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Language difficulty as a factor related to learner errors in financial mathematics

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Abstract
This study examined the relationship between language difficulties and learner's errors in financial mathematics, and it employed a quantitative research approach using a case study design. The population for the study consisted of all grade 10 mathematical literacy (ML) learners from three selected schools in East London District, from which 40 learners and two ML teachers were randomly selected from each school. Data was collected using a content-based questionnaire and structured questionnaires. Descriptive statistics were used to analyze the questionnaires. Hence, the findings revealed a significant relationship between language difficulties and the errors committed by learners. As a result of the findings, the study recommended, among other things, that teachers incorporate error analysis into the planning of their lessons to understand why students make mistakes and how to prevent them.

Keywords: errors, financial mathematics, language difficulty, mathematical literacy, skills

INTRODUCTION: BACKGROUND

Mathematics education is an essential component of the South African education system. South Africa implemented a new curriculum in 2006, and mathematics learning was compulsory in the national curriculum statements (NCS) in the further education and training (FET) band. As a result, all students are expected to take either mathematical literacy (ML) or Mathematics as one of the school's primary subjects. The introduction of ML was an attempt to teach the general public about debts, loans, taxes, interest rates, graphs, and other mathematical concerns that people face daily. The new curriculum was also implemented to transform society by equipping them with mathematical knowledge that can be applied in real life.

Thus, given the rapid advancement of information and technology and the shift in society's expectations (Arseven, 2015; Olawale & Mutongoza, 2021), it becomes necessary to put new models, approaches, and methods in place in the educational system. Arseven (2015) refers to models as tangible entities, objects, and pictures in which some conditions of general ideas wanted to be developed are represented. Thus, mathematical modelling has become an essential part of the South African mathematics curriculum to make mathematics teaching and learning easier, more straightforward, and more applicable to real-life circumstances.

According to the Department of Education (DoE, 2011), examples used in mathematics solving should involve real-life problems in every concept. Mathematics problems should involve scientific, health, political, economic, social, and cultural issues (DoE, 2011, p. 8). Therefore, the view of ML that is tied to socio-economic demands in terms of skill marketability might be referred to as 'functional,' in that it simply refers to an individual's ability to respond to the needs and constraints of society (Arseven, 2015; Bishop et al., 2003). This concept of ML thus aspires to see the world through mathematical lenses. Similarly, higher-order thinking and acquiring and applying broader problem-solving skills are emphasized over basic mathematics skills.

In learner's examination, financial mathematics has a 35% weighting of all topics, which signifies its importance and value in ML studies (DoE, 2011). Financial mathematics encompasses several fundamental mathematical abilities, including numerical computations, conveying solutions,

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Contribution to the literature

- Insight into the adoption of Polya's problem-solving techniques in the teaching and learning ML in South African schools.
- Incorporated error analysis in the teacher-education program curriculum will help reduce learner errors in mathematics.
- Enhanced collaboration between ML teachers and English language teachers.

calculating, and understanding data (Khalo & Bayaga, 2014). However, in these basic mathematics skills, students lose points on assessment tasks. While financial mathematics is regarded as a fascinating subject and also a field of mathematics with applications in everyone's daily lives (Makonye et al., 2014), many learners still find it challenging to comprehend the fundamentals of financial mathematics, which leads them to commit several errors.

Thus, Peng (2009) stated that one of the problems learners faces in learning mathematics is mathematical errors. As a result, learners of different age levels have issues in mathematics, no matter their performance. Brodie (2005) refers to errors as pervasive, persistent, systematic, and constant mistakes learners make within various contexts. The author posits that research on learner errors has a long history in mathematics education. In the critical theoretical understanding of mathematics teaching and learning, Brodie (2014) argues that learner errors are a logical and understandable element of the learning process. Hence, given that mathematical errors are a vital component of teaching and learning for mathematics teachers, identifying aspects connected to learners' errors in financial mathematics. This will enable them to build corrective and preventative methods.

Statement of the Problem

On the one hand, several researchers (Brodie, 2014; Riccomini 2005; Sherman et al., 2005; Yang et al., 2011) argues that error analysis is a vital skill for teaching mathematics to non-native English speakers. On the other hand, researchers such as Carey (2004), Gelman and Butterworth (2005), and Herholdt and Sapire (2014) disagree on whether language is the cause of mathematics difficulty for learners learning in a language other than their native tongue. This makes error analysis relevant in the context of South Africa, where the majority of learners begin learning mathematics in a language other than their native tongue in grade four (Herholdt & Sapire, 2014). As a result, the necessity of a curriculum that supports systematic mastery of mathematical language, conceptual development, and comprehension was emphasized by Yang et al. (2011).

