounded in sound CK. Complementing this is knowledge of learners' conceptions and misconceptions, which involves anticipating and recognizing common errors in Euclidean geometry, such as misunderstandings related to cyclic quadrilaterals or angle properties. This sub-domain emphasizes teachers' diagnostic capabilities and their capacity to adjust instruction in response to learners' thinking (Hill & Chin, 2018; Moosapoor, 2023), again reflecting the practical application of KCT. Building on explanation and diagnosis, knowledge of instructional strategies for geometry focuses on the design and sequencing of tasks, the guidance of proof construction, and the facilitation of reasoning processes that promote deep learning. This includes the purposeful use of diagrams and visualizations to support deductive reasoning in line with curriculum expectations (Venkat & Spaull, 2015), with KCT underpinning the connection between CK and instructional practice.

Together, these sub-domains represent the core competencies required for effective teaching of Euclidean geometry. Examining them enables the study to assess not only teachers' CK but also their ability to integrate this knowledge with learners' thinking and classroom practice. This perspective is consistent with research indicating that high-quality geometry instruction depends on strong content understanding alongside the capacity to respond to learner misconceptions in real time (Fukaya et al., 2024; Sapire et al., 2016). Informed by these interrelated components, the framework also guided the development of the PCKM, which integrates CK, PCK, and diagnostic teaching skills. The model foregrounds understanding learner thinking, linking instruction to prior knowledge, and adapting teaching strategies to support conceptual mastery in Euclidean geometry, while offering a developmental perspective on how teacher knowledge evolves through experience, reflective practice, and targeted professional development (Gudeta, 2022; Tatira, 2020).

Within this study, the conceptual framework directly informed the design of the task-based interview instrument, which focused on beginning teachers' ability to diagnose learner misconceptions and apply appropriate instructional strategies. It also guided the analysis of teachers' responses, allowing for a systematic examination of how CK, understanding of learner thinking, and pedagogical skill—particularly the practical application of KCT—interact in the teaching of Euclidean geometry.

METHODOLOGY

This study adopted a qualitative interpretive design to explore the PCK of beginning grade 11 mathematics teachers in Euclidean geometry. The design was appropriate because it sought to understand how beginning teachers recognized learner misconceptions, justified instructional decisions, and enacted PCK in authentic classroom contexts rather than to measure predetermined variables (Creswell & Creswell, 2017). This design enabled an in-depth examination of how beginning teachers identified learners' misconceptions, explained their instructional responses, and applied PCK during the task-based interviews.

Research Sites

The study was conducted in 15 public secondary schools located in the Amathole East and OR Tambo Inland districts of the Eastern Cape, South Africa. Eight schools were situated in Amathole East and seven in OR Tambo Inland. All schools offered grade 11 mathematics and served predominantly rural communities. The schools were no-fee public schools operating in under-resourced contexts, making them appropriate sites for investigating the PCK of beginning mathematics teachers.

Study Population and Sampling

The study population comprised 92 beginning mathematics teachers employed in public secondary schools across the Amathole East and OR Tambo Inland districts of the Eastern Cape, South Africa. For the purposes of this study, beginning teachers were defined as qualified teachers with three years or less of teaching experience who were actively teaching grade 11 Euclidean geometry. From this population, 15 teachers were purposively selected based on their willingness to participate, accessibility, and responsibility for teaching grade 11 Euclidean geometry.

A two-stage sampling strategy was employed. During the first stage, all 15 participants completed a diagnostic geometry assessment. The assessment was not used to answer the research questions but to identify variation in participants' CK and inform maximum variation purposive sampling. During the second stage, six teachers were purposively selected from the assessment results to represent high-, moderate-, and low-performing groups, thereby enabling comparison of PCK across differing levels of CK. Details of the diagnostic assessment, scoring procedures, and performance categories are presented in the following subsection.

The final sample comprised two high-performing, two moderate-performing, and two low-performing teachers. Selecting participants across these performance groups enabled comparison of how teachers with differing levels of CK identified learner misconceptions, interpreted learners' reasoning, and selected instructional strategies during the task-based interviews. The sample size of six participants was appropriate for the interpretive nature of the study, where the emphasis was on obtaining rich, in-depth accounts of teachers' pedagogical reasoning rather than producing statistically generalizable findings (Creswell & Creswell, 2017). Maximum variation sampling enhanced the diversity of perspectives represented across the three performance groups, while data collection continued until data saturation was reached, with the final interviews yielding no substantially new insights into how participants identified learners' misconceptions, explained their instructional responses, or enacted PCK.

Diagnostic geometry assessment (sampling instrument)

The diagnostic geometry assessment was administered to all 15 participants before the task-based interviews and served solely as a sampling instrument rather than as a source of data for addressing the research questions. Its purpose was to determine variation in participants' CK of grade 11 Euclidean geometry and to facilitate maximum variation purposive sampling.

