MODESTUM

EURASIA Journal of Mathematics, Science and Technology Education, 2026, 22(7), em2853
ISSN:1305-8223 (online)

OPEN ACCESS

Research Paper

https://doi.org/10.29333/ejmste/18737

Making meaning of formulas in physics-lessons: Describing teacher's speech with a verbalization-model for formulas

Wiebke Hinrike Kuske-Janßen [1], Gesche Pospiech [2*]

[1] Gymnasium Ulricianum, Aurich, GERMANY
[2] Technische Universität Dresden, Dresden, GERMANY

Received 03 December 2025 ▪ Accepted 10 May 2026

Abstract

This study examines via qualitative content analysis how teachers use and teach formulas in physics lessons, focusing on linguistic varieties (special language, educational language, and everyday language) and their classification according to a level model of formula verbalization. The research design allows a detailed reconstruction of how teachers convey formulas linguistically in class and which content-related and methodological priorities they set in doing so. A total of 10 physics teachers in Saxony schools were observed over a period of five to ten lessons per teacher and their language audio-recorded. The relevant parts of the lessons were transcribed and analyzed with qualitative content analysis according to Kuckartz (2016). The resulting category system provides the first systematic description of teachers' use of language when dealing with formulas in the classroom and identifies deficits in teaching structural mathematical-physical connections. The results contribute to deriving implications for teacher training, in particular for promoting understanding-oriented formula teaching.

Keywords: physics education, formula verbalization, teacher language use, mathematical-physical structures, teacher professional development, making meaning of formulas

INTRODUCTION

Formulas are of central importance to physics and physics education. In general, formulas enable us to express physical laws concisely, describe proportions, make predictions, and calculate unknown quantities. A broad understanding in these aspects of formula use goes far beyond purely technical applications: formulas have their own semantics, condense empirical or theoretical insights, and play a specific role in science as a form of representation. Dealing with this comprehensive formula semantics in the context of teaching in a classroom is an important and difficult task for teachers and educators all through the educational career. Herewith, only recently the handling of formulas by learners and teachers came into the focus of physics education research (Pospiech & Karam, 2023). This might be because the use of formulas is mostly restricted to college or university teaching, and only partly also in high school. However, already in secondary school formulas should play a role in facilitating students' insight into the scope of physics, the physical method and how physics is done. Despite this importance, it is observed that students have difficulties attributing meaning to formulas (Bagno et al., 2008). This alludes to a discrepancy between the desired meaningful learning (an understanding of the content of formulas) and empirical findings about students' abilities. Therefore the question arises which ways of making meaning teachers offer their students in physics lessons.

In providing students with an insight into the function and role of formulas, the teachers play a decisive role by the manner in which they introduce, use and speak about formulas. In identifying how teachers work with formulas in the classroom, we take seriously the saying that mathematics is the language of physics. This identification task requires a linguistic model of the possibilities of how to work with formulas. Such a linguistic model then serves as an analytical framework: it reveals the diversity of possible bridges between symbolic precision of a formula and student-oriented comprehensibility and it allows to analyze the occurrence of these bridges in the classroom. So, in the

✉ wiebke_janssen@gmx.de ✉ gesche.pospiech@tu-dresden.de **(*Correspondence)**

Contribution to the literature

- Making meaning of formulas is an essential part of teaching and understanding physics. Mostly, the understanding of formula is analyzed in the context of problem-solving showing deep difficulties of students. However, there is only little known about the teaching of formula in classroom where verbal communication plays an important role. So, the need arises to have a systematic way for consciously communicating verbally about formulas and describe this process.
- In this paper a model for different levels of verbal communication with respect to introduction of formulas is derived and validated.
- This level model closes a gap in the literature on making meaning of formulas in physics teaching and can be the basis for further research in this field.

following the theoretical foundations for such a linguistic model are explained and the so-called "level model" is derived, then applied, and validated in the classroom. This level model then presents an instrument to describe the link between formulas and verbal language in physics classrooms and thereby hints to possible impacts on teaching students towards a deeper understanding of formulas.

HISTORICAL AND EPISTEMOLOGICAL FOUNDATIONS

The question of the relationship between mathematical expression and verbal language has long preoccupied physicists. Faraday (1990a) summed up this tension in 1857 in his correspondence with Maxwell:

> When a mathematician engaged in investigating physical actions and results has arrived at his conclusion, may they not be expressed in common language as fully, clearly, and definitely as in mathematical formulae? (p. 552).

This debate reflects what constitutes physical knowledge. Does knowledge end with mathematical formalism, or must physics question its ontological implications for our reality–for example, in the controversy surrounding the interpretation of quantum mechanics (Friebe et al., 2015)? Wigner (1960) formulates:

> The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve.

In "What did mathematics do to physics?", Gingras (2001) analyzes the profound influence of mathematics on physics. Since Newton's (1678) "Principia mathematica", mathematization has had three consequences: social de-democratization (access only for those proficient in mathematics), an epistemological shift toward causal mathematical descriptions instead of verbal explanations, and ontological abstraction, in which substances (such as ether) were replaced by formal equations. This development illustrates that mathematics is not only an auxiliary tool but also shapes the structure of physical knowledge.

de Ataíde and Greca (2013) identify three historical stages of this interlocking: Initially, mathematics served to create analogies between real objects and ideal models. Later, it established itself as a descriptive language for natural laws. Today, it actively generates new physical theories–for example, through mathematical symmetry considerations in particle physics. The "unreasonable effectiveness of mathematics in the natural sciences" (Wigner, 1960) remains an epistemological mystery: Why are man-made mathematical structures so perfectly suited to describing nature? In the following we describe some aspects of this interplay.

Functions and Roles of Mathematics in Physics

The metaphor of mathematics as the "language of physics" (Galilei, 1896, as cited in Krey, 2012, p. 38) is widely spread and has practical implications. As Falkenburg (1999) emphasizes, modern physics primarily formulates laws in mathematical symbols, which are interpreted using quantitative concepts. This abstraction is far removed from everyday understanding, thus presenting a hurdle for learners. But since mathematics fulfills multiple functions in physics we have to emphasize it in the learning of physics. As Krey (2012) and Krey and Karam (2016) point out, it serves to reduce cognitive load by making abstract concepts manageable through sign systems. As a medium of communication, it enables precise exchange among experts, provided they have mastered the conventional symbols. Its precision and objectivity support scientific validity through logically comprehensible arguments and intersubjective verifiability. In addition, idealization allows us to focus on essential aspects of complex phenomena. An often overlooked aspect is aesthetics: Dirac (1977) emphasized that physical laws must follow "beautiful equations", although this aesthetic dimension usually remains hidden from laypeople.

From the educational perspective, Galili (2018) models the relationship between the two disciplines in the discipline-culture model. While mathematics examines abstract objects logically and deductively, physics aims to provide causal explanations of real phenomena using experiments. Both share methods in the "body" of their disciplines but differ in their epistemological "core" (Galili, 2018, p. 22). Thus, the physical context, especially in the context of learning, is both a challenge (linking mathematical results with physical contexts) and aims at cognitive facilitation. Palmgren and Rasa (2024) stress the intertwining of conceptual and mathematical aspects of physical theories and conjecture that emphasizing this strong relation in teaching would enhance understanding (see also Pospiech, 2019).

PHYSICS EDUCATION PERSPECTIVE

The close connection between physics and mathematics presents teaching with didactic dilemmas. Feynman (1997) stated:

> Physics is not mathematics, and mathematics is not physics [...] In physics, you have to understand the connection between words and the real world (p. 72).

However, empirical studies show that learners often fail to interpret the content of formulas. Bagno et al. (2008) show that high school students often understand formulas only as calculation rules. Tuminaro and Redish (2007) identify a "plug-and-chug" behavior among students–the mechanical insertion of values without conceptual reflection. To capture the corresponding difference, Pietrocola (2008) differentiates between technical and structural skills. The former include algorithmic operations (e.g., formula transformations), while the latter involve the conceptual linking of mathematical structures with physical contexts (e.g., interpreting $a = F/m$ as acceleration under a force). de Ataíde and Greca (2013) extend this work and distinguish between three roles of mathematics: as a tool, it serves numerical calculations; as a language (translator), it translates physical content into symbols; as a structuring element (structure), it shapes physical thinking. This structural role is evident in Faraday's (1990b) demand that every arithmetic operation must be physically interpretable (p. 672).

Studies by Uhden (2012) and Bing and Redish (2009) show that focusing exclusively on technical skills leads to a superficial understanding. Didactic approaches such as Skemp's (1976) relational understanding or Prediger's (2009) content-based thinking therefore advocate the integration of both dimensions. In accordance with this approach, Wagenschein (1962) warned against the premature use of formulas in teaching: a purely technical approach would deprive students of an understanding of the content of the "physics condensed in the formula" (p. 171).

Research underscores the need not to reduce formulas in teaching to their algorithmic dimension. Rather, their semantic depth—as carriers of theoretical concepts and epistemological principles—must be explicitly addressed. Only in this way can we prevent learners from perceiving physics, as Redish (1994) ironically puts it, as a "collection of dead leaves" rather than a "living tree."

Mathematical Representations

In physics, multiple mathematical representations are used, starting with numbers for measuring physical quantities, plotting measured values in a graph or using formulas. All these representations are necessary for both understanding and communicating physics. In addition to the skill of understanding and using a representation correctly, students need to develop a "meta-representational competence" (diSessa, 2004) which means beyond the correct use of different representations their linking with each other (Ainsworth, 2008), e.g., the linking of a formula with verbal language. This translation and linking process can also deepen the understanding of the physical content and thus fulfills a didactical purpose by its own. Leisen (2005b) emphasizes changes in representations, especially with different levels of abstraction in the learning-process, and Bolte and Pastille (2010) propose an "activation rectangle" that links phenomena, texts, and graphics. Among the mathematical representations formulas play a paramount role.

Formulas in Physics Teaching

Formulas are a central element of physics teaching, but their multifaceted role is often reduced to technical aspects. The following description is based on work by Romer (1993), Karam and Krey (2015), Pospiech et al. (2019).