However, the introduction of an outcomes-based education paradigm as a new subject in the South African curriculum in 2006 presented significant obstacles to teachers and students. One of the difficulties is that teachers are not well equipped to plan and conduct instructional interventions based on student errors (Herholdt & Sapire, 2014; Riccomini, 2005). Another issue is the expectation that students in ML have eliminated the mistakes they make while solving problems in the subject (Herholdt & Sapire, 2014). According to the Diagnostic Report (2014), the number of candidates writing ML decreased by 12,043 candidates. Candidates who passed at the 30% level decreased by 3.0%, while those who passed at the 40% level decreased by 2.9%.

Based on the above overview, even with the best teaching and learning strategies, learners continue to make similar mistakes when they are taught. Thus, exploring learners' epistemological challenges through error analysis and diagnosis is an integral part of good teaching since it educates researchers, teachers, and students about the difficulties learners have with certain mathematical subjects (Makonye & Luneta, 2013). Hence, the need for the study.

Purpose of the Study

The purpose of the study was to examine if there exists a relationship between language difficulties and financial mathematics errors made by learners.

Research Question

Is there a relationship between language difficulties and financial mathematics errors made by students?

Research Hypothesis

The following hypotheses were developed and tested in response to the study question:

1. **H0:** There is a significant relationship between language difficulties and errors committed by learners in Financial Mathematics.
2. **$H0_1$:** There is no significant relationship between language difficulties and errors committed by learners in Financial Mathematics.

Theoretical Framework

This study is underpinned by the problem-solving model developed by Polya (1945). This model consists of four phases, which are:

1. Understanding the problem,

2. Devising a plan,
3. Carrying out the plan, and
4. Looking back.

Strong knowledge of the phases mentioned above would help eliminate errors made by learners. Thus, Polya's (1945) problem-solving phase is a relevant model for learning ML. During problem-solving, the following principles are to be considered (Polya's, 1945, p. 11):

First principle: Understand the problem

The learners should first understand the problem. Without an understanding of the problem, the learners might seem incompetent. Therefore, the teacher should ensure that learners understand the problem they want to solve first.

Second principle: Devise a plan

There are different methods of solving problems. The skill in choosing the suitable method is best discovered when solving many problems (Polya, 1945, p. 13).

Third principle: Carry out the plan

It is easier to carry out a plan than to create one because only patience is required when carrying out the devised plan. For instance, having a correct substitution is very important when using a formula. Polya (1945, p. 14) stated that "Consistency throughout the algorithms employed to arrive at the final answer is of the utmost importance in this step."

Fourth principle: Look back

This principle allowed reflection on the work, which enables predicting the appropriate method and strategy for solving the problem. If the devised plan fails to provide the desired result, you must ignore it and try again until you find the correct answer.

Polya's (1945) theory on problem-solving principles is suitable for this study because it can influence ML teaching and learning processes. The four-step principles can guide teachers and learners and assist the teacher in dedicating more time to learners' work and thus reduce the number of mathematical errors committed. Hence, this study seeks to examine the underlying factors related to the type of errors committed and how the stages of Polya's (1945) principles can be applied to minimize errors committed by learners.

LITERATURE REVIEW

Understanding the Learning Process of Mathematics

There are two categories of understanding in mathematics' learning process: instrumental and relational understanding. *Instrumental understanding* is demonstrated by an individual who does not understand the rule used in solving a particular problem (rules of division of a number by a fraction you turn it upside down and multiply). The second is the *Relational understanding*–which occurs when one has assembled conceptual mathematics structure. According to Soendergaard and Cachaper (2008, p. 15), "working memory is especially critical to mathematics learning because mathematics learning places frequent demands on working memory." As a result, students must memorize intermediate products of calculations to solve problems. Given that it has been discovered that effective working memory is linked to successful mathematics learning (Nur et al., 2018). Nonetheless, in financial mathematics, connected problems are more common, particularly in the sections on taxation, income, and expenditure.

According to Soendergaard and Cachaper (2008, p. 16), "relational understanding/thinking occurs when one has built a conceptual structure (schema) of mathematics and therefore both know what to do and why when one solves a mathematical problem." For example, when addressing issues involving simple and compound interest, the interest may be increased monthly for four years; this necessitates relational reasoning to determine the value of n. Therefore, there is a need to develop relational thinking through teaching and learning in the classroom. This contributes significantly to the reduction of mathematical errors made by learners.

Error Analysis in Mathematics Education

Error analysis is used to determine the effectiveness of a particular method used by learners in solving problems; it can also be used if learners lack a basic concept of a problem. It is essential for mathematics educators to understand that analyzing students' mathematical errors is important for teaching and learning. It helps them put preventative and corrective measures in place (Peng, 2009). According to Murray (2012), learners detest and fear word problems in mathematics since they are more complex than doing a simple calculation. However, ML is contextually based; thus, it consists primarily of word problems, which by their nature explain the existence of persisting learner errors therein. Sheinuk (2010, p. 12) states that "pre-service teachers who confront own mathematical errors, misconceptions, and strategies to recognize their subject matter knowledge, have an opportunity to develop rich content knowledge."