The assessment consisted of eight items worth a total of 36 marks. The items assessed key grade 11 Euclidean geometry concepts prescribed in the South African CAPS curriculum, including cyclic quadrilaterals, angle relationships, the tangent-chord theorem, circle geometry, geometric proof, and deductive reasoning. The assessment was aligned with Ball et al.'s (2008) MKT. Six items assessed SCK because they required participants to justify geometric reasoning, construct formal proofs, and interpret learner thinking in teaching-related contexts, whereas two items assessed CCK

Table 1. Alignment of the task-based interview tasks with PCK sub-domains and intended focus

Task	Primary PCK sub-domain	Intended focus
Task 1: Theorem explanation	KCT	Articulation of geometric proofs and explanation of complex geometric concepts.
Task 2: Learner error analysis	KCS	Identification and diagnosis of learners' misconceptions and reasoning errors.
Task 3: Instructional remediation	KCT	Design and sequence instructional strategies to address misconceptions and promote deductive reasoning.

through routine geometric problem solving and application of standard theorems.

Before the main study, the assessment was piloted with five beginning mathematics teachers from schools outside the study sample. Feedback from the pilot resulted in the removal of two sub-questions to reduce the completion time from approximately 50 minutes to 30 minutes and the inclusion of additional response space beneath each question. To strengthen content validity, two mathematics education specialists independently reviewed the assessment for alignment with the grade 11 CAPS curriculum, the study objectives, and the intended mathematical knowledge domains.

Participants' scores ranged from 8 to 28 out of 36 (22%-78%). Based on these results, participants were categorized into three performance groups:

- High-performing: 67%-78%
- Moderate-performing: 47%-64%
- Low-performing: 22%-36%

Two teachers were purposively selected from each performance group, resulting in a final interview sample of six participants. This maximum variation sampling strategy enabled comparison of PCK across teachers demonstrating differing levels of CK. The complete diagnostic assessment and sample marking guide are provided in **Appendix A**.

Data Collection Instrument

The task-based interview instrument comprised three tasks designed to elicit evidence of participants' PCK in relation to the study's research questions. Each task was primarily aligned with a particular PCK sub-domain derived from Ball et al.'s (2008) MKT framework while also drawing on complementary aspects of teachers' mathematical and pedagogical knowledge. The tasks targeted different dimensions of beginning teachers' instructional reasoning. **Table 1** summarizes the primary PCK alignment, the intended focus of each task, and the aspects of teachers' reasoning explored during the interviews.

The interview tasks were reviewed by two mathematics education specialists to evaluate their alignment with the research questions, the conceptual framework, and the grade 11 Euclidean geometry curriculum. Their feedback informed minor revisions to the wording and sequencing of several prompts before data collection commenced. The refined instrument was then piloted with two beginning mathematics teachers from a neighboring district who were not part of the final study sample. The pilot evaluated the clarity of the interview tasks, the appropriateness of the probing questions, the time required for completion, and the instrument's ability to elicit evidence of how teachers identified learners' misconceptions and applied PCK to address themca. Based on the pilot findings:

1. The wording was revised to remove leading prompts about learner errors in cyclic quadrilateral proofs.
2. The interview duration was extended from approximately 15 to 30 minutes to allow participants sufficient time to complete the written tasks and explain their reasoning.

Data Analysis

Data were analyzed using a hybrid approach to thematic analysis, combining deductive coding informed by Ball et al.'s (2008) MKT framework with inductive coding to identify themes emerging from the data (Fereday & Muir-Cochrane, 2006). The analysis proceeded in four stages. First, the interview recordings were transcribed verbatim and read repeatedly to achieve familiarity with the data. Second, deductive coding was undertaken using the three PCK sub-domains proposed by Ball et al. (2008) as the initial coding framework. Third, inductive coding was conducted within these categories to identify patterns and insights that were not predetermined by the conceptual framework. Finally, related codes were synthesized into broader themes that captured similarities and differences in how high-, moderate-, and low-performing teachers identified learners' misconceptions and applied PCK to address them.

To enhance the credibility of the analysis, two transcripts were independently coded by a second researcher. Coding differences were discussed until consensus was reached, after which the agreed coding framework was applied to the remaining transcripts.

Ethical Considerations

Ethical approval was obtained from the relevant institutional ethics committee before data collection (approval No. FEDFREC 07-06-24-02). Permission to conduct the study was granted by the Eastern Cape

Department of Education and participating school principals. Participation was voluntary, and written informed consent was obtained from all participants. Teachers were assured of confidentiality and anonymity, and pseudonyms (TA-TF) were used throughout the study. All electronic data were password protected and stored securely, while hard-copy materials were kept in a locked cabinet accessible only to the researcher.

Trustworthiness

Trustworthiness was enhanced through several complementary strategies throughout the research process. Before the main study, the research instruments were piloted to assess their clarity, relevance, and ability to elicit evidence of beginning teachers' PCK. Revisions arising from the pilot study and expert review improved the alignment of the interview tasks with the research questions and conceptual framework. During data collection, participants' written responses to the task-based activities were examined alongside their verbal explanations during the interviews, enabling the researcher to compare multiple sources of evidence when interpreting how teachers identified learners' misconceptions and selected instructional responses. Participants were also given the opportunity to review their interview transcripts to verify that their views had been accurately represented. Detailed descriptions of the study context, participant characteristics, and sampling procedures are provided to enable readers to judge the applicability of the findings to similar educational contexts.