Formula semantics and making meaning

Formulas condense semantic information into a very small space (**Figure 1**). Understanding them requires more than just knowledge of formula symbols. Greca and Moreira (2001) distinguish between the semantics as belonging to physics and the mathematical syntax of a formula. Sherin's (2001) "symbolic forms" decode the implicit semantics of mathematical operators. Thus, "+" is decoded as base plus change (e.g., $v = v_0 + at$) or parts of a whole (e.g., $E = ½ mv^2 + mgh$). Such interpretations differ from mathematical conventions: in physics, units contribute to meaning, and the equal sign often implies causality (Heck & Buuren, 2019). As an example, a semantic network for $s = ½ at^2$ includes kinematic principles, rates of change, and everyday references such as braking distances. Romer (1993) refers to "hidden text"–epistemological status, conditions of validity, limiting cases, and experimental validation. The

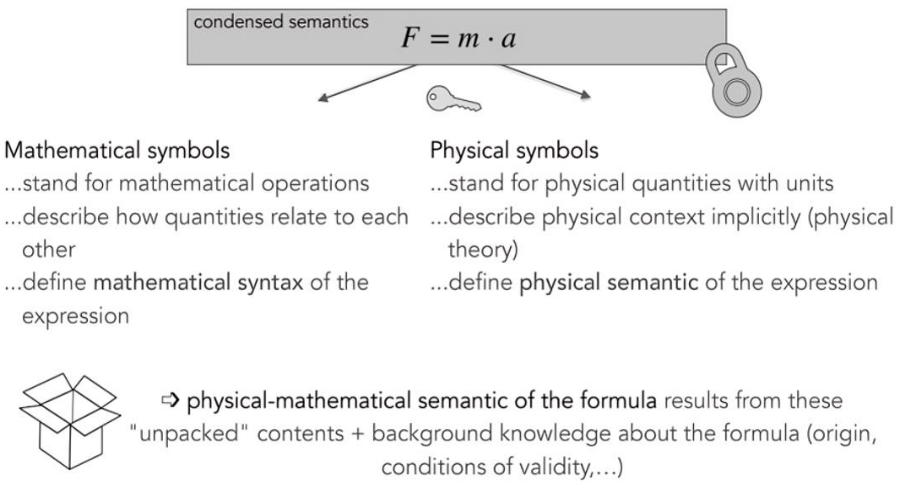

Figure 1. The condensed semantics of a formula (translated from Kuske-Janßen, 2020)

necessity of including the connection of mathematics and physics explicitly in instruction is stressed by Palmgren et al. (2025). Redish and Kuo (2015) reconstruct formula meanings using cognitive linguistic concepts: embodied cognition (anchoring in haptic experiences), contextualization (situational embedding), and encyclopedic knowledge (networking with prior knowledge).

Epistemological differentiation of formulas

In his analysis, Romer (1993) emphasizes that physical equations have different logical status despite the common "equality sign." He distinguishes between three categories: definitions (tautological statements, e.g., $R = U/I$ for electrical resistance), expressions of fundamental laws of nature (such as Newton's laws of motion), and specialized, application-related results (such as $R = \rho \cdot l/A$ for the resistance of a thin long wire). He also emphasizes the epistemological distinction between whether a formula represents an empirically discovered regularity or was logically derived from more fundamental principles (Romer, 1993, p. 129). This differentiation is relevant for teaching because it helps students understand the ontological basis of formulas—for example, why there are different expressions for the same quantity, e.g., concerning the capacitance C of a plate capacitor, there are different expressions ($C = Q/U$ vs. $C = \varepsilon_0 \cdot \varepsilon_r \cdot A/d$) each describing different aspects of its meaning and its calculation.

Karam and Krey (2015) systematize this approach using four epistemological categories: principles (fundamental, experimentally supported truths such as conservation of energy), definitions (necessary quantity specifications such as $p = m \cdot v$), empirical regularities (experimentally determined relationships such as the Balmer formula) and derivations (formal deductions from principles, e.g., $E = mc^2$) (Karam & Krey, 2015, p. 664). This classification not only serves to meta-interpret individual formulas but also promotes reflection on the role of mathematics in physics–a core aspect of "meta-representational competence".

Teaching perspective: attitudes and teaching strategies

The implementation of these concepts in the classroom depends largely on the attitudes of teachers. Pospiech et al. (2019) adapted the pedagogical content knowledge model developed by Magnusson et al. (1999) to capture the interaction between mathematics and physics. In interviews with 23 "expert teachers" (8 from Israel, 15 from Germany), it was found that only about half of them had an elaborate understanding of the structural role of mathematics—for example, as a means of generating knowledge. The other half reduced mathematics to a technical auxiliary science (Pospiech et al., 2019, pp. 278-279). Although all respondents emphasized the need to interpret mathematical expressions physically, examples primarily referred to graphs; only a few explicitly mentioned formulas.

The study identified four types of teachers based on their stated teaching principles: concept-oriented (physics before mathematics), mathematics-oriented (focus on formula technique), application-oriented (practical use), and multi-layered (combined approach) (Pospiech et al., 2019, pp. 280-281). This typology correlates with other studies: de Ataíde and Greca (2013) found three attitudes among teacher training students (mathematics as a tool, as a language, or as a structure) that are reflected in "theorems in action"–for example, "operational mathematics" (trial-and-error-based) vs. "conceptualization" (conceptual linking). Thoms (2011) distinguished between "formula users" (deductive, calculating) and "formula processors" (inductive, explanatory) in questionnaires.

According to Lehavi et al. (2019), teachers specifically use four "teaching patterns": exploration (analysis of mathematical relationships to explore systems), construction (model development from experiments), broadening (cross-cutting principle analysis), and application (problem solving with known laws). These patterns illustrate that formulas are not only calculation tools but also means of explanation and insight—a view supported by surveys of 244 teachers in Germany (Strahl et al., 2013). Here, participants cited functions such as knowledge storage, modeling, and prediction, with arithmetic tasks making up only part of the spectrum.

The developing view of pre-service teachers during their training on the role of mathematics in physics was described in de Winter and Airey (2022). Their study hints to the necessity of preparing teachers for the requirements of the classroom in facilitating the use of mathematics in physics.

Learning activities and cognitive challenges

Despite the described diversity, technical activities often dominate in the classroom. Uhden (2012) analyzed textbooks in Germany and found that 80% of the tasks were computational in nature ("given-sought-solution" scheme), while changes in representation were rarely required. Geyer (2020) classified activities involving algebraic expressions and transfer between different representations into four areas: information extraction (e.g., local/ global dependencies), construction (building formulas), working with representations (transforming, reasoning, controlling), and embedding (contextual linking). Calculation was listed separately as a cross-representation activity.

Particularly relevant is the interpretation of formulas, which is systematized in methods such as Romer's (1993) "reading equations" or Bagno et al.'s (2011) seven-step procedure in the process of interpreting a formula:

(1) naming the formula symbols,

(2) checking units,

(3) conditions of validity,

(4) graphical representation,

(5) analysis of extreme cases,

(6) meaning of compound terms, and

(7) verbal summary.

Textbooks often reduce this to considerations of proportionality, while epistemological reflections (origin of the formula) are neglected. However, these works show that it is important and not trivial to introduce students to working meaningfully with formula.

Learning difficulties and epistemological barriers

Empirical studies reveal serious difficulties among students. Bagno et al. (2011) tested 72 high school students: only 13% were able to correctly verbalize the meaning of $F = m \cdot a$; 41% did not give an answer. Students often limited themselves to translating mathematical symbols ("sum of forces equals mass times acceleration") without explaining physical processes. Strahl et al. (2010) and Pospiech and Oese (2013) found similar deficits, with technical aspects (rearranging, calculating) dominating.

The causes lie in a lack of conceptual transfer between mathematics and physics. Rebello and Cui (2008) as well as Kimpel (2018) show that mathematical knowledge often remains context-bound, implying that students have difficulties transferring their mathematical knowledge to the applications in science. Sherin (2001) identified "symbolic forms" (content interpretations of mathematical operators) that lead to misunderstandings when misapplied–for example, when $s = s_0 + v \cdot t$ is mistakenly interpreted as competing terms rather than as base plus change. In addition, "epistemic framing clusters" (Bing & Redish, 2009) such as calculation (algorithm focus) or invoking authority (teacher citation) might hinder structural thinking.

There exists a discrepancy between the epistemological complexity of physical formulas and their reduced implementation in the classroom. While frameworks such as those of Romer (1993) or Karam and Krey (2015) emphasize structural and epistemological dimensions, technical activities dominate in practice—reinforced by teacher attitudes and textbook designs. Learning difficulties result from insufficient linking of mathematical and physical concepts, which manifests itself in deficient verbalization skills and epistemological misframing. Didactic approaches such as those of Bagno et al. (2011) or Karam and Krey (2015) show that consciously addressing the roles of formulas (e.g., through derivations, contextualization) can promote structural understanding. Many studies focus on the use of formulas in problem solving and related deficiencies of students. In this context Gifford and Finkelstein (2021) use a model of mathematical sense-making by undergraduate physics students.

However, there are only few studies analyzing the opportunities of sense-making provided by teachers in classroom during the introduction of formulas. Zhao et al (2021) develop a model of sci-math sense-making and show in a case study that teachers implement different types of sense-making in teaching undergraduate biology students.

Future research should explore how teaching patterns such as exploration or broadening can be used more extensively to teach students meta-representational competence in dealing with formulas.

Verbal Communication in Physics Lessons

In physics lessons, communication is the central mechanism for knowledge transfer and learning processes (e.g., Wulff, 2024). It takes place between teachers and students as well as among learners themselves. According to this understanding, language is not merely a means of transmitting information, but a means of actively negotiating meanings. In linguistic interaction, knowledge is reconstructed, one's own understanding is reflected upon, and adjusted through feedback. Language makes it possible to offer information, negotiate meanings through questions, and confirm interpretations (Kulgemeyer, 2010). This process of negotiating meaning is fundamental to learning processes, but it also presents challenges.

Linguistic principles illustrate the complexity of communication. According to Peirce (1931-1935), meaning arises through the semiotic triangle: a sign (e.g., a word) stands for an object and generates an "interpretant" (individual concept) in the user. Meanings are therefore not static but are constructed subjectively–two people associate different concepts with the same sign. De Saussure's theory of linguistic signs emphasizes the role of the language user: a sign consists of sign content ("signifié", e.g., the concept "tree") and sign expression ("significant", e.g., the sound image "tree"). Without users, a sign has no meaning (Linke et al., 2004). Heywood and Parker (2010) apply this to physics education: language produces reality by negotiating meanings in dialogical processes. This view emphasizes the role of the learner as an active constructor and contrasts with the idea of an objectively "transferable" physical truth.

Communication difficulties in physics lessons are often due to differences between specialized and everyday language (Apolin, 2002). Physics lessons aim to make students competent in special language–both receptively (understanding) and productively (application). This is anchored in the subject itself, as special language is an integral part of physical discourse, and generally is supported by German educational standards of curricula (Kultusministerkonferenz [KMK], 2024). At the same time, teaching must take place in understandable language in order to build conceptual understanding and not convey technical terms as "empty shells." This tension leads to a variety of terms in research: Leisen (2005a) distinguishes between everyday language, classroom language, and special language; Wagenschein (1988) speaks of spoken native language, written everyday language, and special language; Muckenfuß (1995) uses colloquial versus technical language; Rincke (2007) prefers every day, educational, and special language. For this work, the terms everyday language (functional for everyday topics), educational language (mediating level between every day and special language), and special language (scientific context) are used, based on variety linguistics.

Characteristics of language levels

- Special language is characterized by precision, explicitness, and anonymity. It uses defined terms (e.g., "resistance" instead of "oppositional force"), complex syntax (long sentences, nominalizations), and artificial symbols (formula symbols such as "R"). Its economy enables precise information exchange among experts, but often presents barriers for learners (Baumann, 1998; Hoffmann, 1987).
- Everyday language is conceptually oral, with simple syntax and variable meanings (e.g., individual "tree" concept). It is familiar to students, but can be heterogeneous due to social differences (dialects, sociolects) within the learning group.
- The language of instruction, educational language, functions as a "workshop language" (Leisen, 2005a): it integrates technical terms into simple sentences, is action-oriented, and less precise than special language. It enables the negotiation of meaning and is didactically essential for building bridges to prior knowledge.

Findings in subject didactics

Empirical studies show that special language creates barriers to learning. Students who are initially taught in everyday language develop a better conceptual understanding than those who are directly confronted with special language (Brown & Ryoo, 2008). Apolin (2002) demonstrated in an intervention study that simplified textbook texts significantly increase understanding. Rincke (2007) emphasizes that the transition to special language is not a linear process: learners "stagger" between every day and technical concepts, which requires time and meta-cognitive reflection. A meta-discourse on language—e.g., a contrastive comparison of technical and everyday terms—promotes both language and technical competence (Lemke, 1990; Rincke, 2010).

