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Let two plus two not always equal four: A critical pedagogical exploration of mathematical certainty and context

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Abstract

This mixed-methods study was designed to investigate how engagement with modular arithmetic influences senior high school students' beliefs about mathematical certainty, creativity, and reflective thinking. Grounded in constructivist learning theory and critical mathematics education, the study conceptualizes mathematical truth as context-dependent, rule-governed, and open to epistemic inquiry rather than as an absolute given. Conducted with 50 grade 12 learners from Stirling Schools in Erbil and Kirkuk during the 2024-2025 academic year, the study employed a convergent design integrating pre- and post-Likert-scale surveys with reflective journals. The intervention challenged the universal arithmetic statement "2 + 2 = 4" by introducing congruence modulo *n*, highlighting that mathematical outcomes depend on specific axiomatic rules. Quantitative data analysis revealed significant improvements in beliefs about mathematical certainty, creativity, and critical thinking ($p < .001$), with no gender-based differences noted. Thematic analysis of qualitative reflections identified six interconnected themes: recognition of context-dependent mathematical truth, awareness of underlying structures, epistemic curiosity from cognitive conflict, emergence of creativity in reasoning, understanding that rules shape meaning, and strengthened mathematical identity. The findings suggest that exposure to alternative mathematical systems fosters conceptual flexibility, metacognitive engagement, and a humanistic view of mathematics. The study has important implications for 21st century mathematics education, advocating for critical-pedagogical approaches that enrich classroom inquiry, promote creativity, and challenge absolutist notions of truth. Limitations and recommendations for future research, including longitudinal designs and cross-cultural comparisons, are also discussed.

Keywords: modular arithmetic, mathematical certainty, critical mathematics education, constructivist learning, epistemic beliefs

INTRODUCTION

Mathematics is often viewed as a field of absolute certainty. It is regarded as a realm of universal truths, unchanging rules, and precise logic. While this traditional perspective is important for scientific progress, it can unintentionally limit students' ability to think flexibly and creatively if accepted without question. In many educational systems, mathematics is mainly taught as a set of procedural skills and factual statements that students are expected to learn and replicate. This method can hinder opportunities for inquiry, imagination, and deeper understanding, reducing mathematics to a static body of knowledge instead of recognizing it as a dynamic process of human thought (Ibrahim et al., 2024; Putri et al., 2025).

Recent scholars have called for a more balanced approach that maintains mathematical rigor while also valuing creativity, reflection, and cultural importance (Copur-Gencturk & Li, 2023; Hoang et al., 2023). From a pedagogical perspective, this inquiry focuses on how mathematics is taught and learned in classrooms,

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Contribution to the literature

- This study offers empirical evidence that exposing high school students to alternative mathematical systems, like modular arithmetic, shifts their beliefs about mathematical certainty. Findings indicate a move away from absolutist views toward contextual, rule-based, and fallibilist understandings of mathematics, addressing a gap in the literature where such classroom-based evidence is scarce.
- The study presents modular arithmetic as a critical pedagogical tool that fosters cognitive conflict, creativity, and reflective engagement. It builds on constructivism and critical mathematics education by demonstrating how this accessible context stimulates intellectual curiosity, epistemic openness, and structural reasoning, which are essential skills for 21st century learning.
- The study highlights students' growth in mathematical identity, confidence, openness, and questioning, contributing to the literature on mathematics as a humanizing discipline. Findings show that alternative number systems can enhance students' relationships with mathematics, fostering identity development and equitable participation, regardless of gender. This aligns with broader calls to rehumanize mathematics teaching and connect it with philosophy, culture, and epistemology.

particularly whether instructional practices encourage questioning, sense-making, creativity, and student agency, or whether they reinforce procedural compliance and fixed answers. In this sense, pedagogy concerns the design of learning environments that support exploration, dialogue, and reflective engagement.

From a philosophical perspective, the inquiry shifts toward the nature of mathematical knowledge itself, asking what it means for a mathematical statement to be considered "true," "certain," or "universal." Within this view, the question "Should 2 plus 2 always equal 4?" is not merely a classroom provocation but a deeper epistemological challenge to absolutist conceptions of mathematical truth. Through such questions, mathematics is seen as a living and evolving cultural pursuit influenced by human thinking, history, and social engagement (Hersh, 1998; Schoenfeld, 2016).

It is suggested that mathematical certainty is not an inherent trait of numbers but rather a result of the conceptual systems within which those numbers exist. In specific contexts, such as congruence modulo systems, two plus two does not equal four but instead 0 (mod 4), and 1 (mod 3). Recognizing this contextual flexibility encourages students to see mathematics not as a fixed set of truths but as a space for investigation, meaning creation, and interpretation. This view aligns with the humanistic and constructivist approaches to mathematics education (Aslan Tutak et al., 2011; Vintere, 2018), which highlight that knowledge is actively constructed through experience, dialogue, and reflection.

The current study questions traditional ideas of mathematical absolutism by encouraging students to engage with alternative mathematical systems critically and examine how meaning in mathematics shifts with context. It presents mathematics as both a logical and cultural practice, one through which learners can question, reimagine, and humanize knowledge.

Conventional approaches to mathematics education reinforce the idea that mathematical truths are fixed and unchangeable. The emphasis on procedural instruction and the expectation of only one correct answer often discourages students from questioning or exploring the origins of mathematical meaning. Such rigidity may conflict with democratic and humanistic ideals in education, which promote critical awareness and intellectual autonomy. This study, therefore, addresses the issue of decontextualized mathematical certainty and explores how a pedagogical approach that challenges absolute truths can encourage creativity, inquiry, and reflective understanding among senior high school students.

Traditional mathematics classrooms emphasize accuracy but seldom foster curiosity about how or why mathematical truths are valid. By incorporating principles of critical pedagogy (Freire, 1970; Skovsmose, 1994) and constructivist learning, this study suggests that questioning mathematical certainty can catalyze deeper cognitive engagement. When students recognize that statements like 2 + 2 = 4 rely on definitional and contextual frameworks, they begin to see mathematics as a meaning-making activity rather than just a matter of rule-following. The study thus views the classroom as a space for intellectual emancipation, where students can connect mathematical reasoning to broader philosophical, cultural, and epistemological contexts.

This research contributes to the growing field of critical mathematics education by showing how even simple arithmetic truths can serve as tools for reflection and empowerment. It broadens the discussion of mathematical meaning by demonstrating that concepts such as equality, truth, and logic can be reinterpreted through cultural and contextual perspectives. For educators and curriculum designers, the findings provide a pedagogical framework that encourages inquiry, flexibility, and the acceptance of multiple viewpoints in mathematics learning. For students, the study promotes epistemic curiosity and confidence to approach mathematics as an interpretive rather than purely procedural discipline.

In a world increasingly shaped by algorithms and data, the authority of mathematical certainty has become more widespread than ever. However, educational reform movements highlight the importance of creativity, equity, and critical thinking. This research supports those efforts by using a simple mathematical question, *'Should 2 + 2 always equal 4?'* as a metaphor for reexamining deeply held beliefs about truth and knowledge. Its innovation comes from combining philosophical inquiry with classroom experimentation, showing that questioning mathematical certainty can foster both cognitive and emotional growth among learners.

This study seeks to address the following questions:

1. How do students interpret and reconstruct mathematical truths when exposed to alternative systems such as modular arithmetic?
2. How does this critical-pedagogical approach affect students' creativity, attitudes, and motivation toward learning mathematics?
3. Are there gender-based differences in students' responses to the exploration of mathematical certainty and contextual reasoning?

LITERATURE REVIEW

Mathematical Meaning and Language

Mathematical meaning is not static but emerges through social interaction and cognitive construction (Abrahamson, 2021; Roth, 2020). The expression 2 + 2 = 4 is a conceptual structure whose meaning varies by context and by the learner's prior cognitive frameworks. Mason et al. (2009) emphasize that encouraging students to appreciate structural relationships deepens understanding. Without attention to structure, students develop procedural rather than conceptual fluency. Duval (2006) similarly identifies the challenge of helping learners access abstract mathematical objects through varied representational and cognitive systems.