To promote consistency in the analysis, a clear audit trail documenting data collection, coding decisions, theme development, and analytical memos was maintained throughout the study. The coding framework and emerging interpretations were discussed with an experienced mathematics education researcher during peer debriefing sessions, allowing assumptions to be challenged and interpretations to be refined. In addition, the researcher maintained a reflexive journal throughout the study to acknowledge, reflect on, and monitor the potential influence of the researcher's experiences, assumptions, and interpretations during data collection and analysis.

RESULTS AND FINDINGS

The interview findings presented in this section are based on the six beginning teachers who were selected following the diagnostic geometry assessment. As described in the sampling procedure, the participants represented three levels of CK: two high-performing teachers (TA and TB), two moderate-performing teachers (TC and TD), and two low-performing teachers (TE and TF). Their diagnostic assessment scores are presented in **Table 2**.

Table 2. Diagnostic geometry assessment scores of the interview participants

Participant	Test score	Performance band
TA	78%	High
TB	78%	High
TC	69%	Moderate
TD	67%	Moderate
TE	36%	Low
TF	33%	Low

These performance categories provided the basis for comparing teachers' reported pedagogical reasoning during the task-based interviews. It should be noted that the interview findings reflect teachers' reported instructional intentions and pedagogical reasoning in response to hypothetical classroom scenarios rather than observed classroom practice. The findings are organized according to the three interview tasks and interpreted in relation to three analytical categories derived from the conceptual framework: knowledge of explanations, KCS, and KCT.

Interview Task 1: Teaching Geometric Proofs

Task prompt

> A grade 11 learner can recall geometry theorems but struggles to apply them when solving proof-based riders. How would you support this learner to construct a valid geometric proof? How would you guide their initial reasoning and respond if an incorrect theorem is selected?

This task examined teachers' reported approaches to integrating CK with pedagogical strategies to scaffold learners' construction of geometric proofs.

High-performing teachers (TA, TB)

High-performing teachers generally emphasized interactive dialogue and learner discovery rather than direct demonstration. TA described beginning with learner exploration:

> "I would first let the learner look at the diagram and tell me what they notice before choosing any theorem."

TA further elaborated:

> "After that, I guide them to justify each step using reasons, not just the answer."

Similarly, TB described using targeted questioning to address misconceptions during the proof process:

> "If the learner chooses the wrong theorem, I ask questions so they can see why it does not fit and then reconsider."

These responses suggest an emphasis on guiding learners from informal observation towards formal deductive reasoning.

Moderate-performing teachers (TC, TD)

Moderate-performing teachers favored more direct, teacher-led modelling, citing time constraints and the difficulty of geometric proof. TC explained:

> "I normally show them the proof step by step, and then they practice similar ones ... Proofs are difficult, and there is not always enough time for learners to discover."

TD described a combination of questioning and explicit guidance:

> "I ask them why they chose that theorem, but I still guide them through most of the proof."

These responses suggest a preference for procedural guidance and direct instruction, with fewer opportunities for learners to develop independent reasoning.

Low-performing teachers (TE, TF)

Low-performing teachers proposed predominantly transmissive approaches. TE stated:

> "I explain the proof on the board and learners copy it into their books."

Similarly, TF commented:

> "They must study the examples in the textbook and follow the same steps."

These reported strategies emphasize reproducing procedures rather than engaging learners in the logical structure of geometric proof.

Interview Task 2: Proving a Specific Theorem

Task prompt

> A learner claims that the angle subtended by a diameter of a circle is a right angle because it "looks like" a square corner. How would you guide the learner to construct a formal proof and justify each step without relying on visual appearance?

This task explored teachers' reported strategies for moving learners beyond empirical or visual reasoning towards formal deductive proof.

High-performing teachers (TA, TB)

Both high-performing teachers emphasized the importance of formal geometric reasoning. TA explained:

> "Since the lines from the center are radii, the triangle is isosceles, so these angles are equal."

TB explicitly rejected visual justification:

> "It must be proven using angle properties, not because it looks like ninety degrees."

These responses indicate an intention to ground explanations in formal geometric reasoning.

Moderate-performing teachers (TC, TD)

Moderate-performing teachers demonstrated mixed reasoning. TC stated:

> "I usually tell learners to imagine a square inside the circle."

TD acknowledged uncertainty in explaining the proof:

> "I know the angles are equal, but sometimes it is difficult to explain why clearly."

These responses suggest inconsistencies in translating CK into coherent pedagogical explanations.

Low-performing teachers (TE, TF)

Low-performing teachers relied on incomplete reasoning or empirical verification. TE explained:

> "You just show that the angle is ninety degrees because of the circle."