There are five main descriptions of learner's errors: reading comprehension error, reading error, transformational error, procedural error, and encoding error (Khalo & Bayaga, 2014). Thus, error analysis deals with the analysis of learners working steps towards finding a solution to a problem and studying best practices for remediation (McGuire, 2013). As a result, for a teacher to analyze learner's errors, s/he would need to be well-versed in mathematical content and the degrees of mathematical understanding of the students (Herholdt & Sapire, 2014; McGuire, 2013).

English as an Instructional Language for Mathematics Learning

One of the barriers to mathematical learning is attributed to the fact that English is foreign to most South African learners as it is an additional language and not their home language (mother tongue). They are therefore learning mathematics as well as English at the same time. According to Ga'ndara and Contreras (2009), mathematics instruction for English learners (ELs) should:

1. treat language as a resource, not a deficit,
2. address much more than vocabulary and support ELs' participation in mathematical discussions as they learn English (Moschkovich, 2010), and
3. draw on multiple resources available in classrooms (Moschkovich, 2010, p. 174).

Such resources include objects like drawings, graphs, gestures, and home language and experiences outside of school (Moschkovich, 2010, p. 174).

Thus, teachers need to consider the afore-stated approaches in their teaching of mathematics and ML. According to Moschkovich (2010), language is critical for understanding the two subjects. Continued discussions in class also assist learners in gaining the debating skills, equipping them with abilities to answer Bloom taxonomy level four questions (justifications; judgment, decision making). Moschkovich (2010, p. 180) posit that "one of the goals of mathematics instruction for ELs should be to support all students, regardless of their proficiency in English in participating in discussions that focus on important mathematical concepts and reasoning, rather than on pronunciation, vocabulary, or low-level linguistic skills."

Research has shown that mathematical communication is not just a vocabulary (Moschkovich 2010; Nel, 2012). Though vocabulary is necessary, it is not enough. As a result, language exercises and practices cannot be considered the most effective method of teaching mathematics. Experts in second-language acquisition and vocabulary have described vocabulary acquisition in a first or second language as occurring most successfully in instructional contexts that are language-rich, requiring students to use words in multiple ways over extended periods (Blachowics & Fisher, 2000; Presley, 2000).

To focus on the mathematical meanings, learners prefer to construct rather than focusing on mistakes they make or obstacles they face. As a result, curriculum materials and professional development programs will need to assist instructors in recognizing growing mathematical reasoning that students construct in, though, and with emerging language and learning to employ different representations. The cultural model of South Africa emphasizes the belief that English language acquisition is made up of the significant content of schooling (Setati, 2008, p. 113), which is not in line with the content about giving epistemological access. It is also not in line with language in education policy (LiEP) and research in South Africa, which encourages multilingualism and use of the learners' home language" (Setati, 2008, p. 113).

Lourens et al. (2012) state that errors result from a 'consistent conceptual framework based on earlier acquired knowledge, called misconceptions, and makes sense to learners in their current thinking. Thus, learners do not just make errors, but these errors occur because they make sense to them due to the conceptual link to the knowledge they acquired previously (Lourens et al., 2012). As a result, errors make sense to individuals who make them, and as such, errors should be welcomed rather than disregarded or simply mended in mathematics teaching and learning (Lourens et al., 2012, p. 4). Therefore, it is important to understand how errors and misconceptions can inform instructional practice because errors indicate that the desired outcome was not achieved and something else has to be done.

Language and Mathematics Learning

It seems plausible to conclude that a learner's capacity to understand the language of teaching and their reading comprehension level are important factors in effective learning. Learners should be familiar with the vocabulary, symbols, mathematical concepts, and terminologies used when solving word problems (Moschkovich, 2010). Unfortunately, English is a foreign language to some learners, which has posed a challenge in their learning. According to Radatz (1979), learners' errors are frequently caused by a misunderstanding of the semantics of mathematical text.

> "For many pupils the learning of mathematical concepts, symbols, and vocabulary is a 'foreign language' problem. In solving word problems, pupils must refrain from using the manifold background of a word's meaning in natural language. A misunderstanding of the semantics of mathematical text is often the source of pupils' errors" (Radatz, 1979, p. 165).

Poor language abilities, such as speaking, writing, and reading, are sometimes linked to reasons behind learners' poor performance in mathematics and ML. However, mathematics on its own has a set of vocabulary, symbols, and language patterns (Nel, 2012). This has posed a significant barrier for students learning ML, as they have struggled to comprehend some of the mathematical terms. As a result, learners may be unable to interpret the information required to solve a problem due to a lack of reading comprehension skills. Learners may also struggle to comprehend the mathematical notation required to address a problem (Baldwin & Yun, 2012). Hence, mathematics has its unique vocabulary needed to comprehend the meaning of the stated mathematical problem to arrive at an appropriate problem-solving method.