Recent reforms in mathematics education stress that students must move beyond the consumption of knowledge toward active participation in meaning-making (Boaler, 2016; McTighe & Silver, 2020; Valquaresma, 2024). Research by Atweh et al. (2012) and Lomas et al. (2012) highlights that mathematical truths are historically, culturally, and emotionally situated. This perspective aligns with Sfard's (1998, 2000) metaphors of acquisition and participation, which view learning as a social process of discourse rather than the accumulation of static facts.

Alternative Mathematical Systems

Alternative mathematical frameworks such as modular arithmetic, fuzzy logic, and non-Euclidean geometries demonstrate that truth in mathematics depends on definitions and context (Karataş, 2018; Lakoff & Núñez, 2000). Modular arithmetic is a system of arithmetic operations for integers, which differs from conventional arithmetic in that numbers "wrap around" after reaching a particular value known as the modulus. The modern concept of modular arithmetic, congruence modulo n, was developed and formalized by Carl Friedrich Gauss (1777-1855). He introduced the notation:

$$a \equiv b(mod\ n). \quad (1)$$

Gauss developed the whole theory of congruences in his foundational book *Disquisitiones arithmeticae* (Gauss, 1801). The elements of the ring Z/nZ arise from integer division modulo n; two integers are considered equivalent if they differ by a multiple of n. Therefore, Z/nZ consists of the residue classes $\{0, \dots, n-1\}$. If n is a prime number, then it can be shown that every non-zero residue class has a multiplicative inverse. Therefore, it is evident that Z/nZ forms a field (Dummit & Foote, 2004). For example, in mod 4 arithmetic, 2 + 2 ≡ 0. This shows that mathematical operations can yield multiple valid outcomes depending on the axiomatic system. Similarly, geometries like Galilean and Minkowski spaces redefine the concept of distance, showing that mathematics evolves as contexts shift (Saralar-Aras & Kurudirek, 2025).

Papert (2000) argued that mathematics should be experienced as a process of *doing* and *creating*. His notion of "microworlds" allowed learners to experiment within alternative logics, thereby internalizing mathematical meaning through exploration. Maaß (2010) adds that transitioning between real-world contexts and abstract representations (termed *mathematical modeling*) is central to authentic learning. These frameworks collectively demonstrate that mathematical meaning is plural, contingent, and open to negotiation.

In mathematics education, alternative mathematical systems are commonly introduced as pedagogical tools to deepen conceptual understanding rather than as replacements for conventional arithmetic. Modular arithmetic, for example, is widely used in secondary and tertiary education to support learning in number theory, cryptography, computer science, clock arithmetic, and cyclic processes (Dummit & Foote, 2004). In classroom settings, it is frequently employed to illustrate equivalence relations, structure-preserving rules, and the role of axioms in defining valid operations. Educational research shows that such systems help students recognize that mathematical meaning depends on defined rules rather than surface-level computation, thereby strengthening structural reasoning and conceptual flexibility.

Despite its pedagogical value, the use of alternative mathematical systems also presents limitations in teaching and learning contexts. If introduced without careful scaffolding, students may experience confusion or misinterpret modular results as contradictions rather than context-dependent truths. There is also a risk that students may overgeneralize alternative systems, leading to misconceptions if boundaries between conventional arithmetic and specialized systems are not clearly articulated. Additionally, curriculum constraints, assessment practices focused on procedural accuracy, and limited instructional time may restrict teachers' ability to engage deeply with such conceptual explorations. For these reasons, alternative mathematics is most effective when used selectively, with explicit emphasis on purpose, context, and conceptual boundaries.

Constructivist and Critical Learning Perspectives

Piaget (1964) and Vygotsky (1978) laid the foundation for constructivist theories, emphasizing that knowledge is constructed through active engagement rather than passive reception. Error and exploration are natural components of learning, and understanding grows through questioning rather than memorization (Skemp, 2006; Star, 2014). Within this tradition, students learn mathematics by reflecting on their own reasoning processes, thereby bridging *instrumental* and *relational* understanding.

Ethnomathematics extends this constructivist perspective by recognizing the cultural and political dimensions of mathematical knowledge. It argues that mathematical practices are culturally embedded and offer multiple legitimate pathways to understanding (D'Ambrósio & Knijnik, 2019). Integrating such perspectives encourages learners to view mathematical inquiry as both intellectual and social, promoting equity and inclusion within the classroom.

Philosophical and Theological Dimensions of Knowledge

Mathematical certainty, creativity, and reflective thinking form the conceptual core of this study and are treated as interrelated rather than isolated constructs. Mathematical certainty is understood not as an absolute or immutable property of mathematical statements, but as a conditional form of validity that emerges within specific axiomatic systems and rule-based frameworks (Ernest et al., 2016; Lakatos et al., 1976; Hersh, 1998). Creativity in mathematics is conceptualized as the capacity to explore alternative structures, generate conjectures, and reimagine familiar ideas under new constraints, particularly when students engage with non-routine or unfamiliar mathematical systems (Mann, 2006; Sriraman, 2022). Reflective thinking refers to learners' ability to examine underlying assumptions; question established rules and reconsider the meanings of mathematical truths through deliberate inquiry and metacognitive awareness (Dewey, 1933; Schön, 1983; Schoenfeld, 2016). Together, these notions position mathematics as a dynamic, sense-making activity in which understanding develops through inquiry, reinterpretation, and epistemic openness rather than procedural repetition.

Throughout history, diverse philosophical and theological traditions have grappled with the nature of truth and certainty. In Islamic thought, Al-Ghazali questioned the limits of rational knowledge and emphasized the integration of reason and spirituality (Griffel, 2009). Similarly, Saint Augustine's reflections on divine intellect in Christian philosophy and the interpretive traditions in Jewish Talmudic study reveal that multiple perspectives on truth coexist (Lee & Ko, 2024; Pinkas, 2023). Buddhist teachings, through koan and paradox, employ uncertainty as a pedagogical strategy (Trew, 2021). These traditions collectively underscore that truth, whether mathematical or metaphysical, is always mediated by human interpretation.

Insights from the philosophy of science further reinforce the view that certainty is often conditional rather than absolute. In physics, Heisenberg's uncertainty principle demonstrates that fundamental limits exist on what can be known simultaneously about a system, challenging classical assumptions of complete determinacy (Heisenberg, 1927; Jammer, 1974). While this principle operates within a physical domain, its epistemological implication (that knowledge is constrained by frameworks, measurement, and perspective) offers a powerful parallel to mathematical reasoning. Similarly, Einstein's (1905, 1949) theory of relativity shows that quantities such as time and space are not fixed absolutes but depend on the observer's frame of reference. These scientific perspectives align with the mathematical insight that truths are valid within defined systems rather than universally invariant. Together, they support an educational stance in which uncertainty, context, and perspective are understood as productive conditions for inquiry, reflection, and deeper understanding. Thus, the proposition *"Let two plus two not always equal four"* transcends arithmetic; it becomes a metaphor for epistemological humility and educational transformation.

The Research Gap

Despite increasing scholarly interest in constructivist and critical approaches to mathematics education, most high school curricula still treat mathematical knowledge as absolute and context-free. Previous studies have emphasized the importance of creativity, inquiry, and the sociocultural dimensions of mathematics (Boaler,

Table 1. Composition of participants across the four schools

	Erbil	Kirkuk	**Total**
Boys	13	12	**25**
Girls	13	12	**25**
Total	**26**	**24**	**50**

2016; D'Ambrosio & Knijnik, 2019; Sfard, 2000). However, few have explicitly examined how students might engage with alternative mathematical truths, such as those in modular arithmetic or non-Euclidean geometries, as tools to foster epistemic awareness. Existing research often fails to challenge foundational beliefs about mathematical certainty, thereby maintaining a single, universal narrative of statements such as "2 + 2 = 4." There is a notable gap in empirical research on how questioning this certainty can promote deeper conceptual understanding, creativity, and reflective thinking among high school students. Closing this gap is essential to reimagining mathematics education as a space for dialogue, context, and meaning-making rather than merely emphasizing procedural correctness.