TF suggested:

> "If learners measure the angle, they can see it is a right angle."

These responses rely primarily on observation rather than formal deductive proof.

Interview Task 3: Addressing Learner Misconceptions

Task prompt

> A learner states, "A quadrilateral must have a visible circle drawn around it to be cyclic." How would you respond to this learner to address their thinking and correct the misconception?

This task examined teachers' reported strategies for identifying and addressing learner misconceptions, with particular emphasis on KCS.

High-performing teachers (TA, TB)

High-performing teachers described using diagnostic counterexamples. TA explained:

> "I would draw a quadrilateral without a circle and ask the learner to check the opposite angles. If they add up to 180°, then it is cyclic even if no circle is drawn."

Similarly, TB focused on the defining geometric property:

> "The circle is not the condition. The condition is that opposite angles are supplementary."

These responses indicate an intention to challenge learners' visual misconceptions through conceptual reasoning.

Moderate-performing teachers (TC, TD)

TD focused directly on angle relationships:

> "I would tell the learner to look at the angles first, because if they are supplementary, then the quadrilateral is cyclic."

TC offered a less explicit response:

> "Cyclic quadrilaterals have special angles, so the learner must look at the angles."

Although both teachers recognized the relevant property, TC's explanation did not directly challenge the learner's misconception.

Low-performing teachers (TE, TF)

Low-performing teachers proposed responses that either reinforced the misconception or contained mathematical inaccuracies. TE stated:

> "I would show them the angles in the diagram and explain that cyclic quadrilaterals have equal angles."

TF relied on visual confirmation:

> "It is easier if you draw the circle so the learner can see it is cyclic."

These responses suggest limited diagnostic awareness and continued reliance on visual representations.

Cross-Case Analysis

The qualitative analysis indicated a broad alignment between teachers' baseline test performance and their reported pedagogical reasoning during the interviews. High-performing teachers (TA and TB) consistently described learner-centered scaffolding and deductive explanations. Moderate-performing teachers (TC and TD) reported pedagogical approaches reflecting partial content understanding and a preference for direct instruction, whereas low-performing teachers (TE and TF) relied primarily on procedural demonstration and empirical verification.

Although these patterns were generally consistent, some variation emerged within the performance groups. For example, despite demonstrating strong deductive reasoning in task 1 and task 3, TB acknowledged that during high-stakes examination preparation they often revert to direct, rule-based teaching similar to that described by TC and TD because of curriculum pacing demands. Conversely, although TC generally favored teacher-led instruction, their response to Task 1 reflected an emerging awareness of guided questioning, even if time constraints prevented them from prioritizing this approach. These variations suggest that teachers' reported PCK is shaped not only by their CK but also by contextual factors such as instructional time.

Overall, the findings suggest that differences in teachers' subject-matter knowledge were reflected in their reported instructional decisions, their approaches to diagnosing learner thinking, and the strategies they proposed for teaching Euclidean geometry. Across the three interview tasks, variation was evident within the PCK sub-domains of KCT and knowledge of KCS, supporting the analytical framework adopted in this study. Because these findings are based on teachers' reported pedagogical reasoning rather than observed classroom practice, further classroom observation is needed to examine how these reported intentions are enacted during geometry instruction.

DISCUSSION

The primary purpose of this study was to examine how beginning mathematics teachers mobilized CK and PCK when identifying and responding to learner misconceptions in Euclidean geometry. Rather than merely distinguishing differences in teacher performance, the findings provide insight into how beginning teachers integrate mathematical and pedagogical knowledge when identifying and responding to learner misconceptions in Euclidean geometry. This discussion therefore interprets these patterns in relation to existing theory and considers their implications for teacher education and the proposed Phaliso's classroom knowledge model (PCKM).

Explaining Variations in Diagnostic and Responsive Teaching

The findings suggest that differences in teachers' responses to learner misconceptions were influenced not only by the amount of mathematical knowledge they possessed but also by how effectively they integrated that knowledge with an understanding of learners' thinking. Beginning teachers are still developing professional knowledge and frequently draw on their own experiences of learning mathematics when making instructional decisions (Matthews, 2020). Consequently, they may rely on familiar procedural approaches rather than adapting instruction to learners' conceptual difficulties.

The stronger performance of some teachers can be explained by their ability to coordinate SCK, KCS and KCT, suggesting that effective diagnosis of misconceptions depends less on isolated knowledge possession and more on the integration of knowledge domains. Rather than viewing learner errors simply as incorrect answers, they interpreted misconceptions as evidence of underlying conceptual misunderstandings. For example, they recognized that learners who incorrectly applied properties of cyclic quadrilaterals were often overgeneralizing previously learned geometric relationships. This finding supports Ball et al.'s (2008) argument that effective teaching depends on the integration of mathematical knowledge with knowledge of how learners think about mathematical ideas.