Murray (2012:49) states that "the major part of developing an understanding of Mathematics involves learning to handle the set of mathematics language patterns, symbols, and vocabulary to make connections between them." Similarly, Murray (2012) claims that there is a risk that functional literacy (including reading comprehension) is overly simplified without considering the many factors that can prevent a learner from grasping what they are reading. The author outlined the obstacles to comprehending word problems, mostly related to ML (Murray, 2012). As a result, the problem solver must be familiar with appropriate mathematical terminology, symbols, notations, and models in order to improve the right transformation of problem-solving statements into comparable algebraic equations.

RESEARCH METHODOLOGY

This study is underpinned by a positivist paradigm and employs a quantitative research approach to discover and confirm causal and effects. According to Kaboub (2008, p. 343), "positivists' paradigm asserts that real events can be observed empirically and explained with logical analysis." Hence, the positivist paradigm characterizes the claim that science offers us the most explicit possible knowledge ideal. For the study, three schools purposively selected from the East London district in the province of the Eastern Cape, South Africa, participated. The selected schools are urban schools wherein all grade 10 ML learners formed a study population.

Sample Size and Justification

The three schools from the afore-mentioned district were purposively selected. 40 learners were randomly chosen from each school to participate, giving 120 learners who participated in the study. The study also included six ML educators, i.e., two educators from each school. According to Cohen et al. (2007, p. 97), "determining the size of the sample will have to take account of attribution and respondent mortality, i.e., that some participants will leave the research or fail to return questionnaires."

In order to ensure the results' dependability and trustworthiness, simple random was employed to choose the respondents. The researcher compiled a list of each School's research population, with a three-digit number next to each name. The researcher selected the required number (i.e., 40) randomly from the list of the population. "One problem associated with this particular sampling method is that a complete list of the population is needed, which is not always readily available" (Cohen et al. 2007, p. 100). This was not the case in this study because the research sites are schools, and class lists were readily available, allowing a composite list for each School to be compiled.

Data Collection Instrument

Data was collected using a content-based questionnaire, structured questionnaires, and a review of the document published by the Department of Education. A content-based questionnaire allowed the respondents to commit errors based on financial mathematics problems. In contrast, the structured questionnaire was used to unveil the underlying reasons for the errors committed in the written test. In addition, documents were reviewed to investigate common types of errors made by learners such as

1. errors resulting from wrong thinking or rigidity,
2. errors resulting from the application of irrelevant rules or tactics,
3. errors resulting from a lack of mastery of prerequisite skills, facts, and concepts, and
4. errors resulting from linguistic difficulties.

The data was collected from the respondents using a content-based questionnaire and a follow-up questionnaire with dichotomous questions and rating scale questions on a Likert scale (grade 10 ML learners of the participating schools). In addition, ML educators were allowed to develop a memorandum (marking guide) mark and interact with the content-based questionnaire. After that, a follow-up questionnaire, which sought to uncover the nature and the underlying reasons related to learner errors from the educators' perspective and the instructional approaches they use in their classes, was given to the educators. Learners were given a content-based questionnaire on financial mathematics to discover and uncover the underlying reasons for errors committed in financial mathematics. The questionnaires were based on the significant differences between

1. language difficulties,
2. prerequisite skills,
3. incorrect facts and concepts, and
4. irrelevant rules and strategies by learners in financial mathematics.

Data Analysis

Quantitative analysis with descriptive statistics, which describe the distribution, the relationship among variables, and frequency variability, was also used to analyze the second type of questionnaires. Statistical

Table 1. Distribution of respondents' gender per school

	Male		Female	
	Frequency	%	Frequency	%
School A	12	12.4	19	19.6
School B	10	10.3	28	28.9
School C	10	10.3	18	18.6
Total	32	33.0	65	67

Table 2. Age distribution of the respondents in each school

	15 years		16 years		17 years		18+ years	
	Freq	%	Freq	%	Freq	%	Freq	%
School A	6	6.2	9	9.3	14	14.4	2	2.1
School B	1	1.0	5	5.2	18	18.6	14	14.4
School C	0	0	2	2.1	1	1.0	25	25.8
Total	7	7.2	16	16.5	33	34.0	41	42.3

treatment includes simple frequencies and percentages of correctly answered questions and errors identified as common among the samples. For correlation coefficient analysis, the statistical package of social sciences (SPSS) version 21 was used to measure the link between variables in each of the aforementioned study questions.

Ethical Approval

Ethical clearance was obtained from the University of Fort Hare Ethics Committee for which the reference number is ADU031SKHA01.

RESULTS & DISCUSSION OF FINDINGS

Table 1 shows the distribution of respondents' gender per school. From the three schools (A, B, & C) that participated in the study, a total of 97 respondents (grade 10 ML learners) formed part of the study. Female learners (i.e., girls) turned out to be the majority of the sample, with 65 (67%) respondents being girls and 32 (33%) being males. For instance, the breakdown of female respondents in the three schools are, as follows: School A 19.6%; School B 28.9%, and School C 18.6%, and that did not come as a surprise as it is the general phenomenon in our societies.