METHODS

Research Design

This study adopted a convergent mixed-methods design (Creswell & Creswell, 2017), combining quantitative surveys with qualitative reflections to explore both cognitive and attitudinal aspects of learning alternative mathematical systems in high school. Quantitative data assessed changes in attitudes toward mathematical certainty, while qualitative data highlighted students' interpretive processes and meaning-making during the lessons. Both methods were analyzed separately and then integrated for a comprehensive understanding.

Participants and Sampling

The participants were a convenience sample of 50 grade 12 students (25 boys, 25 girls) from Stirling Schools in Erbil and Kirkuk, Iraq, during the 2024-2025 academic year. Erbil and Kirkuk are two distinct urban centers in northern Iraq that differ in sociocultural composition and educational context. Erbil, the capital of the Kurdistan Region, is characterized by relatively stable educational infrastructure and greater exposure to private and international schooling models. Kirkuk, by contrast, is a culturally diverse city with a more heterogeneous student population and a schooling environment shaped by broader social and linguistic diversity. Including participants from both cities allowed the study to capture a broader range of student experiences while maintaining curricular consistency across the participating schools. Participants were recruited from four schools: two exclusively for boys and two exclusively for girls. Participation was voluntary and based on informed consent. The distribution of participants across the four schools is shown in **Table 1**.

Participants were selected using convenience sampling based on availability and willingness to participate in the instructional program. The equal number of male and female participants was not the result of stratified sampling but reflected the structure of the participating schools, which are gender-segregated, and the voluntary nature of participation. An equal number of students was invited from each school to ensure balanced representation across sites and to avoid overrepresentation of any single campus. This approach supported equitable participation while maintaining consistency in instructional delivery and research procedures.

Instruments and Issues of Validity and Reliability

Quantitative instrument(s)

A structured Likert-scale questionnaire (**Appendix A**) was developed to assess students' conceptions of mathematical certainty, creativity, and reflective attitudes toward learning mathematics. The instrument included 12 items rated on a five-point scale (1 = strongly disagree to 5 = strongly agree), divided into three subscales aligned with the research questions. The first subscale *(beliefs about mathematical certainty; items 1-4)* assessed students' views of mathematics as either absolute or context-dependent. The second subscale *(creativity and critical thinking; items 5-8)* evaluated students' willingness to question, use imagination, and think flexibly. The third subscale *(attitudes and reflection; items 9-12)* reflected emotional responses to learning different mathematical systems and self-reflective engagement.

Item 1 and item 2 were reverse-coded to address acquiescence bias because they expressed absolutist beliefs *(e.g., mathematics always produces one correct answer)*. The reverse transformation (6 - X) ensured that higher total scores indicated more flexible, creative, and reflective views of mathematical truth. The questionnaire was administered prior and post the intervention on modular arithmetic, enabling the assessment of conceptual and attitudinal changes resulting from exposure to alternative mathematical ideas.

Qualitative instrument(s)

To supplement the survey data, participants were asked to keep reflective journals where they wrote short essays prompted by the following open-ended questions:

- What new insight did you gain about "truth" in mathematics after learning modular arithmetic?

Table 2. Cronbach's alpha reliability coefficients for each subscale

Subscale	N	α
Mathematical certainty	4	0.78
Creativity and critical thinking	4	0.83
Attitudes and reflection	4	0.86
Overall instrument	**12**	**0.84**

Note. N: Number of items & α: Cronbach's alpha

- How did this activity change how you think about "right" and "wrong" in mathematics?
- Should two plus two always equal 4? Describe a moment when your view of "2 + 2 = 4" expanded or changed.

The qualitative data collected through these reflective journals provided valuable insights into how students engaged with the philosophical aspects of mathematical certainty.

Validity

To establish content validity, the instrument was reviewed by two mathematics education experts with experience in constructivist and critical pedagogy. Their feedback helped refine ambiguous items and ensured alignment with the theoretical constructs of mathematical certainty, creativity, and reflective learning. Construct validity was supported by the instrument's three-factor design, each corresponding to a research dimension grounded in Freirean critical pedagogy and constructivist learning theory (Piaget, 1964; Skovsmose, 1994; Vygotsky, 1978). Furthermore, combining quantitative and qualitative data sources enhanced the interpretive trustworthiness of findings. Student feedback and member checking were used to verify the accuracy of emerging qualitative themes.

Reliability

Internal consistency reliability of the questionnaire was assessed using Cronbach's alpha for each subscale and the overall instrument. The questionnaire underwent pilot testing with a convenience sample of thirty grade 12 students who were not involved in the main study, to verify item clarity and scale stability. The reliability coefficients ranged from 0.78 to 0.86 (**Table 2**), exceeding the 0.70 threshold (Hair et al., 2021), indicating acceptable reliability for research purposes.

Overall, the mixed-method tools captured both the quantitative changes in beliefs and attitudes and the qualitative depth of students' reflections on mathematical certainty and contextual reasoning. Using parallel quantitative and qualitative measures ensured a balanced, credible, and multidimensional assessment of the pedagogical intervention's impact.

Data Collection and Implementation Procedures

The study was conducted in four Stirling schools during the 2024-2025 school year. The participants were grade 12 students who volunteered for a short-term instructional program designed to challenge traditional ideas of mathematical certainty by studying modular arithmetic.

Preparation and ethical clearance

Before implementation, ethical approval and written permissions were obtained from the Stirling Schools administration and the students' parents. The purpose, procedures, and confidentiality protocols were clearly explained to both students and guardians. Participation was voluntary, and students were informed of their right to withdraw at any stage without penalty.

The program was implemented by one of the researchers, who also served as an education coordinator and mathematics subject specialist within the Stirling Schools network. His dual role ensured both pedagogical consistency and fidelity to the research design while maintaining professional ethical standards.

Structure and implementation

The instructional program spanned two consecutive weeks and involved three 2-hour sessions per school, conducted after regular class hours to avoid disrupting the standard curriculum. Each session was designed around modular arithmetic as a conceptual gateway to explore the idea that mathematical truths can vary depending on the chosen system. The lessons employed inquiry-based and dialogic teaching strategies rooted in critical and constructivist pedagogy. Students engaged in open-ended questioning, collaborative discussions, and problem-based learning. The key concept introduced was *congruence modulo n*, where statements such as $2 + 2 \equiv 0 \ (mod\ 4)$ and $2 + 2 \equiv 1 \ (mod\ 3)$ illustrate that mathematical validity depends on the system in which operations occur. Students were encouraged to compare this contextual truth to the universal arithmetic statement, *two plus two equals four*.

Each session followed a consistent structure:

1. **Conceptual activation:** Review of prior arithmetic concepts and introduction to modular arithmetic with concrete examples.
2. **Exploration and discussion:** Group problem-solving and interpretation of modular results; guided dialogue on how context alters meaning in mathematics.
3. **Reflection:** Students completed brief reflective journals addressing their new understanding of mathematical 'truth' and their emotional and cognitive responses to alternative systems.

Data collection procedures

Data were collected in two phases using a mixed-method approach:

1. **Phase 1 (pre-instruction):** Students completed a Likert-scale survey assessing their beliefs about mathematical certainty, creativity, and reflective attitudes prior to modular arithmetic instruction.
2. **Phase 2 (post-instruction):** After the modular arithmetic lessons, the same survey was re-administered. Additionally, students submitted reflective journals that provided qualitative insights into conceptual and attitudinal changes.

Throughout the sessions, the instructor maintained systematic observation notes, documenting instances of questioning, reasoning, and engagement. This triangulated both the quantitative shifts in attitudes and the qualitative evidence of reflective learning.

Instructional environment and consistency

To ensure instructional consistency across the four campuses, the same researcher conducted all lessons using a unified lesson plan and set of materials. Lesson objectives, examples, and discussion prompts were standardized. All sessions were held in small group formats (6-8 students per group) to foster participation and deeper interaction.

The instructor's background as an education coordinator and mathematics specialist supported high-quality implementation, enabling nuanced facilitation of discussions about mathematical meaning while maintaining alignment with ethical teaching practices.