In contrast, teachers who demonstrated weaker performance focused primarily on procedural correctness and often treated errors as isolated mistakes requiring correction rather than opportunities to explore learners' reasoning. Their explanations relied heavily on textbook procedures and worked examples, with limited attention to the conceptual origins of misconceptions. Similar patterns have been reported in studies showing that teachers with limited SCK frequently depend on procedural explanations when confronted with learner errors (Nojiyeza, 2019; Venkat & Spaull, 2015; Zuya & Kwalat, 2015). These findings suggest that limited SCK constrains teachers' ability to unpack mathematical ideas and provide explanations that promote conceptual understanding.

Data Variations and Emerging Insights

This pattern represents a tension within the data because higher CK performance did not always correspond with stronger diagnostic capability. Some participants demonstrated acceptable procedural competence during the diagnostic assessment but experienced difficulty explaining why learners developed particular misconceptions or how those misconceptions could be addressed instructionally during the interviews. This finding suggests that CK, although essential, is not sufficient on its own to support responsive teaching. Teachers may understand mathematical procedures themselves yet experience difficulty transforming that knowledge into explanations that address learners' thinking. This distinction reflects Shulman's (1986) argument that PCK represents a unique form of professional knowledge that extends beyond disciplinary expertise. It also reinforces Ball et al.'s (2008) proposition that knowledge of learners and knowledge of teaching develop alongside, rather than automatically from, mathematical CK. Consequently, strengthening teachers' CK alone is unlikely to produce meaningful improvements in diagnostic teaching unless opportunities are also provided to develop knowledge of learner thinking and instructional decision-making.

Development of Phaliso's Classroom Knowledge Model

One of the principal contributions of this study is the proposal of PCKM, a conceptual framework developed to explain how beginning mathematics teachers appear to develop PCK when identifying and responding to learner misconceptions in Euclidean geometry. Rather than replacing existing theoretical frameworks, the PCKM extends the work of Shulman (1986) and Ball et al. (2008) by illustrating a possible developmental progression through which beginning teachers integrate CK with pedagogical reasoning.

The methodological basis of the PCKM warrants clarification. The model did not emerge from inductive analysis alone, nor was it imposed directly from an existing theoretical framework. Instead, the study adopted a hybrid thematic analysis in which the initial coding was guided deductively by Ball et al.'s (2008) MKT framework, while iterative comparisons across participants allowed additional patterns to emerge from the data. These recurring patterns revealed differences not only in the amount of knowledge teachers possessed but also in how they mobilized that knowledge to interpret learners' mathematical thinking and make instructional decisions. The PCKM therefore represents a theoretical synthesis grounded in both the empirical findings and established knowledge frameworks. To illustrate these developmental patterns, **Figure 1** presents the PCKM. The model summarizes the progression observed across participants, from teachers whose knowledge was primarily procedural to those who demonstrated integrated pedagogical reasoning. It should be interpreted as a conceptual representation of the patterns identified in this study rather than as a fixed sequence through which all beginning teachers' progress.

Figure 1 illustrates three conceptual phases that describe how PCK appeared to develop among the beginning teachers who participated in this study. The first phase, foundational knowledge building. This phase was evidenced by participants who achieved procedural success in geometry tasks but struggled during interviews to explain the reasoning behind learner errors or propose alternative instructional responses. These teachers relied predominantly on recalling geometric definitions, theorems, and textbook