Table 2 illustrates the age distribution of the respondent from each participating school. According to the South African Schools Act, 84 of 1996, the statistical age norm per grade is the grade number plus 6. Therefore, the grade 10 learners, according to the information presented above, would be grade 10+6=age 16, which would be the correct age.

Thus, a total of 7.2% of the respondents are aged 15 years in all the three schools that participated in the study. Only 16.5% are 16-year-old learners, which is the appropriate age for the grade 10 learners. About 76.3% of the respondents are over the required age, with 34% being aged 17 years and 42.3% above 18 years, which is a stipulated schooling age (South African Schools Act 84 of 1996 Amended 1998).

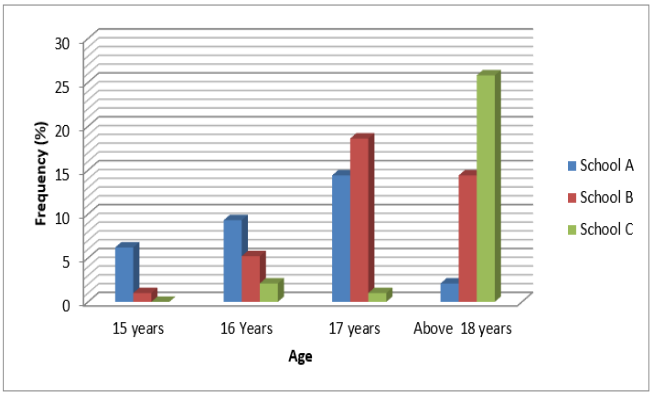

Figure 1. Age distribution of respondents per school

Figure 1 illustrates the age distribution of the learners, as shown in **Table 2**. In School A, six (6.2%) respondents are 15 years old; in school B, only one (1%) respondent is 15 years old, whereas, in School C, there is no respondent of such an age. It should be noted that this refers only to the sample used, not the entire school population.

School A has only two (2.1%) respondents above 18 years, which is the minority compared to the other two schools. School B has 14 (14.4%), and school C has 25 (25.8%) respondents above 18 years of age, and that suggests that above 18 years were the majority in the two samples. In search for the underlying reasons why there are learners above 18 years of age in grade 10, several reasons could come to the surface. The analysis of the content-based questionnaire was presented below based on the underlying causes of errors.

Errors Caused by the Application of Irrelevant Rules and Strategies

The content-based questionnaire consisted of three questions. Question 1 was based on financial documents and consisted of six sub-questions (1.1-1.6).

Respondents were given the opportunity to solve financial mathematics problems using a content-based questionnaire. The respondents illustrated various approaches below based on a variety of questions.

Question 1: Mr. Cetywayo borrowed an amount of R8000 from the bank at 11% simple interest for five years. How much interest will he pay back in total at the end of the term?

The question required learners to calculate the interest earned on an invested amount after a given period. Below is an example of a learner's response to the afore-stated question.

Figure 2 shows a learner's response to question 1. Thus, according to DoE (2011, p. 54), "Learners are not expected to work with any formulae here." The learner used a formula that was not given and was not expected to use a formula to work out this question. Even though the formula was used, the learner incorrectly substituted

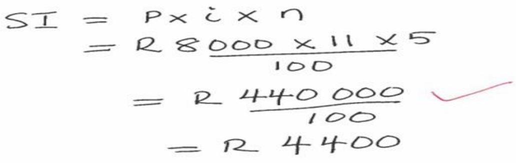

Figure 2. Response to question 1

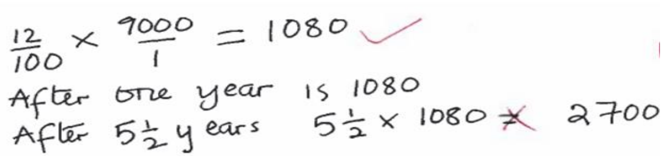

Figure 3. Response to question 2

Table 3. Comparison of learners' response to question 1

Question	School A		School B		School C	
no	Freq	%	Freq	%	Freq	%
1.1	31	100.0	29	76.3	28	100.0
1.2	31	100.0	29	76.3	22	78.6
1.3	30	96.8	26	68.4	24	85.7
1.4	13	41.9	17	44.7	13	46.4
1.5	20	64.5	15	39.5	9	32.1
1.6	22	71	5	13.2	9	32.1
Ave. %		79.03		53.07		62.48

Table 4. Comparison of learners' response to question 1

Question	School A		School B		School C	
no	Freq	%	Freq	%	Freq	%
2.1	20	64.5	23	34.2	16	57.1
2.2	13	41.9	16	42.1	16	57.1
2.3.1	17	54.8	24	63.2	26	92.9
2.3.2	11	35.5	1	2.6	3	10.7
Ave. %		49.2		35.5		54.5

the formula (i.e., i=interest rate, which is expressed as a percentage) was substituted as 11, not 11% as required.