Data Analysis

All survey data were coded and analyzed using IBM SPSS statistics (version 29) and Microsoft Excel. Medians and median gains were computed for each subscale: mathematical certainty, creativity and critical thinking, and attitudes and reflection. Before performing inferential analyses, normality of the composite subscale scores was evaluated using the Shapiro-Wilk test. This test examines whether data distributions significantly deviate from normality (Shapiro & Wilk, 1965). Pre-test distributions were approximately normal ($p > .05$), while some post-test distributions showed significant deviations ($p < .05$), suggesting mild positive skewness due to the overall improvement in students' scores.

To ensure analytical robustness, non-parametric Wilcoxon signed-rank tests were conducted. These tests do not assume normality and evaluate whether the median of post-test scores differs significantly from pre-test medians. Reverse coding was applied to Items 1 and 2 of the Likert-scale instrument (which reflected absolute beliefs about mathematics) using the transformation $6 - X$. This ensured that higher scores consistently represented more contextual, critical, and reflective conceptions of mathematics.

Qualitative analysis

A thematic analysis was conducted using inductive coding (Braun & Clarke, 2006). Recurring themes included contextual reasoning, creative insight, cognitive flexibility, and emotional engagement. Triangulation of data sources ensured validity and depth of interpretation.

Ethical Considerations

Ethical approval was received from the Stirling Schools Human Research Ethics Committee, protocol number E-25/009, dated 04/04/2025. Furthermore, participation was voluntary, with informed consent obtained from students and guardians. Anonymity and confidentiality were maintained. Participants were informed of their right to withdraw from the study at any time without providing a reason. Written materials were used exclusively for research purposes.

RESULTS

Quantitative Results: Pre-Post Intervention Surveys

Fifty grade 12 students completed pre- and post-intervention Likert-scale surveys measuring three constructs: mathematical certainty, creativity and critical thinking, and attitudes and reflection. After exposure to modular arithmetic as an alternative mathematical system, a pre- and post-comparison was conducted to determine whether there was a significant change in responses across all subscales. To identify the appropriate statistical tests for analyzing the data, Shapiro-Wilk tests were performed to assess the normality of the composite subscale scores before and after the intervention.

Normality assumption: Shapiro-Wilk test results

Pre-test distributions of scores in the mathematical certainty, creativity, and critical thinking subscales were approximately normal ($p > .05$) (**Table 3**). In contrast, pretest scores for the attitudes and reflections scales violated the normality assumption. The post-test scores across the three variables of interest showed significant Shapiro-Wilk results ($p < .05$), indicating a departure from normality as students' scores increased following the implementation of the instruction. Consequently, parametric statistical analyses were abandoned in favor of nonparametric statistical methods. Wilcoxon signed-rank tests were computed to assess the statistical significance of the observed effects.

Table 3. Shapiro-Wilk normality test results for each subscale

Subscale	Shapiro-Wilk Pre (W)	p (pre)	Shapiro-Wilk Post (W)	p (post)
Mathematical certainty	0.96	0.06	0.94	0.011*
Creativity and critical thinking	0.96	0.07	0.91	0.0008*
Attitudes and reflection	0.95	0.03*	0.92	0.003*

Table 4. Wilcoxon signed-rank test results for pre- and post-intervention scores

Subscale	Median pre	Median post	Δ (post-pre)	Wilcoxon W	p
Mathematical certainty	2.25	3.75	1.50	0.00	< .001
Creativity and critical thinking	3.00	4.00	1.00	0.00	< .001
Attitudes and reflection	3.00	4.00	1.00	0.00	< .001

Table 5. Mann-Whitney U test results for gender-based differences

Subscale	Female median Δ	Male median Δ	Δ (F-M)	Mann-Whitney U	p
Mathematical certainty	1.50	1.25	0.25	313.50	0.99
Creativity and critical thinking	1.00	1.00	0.00	272.00	0.43
Attitudes and reflection	1.25	1.25	0.00	329.50	0.75

Non-parametric analysis: Wilcoxon signed-rank test results

Given that some post-test subscales showed significant deviations from normality in the Shapiro-Wilk tests, non-parametric Wilcoxon signed-rank tests were conducted. The Wilcoxon-signed rank test does not assume normality and compares median scores between pre- and post-intervention measurements. Based on the results presented in **Table 4**, the Wilcoxon signed-rank tests confirmed significant differences between pre- and post-intervention scores for all subscales: mathematical certainty, creativity and critical thinking, and attitudes and reflection. Median scores increased substantially from pre- to post-test, indicating that students' beliefs, creativity, and reflective attitudes improved after exposure to modular arithmetic. For mathematical certainty, a Wilcoxon signed-rank test indicated a significant increase from pre-test (median = 2.25) to post-test (median = 3.75), W = 0.00, p < .001. Similarly, creativity and critical thinking increased from pre-test (median = 3.00) to post-test (median = 4.00), W = 0.00, p < .001. Attitudes and reflection also showed a significant improvement from pre-test (median = 3.00) to post-test (median = 4.00), W = 0.00, p < .001.

These non-parametric results demonstrate that the critical-pedagogical intervention with modular arithmetic significantly enhanced students' understanding of mathematical concepts, their creative and critical engagement, and their reflective attitudes toward learning mathematics. Additional analysis was conducted to determine if these results varied by gender. The Mann-Whitney U test was used for this analysis.

Gender-based comparison: Mann-Whitney U test results

To address the third research question, Mann-Whitney U tests were conducted to determine whether there were significant gender differences in pre-post changes across the three subscales. Since the data violated normality assumptions, a nonparametric test was used to compare the median difference (Δ) scores between female and male students. Results of the Mann-Whitney U tests indicated no statistically significant gender differences in the magnitude of change across the three subscales (p > .05) (**Table 5**). Both male and female students showed similar improvements in their beliefs, creativity, and reflective attitudes following the modular arithmetic intervention.

The difference in improvement between female (median Δ = 1.50) and male (median Δ = 1.25) students on mathematical certainty was not significant, U = 313.50, p = 0.99. Similar non-significant results were observed for creativity and critical thinking (female median Δ = 1.00, male median Δ = 1.00), U = 272.00, p = 0.43, and attitudes and reflection (female median Δ = 1.25, male median Δ = 1.25), U = 329.50, p = 0.75.

These results indicate that the critical-pedagogical approach using modular arithmetic was equally effective for both genders. Female and male students benefited from the intervention, showing significant and similar improvements in conceptual flexibility, creativity, and reflective attitudes. The lack of gender differences suggests that the critical pedagogical approach was fair and effective across all groups.

To answer the first research question and deepen understanding of how modular arithmetic influenced participants' perceptions of mathematical truths, the following section analyzes data from students' reflective journals.

Qualitative Results: Reflective Journals

Analysis of the ten reflective journals (RJ1-RJ10) (see **Appendix B**) revealed a consistent pattern of epistemological transformation as students engaged with modular arithmetic. The core finding indicates that exposure to alternative mathematical systems prompted students to reinterpret and reconstruct mathematical truth as context-dependent, rule-governed, and structurally defined. Six interrelated themes emerged from the data.

Theme 1. Mathematical truth as context-dependent rather than absolute

Students reported a clear shift from an absolutist view of mathematics to a conditional, system-based understanding of truth. Many expressed surprise that statements they had previously considered universal, such as 2 + 2 = 4, were not necessarily valid in alternative number systems, such as modular arithmetic. For example, one student reflected, *"I started to think that the concepts of 'right' and 'wrong' … change according to mathematical context"* (RJ1), while another acknowledged a broader epistemic shift, stating, *"I believed that math was perfect and unquestionable… But when we studied modular arithmetic, I realized that the concept of 'right' and 'wrong' in math depend[s] on assumptions"* (RJ2). Other students further reinforced this evolving awareness of conditional truth. RJ2 noted succinctly that *"mathematical truths are conditional,"* emphasizing that correctness arises from the rules governing the structure rather than universal norms. Similarly, RJ8 echoed this relativistic perspective: *"something that is 'true' in one language may not be true in another,"* highlighting the role of context in defining mathematical meaning. RJ10 extended this reasoning by situating mathematical truth within human-defined structures, writing, *"The so-called absolute truths in mathematics depend on structures defined by intelligent minds."* Collectively, these reflections demonstrate students' growing recognition that mathematical truth is not fixed or absolute, but emerges from the axioms, conventions, and frameworks that define the system in use.