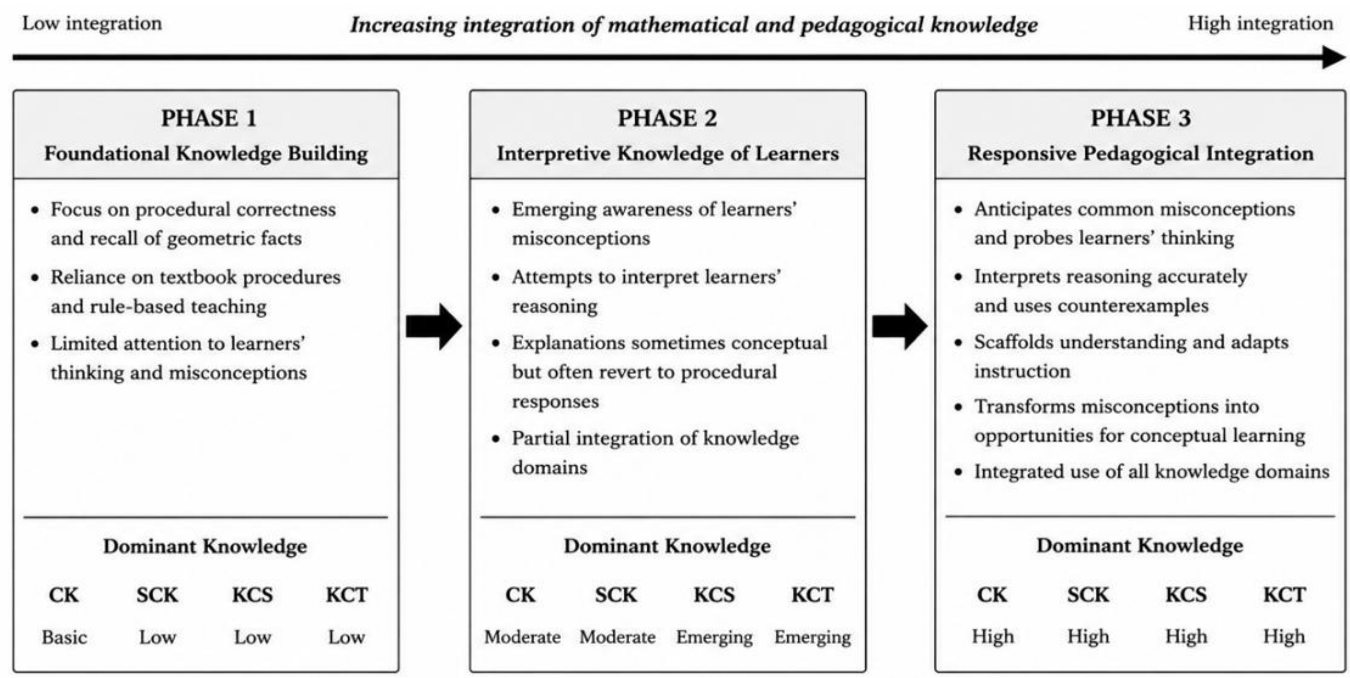

Figure 1. PCKM: Development of beginning teacher' PCK in Euclidean geometry (Phaliso, 2022)

procedures, while showing little evidence of interpreting the conceptual basis of learner errors. Their responses suggested that although foundational mathematical knowledge had been established, it had not yet been transformed into pedagogically meaningful knowledge for teaching.

The second phase, interpretive knowledge of learners, emerged from participants who demonstrated growing awareness of learner misconceptions and attempted to interpret learners' reasoning. Although these teachers recognized conceptual difficulties, they frequently experienced challenges in translating their diagnostic insights into effective instructional responses. Their explanations often alternated between conceptual reasoning and procedural demonstrations, suggesting an intermediate stage in which knowledge of learners was developing but remained only partially integrated with pedagogical decision-making.

The third phase, Responsive Pedagogical Integration, was evident among participants who consistently integrated SCK, KCS, and KCT. These teachers anticipated common misconceptions, used probing questions, counterexamples, and scaffolded explanations, and adapted their instruction in response to learners' mathematical thinking. Rather than simply correcting errors, they transformed misconceptions into opportunities for conceptual learning. This phase therefore represents the highest level of pedagogical integration observed within the present study.

Importantly, these three phases should not be interpreted as fixed or universally sequential stages through which every beginning teacher progresses. Instead, they represent conceptual patterns identified across participants with differing levels of pedagogical expertise. The phases describe how PCK appeared to be organized within this sample rather than prescribing a universal pathway of teacher development. Consequently, the proposed PCKM should be regarded as a conceptual framework that offers one possible explanation of beginning teachers' professional learning in Euclidean geometry.

The horizontal arrow in **Figure 1** represents increasing integration of mathematical and pedagogical knowledge. Low integration refers to situations where teachers possess knowledge domains that operate largely in isolation, resulting in procedural instruction and limited responsiveness to learner thinking. High integration reflects the coordinated use of SCK, knowledge of learners, and knowledge of teaching to diagnose misconceptions and implement responsive instructional strategies.

Dominant knowledge levels or the levels of knowledge development in the PCKM are represented as a continuum from basic to high integration (**Figure 1**). At the basic level, teachers demonstrate limited understanding of mathematical concepts and procedures and rely mainly on memorized rules. At the low level, teachers show some procedural knowledge but struggle with conceptual explanations, identifying misconceptions, and connecting mathematical ideas to learners' difficulties. At the emerging level, teachers demonstrate developing understanding and begin to integrate mathematical knowledge with learners' thinking and instructional approaches, although inconsistencies remain. At the moderate level, depending on your scale, teachers demonstrate adequate knowledge, recognize misconceptions, and apply appropriate pedagogical strategies with some integration among knowledge domains. At the high level, teachers demonstrate well-integrated knowledge, anticipate learners' misconceptions, provide meaningful explanations, and adapt instruction based on learners' reasoning.

Implications of Phaliso's Classroom Knowledge Model

The proposed PCKM has several implications for mathematics teacher education and future research. First, the findings suggest that the development of PCK is not achieved simply through strengthening CK. Rather, responsive teaching appears to develop when teachers learn to integrate mathematical understanding with knowledge of learners' misconceptions and appropriate instructional strategies. Professional development programs may therefore benefit from supporting beginning teachers to develop these knowledge domains in an integrated manner rather than treating them as independent competencies.

Second, the findings suggest that the model may offer insights into the importance of explicitly developing teachers' diagnostic reasoning. The findings indicate that teachers who possessed acceptable mathematical knowledge did not always demonstrate the ability to interpret learners' misconceptions or adapt instruction accordingly. This suggests that teacher education programs should provide structured opportunities to analyze learner errors, discuss alternative solution strategies, and design instructional responses that promote conceptual understanding.