As a result, of ignoring the percentage (%), the learner left the answer at R440,000 instead of dividing by 100 to give the correct answer, which is R4,400. In some cases, learners would calculate the repayable amount, which is loan amount plus interest, instead of calculating the interest. This type of error is described as an error due to applying irrelevant rules or strategies. In analyzing question one from the three participating schools, the results regarding the performance/responses are summarized in **Table 3**.

Table 3 shows the comparison of learners' responses to question 1. **Table 3** revealed that schools A and C performed well (i.e., 100%) on question 1.1, but surprisingly school B could not do so well (76.3%). Based on the average percentages illustrated in **Table 3**, school A (previously colored school) answered question 1 better than the other schools with an average of 79.03%. The researchers have drawn an inference on school C, which illustrated a moderate performance compared to the other two.

Errors Caused as a Result of Incorrect Facts and Concepts

Question 2 is about interest, and it requires students to perform calculations involving simple and compound interest using manual calculations rather than formulae. The emphasis is on assessing a comprehension of the ideas of compounding calculations, in which the values used in a calculation are based on the answer/values from a preceding calculation. The respondents given a loan amount, interest rate, and the period in years were asked to calculate the repayment value or compare the maturity value at the end of the term. Question 2, which included the simple interest as a topic covered in financial mathematics, consisted of three sub-questions (i.e., 2.1, 2.2, and 2.3).

Question 2: Mr. Cetywayo's friend Malusi received a service bonus of R9,000 and decided to invest it at 12% per year simple interest. Calculate the value of the investment after five years six months.

The question demands learners to work out the investment value after five years six months of investment. Below is an example of a learner's response to question 2.

Figure 3 showed participants' responses to question 2. In **Figure 3**, the learner calculated the interest at 12% per annum, which was correct but did not calculate it for five years six months. For the learner to be able to calculate it for five years six months, the learner had to be able to convert the given period to years which is $5\frac{1}{2}$years or 5.5 years. After working out the interest for 5.5 years, the learner then had to add the interest to the invested amount. The following is the correct method the learner could have applied:

Interest = 12% of R9,000×5.5 years

$= \frac{12}{100} \times R9{,}000 \ \times 5.5\ years$ 124=0.12×R9,000×5.5

= R5,940

Value of the investment = R9,000+R5,940=R14,940

Table 4 shows the comparison of learners' performance in question 2. **Table 4** revealed that schools A, B, and C did not perform averagely well in question 2. School A had an average performance in question 2 with 49.2%, school B had an average performance in question 2 with 35.5%, and school C with the highest average performance in question 2 with 54.5%.

Errors Caused as a Result of Lack of Prerequisite Skills

This question consists of three sub-questions that cover the simple and compound interest.

Question: This question was sub-divided into two, 3.1 and 3.2, working with percentages and based on decision-making. In question 3.1, the learners were required to work out 25% of the given amount. This question sought to uncover the learner's understanding

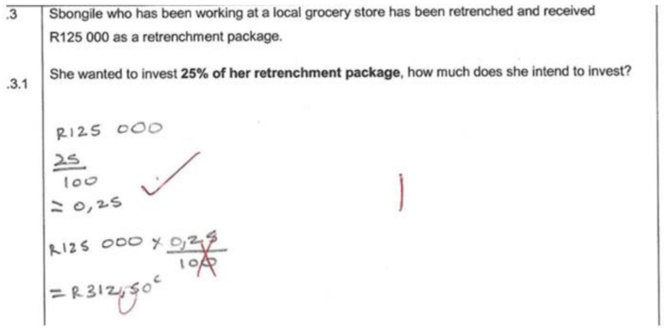

Figure 4. Response to question 3.1

Figure 5. Response to question 3.1

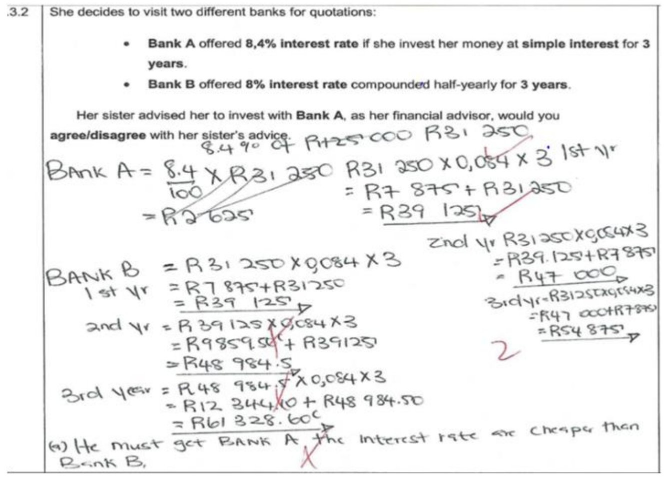

Figure 6. Learner's response to question 3.2

of the percentage (%) as a concept and its mathematical meaning.