Competence models underscore the relevance of linguistic skills. Kulgemeyer and Schecker (2009, 2012) define physics-specific communication competence as the ability to explain facts in a manner appropriate to the audience (prior knowledge and language level) and appropriate to the subject matter (technically correct). Höttecke et al. (2017) show that special language competence correlates strongly with physics grades. Teacher knowledge is central here: Markic (2017) found that teachers recognize special language as a challenge, but often only have intuitive strategies and reduce it to vocabulary learning. Many equate special language with foreign languages, even though it is about concept learning. Therefore an explicit language support seems necessary, also by fostering a meta-discourse: It might be helpful to explicitly address differences between language varieties, to work on collocations (word combinations such as "exercise force") and introduce terms in context–not via lexical definitions (Muckenfuß, 1995; Rincke, 2010). In addition it might be helpful for improving the skills in using special language to actively switch between different representations of the topic in question (Bolte & Pastille, 2010, pp. 40-41). Future studies should further investigate the effectiveness of specific methods (e.g., meta-discourse, changes in representation) and strengthen teaching skills in language-sensitive instruction–a desideratum in teacher training.

LINKING FORMULAS AND LANGUAGE: A LEVEL MODEL OF THE VERBALIZATION OF FORMULAS

On the basis of these theoretical reflections a model of the verbalization of formula, called "level model" in the following, has been developed, tested, and adapted in several studies analyzing the use of verbalization of formula by teachers in classroom.

Theoretical Foundations of the Level Model

The level model developed here represents an attempt to systematically capture the complex relationship between mathematical formulas and verbal language in physics education. It is based on the fundamental assumption that formulas and language represent different forms of physical knowledge, each of which places specific cognitive demands on learners. While formulas offer a high degree of abstraction and precision through their symbolic condensation, verbal language—especially in its more everyday registers—enables step-by-step access to these condensates. The model integrates perspectives from subject didactics and linguistics in order to reflect the complexity of the verbalization process.

From a subject-specific didactic perspective, the model ties in with Bruner's (1974) concept of enactive-iconic-symbolic representation. Bruner (1974) argues that learning processes ideally go through three phases: an inactive phase in which knowledge is experienced through actions; an iconic phase that uses visual representations; and a symbolic phase in which abstract sign systems such as formulas dominate. The key point here is that learners must be guided step by step through these forms of representation in order to understand symbolic representations.

Bruner (1974) emphasizes the "economy" of symbolic representations—their ability to bundle complex information in a compact form—but at the same time points to the need to access them through less condensed representations: "Condensation or summarization are the means by which we fill our seven slots [of working memory] with gold instead of worthless stuff" (Bruner, 1974, p. 18). For physics education, this means that formulas must be understood as part of a multimodal representation network, the understanding of which is promoted by linking them with enactive and iconic representations.

This perspective is complemented by Prediger and Wessel's (2011) classification of forms of representation,

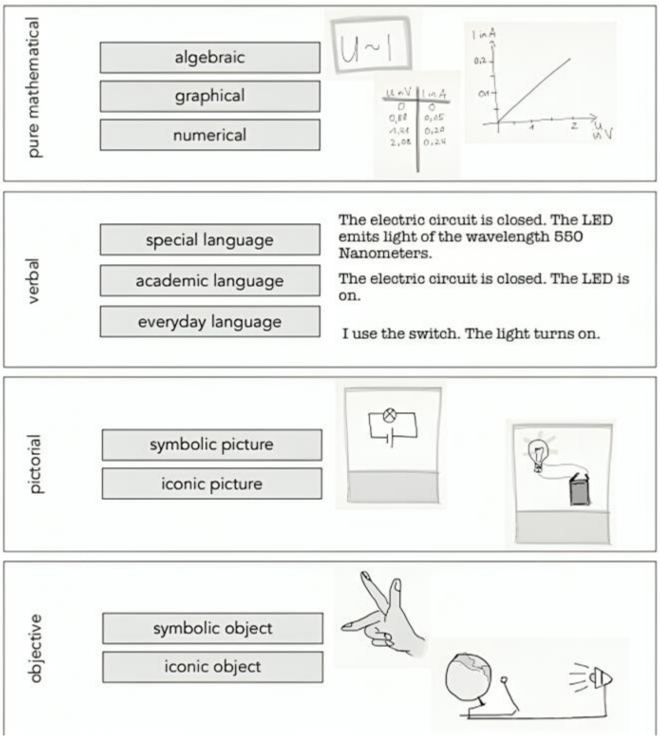

Figure 2. Classification of mathematical representations in physics lessons (Source: Geyer & Kuske-Janßen, 2019)

which distinguishes between symbolic-algebraic, verbal-technical, verbal-everyday language, graphic and concrete representations. For physics teaching, this system has been expanded to include the distinction between iconic (image-based) and symbolic (convention-based) elements (Geyer & Kuske-Janßen 2019, for an example, see **Figure 2**).

The key insight is that mathematical representations can also occur within verbal utterances–for example, when special language describes mathematical relations. Against this background, the targeted linking of formulas with verbal representations appears particularly promising, as language, as a fundamental medium of communication, can flexibly cover different levels of abstraction.

The linguistic foundation is provided by Hoffmann's (1987) theory of the vertical stratification of special languages. Hoffmann (1987) identifies five layers (A-E) that differ in terms of degree of abstraction, linguistic features, and communication context:

- Layer A (artificial symbols, e.g., formulas) for expert discourse;
- Layer B (artificial symbols for elements, natural language for relations) for technical applications;
- Layer C (technical terms in strict syntax) for applied sciences;
- Layer D (technical terms in loose syntax) for the production level;
- Layer E (everyday language with technical terms) for lay communication.

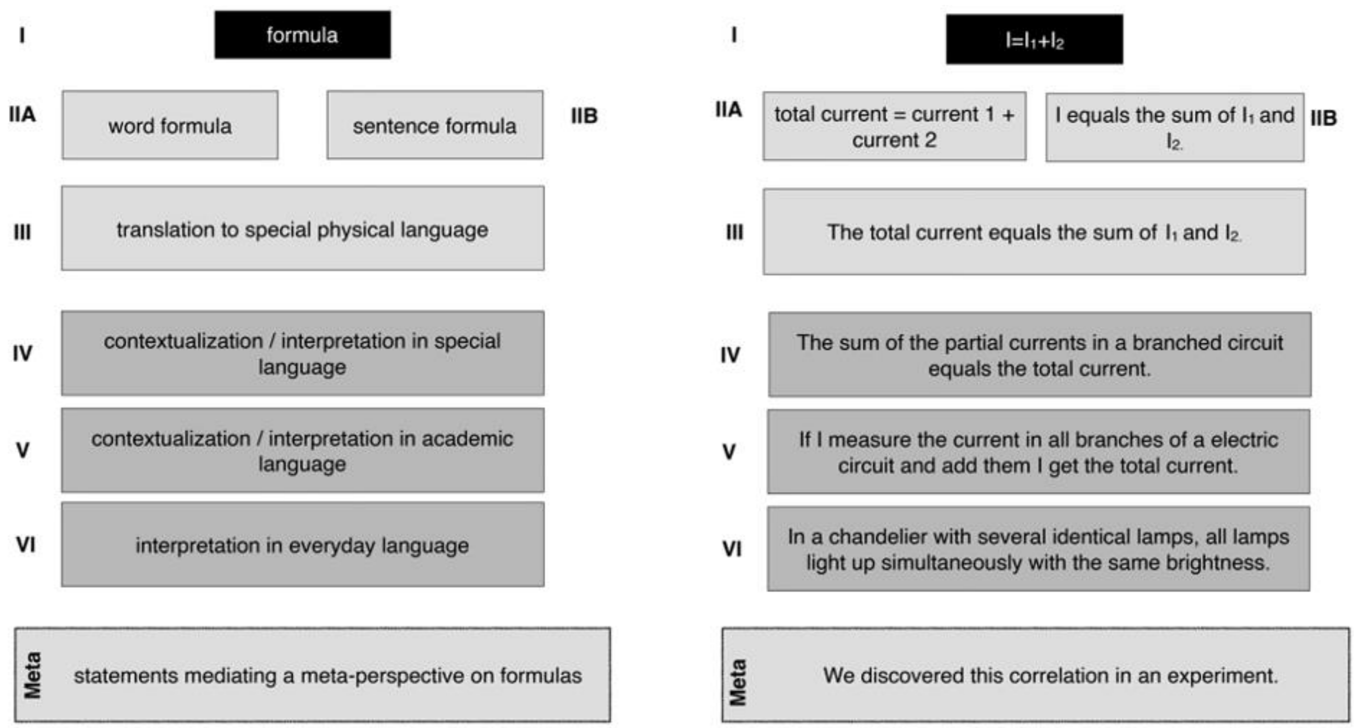

Figure 3. Theoretical levels of verbalizing a formula and application on an example (translated from Kuske-Janßen, 2020)

Description of the Level Model

These foundations have been adopted to develop a level model suitable in the context of physics teaching. Here, the final model is described. The model distinguishes 6 different levels. Level I to level V differ in their degree of abstraction with the highest abstraction in level I (the formula).

- Reformulation of information from the formula (three levels):
  - Level I (symbolic formula) corresponds to layer A from Hoffmann's (1987) theory described before. It shows the pure formula (e.g., $F = m \cdot a$) with maximum semantic density. As Kalverkämper (1998) emphasizes, formula symbols function as "instructions to the recipient to apply their prior knowledge to the text comprehension process" (p. 15).
  - Level II (formula sentence with formula symbols) reflects layer B and is differentiated into IIA and IIB based on textbook analyses. Level IIA (word formula) translates formula symbols into technical terms but retains mathematical symbols (force = mass · acceleration). This representation exists primarily in written form. Level IIB (formula sentence) verbalizes mathematical operators but uses formula symbols ("F is m times a").
  - Level III (technical translation) corresponds to layer C and combines both translations into complete special language ("Force is the product of mass and acceleration").

These three levels are equivalent to each other in carrying the same information (**Figure 3**). This changes in the following levels.

- Contextualization of the formula (three levels)
  - Level IV (special language application/interpretation) uses special language for quantitative or semi-quantitative statements: proportionalities ("The greater F, the greater a"), derivations, experimental explanations, or symbolic forms in Sherin's (2001) sense.
  - Level V (contextualization in educational language) operates with simplified syntax and reduced terminology (e.g., "current flow" instead of "current strength"). This includes qualitative descriptions, model links (particle model in thermodynamics), or naming dependencies without mathematical precision.
  - Level VI (everyday language interpretation) transfers connections into analogies from everyday life, which are always incomplete but create affective access ("persuasion to go for a walk like acceleration of small masses").

These levels which describe the formula from different abstractions are supplemented by statements on the epistemological status of the formula and, hence, presenting meta-statements.

- Meta level: This level addresses formulas from a meta-perspective. It includes classifications (e.g., "definition equation"), conditions of validity, functions ("formulas summarize experiments"), or comparisons with other representations. This level promotes meta-representational competence in the sense of diSessa (2004).

The separation in level II reflects the dual nature of formulas: they contain both physical technical terms (represented by formula symbols) and mathematical relations (represented by operators). The separate consideration of both aspects allows for a more precise description of verbalization strategies.