Finally, although the PCKM is grounded in the empirical patterns identified in this study, its broader applicability remains to be established. The model should therefore be regarded as a theoretical proposition rather than a validated framework for teacher development. Further research involving larger and more diverse samples, longitudinal designs, and classroom observations is needed to examine whether the developmental patterns identified in this study are evident across different educational contexts and mathematical topics.

Limitations

Several limitations should be considered when interpreting these findings. First, the study involved a relatively small purposive sample of beginning mathematics teachers from two rural districts in the Eastern Cape. Consequently, the findings are intended to provide analytical rather than statistical generalization, and readers should consider their transferability to comparable educational contexts. Second, the study incorporated diagnostic assessments and task-based interviews; it did not include classroom observations or longitudinal evidence of enacted practice. Although these data sources provided rich insight into teachers' mathematical and pedagogical reasoning, they did not capture teachers' enacted classroom practices. Future studies incorporating sustained classroom observations could provide a more comprehensive understanding of how teachers identify and respond to learner misconceptions during instruction.

Finally, interview data are subject to the limitations associated with self-reported professional reasoning. Participants may describe instructional approaches that differ from their everyday classroom practices. Future research should therefore triangulate interview data with classroom observations and learner outcomes to strengthen understanding of the relationship between teachers' knowledge and instructional practice.

Overall, the findings suggest that effective responses to learner misconceptions depend on the integration of mathematical knowledge with pedagogical reasoning rather than CK alone. The proposed PCKM provides a conceptual explanation of how this integration appeared to develop among the beginning teachers who participated in this study. Although the model requires further empirical validation, it offers a foundation for future research and mathematics teacher education aimed at strengthening diagnostic and responsive teaching in Euclidean geometry.

CONCLUSION

This study examined how beginning mathematics teachers identify learners' misconceptions in Euclidean geometry and apply their content and pedagogical knowledge to address those misconceptions. The findings suggest that effective diagnostic and responsive teaching depends not only on teachers' CK but also on their ability to integrate SCK, KCS, and KCT. Teachers who demonstrated stronger integration of these knowledge domains were better able to interpret learners' mathematical thinking and implement instructional responses that promoted conceptual understanding.

A key contribution of this study is the development of PCKM. Unlike existing PCK frameworks that primarily describe the knowledge required for effective teaching, the PCKM proposes a conceptual explanation of how beginning teachers appear to develop and integrate these knowledge domains when responding to learner misconceptions in Euclidean geometry. The model therefore complements, rather than replaces, the frameworks of Shulman (1986) and Ball et al. (2008) by emphasizing the developmental progression of pedagogical reasoning observed within this study.

The findings have practical implications for mathematics teacher education. Professional development for beginning teachers should be developmentally sequenced by first strengthening conceptual understanding of key Euclidean geometry topics, then developing teachers' ability to analyze learner misconceptions through diagnostic tasks, and finally providing structured opportunities to design, implement, and reflect on instructional responses that promote conceptual understanding. Such sequencing may better support the integration of mathematical knowledge with pedagogical reasoning than approaches that address these competencies separately.

The study also highlights several directions for future research. Further studies should validate the PCKM with larger and more diverse samples of beginning mathematics teachers across different educational contexts. Longitudinal research is needed to examine how teachers' PCK develops over time, while classroom observations should be incorporated to investigate how the developmental patterns proposed in the PCKM are enacted in authentic teaching practice. Such research will help determine the broader applicability and refinement of the model.

Although the PCKM requires further empirical validation, this study provides a foundation for understanding how beginning mathematics teachers develop the knowledge required to identify and respond to learners' misconceptions in Euclidean geometry. In doing so, it contributes to the growing body of research on PCK by offering a conceptual perspective on the developmental integration of mathematical and pedagogical knowledge among beginning teachers.

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Ethical statement: This study was approved by the Faculty Research Ethics Committee at Walter Sisulu University on 6 June 2024 with approval number FEDFREC 07-06-24-02. Informed consent was obtained from the participating teachers before data collection. Participation was voluntary, and the confidentiality of participants was maintained throughout the research process. Participants' identities were protected through the use of pseudonyms, and identifying information was excluded from the reporting of the findings. Research data were handled confidentially and used for research purposes.

AI statement: Generative AI was not used in this study except to generate the diagram presented in **Figure 1**. The authors reviewed **Figure 1** and took responsibility for its accuracy and relevance to the study. Grammarly was also used for the purposes of spellings, punctuation and sentence construction.

Declaration of interest: No conflict of interest is declared by the authors.

Data sharing statement: Data supporting the findings and conclusions are available upon request from the corresponding author.