Theme 2. Recognition of underlying structures and rules

Students began to describe mathematics as a human-constructed system defined by logical structures and axioms rather than as a body of fixed, universal facts. Several students articulated that understanding mathematics requires attention to the underlying frameworks that govern operations. As RJ1 observed, *"… mathematics is not just memorized information but also about the operations and structures used,"* highlighting the shift from surface-level computation to deeper structural reasoning. This awareness prompted students to question long-held assumptions, with one commenting, *"I used to memorize most mathematical expressions and formulas, but now I question them …'"* (RJ3), and adding that *"something that is wrong in one system might be right in another …"* Other students similarly emphasized the importance of context in determining mathematical meaning. RJ4 noted that *"context matters even in mathematics."* RJ8 reinforced this idea, stating, *"truth in mathematics comes not from the numbers themselves but from the rules we agree upon,"* indicating a transition toward structural and axiomatic thinking. Together, these reflections illustrate students' growing recognition that mathematics operates within defined systems of logic and that modifying underlying rules gives rise to new mathematical structures.

Theme 3. Epistemic curiosity elicited through cognitive conflict

The unexpected nature of modular arithmetic, particularly the confrontation with statements such as *2 + 2 ≡ 1 (mod 3)* triggered curiosity and encouraged deeper reflection among students. The cognitive conflict produced by these unfamiliar results functioned as a powerful stimulus for engagement. As RJ1 described, *"I got excited and began to pay closer attention,"* indicating that the perceived contradiction heightened interest rather than confusion. Similarly, RJ2 noted that *"The question 'Does 2 + 2 always have to make 4?' made me curious rather than certain,"* reflecting a shift from passive acceptance to active inquiry. This sense of surprise was also evident in RJ6, who admitted, *"I thought it was a joke… even wanted to make a joke myself when the teacher explained that 2+2=4 is true in normal arithmetic but 2+2=0 in mod 4, my mathematical thinking reached a new dimension,"* suggesting that initial disbelief quickly became an entry point into meaningful conceptual exploration. RJ7 further expressed fascination with how modular arithmetic redefines familiar ideas, stating,

> "The expression 7 ≡ 2 (mod 5) fascinated me because it showed that equality can have different meanings in different systems." RJ2 remarked: "I wonder if such fascinating situations exist on other areas of mathematics too, I am very curious to find out."

Collectively, these reflections reveal that cognitive dissonance served not as a barrier but as a productive catalyst for learning, prompting students to interrogate underlying assumptions and engage more deeply with mathematical structures.

Theme 4. Emergence of creativity and imagination in mathematics

Several students' reflective journals described mathematics as becoming "creative," "surprising," or "imaginative" through the exploration of modular arithmetic. Students expressed that encountering non-standard results prompted them to see mathematics as more dynamic and conceptually rich than they previously believed. RJ5 captured this shift by noting, *"mathematics is not only about finding the correct answer but can also be full of creativity and imagination,"* emphasizing a move away from procedural thinking toward a more exploratory orientation. RJ1 similarly described this experience as *"creative and surprise-filled…no less exciting* *than an action movie,"* suggesting that alternative systems introduced novelty and intellectual playfulness. RJ9 highlighted how modular arithmetic fostered deeper cognitive engagement, stating, *"modular arithmetic showed me that mathematics is much richer than I thought. I feel my creativity has increased; now I want to ask more 'what if' questions,"* indicating that the activity promoted flexible thinking and conceptual innovation. Collectively, these reflections illustrate that modular arithmetic encouraged students to view mathematics as a creative and evolving discipline rather than a rigid collection of fixed procedures.

Theme 5. Realization that rules shape logic and meaning

Students came to understand that the meanings of operations and equalities change with the rules of the number system, thereby revealing a growing appreciation for the formal structures underlying mathematics. RJ7 captured this realization succinctly, stating that mathematics is *"not always absolute, it depends on the rules we set,"* emphasizing the idea that mathematical validity is rooted in the axioms and conventions chosen for a given system. The comment, *"I realized we were redefining the meaning of the word 'equal',"* by RJ5 highlights the transformative shift from viewing equality as absolute to recognizing it as context-dependent. Other reflections reinforced this expanding awareness. RJ1 remarked that *"what was happening wasn't normal anymore,"* demonstrating how modular arithmetic disrupted familiar expectations and prompted reconsideration of standard operations. RJ8 extended this idea by drawing a parallel beyond mathematics, noting, *"just like in life, perspective matters in mathematics,"* suggesting a broader conceptual shift toward relativism and structural awareness. Collectively, these reflections illustrate that students developed a deeper understanding of mathematical formalism, recognizing that operations, equalities, and truths acquire meaning only within the specific rules and structures that define a number system.

Theme 6. Growth in mathematical identity and epistemic openness

Students described meaningful personal transformations, including greater openness, curiosity, and a willingness to question long-held assumptions. RJ10 expressed this shift clearly, noting that "'2 + 2 *is not always 4' has become a symbol of open-minded thinking,"* suggesting that modular arithmetic catalyzed a broader epistemic flexibility. Several others echoed this sense of intellectual empowerment. RJ6 reflected, *"My perspective on mathematics stopped being rigid, now I think it's about discovery and reasoning,"* while RJ7 noted that, *"My horizons have widened,"* indicating that exposure to alternative systems expanded their sense of what mathematics is about. RJ9 highlighted the depth of the learning experience, stating, *"I understood its deeper meaning … it was about learning to see alternative logical systems."* Together, these reflections demonstrate that students began to develop more sophisticated mathematical identities characterized by critical thinking, openness to unconventional ideas, and an appreciation of diverse logical structures. Collectively, these themes indicate that modular arithmetic can serve as a powerful pedagogical tool for reshaping learners' understanding of what constitutes mathematical truth and for fostering epistemological growth.

Figure 1 presents a visual map summarizing the emerging themes. A notable finding is that introducing high school students to alternative mathematical systems, such as modular arithmetic, encouraged them to reconstruct and interpret mathematical truths as context-dependent and rule-based. This exposure led to a significant shift in the students' understanding of mathematics, moving away from absolutist views toward more fallibilist or constructivist perspectives.

DISCUSSION

This mixed-methods study investigated how engaging with modular arithmetic as an alternative mathematical system influences high school students' perceptions of mathematical truth, creativity, and reflective thinking. The integration of quantitative and qualitative findings provides robust evidence that challenging the absolutist statement "2 + 2 = 4" can transform students' epistemic beliefs about mathematics. The results collectively revealed a shift from fixed, procedural conceptions of mathematics toward more flexible, contextual, and reflective understandings.

Reconceptualizing Mathematical Truth: From Absolutism to Contextualism

Quantitative findings showed statistically significant increases in students' agreement with statements regarding the contextual nature of mathematical truth. This empirical shift was supported by qualitative reflections illustrating profound epistemological change. Students articulated that mathematical truth "depends on the rules we set" and that meaning varies across systems, echoing fallibilist perspectives in the philosophy of mathematics (Ernest et al., 2016; Lakatos et al., 1976).