The levels differ fundamentally in three dimensions:

1. Language register: From formal special language (level I-level IV) to simplifications used in teaching (V) to everyday language (VI).
2. Semantic completeness: Level I-level III are equivalent in terms of information; level IV-level VI focus on specific aspects or expand the context.
3. Cognitive demands: The levels require different transfer skills (interpretation, application, contextualization).

It is critical to recognize that verbalizations can never capture the complete meaning of a formula—as Muckenfuß (1995, p. 259) emphasizes, a formula "cannot be replaced by language any more completely than a work of art". This reflects the hermeneutic dilemma of physics: On the one hand, technical understanding requires interpretative approaches, but on the other hand, their subjectivity contradicts the ideal of empirical clarity. Heywood and Parker (2010, p. 103) highlight this tension: "Hermeneutics allows for multiple interpretations [...] this rests uneasily on correspondence theories of truth in the natural sciences".

For teaching practice, the model offers less normative guidelines and more of an analytical framework: it reveals the diversity of possible bridges between symbolic precision and student-oriented comprehensibility. Teachers can thus reflect on which levels they prioritize in their teaching–whether, for example, they use mathematical derivations (level IV) or analogies from everyday life (level VI) more intensely. The empirical analysis will show how these decisions structure the process of formula comprehension.

RESEARCH DESIGN

This study examines how teachers convey formulas linguistically in physics lessons. From the described theoretical background the research desideratum is derived to analyze how teachers deal with formulas linguistically in class and what content and methodological priorities they set in doing so. The central goal is to get an appropriate insight and develop an instrument to describe the link between formulas and verbal language in physics classrooms and thereby get a grasp on possible impacts on teaching students a deeper content understanding of formulas. The central research question is "In what form and how often do teachers verbalize formulas in physics lessons?"

This main question is specified by three sub-questions:

1. What levels of verbalization do teachers use?
2. How often are formulas used or interpreted linguistically (level IV to level VI) compared to simply "speaking" formulas (level I to level III)?
3. In what form are formulas used, interpreted, and contextualized linguistically by teachers?

Research Design

To answer these questions, a qualitative research design with contrastive sampling was chosen. The verbalization model that was used as a descriptive tool in this state of the research was previously validated by analysis of textbooks. Additionally, it was complemented and adjusted using both deductively and inductively found categories during the process of the qualitative content analysis. The study examined physics lessons in 8th grade at Saxon high schools and middle schools[1] , focusing on electricity, specifically on the treatment of electrical resistance. The focus was on grade 8, as the Saxony curriculum provides for a significant increase in the mathematization of physics content at this level. Furthermore, the limitation in content was made because the abstract nature of electricity as a subject requires special linguistic teaching strategies and teachers need to pay particular attention to students' learning difficulties in this area. The relevant formulas are $R = U/I$ (defining equation) and $R = \rho \cdot l/A$ (resistance law), supplemented by Ohm's law $I \sim U$ under certain conditions (special devices and constant temperature).

The research methods included:

- Classroom observations: 74 lessons with 10 teachers were observed and audio-recorded, with 5-10 hours per teacher on the topic of electrical resistance. This resulted in a total of 79 hours of recordings.
- Data collection: The teachers' language was recorded using lapel microphones; nonverbal actions, blackboard notes, and lesson progress were documented synchronously with smart pens. In addition, guided interviews with the teachers and short questionnaires on their personal characteristics (age, professional experience, subject combination) were collected.

[1] In Germany students are mostly separated in grade 5 by their academic performance. At "Gymnasium"(translated by high school, students are prepared for a higher education at university whereas at "Mittelschule"(translated by middle school) students are mostly prepared for a vocational training.

- Data preparation: Relevant passages (1964 minutes = 32.7 hours) were transcribed, excluding phases without formula reference (e.g., student experiments). The transcripts followed a simplified transcription system.

Participants

A total of 10 physics teachers (4 male high school-teachers, 6 middle school-teachers of which 4 were female, 2 male teachers) in Saxonian schools were observed over a period of five to ten lessons per teacher. The sample was deliberately designed to be heterogeneous in order to reflect a broad spectrum of teaching practices. It included teachers of different age (29-61 years), gender, professional experience (2-34 years), and school-specific expertise. 9 teachers taught mathematics as a second subject, one teacher taught a non-science subject.

Comparing the two types of schools made it possible to analyze differences in how different levels of mathematization are handled: While both formulas are mandatory at high school the relationship is taught semi-quantitatively (e.g., through "the more...the more" statements) at middle school.

Evaluation Methodology

The teachers were assigned arbitrary names during analysis. These names have nothing in common with their real names.

The evaluation was carried out using qualitative content analysis according to Kuckartz (2016). The transcripts were coded using MAXQDA 11 software. Coding units were units of meaning; double coding was permitted. During analysis deductive categories (based on the level model) and inductive categories (developed from the material) were used. The inductively derived final category system comprised three main categories with relation to the level model:

1. Formula is spoken (level I-level III: pure reproduction).
2. Formula is applied or interpreted (level IV-level VI: mathematical-physical or contextual interpretation).
3. Formula is discussed (meta-level: e.g., didactic explanations).

Subcategories (e.g., "proportionality statements" under "application/interpretation") were derived inductively from the data material. This coding-process both complemented and validated the level-model and showed a saturation of the categories. Saturation was indicated by a massive decreasing number of new categories during the coding-process and the inter-coding-process. The system of categories was developed coding one quarter of the data. After this only small adjustments of the system of categories were necessary.

To ensure quality criteria, intercoder and intra-coder reliability were tested. 10% of the data was inter-coded by an intercoder (agreement: 66%) and 10% was reevaluated by the principal investigator after 10 months (agreement: 85%). Discussions of borderline cases and peer validation through discussion within the research team ensured interpretive consistency. A further consistency check was made by comparison with the theoretical framework and preliminary studies.

Documentation of the research process, public accessibility of the survey instruments (guidelines, transcription rules) is provided in Kuske-Janßen (2020).

RESULTS: FORMULA USE IN PHYSICS LESSONS: WRITTEN AND ORAL PRACTICES

The focus was on three central formulas:

- $R = U/I$ (definition of resistance)
- $I \sim U$ (Ohm's law)
- $R = \rho \cdot l/A$ (resistance law)

In the qualitative data analysis, a category system was developed, which was then structured using the level model. 3 main categories were divided into the different levels of verbalization:

- Writing and speaking the formula
  - Level I: Purely writing down the formula (e.g., $R = U/I$).
  - Level IIA: Word formulas (e.g., "resistance = voltage divided by current").
  - Level IIB: Formula sentence (e.g., "R is U divided by I")
  - Level III: Technical translation (e.g., "electrical resistance is the quotient of voltage and current").
- Applying and interpreting the formula
  - Level IV: Application and mathematical interpretation (= special physical language) (e.g., "the greater U, the greater R at constant I").
  - Level V: Contextualization an application in educational language (e.g., "the less current I have the higher resistance must be there ")
  - Level VI: Contextualization and analogy formation (e.g., "resistance acts like a bottleneck in a water pipe").

In textbook analyses (three master's theses) and a teacher training course (25 participants), the model proved suitable for describing linguistic patterns. However, there were some ambiguities between level IV and level V, which were clarified in the main study through inductive category formation. The textbook analyses also revealed that verbalizations rarely covered all levels and that mathematical implications (level IV) often dominated over every day analogies (level VI).

Analysis of "Writing and Speaking the Formula" (= Level I-Level III)

The results reveal clear differences in the frequency and type of use with which formulas are written and spoken. While some teachers use formulas extremely sparingly–both in writing and orally–others integrate them intensively into their lessons. For instance, the data from Mr. Weber (6 written, 15 spoken formulas) and Ms. Gerber (15 written, 19 spoken) exemplify a minimalist approach. In contrast, Ms. Müller (18 written, 68 spoken) and Mr. Meyer (13 written, 45 spoken) strongly favor spoken formulas.

It is noteworthy that the number of written formulas remains comparable for all teachers observed: the average is 15, the minimum is 6, and the maximum is 26. Mr. Lenz is the only exception, as he writes more formulas (26) than he speaks (20). This suggests that he sometimes only presented formulas visually (e.g., on the blackboard or on worksheets) but did not explain them verbally.

Interestingly, both, the teacher with the highest frequency of formulas (Ms. Müller) and the teacher with the lowest frequency (Mr. Weber) teach at middle schools, even though the middle school curriculum does not explicitly require the law of resistance to be taught. Nevertheless, in this sample there is no systematic difference between the types of schools, which relativizes the assumption that formulas are given greater weight at high schools per se.

Results on use of level I

The category "formula", corresponding to level I of the level model, includes formulas written on the blackboard or on worksheets. It appears primarily during the introduction of new concepts, during calculations on the blackboard.. In addition to the basic form, there are rearranged variants (I-rearranged), units in equations (I-unit), and the special case U/I = const. Basic forms (I-quantity) clearly dominate, while the rearranged variants (I-rearranged), units in equations (I-unit), and special cases such as $U \sim I$ occur rarely.

The definition formula $R = U/I$ is noted most frequently, which is due to its central role in calculations related to the resistance or Ohm's law. Variants from the basic notation are rare: reversal ($U/I = R$) occurred only once, three times the partial notation (U/I), and eleven times the use of indices in specific calculations $R_{ges} = I_{ges} \cdot U_{ges}$. Among the rearranged forms $U = R \cdot I$ and $I = U / R$ are the most common.

Equivalence of units is noted with similar frequency as definitions (1 VA = 1Ω) or descriptions (1Ω = 1 VA), the latter sometimes in a calculation context. The expression U/I = const. is used in preparation for or discussion of the validity of Ohm's law.

The law of resistance (in a long wire) appears in three notations:

$R = \rho \cdot l/A$ (78%, 21 of 27 codes), $R = (\rho \cdot l)/A$ (18.5%), and $R = \rho \cdot (l/A)$ (3.7%).

Rearranged formulas for the calculation of the length vary, with $l = (R \cdot A)/\rho$ being the most common variant. Formulas rearranged according to ρ and A are less common. Ohm's law is noted as $I \sim U$ (7 times) or $U \sim I$ (4 times), without further subcategories.

A comparison of teachers shows that the number of formulas noted is similar despite different curriculum requirements for high schools and middle schools. Both types of schools have teachers with very few (Mr. Schmitt: 8, high school; Mr. Weber: 6, middle school) and a great amount of formulas written down (Mr. Lenz: 25, high school; Ms. Müller: 16, middle school). Since this is not a representative sample, no general conclusions can be drawn, but the trend indicates a similar importance of formulas in both types of schools.

Results on use of level II

Level IIA: The category "word formula" includes formulas with written-out terms instead of formula symbols, mainly in the introduction. Examples include the definition formula for resistance such as: "Electrical resistance = voltage at the component/current in the component" and the resistance law, but not Ohm's law. The notations vary slightly in the terms used. With only 7 codes, this category is very rare and is only used by 5 of the 10 teachers, making it less common than other categories. It occurs exclusively in the introduction of formulas. Their marginal prevalence underscores the fact that this form of presentation plays a subordinate role in everyday teaching.

Level IIB: The category "Formula sentence with formula symbols" covers verbal statements in which at least one physical quantity is spoken as a formula symbol. With 188 coding, this is the most common category.