Figure 4 shows a learner's response to question 3.1, and the learner illustrated a complete understanding of this mathematical concept and was able to write 25% as $\frac{25}{100}$ or 0.25. The mistake this learner committed was then dividing 0.25 by 100 even though the learner got 0.25 by dividing by 100. The learner was correct by multiplying R125,000 by 0.25 but did not divide by 100, so the correct answer would be R31,250.

Figure 5 also shows a learner's response to the same question (i.e., 3.1). This learner did not know what to do with the 25% included in the question. Instead of multiplying by 25%, he/she decided to divide by 25%. This illustrates the lack of prerequisite skills from the learner as the learner is supposed to know that 25% of R125 000 is 25%×R125,000 from the lower grades.

Question 3.2 is a Bloom's taxonomy level 4 question, which requires reasoning and reflection. Learners were required to compare two bank quotations and decide which would yield more interest on an invested amount. For a learner to be able to compare, the learner should be able to calculate both the simple and compound interest. The invested amount was calculated in 3.1 as 25% of the retrenchment package of R125,000.

Since it is a continuation, a learner who did not calculate 3.1 correctly will not be able to arrive at a correct answer in 3.2. It required a multi-step procedure to arrive at the final solution for each bank. **Figure 6** shows a learner's response in school B to question 3.2.

Below are the correct algorithms that were expected in answering question 3.2:

Bank A = $\text{R}31{,}250+\text{R}31{,}250\times\frac{8.4}{100}\times 3 = \text{R}39{,}125$

Bank = 1^{st} year $= \text{R}31{,}250+\text{R}31{,}250\times\frac{4}{100} = \text{R}32\,500$

$= \text{R}32{,}500+\text{R}32{,}500\times\frac{4}{100} = \text{R}33{,}800$

2^{nd} year $= \text{R}33{,}800+\text{R}33{,}800\times\frac{4}{100} = \text{R}35{,}152$

Table 5. Comparison of learners' response to question 3

Question no	School A Freq	School A %	School B Freq	School B %	School C Freq	School C %
3.1	0	0.0	0	0.0	0	0.0
3.2	4	14.3	0	0.0	2	6.5
3.3	2	7.1	0	0.0	1	3.2
Ave. %		7.1		0.0		3.2

$= \text{R}35{,}152+\text{R}35{,}152\times\frac{4}{100} = \text{R}36{,}558.08$

3^{rd} year $= \text{R}36{,}558.08+\text{R}36{,}558.08\times\frac{4}{100} = \text{R}38020.4032$

$= \text{R}38{,}020.4032+\text{R}38{,}020.4032\times\frac{4}{100} = \text{R}39{,}541.219328$

$\approx \text{R}39{,}541.22$

Disagree; she should invest with Bank B, as Bank B would yield more interest than Bank A.

Below is the summary of the learners' performance on question 3 by the three sampled schools.

The comparison between the three schools in **Table 5** revealed that the afore-mentioned question was poorly answered; none of the learners in school B were able to arrive at a correct answer on any of the third questions. Learners would just write anything without applying their minds to any of these questions. This was demonstrated by the irrelevance of the answers given by learners and the irrelevant methods applied. Comparing the average percentages, school C performed better than the other schools, but their averages were very low.

Errors Caused by Language Difficulties

The assumptions of homogeneity of the variables of hypothesis 1 were met as 1.184÷0.776=1.526, which is not greater than two.

Table 6 illustrates the learners' responses to the questions addressing Hypothesis 1 in relation to language difficulty. Learners who read a question once and write down the answer have a standard deviation of 1.18. This indicates that, on average, each rating of the afore-stated question is approximately a.18 points away

Table 6. Descriptive analysis of Learner responses in relation to language difficulties

	SDs	D	SoU	A	SA	SD
I read a question once and write down the answer	25.5	43.6	10.6	13.8	6.4	1.184
I have to read a question at least twice to understand it	2.1	4.2	4.2	50.5	38.9	0.870
I have the answer without reading the whole question	64.1	27.1	5.4	2.2	1.1	0.791
I look for number (numerical values) in question & read only last part of question	40.2	46.7	10.9	1.1	1.1	0.776
I write down any answer if I do not understand the question	26.9	35.5	19.4	14.0	4.3	1.145

Note. SDs: Strongly disagree; D: Disagree; SoU: Sort of (unsure); A: Agree; SA: Strongly agree; & SD: Standard deviation

Table 7. The relationship between language difficulties and errors committed by learners in financial mathematics

Model	Sum of squares	df	Mean square	F	R^2	Adj. R^2	Significance
Regression	52.899	8	6.601	2.001	0.232	0.116	0.064
Residual	174.868	53	3.299				
Total	227.677	61					

Table 8. Results on tested variables related to hypothesis 1

Variables	B	Standard error	t-test	Significance	Decision
Constant	3.226	3.443	0.937	0.353	
Age	-3.15	0.321	-0.982	0.330	NS
Gender	0.203	0.563	0.360	0.720	NS
Language	-0.222	0.099	-2.236	0.030	Significant

from 6.4% (sample average). About 20.2% of the respondents agreed to read a question once and write down the answer. A majority of 69.1% disagreed with reading a question once and writing down the answer, whereas only 10.6% were unsure. Only 6.4% of the respondents strongly agreed to read the question once and write down the answer.