Recent literature aligns with these insights. Contemporary scholars continue to argue that mathematical meaning is socially constructed and context-dependent (Brantlinger, 2022; Sjaastad, 2025). Likewise, recent studies in mathematics education (Hoang et al., 2023; Copur-Gencturk & Li, 2023) emphasize moving beyond static notions of correctness toward reasoning within systems and structures. Students' reflections, such as RJ5's statement suggesting

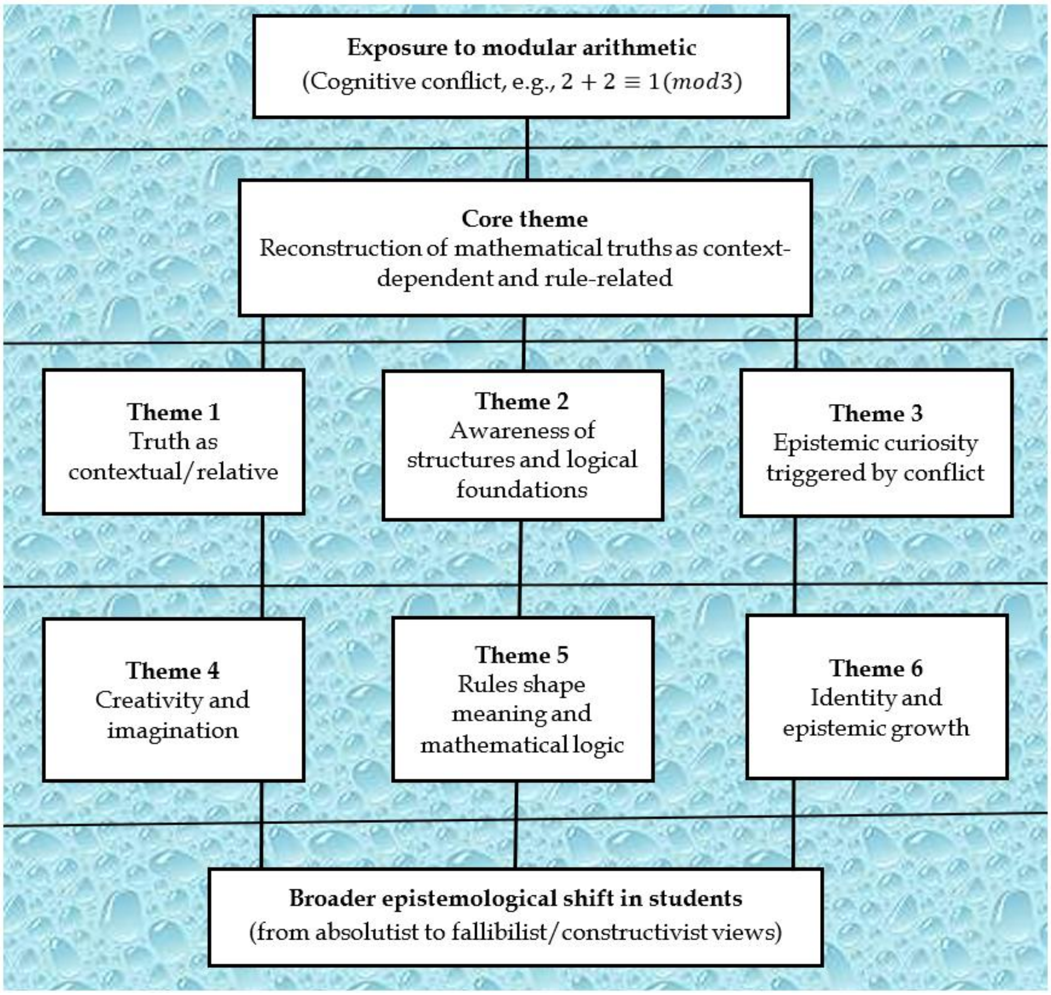

Figure 1. Visual thematic map (Source: Authors' own elaboration)

that equality can have different meanings in different systems, closely parallel current research highlighting the importance of structural understanding for deep learning (Mason et al., 2009; Tall, 2021).

Thus, exposure to modular arithmetic successfully destabilized absolutist epistemologies and replaced them with contextually grounded, reflective perspectives.

Growth in Structural and Axiomatic Awareness

Students in the qualitative phase increasingly understood that mathematical outcomes come from axioms and definitions. Improvements in the "creativity and critical thinking" subscale were reflected in reflective journals where students described mathematics as "rules and structures" rather than universal facts. These findings align strongly with established theoretical models. One such model is Sfard's (1998) dual-process model of mathematical learning. This model highlights the shift from operational to structural conceptions. Another relevant framework is Tall's (2021) perceptual learning. In this framework, learners combine process and concept to develop a more mature understanding of mathematics.

Additionally, Duval's (2006) theory of semiotic representation is significant. Recent work also advocates for deeper structural awareness among pre-university learners (Putri et al., 2025).

Engagement with modular arithmetic provided students with a concrete context for recognizing how altering axioms yields new logical systems. This supports claims that exposure to alternative mathematical frameworks fosters conceptual flexibility (Gilmore, 2023; Saralar-Aras & Kurudirek, 2025).

Cognitive Conflict as a Catalyst for Epistemic Curiosity

Engagement with modular arithmetic intentionally invoked cognitive conflict by challenging students with counterintuitive results such as $2 + 2 \equiv 0 \pmod 4$. Students described the experience using words such as "surprise," "strange", "wondering", "a lot of fun", "fascinating", "exciting," and "I now enjoy asking why something works." These emotional responses align with Piaget's disequilibrium theory, which states that cognitive conflict promotes the development of new mental structures, as well as with modern interpretations of "productive struggle" in mathematics learning (Boaler, 2016). The excitement and curiosity students expressed reflect Harel and Sowder's (1998) concept of "intellectual need," suggesting that learners engage more deeply when faced with meaningful contradictions that require explanation. Recent research (Ibrahim et al., 2024; Putri et al., 2025) confirms that problem situations and challenging initial assumptions can increase engagement and foster more profound reasoning. This aligns with RJ6's comment that what initially seemed like a "joke" eventually became an opportunity for genuine conceptual insight.

Emergence of Creativity, Imagination, and Reflective Thinking

Modular arithmetic, by challenging traditional expectations, seems to have created space for creative engagement. Both datasets showed that students started to see mathematics as more creative rather than just procedural. Substantial improvements in students' reflective attitudes complemented qualitative descriptions of mathematics becoming "full of imagination" or "a place to explore and question." Recent literature more often emphasizes creativity as central rather than peripheral to mathematics learning (Mann, 2006; Sriraman, 2022). Current research (Hoang et al., 2023; Ibrahim et al., 2024) also supports that exploratory environments foster creative mathematical thinking. By introducing unfamiliar systems, the intervention aligned with Papert's (2000) concept of "microworlds" that encourage playful experimentation.

Development of Mathematical Identity and Epistemic Openness

Students described becoming "more open," "less afraid to question," and "more confident in expressing mathematical ideas." This psychological growth signals a significant affective shift aligned with modern theories of mathematical identity (Boaler, 2016; Schoenfeld, 2016).

These findings also resonate with recent research demonstrating that learners' beliefs about mathematical certainty significantly shape their engagement and sense of agency (Ghufron et al., 2020; Refinal et al., 2024). By interrogating foundational claims like "2 + 2 always equals 4," students developed identities as capable reasoners rather than passive recipients of truth.

Notably, no gender differences emerged across any subscales, indicating that the intervention promoted equitable participation, consistent with contemporary calls for gender-fair mathematics pedagogies (Copur-Gencturk & Li, 2023; Gervasoni et al., 2012).

Implications of Findings for 21st Century Mathematics Education

The findings of this study have important implications for 21st century mathematical pedagogy.

Cultivating conceptual flexibility in a data- and AI-driven world

Modern environments demand learners who can reason across different systems, such as modular arithmetic, algorithms, or abstract algebraic structures. Encouraging students to question assumptions fosters the conceptual flexibility essential in fields like cryptography, computer science, and digital technology security.

Enhancing critical and creative mathematical thinking

Results suggest that exploring alternative systems promotes flexible reasoning, tolerance of ambiguity, the ability to examine definitions, and creative exploration of multiple representations. These competencies align with global priorities in mathematics education, as outlined by the OECD (2024) and UNESCO (2023).

Rehumanizing mathematics classrooms

Students reported increased enjoyment, confidence, and intellectual curiosity. As contemporary scholars argue (Boaler, 2016; D'Ambrosio & Knijnik, 2019), human-centered, inquiry-rich environments challenge the rote, procedural approaches still common worldwide.

Bridging mathematics with philosophy, culture, and ethics

Because students explored the concept of "truth," the study contributes to broader interdisciplinary efforts to integrate mathematical, philosophical, and cultural reasoning–an increasingly valued aspect of STEM education.

Limitations of the Study

While the findings are strong, several limitations should be acknowledged. Firstly, the study involved 50 students from four campuses, which may limit the generalizability of the results. Additionally, the sample's cultural homogeneity could limit its applicability to broader populations. Another limitation is the short duration of the intervention, which lasted only two weeks; as a result, the long-term conceptual changes remain unknown. Furthermore, many of the measures relied on student perceptions, making them susceptible to bias. Lastly, the study focused exclusively on modular arithmetic as the alternative mathematical system, and it is possible that other systems, such as non-Euclidean geometry or fuzzy logic, may yield different outcomes. These limitations provide direction for further research.