The definition formula $R = U/I$ dominates with 137 of 188 codes (73%), mostly in its basic form (e.g., "R is equal to U divided by I"). Basic forms (IIB-quantity) are used most frequently, rearranged forms (IIB-rearranged) and units (IIB-unit) less frequently. There are various formulations for $R = U/I$, ranging from complete equations ("R equals U divided by I") to partial expressions ("U divided by I"). The variants "U by I" and "R is equal to U by I" are the most common, each accounting for around 28%. Specific values in calculations ("U-one by I-one") and rearranged formulas largely correspond to the written notations.

In the case of "R is rho times l divided by A" is used almost exclusively, with terms used in teaching.

Rearranged formulas reflect written structures and are used consistently by individual teachers.

Ohm's law is formulated as "I proportional to U" (23 times) or "U proportional to I," (15 times) with the former variant predominating. Two special cases express the proportionality sign iconically as a "wavy line." All teachers use this category, with significant differences in frequency (Mr. Meyer: 39 times, Mr. Weber: 2 times).

Use of level III

The category "formula sentence with terms" (level III) includes verbalizations with written-out terms. With 126 coding, it is less common than level IIB (186) but shows interesting patterns. Its use is related to the length of the formula: the short Ohm's law is spoken more frequently with terms (III: 38 vs. IIB: 17), the definition formula less frequently (III: 43 vs. IIB: 137), and the resistance law almost never (III: 4 vs. IIB: 34).

Exclusively basic forms of the formulas occur in this category (level III); rearranged formulas are always spoken with formula symbols (level IIB).

For $R = U/I$, two basic patterns dominate (together 51%): complete sentences such as "resistance is voltage divided by current" (24 codes) and partial expressions such as "voltage divided by current" (19 codes). Other formulations use verbs ("divide") or nouns ("ratio," "quotient"). The subcategory III-unit describes the unit equivalence (1 Ω = 1 V/A), often when introducing the unit ohm, sometimes with historical or technical notes or in calculations.

There are only four codes for Ohm's law, mostly as instructions or to explain the formula symbols. Ohm's law is expressed as proportionality with three variants: "quantity A is proportional to B" (42%), "A and B are proportional to each other" and "proportionality between A and B".

Direct proportionality is specified in 45% of cases. The direction $I \sim U$ predominates (23 vs. 15 for U ~ I) fitting to the physical semantic of the proportionality because the electric current is being influenced by voltage and not the other way round.

Most teachers prefer level IIB, with the exceptions of Ms. Müller (IIB: 29, III: 39), Mr. Lenz (9 vs. 11), and Mr. Weber (2 vs. 13). On average, teachers use formula symbols 18 times and terms 13 times when speaking.

Teacher-specific preferences

The individual differences are significant: while Mr. Meyer strongly emphasizes formulas verbally (39 codes in IIB vs. 6 in III), Ms. Müller focuses on terminological detail (29 in IIB vs. 39 in III). Mr. Weber hardly uses either category (2 or 13 codes, respectively), which is consistent with his minimalist overall approach. On average, teachers slightly prefer to speak formula symbols (18 times) over terms (13 times). The hypothesis that formulas are used more intensively in high schools cannot be confirmed: both minimum and maximum values occur in both types of schools.

The evaluation showed that mathematical operations such as calculations and the discussion of proportions dominate, while conceptual interpretations such as models or analogies are less frequently explicitly linked to the formulas.

Analysis of "Applying and Interpreting the Formula" (= Level IV-Level VI)

This main category describes how teachers go beyond just writing or speaking the formula, but how they work with them. In this category several subcategories could be identified and are described below.

Calculations as a core element

Calculations are the most common activity, with about one-third of all coded teaching situations falling into this category. Within this category, the subcategory of „calculations without context" dominates clearly: here, formulas are applied in isolation, often as purely mathematical exercises without any connection to real-world problems. For example, students calculate resistance values from given voltage and current data without reflecting on the physical significance of the results. Pseudo-contexts–apparent application scenarios with no real-life relevance–reinforce this trend. In contrast to this are application tasks that are used every day or technical references, such as locating a cable break using the law of resistance. However, such tasks, the subcategory „application tasks", make up only a fraction of the total and are rarely used to illustrate the meaning of a formula.

It is striking that another subcategory „calculations with experimental values" occur almost exclusively for $R = U/I$, for example when measurement data from experiments is used to determine resistance. In the case of Ohm's law, however, such links are missing despite the experimental development of the underlying proportionalities.

The subcategory „rearrangement of formulas" is explored in varying degrees of depth: some teachers treat it superficially ("looking closely" at the equation), while others work through it systematically, including the derivation of "the more ... the more ..." statements. Proportionalities play a particularly important role in the resistance law, where they are used to explore ratios (e.g., "tripling the length leads to triple the resistance"). However, the connection to $R = \rho \cdot l/A$ often remains implicit, as the proportionalities are usually derived from measured values without explicitly taking the step towards mathematical generalization.

Derivation and conceptual approaches

Derivations of formulas are rare overall and differ depending on the type of formula. Ohm's law and the resistance law are usually developed inductively from experiments by a series of measurements of voltage and current leading to the proportionality: $I \sim U$ for Ohm's law, while experiments on the dependence of resistance on conductor length and cross-section lead to the relationships $R \sim l$ and $R \sim l/A$. The synthesis to the resistance law is then often achieved by multiplying the proportionalities, whereby the specific resistance ρ is introduced as a material constant.

The defining formula $R = U/I$ on the other hand, often lacks a true derivation; it is mostly just given, usually derived as a quotient from Ohm's law or directly specified by definition. These differences are also reflected in the epistemological classification: While $I \sim U$ and $R = \rho \cdot l/A$ are presented as empirical laws, $R = U/I$ often appears to be a pure calculation rule.

Measuring principle

The measuring principle—the use of formulas to indirectly determine quantities—is primarily addressed for $R = U/I$. Teachers emphasize that resistance cannot be measured directly but must be calculated using voltage and current. Multimeters are often described as "black boxes" that use the same calculation method internally.

Description of unit

The description of the unit Ohm as *volt/ampere* is occasionally used to interpret content: An Ohm is interpreted as "voltage per current," with the fraction line read as "per" (e.g., "33 volts are required per one ampere of current"). Such statements link mathematical structures with physical meaning, but remain marginal, accounting for only 2% of the coding.

Relationships between quantities and application relevance

The category of relationships between quantities is particularly relevant for the resistance law (56% of coding). Here, qualitative and semi-quantitative statements are used to illustrate the proportionalities. Simple directional statements ("larger cross-sectional area → lower resistance") predominate, while precise formulations under constant conditions ("with the same material") are rare. "The more ... the more ..." statements are often derived from measurement data, for example when deriving the resistance law. Teachers often note proportionalities symbolically ($R \sim l$), less often verbally with technical terms ("The resistance is proportional to the length", level V). Borderline cases such as $R \rightarrow 0$ (short circuit) or $R \rightarrow \infty$ (insulator) are only touched upon, although they would offer starting points for conceptual deepening corresponding to the exploration-pattern described by Lehavi et al. (2019). Here, statements in both technical and educational language appear. It is striking that statements in special language predominate, and proportionalities often even occur as a formula by themselves.

Application or experiment

Applications or experiments are mentioned but rarely explained. For example, teachers refer to resistors connected in series for protecting sensitive components or to the use of thick cables in high-voltage lines without explaining the role of the formula in detail. In the case of resistance law, the standard task of cable break localization serves as an application, but here too, the reference to the formula remains superficial. Experiments are often only described ("We measure voltage and current") without reflecting on the transition to the application of the formula.

Description of individual variables and conceptual approaches

The description of individual quantities including the correct units is the presupposition of understanding the formula completely. The coding focus heavily on the concept of resistance. Over 70% of the statements interpret resistance as an "impediment to the flow of current," often linked to everyday analogies (traffic jam on a road). Voltage is characterized as a "driving" or "force," and current as a "flow" of charge carriers. The specific resistance ρ, on the other hand, is usually reduced to its unit or as a tabulated material constant without deeper conceptual classification.

Model

Models used for illustration purposes primarily employ the particle model: electrons move through an atomic lattice, with resistance being interpreted as "friction" or "obstacles." However, these approaches often remain simplified and do not systematically link microscopic concepts with macroscopic formulas. Analogies (e.g., water cycle, door with gravitational pull) serve to illustrate concepts but are unevenly distributed: some teachers use them extensively, others hardly at all.

Speaking About Formulas in Physics Lessons (= Meta-Level)

This main category describes explanations of formulas at a meta-cognitive level. These meta-statements account for 15% of all coded statements and can be divided into eight subcategories covering different facets of formula reflection. The epistemological classification of formulas dominates the meta-level: 60% of all coding fall into this category.

Importance or advantages of the formula

When it comes to the importance or advantages of formulas (17% of the codes), pragmatic aspects are at the forefront: 27% of the statements emphasize the need to memorize them ("This is a formula you really need to know"), while 29% refer to their relevance for exams ("Always write down the basic formula first–you get points for that"). Only rarely (6%) is the accuracy of mathematical representations emphasized. Functional advantages such as cognitive relief or the idealization of complex phenomena are mostly not discussed.

Conditions of validity

Conditions of validity, such as temperature constancy or material restrictions of metallic conductors, are particularly discussed in relation to Ohm's law (9 out of 10 teachers). For the definition formula $R = U/I$, such restrictions are almost completely absent–an indication of their universal character, which, however, is rarely explicitly mentioned. When discussing resistance law, teachers refer to the temperature dependence of the specific resistance ρ but rarely connect this with fundamental principles of physical modeling.

Scope of application of the formula

The scope of application formulas is discussed primarily in relation to the resistance law (7 out of 10 teachers). Here, teachers contrast experimental and theoretical applications: "Experimentally, one would tend to use $R = U/I$, while in theoretical considerations, the law of resistance would be used." Such comparisons often arise from student questions about the ambiguity of formula symbols–for example, when ρ denotes both density and specific resistance.

Epistemological classification of the formula

Teachers classify formulas here as calculation aids, empirical laws, or definitions. There are striking differences between the formulas: While $R = U / I$ is predominantly referred to as a calculation formula (10 out of 10 teachers) and a defining equation (7 out of 10), teachers consistently classify Ohm's law as an empirical law (7 out of 10), but never as a calculation tool. The resistance law occupies a middle position–it is described both as a calculation formula (8 out of 10) and as a law (7 out of 10). This differentiation implicitly reflects the epistemological nature of the formulas: definition formulas are considered universal, while laws are linked to experimental conditions. Embedding in higher-level theories (e.g., electrodynamics), on the other hand, remains underrepresented.

Comparison of experiment and theory

Comparisons between experiments and theory (12% of codes) are primarily used for error analysis: teachers highlight discrepancies between calculated and measured values ("Damn! Why are we getting a different voltage here?") and discuss measurement inaccuracies. Only in isolated cases are the process of scientific knowledge acquisition addressed, for example when the experimental determination of tabulated resistance values is described.

Analogies to different formulas

Four teachers use analogies to other formulas exclusively for the definition formula $R = U/I$. The comparison with the velocity formula $v = s/t$ is often used to illustrate the concept of ratio formation: "As we did when defining time–distance as velocity, we do the same here with current–voltage as resistance." These analogies aim at understanding the content but are rarely developed systematically.