Thus, reading the question at least twice to understand has a standard deviation of 0.87. This means that, on average, each rating of the afore-stated question is approximately 0.87 points away from 38.9%, which is the sample average. About 89.4% of the respondents agreed to read a question at least twice to understand it, whereas only 6.3% disagreed with 4.2% unsure. It also shows that about 38.9% of respondents strongly agreed to read the question at least twice before answering it.

Learners' having an answer without reading the question displayed a standard deviation of 0.79, closer to the sample mean of 1.1% as a sample average of this question. That indicates that only 1.1% of the respondents strongly agreed to answer questions without thoroughly reading the question first. Learners look for the number (numerical values) in the question and read only the last part of the question displayed a standard deviation of 0.78 closer to the sample mean of 1.1% strongly agreed with I look for number (numerical values) in the question and read only the last part of the question. That indicates that only 1.1% of the respondents strongly agreed with the statement: 'I look for numbers (numerical values) in the question and read only the last part of the question.' About 86.9% of the respondents disagreed with looking for a number (numerical value) in the question and read only the last part of the question, whereas only 2.2% agreed with that statement. About 10.9% were unsure of the response to the statement.

Learners writing down any answer if they do not understand the question illustrates a standard deviation of 1.15, far away from 4.3%, a sample mean of the respondents that strongly agree. About 33.4% of the respondents agreed to write down any answer if they did not understand the question. In contrast, about 62.4% disagreed with writing down any answer when they did not understand the question.

Research question 1: Is there any relationship between language difficulties and errors committed by learners in financial mathematics?

$H0_1$: There is no significant relationship between language difficulties in errors committed by learners in financial mathematics.

Table 7 shows the relationship between language difficulties and errors committed by learners in financial mathematics. However, in **Table 7**, there is a relationship between the language difficulty and errors committed by learners, but the coefficient of determination is low at 0.232. This indicates that other possible variables are not captured in the model, influencing errors committed.

Table 8 shows the results on tested variables related to hypothesis 1. In **Table 8**, only language significantly affects financial mathematics errors. All the other variables (age and gender) with t-test -0.982 and 0.360 respectively revealed an insignificant decision; hence, we reject hypothesis 1, which says there is no significant relationship between language difficulties and errors committed in financial mathematics. The t-test revealed -2.236 in the language difficulties, which illustrated that it reduces scores, hence increasing errors.

Thus, based on the findings, it can be concluded that there exists a relationship between the language difficulties experienced by learners and the errors they commit in financial mathematics. This could result from learners' poor language skills or English being a foreign language. Thus, this study is consistent with the findings of (Moschkovich, 2010; Nel, 2012), who found that poor language abilities, such as writing, speaking, and reading, are frequently linked to poor mathematics and ML achievement. Furthermore, the study discovered that misunderstanding the semantics of mathematical texts is often a source of learners' errors, offering a difficulty to students seeking to grasp some of the terms (Moschkovich, 2010). In addition, findings revealed that gender and age have no significant effect in relation to errors committed in financial mathematics.

CONCLUSION

This study examined language difficulty as a factor related to learner errors in financial mathematics. It also examined student errors, such as those caused by rigid thinking, the application of irrelevant rules or tactics, errors caused by a lack of mastery of required skills, facts, and concepts, and errors caused by language issues. The study concludes that if educators are to assist learners in eliminating common errors made in financial mathematics, they must endeavor to examine each of the learners' written work diagnostically to identify patterns and hence find possible causes and solutions for errors and misconceptions. This can be done by considering Polya's problem-solving techniques because it improves the problem-solving abilities of learners and helps them to evolve into logical thinkers rather than emotional thinkers

Recommendation

Based on the study's findings, we recommend that educators encourage collaboration between language teachers and ML teachers to improve learners' competencies in language and ML, as this is capable of reducing learners' errors in financial mathematics. In addition, instructional design for teaching and learning ML should be based on Polya's problem-solving techniques. This will assist learners in understanding the problem, coming up with a plan to solve the problem, executing the plan, and reflecting on the work completed to predict a relevant strategy to solve similar future tasks. Lastly, teacher-training institutions should include error analysis in their teacher-training curriculum. This will assist pre-service teachers in being exposed to the different error analysis techniques and acquiring relevant skills needed to assist learners in identifying, reducing, and later eliminating both mathematics and ML errors.

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