Recommendations for Future Research

Longitudinal studies are essential for understanding how epistemic beliefs change over months or years, including their effects on achievement, mathematical identity, and course choices. Likewise, comparative studies across different mathematical systems can reveal which methods most effectively promote conceptual flexibility. To improve the applicability of results, conducting cross-cultural and cross-grade replication studies in various countries, curricula, and age groups is important. Additionally, teacher-focused research is vital for examining how educators perceive mathematical truth and how their beliefs influence classroom culture. Lastly, incorporating AI and digital tools into research may offer insights into whether dynamic microworlds, simulations, or coding environments improve epistemic flexibility in ways similar to traditional methods like modular arithmetic.

CONCLUSION

It is shown that when students are exposed to the idea that "2 + 2 need not always equal 4" , significant cognitive, creative, and epistemological changes can happen. Quantitative results confirmed notable improvements in students' views on mathematical certainty, creativity, and reflective thinking. Qualitative results revealed shifts toward contextual reasoning, structural understanding, epistemic curiosity, and the development of mathematical identity.

Together, these insights affirm that mathematics education benefits greatly from approaches that challenge universal truths and invite students to explore the axiomatic foundations of mathematics. By engaging with alternative number systems such as modular arithmetic, students construct deeper meaning, develop flexible reasoning, and form more confident, inquiry-driven mathematical identities.

In a time when mathematics supports digital systems, artificial intelligence, and global problem-solving, developing students who can think critically, question assumptions, and work across multiple frameworks is crucial. This research contributes to an expanding body of scholarship advocating for a conceptual, reflective, and human-centered approach to mathematics education. Ultimately, the results highlight the transformative power of teaching mathematics not as a static collection of facts but as a dynamic, evolving, and creative human activity.

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APPENDIX A: QUESTIONNAIRE

Student Perceptions of Mathematical Truth and Context Purpose

This instrument measures students' beliefs and attitudes toward mathematical certainty, creativity, and contextual reasoning before and after learning modular arithmetic.

Instructions

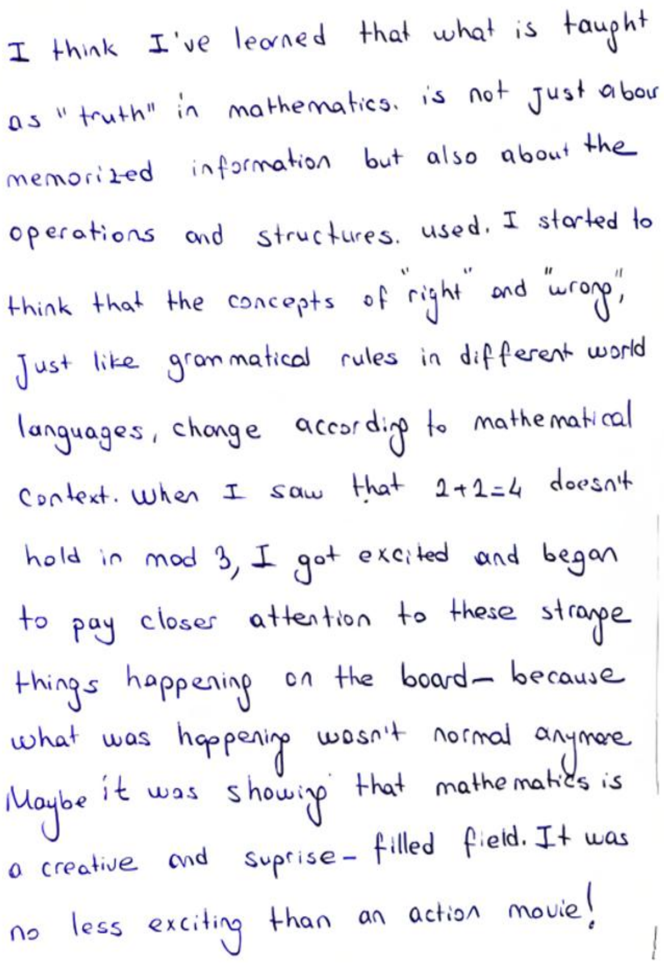
For each statement, circle the number that best represents your opinion (1 = strongly disagree, 2 = disagree, 3 = neutral, 4 = agree, & 5 = strongly agree).

Table A1. Questionnaire

No	Statement	Response scale				
		1	2	3	4	5
Section A–Beliefs about mathematical certainty (research question 1)						
1	Mathematics always produces one correct answer for every problem. (reverse-coded)					
2	The statement "2 + 2 = 4" is valid in all possible mathematical systems. (reverse-coded)					
3	Mathematical truth can depend on the system or context in which it is used.					
4	I can imagine a situation or system where "2 + 2" does not equal "4."					
Section B–Creativity and critical thinking in mathematics (research question 2)						
5	Mathematics allows for creativity and imagination.					
6	Questioning mathematical rules helps me understand mathematics more deeply.					
7	It is acceptable to explore ideas in mathematics that seem "wrong" at first.					
8	I enjoy discussing different interpretations of mathematical problems.					
Section C–Attitudes and reflection on learning (research questions 2 & 3)						
9	Learning systems like modular arithmetic helps me see mathematics differently.					
10	I feel more confident expressing my mathematical ideas about alternative systems.					
11	Mathematics becomes more interesting when I can question and discuss it.					
12	There can be multiple valid ways to understand a mathematical statement.					

APPENDIX B: REFLECTIVE JOURNAL PROMPTS AND SAMPLE JOURNALS

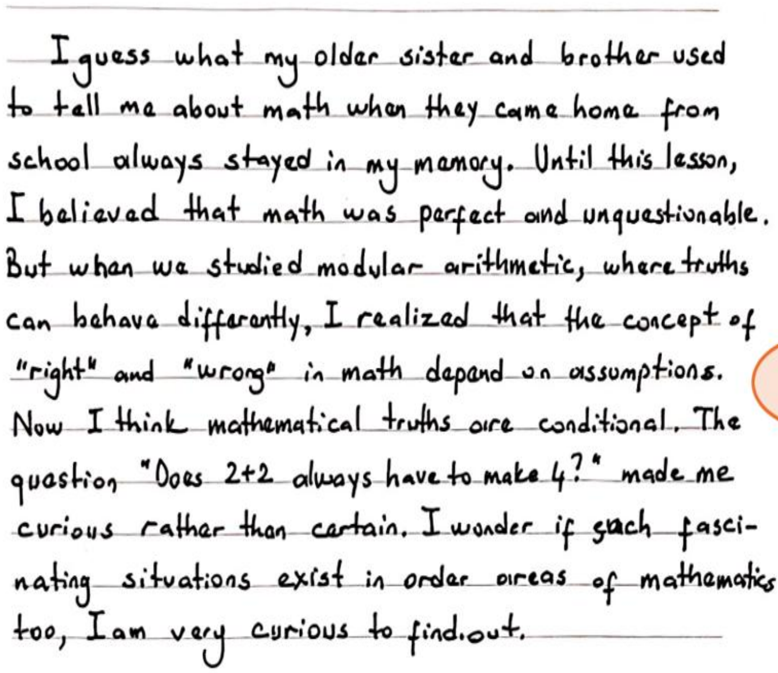
1. What new insight did you gain about "truth" in mathematics after learning modular arithmetic?
2. How did this activity change how you think about "right" and "wrong" in mathematics?
3. Describe a moment when your view of "2 + 2 = 4" expanded or changed.



I think I've learned that what is taught as "truth" in mathematics. is not just abou memorized information but also about the operations and structures. used. I started to think that the concepts of "right" and "wrong", Just like grammatical rules in different world languages, change according to mathematical Context. When I saw that 2+2=4 doesn't hold in mod 3, I got excited and began to pay closer attention to these strange things happening on the board– because what was happening wasn't normal anymore Maybe it was showing that mathematics is a creative and suprise-filled field. It was no less exciting than an action movie!