Mnemonics

Mnemonics (e.g., memory aids for rearranging formulas) and linguistic meta-reflections (21% of the coding) focus on the definition formula. Teachers explain the origin of terms ("resistance with 'i' because something is resisted") or foreign language references ("*R* as in English resistance"). However, such aids often remain superficial and rarely link linguistic aspects with physical concepts.

Meta-reflection on language

In some cases teachers reflect about terms or formula signs in a linguistic way by discussing the orthography of a term or the origin of a formula symbol. An example for the second case is the connection of the symbol *R* with the English word "Resistance".

Implications for teaching

The study reveals a discrepancy between implicit and explicit formula knowledge: although teachers apply differences between formula types in practice (e.g., by treating validity conditions differently), they hardly reflect on their epistemological status. Functions of formulas—such as idealization, cognitive relief, or precision—are only addressed in 6% of meta-statements. Instead, pragmatic aspects such as exam relevance or memorability dominate.

The variability between teachers is also striking: while some operate comprehensively at the meta-level (up to 41 codes for epistemological classification), others limit themselves to minimal statements. This range indicates untapped potential: Systematic comparisons between formulas, explicit discussions of model limitations, or links to scientific historical contexts could deepen the understanding of mathematical representations. The study thus underscores the need for professional development concepts that support teachers in making the multidimensionality of formulas–as tools, media of knowledge, and language systems–visible in the classroom.

ANALYSIS OF THE LINGUISTIC LEVEL IN THE TEACHING OF MATHEMATICAL FORMULAS: CATEGORIZATION AND CHARACTERIZATION OF TEACHER LANGUAGE

This study on the verbalization of mathematical formulas in a teaching context is based on a differentiated level model, the theoretical foundation of which was presented before. This theoretical model was validated using the categories presented later. The levels of verbalization could be differentiated content wise according to the categories. Furthermore, the model is intended to structure the identified categories and thus gain a deeper understanding of how teachers linguistically handle formulas. The level model structures the linguistic handling of formulas into six operational levels, supplemented by a meta-level for reflection on formulas as a means of representation. The first three levels focus on the immediate verbalization of formulas directly translating the information contained in the formula into verbal language. Level IV through level VI describe the contextualized application and interpretation of formulas and are distinguished according to three linguistically distinct varieties, which are characterized by varying degrees of mathematization and technicality as explained before. Finally, the meta-level addresses overarching aspects of formula use, including conditions of validity, epistemological functions, or didactic evaluations, as in the reflection "This formula only applies to ohmic conductors at a constant temperature."

Categories and Levels

The systematic assignment of the analyzed teacher statements in the categories described before to these levels was based on two central criteria that enabled precise categorization. The first determining factor was the degree of quantification, with completely quantitative statements with mathematical precision being assigned to level IV, while qualitative descriptions without numerical references were assigned to level VI. The second criterion concerned linguistic variety, with special language being strictly identified by the exclusive use of defined terms. Educational language was diagnosed when technical terms were replaced by simplified contextual equivalents, while everyday language was characterized by the use of everyday concepts and the absence of precise terms. The borderline cases of this categorization are particularly instructive when looking at the example of unit treatment: technical precision in statements such as "One ohm corresponds to one volt per ampere" led to

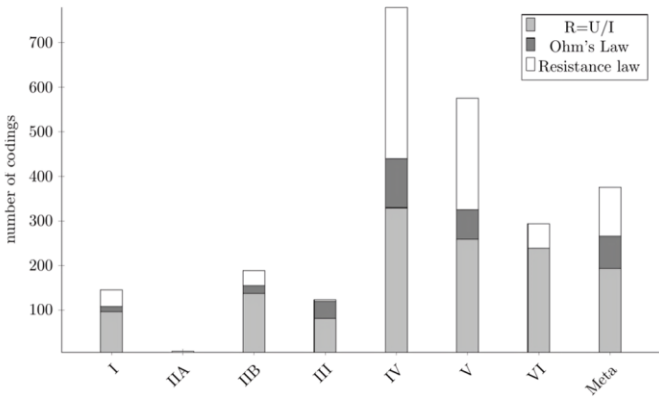

Figure 4. Number of coding by levels of verbalization (translated from Kuske-Janßen, 2020)

classification in level IV, while teaching formulas, e.g., in "U in V" were assigned to level V due to their didactic reduction. Similar distinctions were made in the category "degree of change," where formulations referring to proportional change using technical vocabulary ("to the same degree") were assigned to level IV, while more unspecific descriptions ("roughly equal") were assigned to level V.

The quantitative evaluation of a total of 600 coded text passages revealed a significant dominance of special language: Level IV comprised 48% of all coding and proved to be a defining pattern in all formulas examined. Educational language (level V) was significantly less common at 32%, while everyday language (level VI) played a marginal role in the teaching discourse at only 7%. Remarkably, the definition of resistance $R = U/I$ was an exception, as everyday language analogies were used here to form concepts, for example in descriptions of resistance as "impedance to electric current". The meta-level (9%) focused primarily on application limits and representation functions, with particular emphasis on the reflection of validity conditions and the discussion of model limits. These distribution patterns illustrate that teachers primarily address formulas through mathematizing and special language approaches, while real-life references and explicit language reflection remain significantly underrepresented (**Figure 4**).

Description of Teacher Types

The level model allows also for characterizing the preferences of teachers while introducing or using formula in the classroom. The detailed analysis of individual language use by teachers revealed considerable interpersonal differences, from which four prototypical teaching profiles can be derived.

The group of teachers oriented towards special language, represented by Ms. Berger and Mr. Lenz, concentrated over 60% of their statements on level IV and made almost no use of everyday language (<5% on level VI). At the same time, this group was characterized by below-average use of the meta-level, which indicates a focused concentration on mathematical applications while neglecting reflective elements.

In contrast, teachers such as Mr. Schmitt and Ms. Gerber communicated in a student-oriented manner, consciously using educational and everyday language to reduce cognitive barriers to access. This approach was particularly evident in Ms. Gerber, whose teaching in a middle school class concentrated 55% of her utterances at level V, which can be interpreted as a deliberate adaptation to the cognitive level of the learning group.

A third group of teachers–represented by Mr. Jasper and Ms. Carle–acted through a balanced distribution of language varieties, with no level accounting for more than 35% of the coding. Another characteristic of these teachers was their intensive use of the meta-level, which was expressed in frequent reflections on the function of formulas as a means of representation.

The fourth type consisted of formula-centered teachers such as Mr. Funke, who spoke about formulas more frequently than average (level I-level III >40%) and combined them with special language, while contextualizing approaches in everyday or educational language were used much less frequently.

School type-specific trends underscore these findings in a remarkable way: while high school-teachers consistently prioritized special language and aimed for higher levels of mathematization, middle school teachers systematically adapted their language to the cognitive level of their learning groups, in particular by reducing levels of abstraction and making greater use of simplifications in educational language. Overall, everyday language proved to be significantly underrepresented; only three out of the ten teachers used it regularly while speaking with or about formulas (proportion level VI > 10%), although interestingly, no significant correlation with school type was found. The range of variation in meta-reflection was particularly significant–while some teachers intensively addressed the function of formulas as a means of representation and also explicitly reflected on linguistic aspects, this dimension was ignored by others. This range of didactic approaches highlights untapped potential for teaching practice: especially for learners with little prior knowledge of the subject, systematic bridges between everyday language analogies (level VI) and special language (level IV) could significantly promote cognitive understanding of mathematical formulas. Future research should therefore investigate the extent to which targeted language sensitization measures for teachers can enhance the use of this didactic potential, in particular through conscious code switching between the identified levels.

SUMMARY AND CONCLUSIONS

Most studies on understanding formula in physics refer to college or university level (e.g., Palmgren et al., 2025). Often, the focus lies on the students' problem solving or sense making of mathematics in physics (Gifford & Finkelstein, 2021). However, little focus is on the teaching of the interplay of mathematics and physics, especially in lower secondary school. Here, language plays an important role for teaching physics as enabler of knowledge construction (Wulff, 2024). In this context, this study examines on the basis of a so-called "level model" how teachers speak with and about formulas in physics lessons, focusing on linguistic varieties (special language, educational language, everyday language) and their classification according to a level model of formula verbalization. As an example case, three central formulas from the field of electricity were analyzed: the definition formula $R = U/I$, Ohm's law ($I \sim U$), and resistance law ($R = \rho \cdot l/A$).

The research design allowed for a detailed reconstruction of the linguistic use of formulas in physics lessons. The contrastive sample and multi-method data collection allowed both to analyze the wide range of formula verbalization and to identify typical patterns and school-type-specific differences.

Overall, the research design enabled a detailed reconstruction of how teachers convey formulas linguistically–from symbolic representation to contextual embedding–and which implicit or explicit levels of understanding are prioritized in the process. The analysis of teaching sequences on the topic of electrical resistance shows a heterogeneous picture of how teachers use and teach formulas in physics lessons and provides a complex picture of formula usage: written notations remain quantitatively stable, while oral practices vary greatly.

The dominance of the definition formula $R = U/I$ underscores its function as the "workhorse" of teaching. Overall, the analysis shows a clear focus on mathematical aspects: calculations, proportionalities, and rearranging formulas dominate the lessons. Conceptual interpretations through models, analogies, or embedding in application contexts take a back seat and are rarely explicitly linked to the formulas. In addition, the use of formulas varies greatly between teachers and school types: while middle schools work more frequently with proportionalities, high schools use formal derivations more often. This heterogeneity underscores the need to strengthen conceptual depth and application references in physics teaching in order to promote a balanced understanding of formulas as tools for both calculating and describing physical phenomena. The rare use of word formulas and terminological explanations indicates a preference for concise, symbol-based communication.

The results are based on a detailed categorization of lesson transcripts and show that teachers emphasize different aspects depending on the formula. For example, the definition formula is often classified as a calculation formula and definition, while the resistance law and Ohm's law are more often referred to as empirical laws. The nature and frequency with which validity conditions or areas of application are discussed also varies between the formulas.

It is found that teachers primarily use and interpret formulas in special language (level IV), while everyday language (level VI) is used only marginally. This dominance of special language is reflected in quantitative analyses: Level IV comprises by far the most coding (e.g., in derivations, calculations, or the description of mathematical relationships), while level VI occurs only in isolated cases, such as analogies or everyday explanations. The educational language (level V) often acts as a bridge, especially in the formation of concepts or simplified representations, but shows no systematic connection to physical concepts. This confirms the results of Apolin (2002) and the everlasting need of sensitizing teacher for the difficulties of different varieties of language used in classroom as required by Leisen (2005a).

There are noticeable differences between different types of teachers: some use expressions that are heavily influenced by special language (e.g., in mathematical derivations), while others rely more on varieties that are familiar to students (educational or everyday language), especially in middle school contexts. These differences correlate with the type of school, but also with individual preferences. In addition, the use of formulas varies in terms of content: the definition formula $R = U/I$ is often introduced as a measurement principle or calculation tool, with a focus on units and mnemonics. The resistance law, on the other hand, is often derived empirically and contextualized as a physical law, while Ohm's law is more often spoken than written due to its brevity. Meta-statements about formulas (e.g., epistemological classifications or areas of validity) usually remain implicit and are rarely the subject of explicit reflection.