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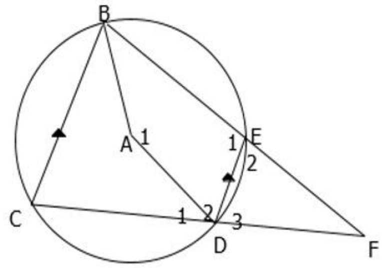
APPENDIX A: GEOMETRY TEST

Name of the teacher: ___________________

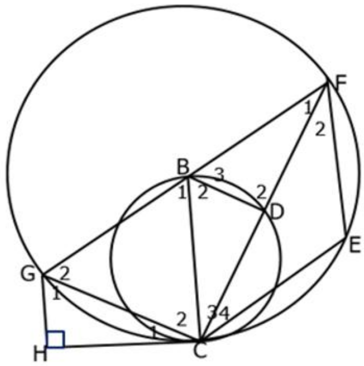
School: ________________________________

Answer ALL questions to the best of your ability in the spaces provided.

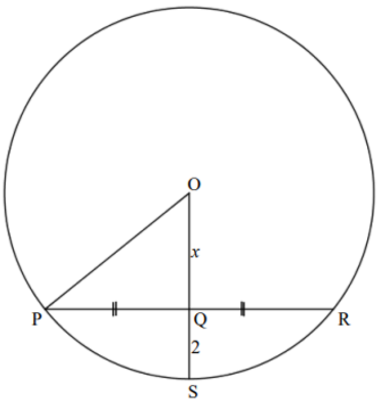
1. In the figure below, BCDE is a cyclic quadrilateral. $BC//ED$ in the circle with center A. BE and CD produced meet at F. $\angle D_3 = x$.



   a. Show that $FE = FD$ (4 points).
   b. If $\angle D_3 = x$, determine the value of $\angle F$, in terms of x (2 points).
   c. Hence, show that $BADF$ is a cyclic quadrilateral (3 points).
2. B is the center of the larger circle $CEFG$. BC is the diameter of the smaller circle CDB. HC is a tangent to both circles at C. $GH \perp HC$, $\angle C_1 = x$.



   a. Prove that CG bisects $\angle BGH$ (5 points).
   b. Prove that $\angle GBD = \angle CEF$ (5 points).
3. Use any two methods to prove the theorem which states that: The line drawn from the center of a circle that bisects a chord is perpendicular to the chord (10 points).
4. In the diagram, O is the center of the circle. Q is the midpoint of chord PR. OQS is the radius of the circle. PR = 8 units, $OQ = x$ units and QS = 2 units.



   a. Determine, giving reasons, the size of $\angle OQP$ (2 points).
   b. Calculate the length of PO (5 points).

Total 36 marks.

Marking Guideline

1.1.1. $\hat{D}_3 = \hat{C}$ (corr ∠s BC//ED)
$\hat{C} = \hat{E}_2$ (ext ∠ of cyclic quad)
$\hat{E}_1 = \hat{D}_3$ (Both ∠s $= x$)
$\therefore$ FD = FE (Sides opp equal ∠s) 4

1.1.2. $\hat{F} + \hat{E}_2 + \hat{D}_5 = 180°$ (∠s in Δ)
$\hat{F} + 2x = 180°$ 2
$\hat{F} = 180° - 2x$

1.1.3. $\hat{A}_2 = 2x$ (∠ at centre = 2∠ at cir)
$\hat{A}_1 + \hat{F}$
$2x + 180° - 2x$ 3
$180°$
$\therefore$ BADF is a cyclic quad (Conv. opp ∠s of a cyclic quad)

1.2.

1.2.1. $\hat{G}_1 = 90° - x$ (∠s in Δ)
$\hat{C}_1 + \hat{C}_2 = 90°$ (dia ⊥ tang)
$x + \hat{C}_2 = 90°$
$\hat{C}_2 = 90° - x$
BC = BG (radii)
$\hat{C}_2 = \hat{G}_2 = 90° - x$ (∠s opp = sides)
$\hat{G}_1 = \hat{G}_2$ (Both $= 90° - x$)
$\therefore$ CG bisect $B\hat{G}H$ 5

1.2.2. $C\hat{E}F = 90° + x$ (opp ∠s of cyclic quad)
$\hat{B}_1 = 2x$ (∠ at centre = 2× ∠ at circumference)
BF = BG (Radii)
$\hat{C}_3 = \hat{F}_1 = x$ (∠s opp = sides)
$B\hat{D}C = 90°$ (Sub by diameter BC)
$\hat{B}_2 = 90° - x$ (∠s in Δ)
$G\hat{B}D = 90° + x$ 5
$\therefore G\hat{B}D = C\hat{E}F$ (Both ∠s $= 90° + x$)

1.3
Method 1: Congruent triangles 5
Method 2: Isosceles triangle method 5

1.4.

1.4.1. $O\hat{Q}P = 90°$ (line from centre to midpoint of chord)

1.4.2. PQ = 4 units (PQ = QR = 4 units)
OP = OS (radii)
$OQ^2 + QP^2 = OP^2$ (Pyth)
$x^2 + 4^2 = (x+2)^2$
$x^2 + 16 = x^2 + 4x + 4$
$x^2 - x^2 + 4x + 4 - 16 = 0$
$4x = 12$ 5
$x = 3$
PO = 3 + 2
= 5 units

TOTAL [36]