RJ1

(Source: Student Reflective Journal Number 1)



I guess what my older sister and brother used to tell me about math when they came home from school always stayed in my memory. Until this lesson, I believed that math was perfect and unquestionable. But when we studied modular arithmetic, where truths can behave differently, I realized that the concept of "right" and "wrong" in math depend on assumptions. Now I think mathematical truths are conditional. The question "Does 2+2 always have to make 4?" made me curious rather than certain. I wonder if such fascinating situations exist in order areas of mathematics too, I am very curious to find out.

RJ2

(Source: Student Reflective Journal Number 2)

I discovered that mathematical truth is not always fixed, it depends on the system, as in modular arithmetic. this activity changed the way I view mistakes: something that seems wrong in one system might be right in another, why not? I used to memorize most mathematical expressions and formulas, but now I question them. for me, the phrase "2 + 2 may not always make 4" means that learning is about questioning, not just accepting.

RJ3

As someone who loves mathematics, after taking the lesson on modular arithmetic, I realized that "truth" in mathematics depends on the system we use, For example, in the mod 3 system, 2 + 2 = 1, which amazed me because I used to think that mathematics always had only one correct answer. This experience made me feel that context matters even in mathematics. Now I see math more as a language rather than a collection of fixed truths.

RJ4

(Source: Students' Reflective Journals Number 3 and 4)

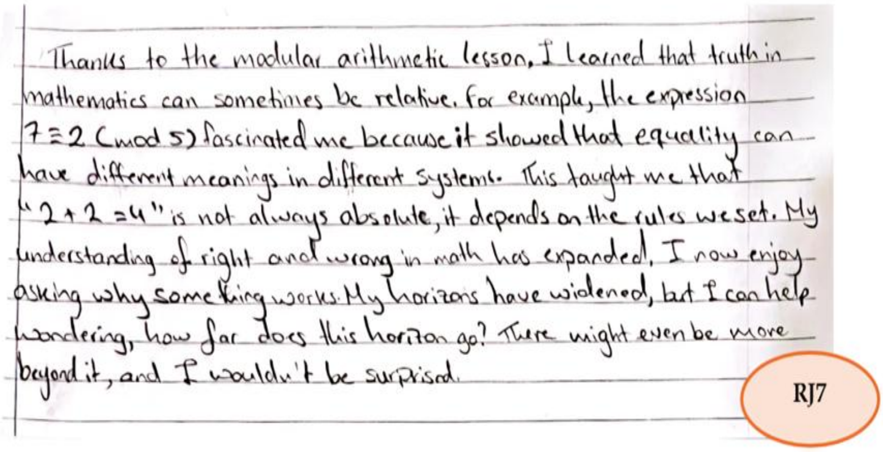
At first, the modular arithmetic lesson seemed strange to me. But when we played with clock arithmetic, like when 11+5=4, it was a lot of fun I realized we were redefining the meaning of the word "equal." I learned that mathmatics is not only about finding the correct answer but can also be full of creativity and imagination. Now I think that truth in mathmatics is built upon the rules we agree on

RJ5

when the new teacher came to class and wrote 2+2=1 on the board, I thought it was a Joke. I was trying hard not to laugh and even wanted to make a Joke myself. when our teacher explained that 2+2=4 is true in normal arithmetic but 2+2=0 in mod 4, my mathematical thinking reached a new dimension. This activity taught me that there can be powerful tools for Learning. my perspective on mathematics stopped being rigid, now I think it's about discovery and reasoning.

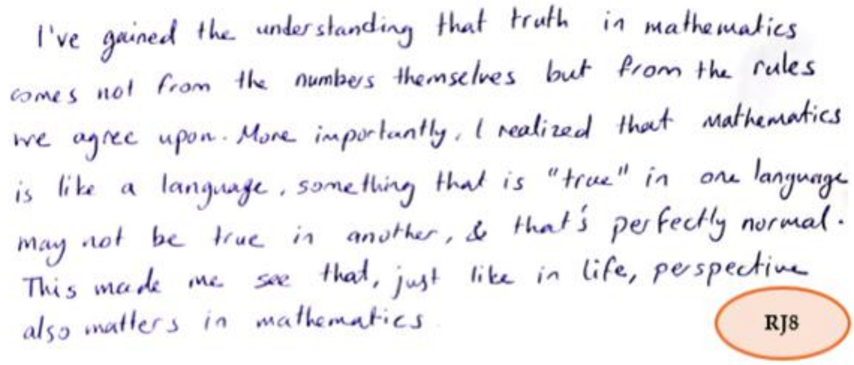
RJ6

(Source: Students' Reflective Journals Number 5 and 6)

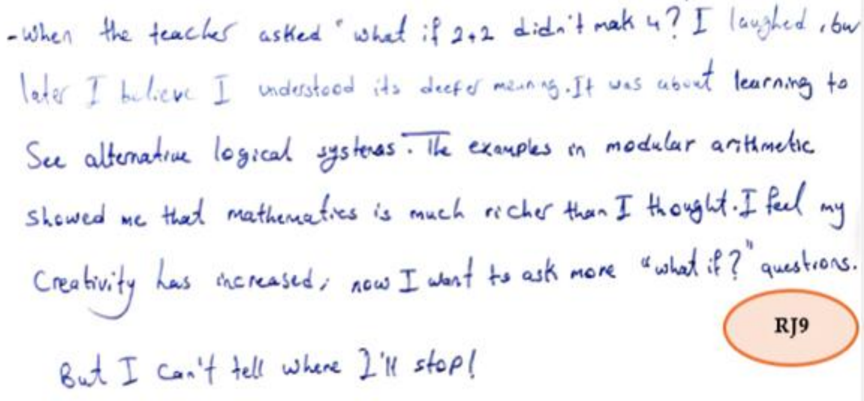

Thanks to the modular arithmetic lesson, I learned that truth in mathematics can sometimes be relative. For example, the expression 7 ≡ 2 (mod 5) fascinated me because it showed that equality can have different meanings in different systems. This taught me that "2 + 2 = 4" is not always absolute, it depends on the rules we set. My understanding of right and wrong in math has expanded, I now enjoy asking why something works. My horizons have widened, but I can help wondering, how far does this horizon go? There might even be more beyond it, and I wouldn't be surprised.

RJ7

(Source: Student Reflective Journal Number 7)

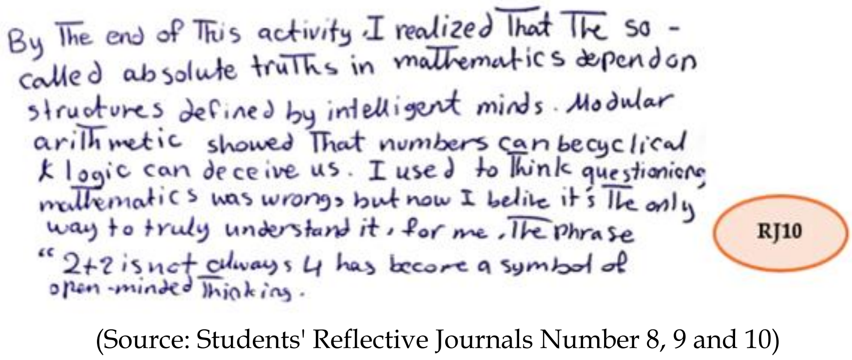

I've gained the understanding that truth in mathematics comes not from the numbers themselves but from the rules we agree upon. More importantly, I realized that mathematics is like a language, something that is "true" in one language may not be true in another, & that's perfectly normal. This made me see that, just like in life, perspective also matters in mathematics.

RJ8


- When the teacher asked "what if 2+2 didn't make 4?" I laughed, but later I believe I understood its deeper meaning. It was about learning to See alternative logical systems. The examples in modular arithmetic showed me that mathematics is much richer than I thought. I feel my Creativity has increased; now I want to ask more "what if?" questions.

RJ9

But I can't tell where I'll stop!


By The end of This activity I realized That The so-called absolute truths in mathematics depend on structures defined by intelligent minds. Modular arithmetic showed That numbers can be cyclical & logic can deceive us. I used to Think questioning mathematics was wrong, but now I belive it's The only way to truly understand it. For me, The phrase "2+2 is not always 4 has become a symbol of open-minded Thinking.

RJ10

(Source: Students' Reflective Journals Number 8, 9 and 10)