The differentiation of technical and structural approaches of mathematics in physics (Pietrocola, 2008) could be shown and adapted to the speaking about formulas. The predominantly technical approach to mathematics, that was criticized by the results of Uhden (2012) and Bing and Redish (2009), was observed: calculations dominate teaching, but are rarely linked to physical contexts. So, the warnings of Wagenschein (1962) seem to remain valid up to this day. Answer sentences are often missing, unit checks are carried out without reflection on content, and proportionalities (e.g., $I \sim U$ vs. $U \sim I$) are treated as mathematically equivalent which might be physically inconsistent. This suggests a neglect of the structural role of mathematics, in which formulas should be interpreted as carriers of physical meaning. Although many aspects of formula comprehension are addressed (e.g., basic mathematical skills or descriptions of quantities), they are rarely explored in depth. The teaching patterns identified by Lehavi et al. (2019) (exploration, construction, broadening, application) can be recognized to some extent but are not used systematically to promote a deeper understanding.

The discrepancy between curriculum requirements (higher formula frequency at high schools) and observed practice is noteworthy: The similarity of the patterns across school types suggests that individual teaching styles carry more weight than institutional conditions. This tendency–despite the limitations of the non-representative sample–could indicate an implicit equivalence of the formulas in everyday teaching in both examined school types.

OUTLOOK

The study provides the first systematic description of teachers' use of language when dealing with formulas and identifies deficits in the teaching of structural mathematical-physical connections. Several perspectives arise for future research:

1. Deepening language analysis: The impact of the identified linguistic varieties on student understanding should be investigated. Experimental studies could examine whether the targeted use of everyday language or meta-discourse (e.g., on the epistemological role of formulas) improves conceptual understanding.
2. Development of teaching concepts: Based on the teacher types, adaptive materials should be designed to help teachers emphasize structural aspects—for example, by explicitly linking formulas to physical causalities (e.g., "Why is I~U more meaningful than U~I?") or by reflecting on units as a bridge between mathematics and physics.
3. Expansion of the formula spectrum: The analysis is limited to three formulas used as well in secondary school as also in middle school. Future work should include more complex formulas in high school (e.g., from quantum physics) to test whether the observed patterns can be generalized or whether grade-specific differences exist.
4. Interdisciplinary collaboration: Since many teachers assume mathematical skills as prior knowledge without contextualizing them physically, collaborative projects with mathematics education are useful. These could develop joint learning paths, for example on proportional reasoning or the meaning of the equal sign in physical contexts.
5. Digitally supported solutions: AI tools or simulations could be used to automate technical calculations, thus creating space for content-related discussions—for example, on the

interpretation of answer sentences or critical reflection on areas of validity.

Finally, the study shows that formulas are still often treated as isolated calculation tools in the classroom. A shift in perspective toward structural interpretation–in which mathematics is understood as the language of physics–could not only deepen understanding of formulas but also help sensitize students to the epistemic role of formal representations in the natural sciences. However, this requires further empirical and conceptual foundations, which this work initiates. Hence, the results contribute to deriving implications for teacher training, in particular for promoting understanding-oriented formula teaching.

Author contributions: WHK-J: conceptualization, data curation, formal analysis, investigation, methodology, validation, visualization, writing – original draft, writing – review & editing; **GP:** conceptualization, supervision, writing – review & editing. Both authors agreed with the results and conclusions.

Funding: No funding source is reported for this study.

Ethical statement: Ethical review and approval were not required for the study on human participants in accordance with the local legislation and institutional requirements. The participants provided their written informed consent to participate in this study.

AI statement: The authors declare that generative AI has not been used in preparing this article.

Declaration of interest: No conflict of interest is declared by the authors.

Data sharing statement: Data supporting the findings and conclusions are available upon request from the corresponding author.

REFERENCES

Ainsworth, S. (2008). The educational value of multiple-representations when learning complex scientific concepts. In J. K. Gilbert, M. Reiner, & M. Nakhleh (Eds.), *Visualization: Theory and practice in science education* (pp. 191-208). Springer. https://doi.org/10.1007/978-1-4020-5267-5_9

Apolin, M. (2002). *Die Sprache in Physikschulbüchern unter besonderer Berücksichtigungvon Texten zur speziellen Relativitätstheorie* [The language used in physics textbooks, with particular consideration of texts on special relativity] [PhD thesis, Universität Wien].

Bagno, E., Berger, H., & Eylon, B. (2008). Meeting the challenge of students' understanding of formulae in high-school physics: A learning tool. *Physics Education, 43*(1), 75-82. https://doi.org/10.1088/0031-9120/43/01/008

Bagno, E., Eylon B., & Berger, H. (2011). How to promote the learning of physics from formulae? In *Proceedings of the GIREP-EPEC & PHEC 2009* (pp. 77-83).

Baumann, K.-D. (1998). Das Postulat der Exaktheit für den Fachsprachengebrauch [The postulate of exactitude for the use of technical language]. In L. Hoffmann, H. Kalverkämper, & H. E. Wiegand (Eds.), *Fachsprachen. Languages for Special Purposes. Ein internationales Handbuch zur Fachsprachenforschung und Terminologiewissenschaft* (pp. 373-377). de Gruyter.

Bing, T. J., & Redish, E.J. (2009). Analyzing problem solving using math in physics: Epistemological framing via warrants. *Physical Review Physics Education Research, 5*, Article 020108. https://doi.org/10.1103/PhysRevSTPER.5.020108

Bolte, C., & Pastille, R. (2010). Naturwissenschaften zur Sprache bringen [Bring up natural sciences]. In *Sprache, Mathematik und Naturwissenschaften* (pp. 26-46).

Brown, B., & Ryoo, K. (2008). Teaching science as a language: A content-first approach to science teaching. *Journal of Research in Science Teaching 45*(5), 529-553. https://doi.org/10.1002/tea.20255

Bruner, J. S. (1974). *Entwurf einer Unterrichtstheorie* [Outline of a teaching theory]. Berlin Verlag Arno Spitz.

de Ataíde, A. R. P., & Greca, I. M. (2013). Epistemic views of the relationship between physics and mathematics: Its influence on the approach of undergraduate students to problem solving. *Science and Education, 22*, 1405-1421. https://doi.org/10.1007/s11191-012-9492-2

de Winter, J., & Airey, J. (2022). Pre-service physics teachers' developing views on the role of mathematics in the teaching and learning of physics. *Physics Education, 57*(6), Article 065007. https://doi.org/10.1088/1361-6552/ac8138

Dirac, P. (1977). Annahmen und Voreingenommenheit in der Physik [Assumptions and biases in physics]. In *Naturwissenschaftliche Rundschau 30* (pp. 429-432).

diSessa, A. A. (2004). Metarepresentation: Native competence and targets for instruction. *Cognition and Instruction, 22*(3), 293-331. https://doi.org/10.1207/s1532690xci2203_2

Falkenburg, B. (1999). Sprache und Anschauung in der modernen Physik [Language and perception in modern physics]. In *Sprache und Sprachen in den Wissenschaften. Geschichte und Gegenwart* (pp. 89-118). de Gruyter. https://doi.org/10.1515/9783110801323.89

Faraday, M. (1990a). Faraday to Maxwell, 13 November 1857 cited by Gingras (2001, p. 397). In P. M. Harman (Ed.), *The scientific letters and papers of James Clerk Maxwell*. Cambridge University Press.

Faraday, M. (1990b). Faraday to Maxwell, 25 March 1857 cited by Gingras (2001, p. 404). In P. M. Harman (Ed.), *The scientific letters and papers of James Clerk Maxwell*. Cambridge University Press.

Feynman, R. P. (1997). *Vom Wesen physikalischer Gesetze* [On the nature of physical laws]. Piper.

Friebe, C., Kuhlmann, M., Lyre, H., Näger P. M., Passon, O., & Stöckler, M., (2015). *Philosophie der Quantenphysik: Zentrale Begriffe, Probleme, Positionen* [Philosophy of quantum physics: Central concepts, problems, positions]. Springer. https://doi.org/10.1007/978-3-642-37790-7

Galilei, G. (1896). *Il Saggiatore*. Vol. 6. Quoted by Krey (2012), p. 38. Edition Nazionale.

Galili, I. (2018). Physics and mathematics as interwoven disciplines in science education. *Science & Education, 27*(1-2), 7-37. https://doi.org/10.1007/s11191-018-9958-y

Geyer, M.-A. (2020). *Physikalisch-mathematische Darstellungswechsel funktionaler Zusammenhänge. Das Vorgehen von SchülerInnen der Sekundarstufe 1 und ihre Schwierigkeiten* [Physical-mathematical shifts in the representation of functional relationships: The approach of secondary school students and their difficulties]. Logos.

Geyer, M.-A., & Kuske-Janßen, W. (2019). Mathematical representations in physics lessons. In G. Pospiech, M. Michelini, & B. Eylon (Eds.), *Mathematics in physics education* (pp. 75-102). Springer. https://doi.org/10.1007/978-3-030-04627-9_4

Gifford, J. D., & Finkelstein, N. D. (2021). Applying a mathematical sense-making framework to student work and its potential for curriculum design. *Physical Review Physics Education Research, 17*, Article 010138. https://doi.org/10.1103/PhysRevPhysEducRes.17.010138

Gingras, Y. (2001). What did mathematics do to physics? *History of Science, 39*(4), 383-416. https://doi.org/10.1177/007327530103900401

Greca, I. M., & Moreira, M. A. (2001). Mental, physical, and mathematical models in the teaching and learning of physics. *Science Education, 86*(1), 106-121. https://doi.org/10.1002/sce.10013

Heck, A., & van Buuren, O. (2019). Students' understanding of algebraic concepts. In G. Pospiech, M. Michelini, & B. Eylon (Eds.), *Mathematics in physics education* (pp. 53-74). Springer. https://doi.org/10.1007/978-3-030-04627-9_3

Heywood, D., & Parker, J. (2010). Language interpretation and meaning. In D. Heywood, & J. Parker (Eds.), *The pedagogy of physical science* (pp. 93-112). Springer. https://doi.org/10.1007/978-1-4020-5271-2_5

Hoffmann, L. (1987). *Kommunikationsmittel Fachsprache. Eine Einführung* [Communication tools, specialized terminology. An introduction]. Akademie-V.

Höttecke, D., Ehmke, T., Krieger, C., & Kulik, M. A. (2017). Vergleichende Messung fachsprachlicher Fähigkeiten in den Domänen Physik und Sport [Comparative measurement of subject-specific language skills in the domains of physics and sports]. *Zeitschrift für Didaktik der Naturwissenschaften, 23*, 53-69. https://doi.org/10.1007/s40573-017-0055-6

Kalverkämper, H. (1998). Fach und Fachwissen [Subject and subject knowledge]. In L. Hoffmann (Ed.), *Fachsprachen. Languages for special purposes* (Vol. 1, pp. 1–24). de Gruyter.

Karam, R. A. S., & Krey, O. (2015). Quod erat demonstrandum: Understanding and explaining equations in physics teacher education. *Science & Education, 24*(5-6), 661-698. https://doi.org/10.1007/s11191-015-9743-0

Kimpel, L. (2018). *Aufgaben in der Allgemeinen Chemie: Zum Zusammenspiel von chemischem Verständnis und Rechenfähigkeit* [Tasks in General Chemistry: On the interplay of chemical understanding and numeracy skills]. Logos.

KMK. (2024): Weiterentwickelte Bildungsstandards in den Naturwissenschaften für das Fach Physik (MSA) [Advanced educational standards in the natural sciences for the subject of physics (MSA)]. *Kultusministerkonferenz*. https://www.kmk.org/themen/qualitaetssicherung-in-schulen/bildungsstandards.html

Krey, O. (2012). *Zur Rolle der Mathematik in der Physik: Wissenschaftstheoretische Aspekte und Vorstellungen Physiklernender* [On the role of mathematics in physics: Epistemological aspects and the conceptions of physics students]. Logos.

Krey, O., & Karam, R. A. S. (2016). Mathematik in der Physik –Muss das sein? Und wenn ja, warum und wozu [Mathematics in physics–Is it necessary? And if so, why and what for]? In G. Pospiech, & R. A. S. Karam (Eds.), *Naturwissenschaften im Unterricht Physik 153/154* (pp. 7-13).

Kuckartz, U. (2016). *Qualitative Inhaltsanalyse: Methoden, Praxis, Computerunterstützung* [Qualitative content analysis: Methods, practice, computer support] (3rd ed.). Beltz Juventa.

Kulgemeyer, C. (2010). *Physikalische Kommunikationskompetenz. Modellierung und Diagnostik* [Physical communication competence. Modeling and diagnostics]. Logos.

Kulgemeyer, C., & Schecker, H. (2009). Kommunikationskompetenz in der Physik: Zur Entwicklung eines domänenspezifischen Kommunikationsbegriffs [Communication competence in physics: On the development of a domain-specific concept of communication]. *Zeitschrift für Didaktik der Naturwissenschaften, 15*, 131-153.

Kulgemeyer, C., & Schecker, H. (2012). Physikalische Kommunikationskompetenz-empirische Validierung eines normativen Modells [Empirical validation of a model for physics-related communication competence]. *Zeitschrift für Didaktik der Naturwissenschaften, 18*, 29-54.

Kuske-Janßen, W. H. (2020). *Sprachlicher Umgang mit Formeln von LehrerInnen im Physikunterricht am Beispiel des elektrischen Widerstandes in Klassenstufe 8 (= Studien zum Physik- und Chemielernen volume 305)* [Linguistic handling of formulas by teachers in physics lessons using the example of electrical resistance in grade 8 (= Studies on physics and chemistry learning volume 305)]. Logos. https://doi.org/10.30819/5183

Lehavi, Y., Mualem, R., Bagno, E., Eylon, B., & Pospiech, G. (2019). Taking the phys-math interplay from research into practice. In G. Pospiech, M. Michelini, & B. Eylon (Eds.), *Mathematics in physics education* (pp. 335-353). Springer. https://doi.org/10.1007/978-3-030-04627-9_15

Leisen, J. (2005a). Muss ich jetzt auch noch Sprache unterrichten?–Sprache und Physikunterricht [Do I now have to teach language too?–Language and physics lessons]. *Naturwissenschaften im Unterricht Physik, 3*, 4-9.

Leisen, J. (2005b). Wechsel der Darstellungsformen. Ein Unterrichtsprinzip für alle Fächer [Changing presentation styles. A teaching principle for all subjects]. *Der Fremdsprachliche Unterricht Englisch, 78*, 9-11.

Lemke, J. L. (1990). *Talking science: Language, learning and values*. Ablex Publishing Corporation.

Linke, A., Nussbaumer, M., & Portmann, P. R. (2004). *Studienbuch Linguistik. 5., erweiterte Auflage. Reihe Germanistische Linguistik 121* [Linguistics Textbook. 5th, expanded edition. Series Germanic Linguistics 121]. Niemeyer. https://doi.org/10.1515/9783110914825

Magnusson, S., Krajcik, J., & Borko, H (1999). Nature, sources, and development of pedagogical content knowledge for science teaching. In J. Gess-Newsome, & N. G. Lederman (Eds.), *Examining pedagogical content knowledge. Science & technology education library, vol 6* (pp. 95-132). Springer. https://doi.org/10.1007/0-306-47217-1_4

Markic, S. (2017). *Chemistry teachers' pedagogical scientific language knowledge* [Paper Presentation]. The 12th Conference of the European Science Education Research Association.

Muckenfuß, H. (1995). *Lernen im sinnstiftenden Kontext. Entwurf einer zeitgemäßen Didaktik des Physikunterrichts* [Learning in a meaningful context: A draft of a contemporary didactics for physics teaching]. Cornelsen.

Newton, I. (1687). *Philosophiae naturalis principia mathematica* [Mathematical principles of natural philosophy]. Cambridge University Digital Library. https://cudl.lib.cam.ac.uk/view/PR-ADV-B-00039-00001/9

Palmgren, E., & Rasa, T. (2024). Modelling roles of mathematics in physics: Perspectives for physics education. *Science & Education, 33*(2), 365-382. https://doi.org/10.1007/s11191-022-00393-5

Palmgren, E., Kokkonen, T., & Bruun, J. (2025). Roles of mathematics in physics education: A systematic review. *Physical Review Physics Education Research, 21*, Article 020602. https://doi.org/10.1103/wwww-gwp8

Peirce, C. S. (1931-1935). *Collected papers of Charles Sanders Peirce. Volume I-VI*. Belknap Press.

Pietrocola, M. (2008). Mathematics as structural language of physical thoughts. *Kansas State University*. https://web.phys.ksu.edu/icpe/publications/teach2/pietrocola.pdf

Pospiech, G. (2019). Framework of mathematization in physics from a teaching perspective. In G. Pospiech, M. Michelini, & B. Eylon (Eds.), *Mathematics in physics education* (pp. 1-33). Springer. https://doi.org/10.1007/978-3-030-04627-9_1

Pospiech, G., & Karam, R. A. S. (2023). Role of mathematics in physics from multiple perspectives. In M. F. Taşar, & P. R. L. Heron (Eds.), *The international handbook of physics education research: Special topics* (pp. 18-28). AIP Publishing. https://doi.org/10.1063/9780735425514_018

Pospiech, G., & Oese, E. (2013). The use of mathematical elements in physics–View of grade 8 pupils. In *Proceedings of the ICPE-EPEC 2013* (pp. 199-205).

Pospiech, G., Eylon, B., Bagno, E., & Lehavi, Y. (2019). Role of teachers as facilitators of the interplay physics and mathematics. In G. Pospiech, M. Michelini, & B. Eylon (Eds.), *Mathematics in physics education* (pp. 269-291). Springer. https://doi.org/10.1007/978-3-030-04627-9_12

Prediger, S. (2009). Inhaltliches Denken vor Kalkül–Ein didaktisches Prinzip zur Vorbeugung und Förderung bei Rechenschwierigkeiten [Content-based thinking before calculation–A didactic principle for preventing and promoting learning difficulties in mathematics]. In S. Fritz, & A. Schmidt (Eds.), *Fördernder Mathematikunterricht in der Sekundarstufe I* (pp. 213-234). Beltz Verlag.

Prediger, S., & Wessel, L. (2011). Darstellen–Deuten–Darstellungen vernetzen. Ein fach- und sprachintegrierter Förderansatz für mehrsprachige Lernende im Mathematikunterricht [Representing–interpreting–connecting representations: A subject- and language-integrated support approach for multilingual learners in mathematics education]. In S. Prediger, & E. Özdil (Eds.), *Mathematiklernen unter Bedingungen der Mehrsprachigkeit. Stand und Perspektiven der Forschung und Entwicklung in Deutschland*. Waxmann.

Rebello, S., & Cui, L. (2008). Retention and transfer of learning from math to physics to engineering. In *Proceedings of the ASEE Annual Conference and Exposition, Conference*. https://doi.org/10.18260/1-2--3778

Redish, E. F. (1994). Implications of cognitive studies for teaching physics. *American Journal of Physics, 62*(9), 796-803. https://doi.org/10.1119/1.17461

Redish, E. F., & Kuo, E. (2015). Language of physics, language of math: Disciplinary culture and dynamic epistemology. *Science & Education, 24*(5-6), 561-590. https://doi.org/10.1007/s11191-015-9749-7

Rincke, K. (2007). *Sprachentwicklung und Fachlernen im Mechanikunterricht. Sprache und Kommunikation bei der Einführung in den Kraftbegriff* [Language development and subject learning in mechanics lessons. Language and communication in the introduction to the concept of force] [PhD thesis, Universität Regensburg].

Rincke, K. (2010). Von der Alltagssprache zur Fachsprache–Bruch oder schrittweiser Übergang [From everyday language to technical language–A break or a gradual transition]? In G. Fenkhart, A. Lembens, & E. Zeitlinger-Erlacher (Eds.), *Sprache, Mathematik und Naturwissenschaften* (pp. 47-62). Studien Verlag.

Romer, R. H. (1993). Reading the equations and confronting the phenomena—The delights and dilemmas of physics teaching. *American Journal of Physics, 61*, 128-142. https://doi.org/10.1119/1.17327

Sherin, B. L. (2001). How students understand physics equations. *Cognition and Instruction, 19*(4), 479-541. https://doi.org/10.1207/S1532690XCI1904_3

Skemp, R. R. (1976). Relational understanding and instrumental understanding. *Mathematics Teaching in the Middle School, 12*(2) , 88-95. https://doi.org/10.5951/MTMS.12.2.0088

Strahl, A., Schleusner, U., Mohr, M., & Müller, R. (2010). Wie Schüler Formeln gliedern–Eine explorative Studie [How students structure formulas–An exploratory study]. *Physik und Didaktik in Schule und Hochschule, 9*(1), 18-24.

Strahl, A., Thiele, S., & Müller, R. (2013). Formeln in Physik(schul)- büchern–Eine quantitative Untersuchung [Formulas in physics (school) books–A quantitative study]. In S. Bernholt (Ed.), *Inquiry-based learning* (pp. 329-331). IPN.

Thoms, L.-J. (2011). *Formelnutzung in der Physik* [Formula usage in physics] [PhD thesis, Technische Universität Carolo-Wilhemina zu Braunschweig].

Tuminaro, J., & Redish, E. F. (2007). Elements of a cognitive model of physics problem solving: Epistemic games. *Physical Review Physics Education Research, 3*, Article 020101. https://doi.org/10.1103/PhysRevSTPER.3.020101

Uhden, O. (2012). *Mathematisches Denken im Physikunterricht/Theorieentwicklung und Problemanalyse. Studien zum Physik- und Chemielernen 133* [Mathematical thinking in physics teaching/theory development and problem analysis. Studies on physics and chemistry learning 133]. Logos.

Wagenschein, M. (1962). Rechtzeitigkeit der physikalischen Begriffsbildung [Timeliness of physical concept formation]. In *Die pädagogische Dimension der Physik* (pp. 166-181). Westermann.

Wagenschein, M. (1988). *Naturphänomene sehen und verstehen* [Seeing and understanding natural phenomena] (2., korrigierte Auflage). Klett.

Wigner, E. P. (1960). The unreasonable effectiveness of mathematics in the natural sciences. *Communications on Pure and Applied Mathematics, 13*(1), 1-14. https://doi.org/10.1002/cpa.3160130102

Wulff, P. (2024). Physics language and language use in physics—What do we know and how AI might enhance language-related research and instruction. *European Journal of Physics, 45*, Article 023001. https://doi.org/10.1088/1361-6404/ad0f9c

Zhao, F. F., Chau, L., & Schuchardt, A. (2021). Blended and more: Instructors organize sensemaking opportunities for mathematical equations in different ways when teaching the same scientific phenomenon. *International Journal of STEM Education, 8*(1), Article 26. https://doi.org/10.1186/s40594-021-00280-5